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Abstract

In this paper we develop, price, and analyze a securitization structure of life insurance policies. By transferring term insurance policies to a special purpose vehicle, all risk is transferred from insurers to the capital market. With mortality rates as the only underlying source of uncertainty, the structure is easy to analyze. We calibrate our model to the Swiss Re/Vita III-deal and find that insurers may transfer mortality risk to the capital market at a reasonable cost.

Keywords: life insurance, securitization, mortality risk.

Introduction

In this paper we construct, analyze, and price a possible securitization structure of life insurance policies, hereafter abbreviated SSLIP. We calibrate it to market data using the Vita III-deal from 2007.

Securitization is a relatively new and important innovation in the history of modern finance. The technique has developed rapidly, and the scope has expanded from mortgage loans to odd cash flow streams such as future royalties from rock music. Admittedly, the financial turmoils and the recession that have followed in the wake of the subprime crisis have put securitization in a dim light.

The idea of securitization is to isolate specific future cash flow streams and make them tradable. In general, all future cash flows have potential for securitization. The rights or obligation to future cash flows may be physically transferred to a single purpose vehicle (SPV) as an off-balance-sheet transaction, or simply just held as collateral against the SPV in an on-balance-sheet transaction. Next, the SPV issues a number of tranches of securities with different seniority to the cash flows. The most junior tranche is often called the equity class, and typically receives what is left after the other tranches have received their parts. This tranche is, thus, the most risky, but also the one with the highest expected return. Normally, one or several of the acknowledged rating agencies are hired to rate the different tranches before they are sold to investors through financial institutions.

So far, the life insurance industry has been outside the scope of mass securitization. There have been some deals in recent years, but most of these have been either closed book, value in force, or XXX deals (see Cowley and Cummins, 2005; or Garnsworthy, 2006 for an overview). The first securitization directly linked to mortality rates was the Swiss Re/Vita deal in 2003, followed by Vita II in 2005 and Vita III in 2007.

With growing demand for life insurance and new and stricter regulatory reserve requirements, the industry needs more free capital. Life insurers will need 80-100 billion of new reserves the next 7-8 years, according to research by KPMG's actuarial service practice in 2005 (see Life Insurance International, 2005). There are regulations on what types of capital classes may be utilized as reserves, and the most common have been equity or subordinate debt, which both are costly. According to Life Insurance International (2006), alternatives as reinsurance and letters of credit are limited in volume, costly, and are only available for a few years at a time. As a consequence, the presence of bankruptcy cost and regulation can justify securitization as a new way to cope with the capital problems, both as a new source of financing and as a risk management tool that can lower the need for holding reserve capital. However, the main advantage of securitization over traditional solutions such as reinsurance and letter of credit, is the possibility to develop long-term solutions.

Most life insurance companies hold large blocks of policies with expected profits to emerge in the future. Securitization may allow insurers to unlock part of these profits today and at the same time shift the risk associated with the policies to the capital market. The released capital may be used to write new policies, which again may be securitized. Securitization can, therefore, make it possible to undertake new business at a higher pace and utilize capital more efficiently. Insurers may focus on selling new policies, while the capital market takes care of investment management and risk bearing. The policy servicing function could either still be handled by the originator, or outsourced to a specialized third party.

Depending on how a securitization is structured, mortality risk or longevity risk may be the main drivers. These risk drivers often have low correlation with other risk drivers investors are exposed to. Securities based on life insurance products, therefore, add little systematic risk to a well diversified portfolio, and investors should demand a low rate of return. The unsystematic risk is diversifiable. Investors may, therefore, easier be able to bear this mortality risk and longevity risk than, e.g., reinsurers, and may, therefore, offer more attractive conditions.

Milevsky, Promislow, and Young (2006) show that when mortality rates are stochastic not all mortality risk is diversifiable. For a more thorough discussion regarding securitization, see Cowley and Cummins (2005).

Cairns, Blake, and Dowd (2006) develop a two-factor model for the development over time of mortality rates that can be used when pricing assets exposed to mortality and longevity risk. The first factor affects mortality-rate dynamics at all ages in the same way. The second factor affects the tilt of the mortality curve. Cairns, Blake, Dowd, Coughlan, Epstein, Ong, and Balevich (2009) analyze eight stochastic mortality-rate models using data from England and Wales (EW) and the United States (US). They find that both factors in the model by Cairns et al. (2006) are important and also that cohort effects are important. Cox, Lin, and Wang (2006) model mortality rates as a jump-diffusion process. They further take into account that changes in mortality rates across countries are correlated. This work is extended by Chen and Cox (2009) to take into account correlation between mortality-rate changes over time. Lin and Cox (2008) develop a model for analyzing securitization of catastrophe mortality risks.

In this paper we focus on how term insurance policies can be securitized. The paper is organized as follows: In section 1 we describe our securitization structure, in section 2 we discuss mortality rate modelling, in section 3 we extract the market price of risk from the Vita III-deal, and in section 4 we analyze the securitization structure we propose in this paper. Finally, we conclude in the last section.

1. The securitization structure

In this section we present a way insurers can securitize term insurance policies, and also how to find a price on these mortality-linked securities. We want to transfer all the mortality risk to the capital market by setting up a so-called risk transfer securitization[1]. This is in principle similar to a closed-book securitization, as the insurer does not have to worry about the risk of holding these policies anymore. Furthermore, the securitization allows life insurers to realize future profits today.

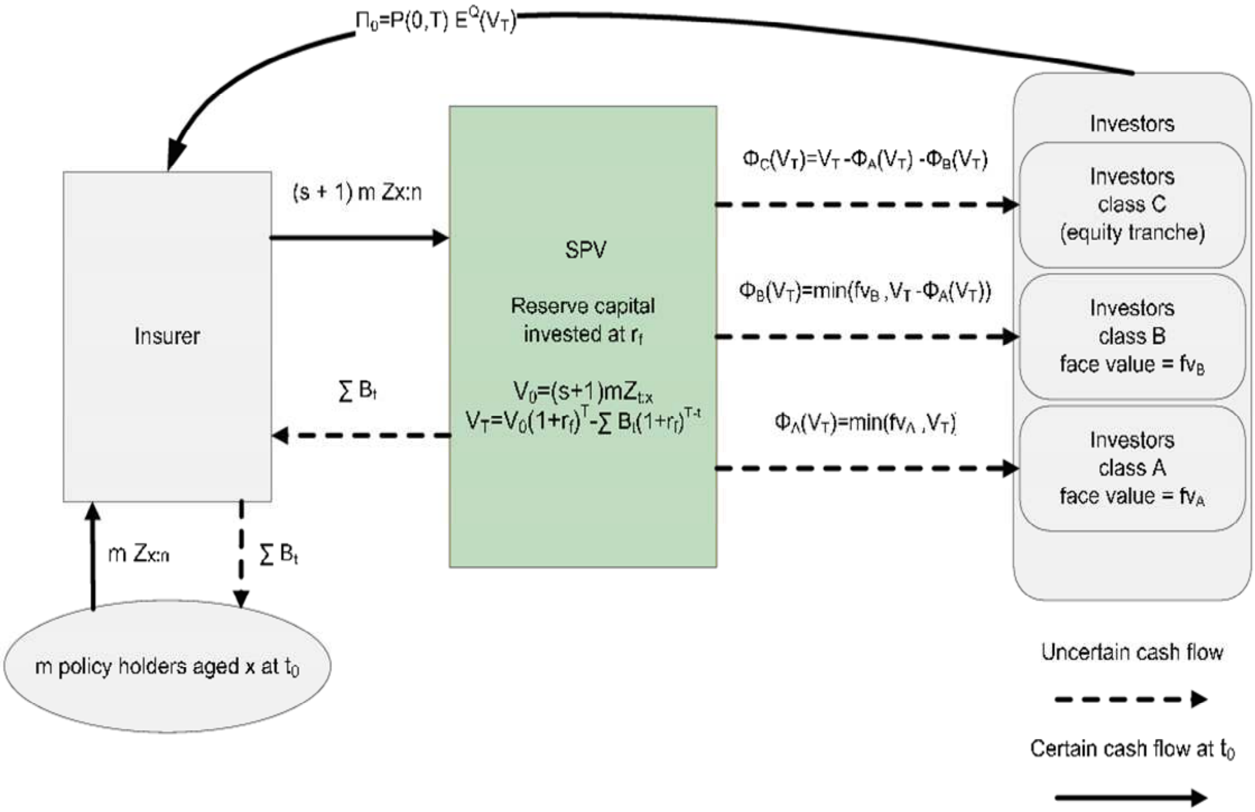


Fig. 1. Illustration of the securitization structure referred to as SSLIP

[1] In practice, an insurer typically only transfers part of the mortality risk to reduce the problem with asymmetric information. See, e.g., Biffs and Blake (2008) for a discussion of asymmetric information and securitization when there is longevity risk.

All cash flows t_0 SSLIP are at time t_0. Investors are, therefore, protected against all counterparty risk. The collateral is invested in risk-free assets. Mortality and benefits (B_t) paid upon the underlying portfolio of policies, are the only risk drivers influencing V_T. To protect policyholders, s (the safety margin) is set such that there is very low probability that the collateral is less than the benefits, B. The terminal value of the SPV, V_T, is divided among the classes after the typical waterfall principle, where A has seniority over B, and C gets what is left. There are no coupon payments from the SPV. Benefits are paid via the insurer or directly to the policyholders.

We present this structure by setting up an example (see also Figure 1). An insurer takes a number, m_0, equal single premium n-year term insurance policies with face value fv_p, sold at the same date, t_0, to people in the same region of a certain gender and age, x. The policies are transferred to a special purpose vehicle (SPV). In addition to the fair premiums[1], $m_0 Z_{x:n}$, the insurer transfers a safety margin of size $sm_0 Z_{x:n}$, to the SPV. The capital of the SPV is invested at the risk-free rate, r_f, and has the value V_T at maturity $T = t_0 + n$ after paying benefits $B_t = q_{x,t} m_{t-1} fv_p$ each year from t_1 to T, where $q_{x,t} = q(x,t)$ is the probability that a person aged x at $t = t_0$, who was alive at $t-1$, dies between $t-1$ and t, and m_{t-1} is the number of policyholders still alive at $t-1$. Hence,

$$V_t = V_{t-1}(1 + r_f) - B_t, \quad t = 1, 2, ..., T, \tag{1}$$

and

$$V_T = V_0 (1 + r_f)^T - \sum_{t=1}^{T} B_t (1 + r_f)^{T-t}, \tag{2}$$

where

$$V_0 = (s + 1) m_0 Z_{x:n},$$

i.e., all single premiums and the safety margin.

The SPV is split into three classes that have different claims on V_T. The two most senior classes, A and B, are set up as zero-coupon bonds that mature at T with face values fv_A and fv_B. The third class, C, can be considered an equity class that receives what is left after A and B have received their face values. The claim functions on V_T for the three classes are:

$$\begin{aligned} &\Phi_A(V_T) = \min(fv_A, V_T), \\ &\Phi_B(V_T) = \min(fv_B, V_T - \Phi_A(V_T)), \text{ and} \\ &\Phi_C(V_T) = V_T - \Phi_A(V_T) - \Phi_B(V_T). \end{aligned} \tag{3}$$

Class A has the payoff structure of a short put with strike price fv_A in combination with a risk-free investment paying fv_A. Class B has the payoff structure of a long bull spread in combination with a risk-free asset, while class C has the payoff structure of a long call. The value of the SPV at time T could in principle become negative if benefits are very large due to extreme changes in mortality rates. The SPV structure is meant to protect the insurer from mortality risk, but also make sure investors cannot lose more than invested. To avoid losses for policyholders, one can either use a guarantor, or increase the safety margin, s. For SSLIP we choose the latter, and set s big enough to make sure $\Pr(V_T > 0) \approx 1$.

At $t = t_0$, investors are offered to buy these claims on the SPV and the proceeds go to the insurer. The value of the claims are[2]:

$$\Pi_{t,j} = P(t,T) E^Q[\Phi_j(V_T)], \quad \text{for } j = \text{A, B, C}, \tag{4}$$

where E^Q is the expected value under the risk-adjusted probability measure Q. The discount factor, $P(t,T)$, is the price at time t for a risk-free zero-coupon bond that pays 1 at T. The cost for the insurer to transfer the risk related to the policies at t_0 is thus $(s+1)m_0 Z_{x:n} - \sum_{j=A,B,C} \Pi_{t_0,j}$. Under the real world probability measure P, we get

$$\Pi_{t,j} = P(t,T) e^{-\delta_j (T-t)} E^P[\Phi_j(V_T)], \quad \text{for } j = \text{A, B, C}, \tag{5}$$

where δ_j can be interpreted as an average risk premium per annum (see, e.g., Cairns et al., 2006). By setting equations (4) and (5) equal we get

$$e^{\delta_j (T-t)} = \frac{E^P[\Phi_j(V_T)]}{E^Q[\Phi_j(V_T)]}, \quad \text{for } j = \text{A, B, C}. \tag{6}$$

[1] We denote the present value of expected future cash flows by $Z_{x:n}$.

[2] We make the same assumption as Cairns et al. (2006, page 701, Assumption 3) that mortality rates and interest rates are stochastically independent. Note in particular that the formulation in equation (4) implies deterministic interest rates since a zero-coupon bond is used for discounting, and the Q-measure and the forward measure, therefore, coincide.

The risk premium, δ_j, is often referred to as *the price* of the investment product j, because it reflects how much more (or less) an investor expects to get compared to investing the money with no risk and earning the risk-free rate.

The yield to maturity (YTM), y_j, can be found for the two bond classes by solving for y_j in

$$e^{y_j(T-t)} = \frac{fv_j}{\Pi_{t,j}}, \quad \text{for } j = \text{A, B.} \qquad (7)$$

2. Mortality rate modelling

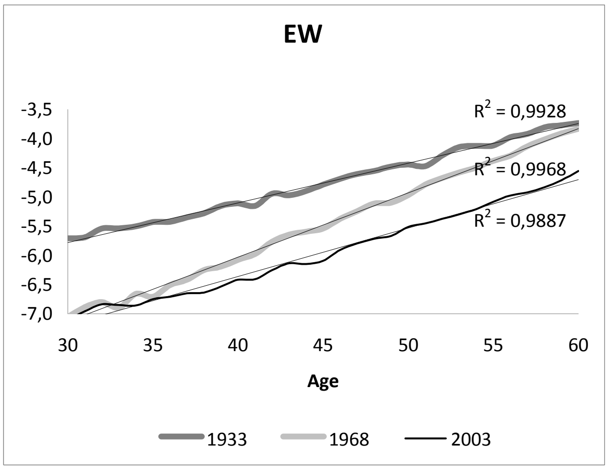
We use information contained in the prices from the Vita III-deal to price SSLIP. To this end, we need a mortality rate model, which can be applied both to the Vita III-deal and to SSLIP. According to Albertini (2006), term insurance policies, the underlying of SSLIP, are mainly bought by middle-aged males. As a consequence, the Vita III index has most weight on males between ages of 31 and 60.

Mortality risk is the only underlying source of risk in SSLIP. We need a model for mortality risk that uses age-specific mortality rates, and not just the population as a whole. The two-factor model presented by Cairns et al. (2006) does that. This model includes two factors, one that affects all ages (a shift in the mortality curve)[1], and one that is linked proportionally to age (a tilt in the mortality curve). From Figure 2 we see that over the years there have been downward shifts in mortality rates and mortality rates for the younger decreased earlier than mortality rates for the older. We also find there to be correlation between the mortality rates for EW and US for these effects. The two-factor model includes data from one region only. Our model should include at least two regions to fit Vita III. Also, the two-factor model is constructed to price longevity bonds, and therefore, only considers the age group of 60-89 years old. For the purpose of this paper, the model should include the age group of 31-60 years old.

Cairns et al. (2009) expand the two factor model by Cairns et al. (2006) in three different ways by adding:

a) a constant cohort effect term;
b) a quadratic age effect term;
c) an age-dependent cohort effect term.

When comparing our data to the data used by Cairns et al. (2009), it is clear that a quadratic term is not as appropriate for ages of 31-60 as it is for ages of 60-89. Figure 2 shows $\text{logit}(q)$ for our age group for three different years with linear trend lines and corresponding R^2 for EW and US. R^2 for the quadratic trend line[2] is slightly higher than R^2 for the linear trend line, but they are both very good. Even if the difference between the two R^2s would prove to be significant, it means very little for the results when adding a stochastic term to predict the future. We, therefore, do not include a quadratic term in our mortality rate model.



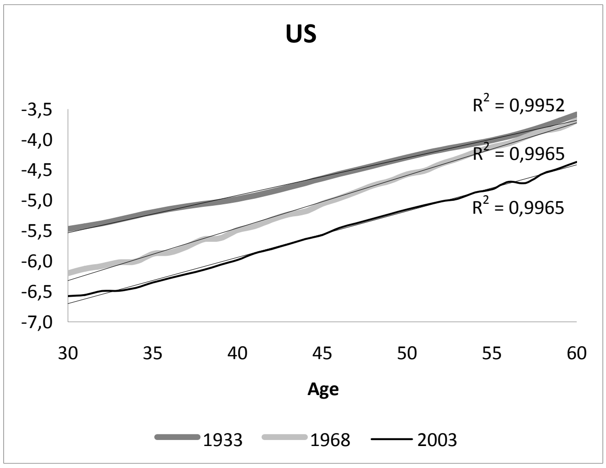


Fig. 2. *Logit*(*q*) for males aged 30-60 years in EW and US with linear trend line and corresponding R^2 for three different years

When analyzing EW and US data from 1933 to 2003 for cohort effects, we find some of the same cohort effects as Cairns et al. (2009) find in EW and US data from 1964 to 2002. For US data we find no cohort effects before 1959, but for EW data the same effects can be traced through all our data[3]. It is also difficult to find any cohort effects for the generations born after 1948 and before 1916. Cairns et al. (2009) show that including cohort effects yields a bet-

[1] With q being the mortality rate, the mortality curve is defined as how *logit*(q) varies with age, x.
[2] R^2 for the quadratic trend line is not shown here.

[3] It is unlikely that cohort effects suddenly appeared in the US in 1959, while it has been present in EW through all our data. However, we choose to not research this further, as validating data is not the focus of this paper.

ter model for data from 1964 to 2002, but we still choose to exclude cohort effects in our model because we want to keep it as parsimonious as possible.

The challenge is to find a mortality rate model that works with both Vita III and SSLIP. The underlying of Vita III is a mortality rate index that is weighted for country, age, and gender to reduce basis risk related to the portfolio of policies in the Vita III SPV. However, in SSLIP we want to be able to look at only one country, one age group, and one gender. We find the original two-factor model presented in Cairns et al. (2006) to be suitable for our purpose, mainly because it allows us to use any age we like as input.

Although the model suits us, we need to expand it in a few ways. To model the dynamics of the mortality rate index used by Vita III, we need to include more than one region and also model the correlation between regions. By including two regions the principle is shown, and if the two regions are the US and EW, most of the index is covered. Cox et al. (2006) also use these two regions when working with Vita I. We now have a four-factor model and can find λ (the market price of risk) for two different countries in Vita III. When we know λs for different countries, we are able to price SSLIP by using the λ corresponding to the country used as input in SSLIP. We ignore gender, and use data for males both when we find λ and when estimating the price of SSLIP. Cox et al. (2006) use total population for both US and EW in their model, and Cairns et al. (2006) use males only, so these simplifications are quite common.

2.1. Specification of the two-region-two-factor mortality rate model. The measure for mortality applied here is the mortality rate $q\ (t, x)$.

$q\ (t, x)$ = the probability that an individual aged x at t_0 dies between $t-1$ and t, for $t = 1, 2, \ldots, T$.

Cairns et al. (2006) show that $\operatorname{logit}(q)$ is close to linear in age for males older than 60 years. Plots in Figure 2 indicate that this holds for males between 31 and 60 years old in both EW and US in 1933, 1968, and 2003. The plots also show that the level and the tilt develop over time. We, therefore, use a stochastic mortality rate model with two stochastic parameters. The model is of the form:

$$\operatorname{logit}(q(t,x)) = A_1(t) + A_2(t)(x-\bar{x}), \qquad (8)$$

or rearranged:

$$q(t,x) = \frac{e^{A_1(t)+A_2(t)(x-\bar{x})}}{1+e^{A_1(t)+A_2(t)(x-\bar{x})}}, \qquad (9)$$

where $\bar{x}$ is the mean age over the range of ages being used in this analysis ($\bar{x} = 45.5$).

For each year t, A_1, and A_2 are estimated using least squares. The plots in Figure 3 show how the parameters develop over time. The level parameter A_1 have a downward drift for both EW and the US. The tilting factor A_2 develops more randomly, and the sign of the drift is not very clear. For both factors we find correlation between EW and US.

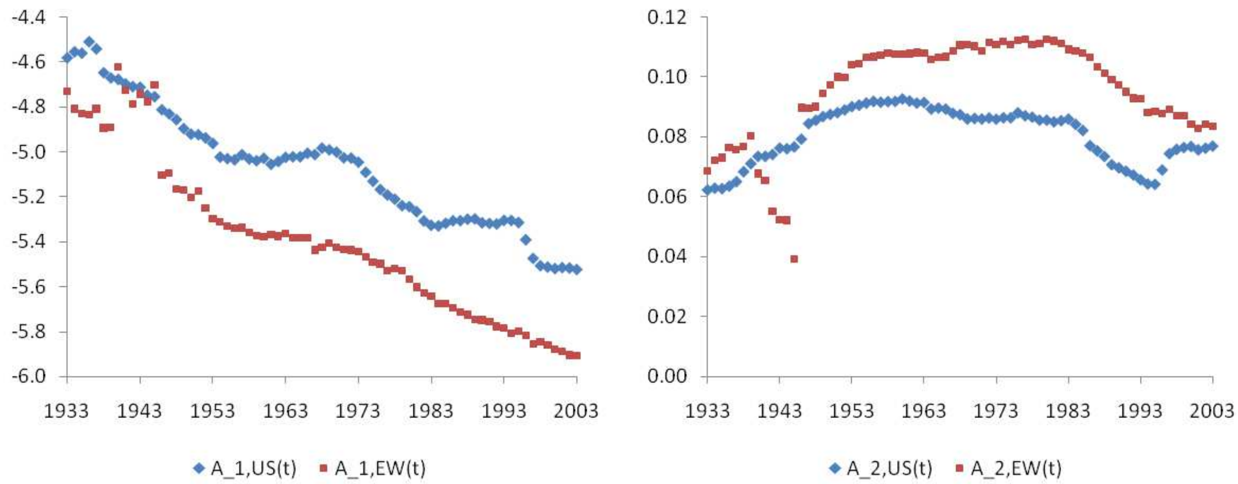


Fig. 3. Developing A_1 and A_2 over time for EW and US

To forecast future mortality rates we need to model how

$$A(t) = [A_{1,\mathrm{EW}}(t), A_{1,\mathrm{US}}(t), A_{2,\mathrm{EW}}(t), A_{2,\mathrm{US}}(t)]' \qquad (10)$$

develops. Here we assume that $A(t)$ is a random walk with drift

$$A(t) = A(t-1) + \mu + CZ(t), \qquad (11)$$

where μ is a constant 4×1 vector, C is a constant 4×4 upper triangular matrix, and $Z(t)$ is a four-dimensional standard normal random variable. This model specification allows for covariance among all four factors in our two-region-two-factor model. If we do not allow for covariance, we are likely to underestimate the risk when applying the mortality rate model on Vita III. Data of first-differences is used to estimate the drift μ and the covariance matrix *V*. $V = CC'$ have infinitely many solutions. Similarly to Cairns et al. (2006) we restrict C to be upper triangular and can, thus, easily derive a unique solution by applying Cholesky decomposition.

2.2. Selection of mortality rate data. As we choose to work with a two-region model and since the objective now is to extract risk premiums from the Vita III-deal, data from EW and US is used to calibrate the model. The Vita III payout structure is based on an index, where death rates from EW and the US account for 80% of the total. Cox et al. (2006) use the same simplification in their analysis of Vita I.

Life tables for EW and for US are published as interim life tables for three years at a time. At Human Mortality Database (2008) one can find mortality rates in life tables for one year at a time for EW and US. EW life tables by year of death are available for 1841-2003, while the same tables for the US only are available for 1933-2004. Also, for EW the volatility of the mortality rates was larger before the mid 1950s than the volatility for the next 50 years. To have data from the same time period for both regions we use data from 1933-2003. In the calibration of the model, data is limited to contain mortality rates for males aged of 31 to 60.

2.3. Calibration of the two-region-two-factor model and considerations. When mortality rates for males between 31 and 60 years old for EW and US between 1933 and 2003 are used to calibrate the model, we get the following results:

$$\hat{\mu} = \begin{bmatrix} \hat{\mu}_{1,\text{EW}} \\ \hat{\mu}_{1,\text{US}} \\ \hat{\mu}_{2,\text{EW}} \\ \hat{\mu}_{2,\text{US}} \end{bmatrix} = \begin{bmatrix} -1.684E-02 \\ -1.348E-02 \\ 2.150E-04 \\ 2.101E-04 \end{bmatrix}, \quad \text{and} \tag{12}$$

$$\hat{V} = \begin{bmatrix} 4.042E-03 & 4.503E-04 & -3.411E-04 & -5.065E-06 \\ 4.503E-04 & 6.483E-04 & -3.567E-05 & -2.378E-05 \\ -3.411E-04 & -3.567E-05 & 4.579E-05 & 2.181E-06 \\ -5.065E-06 & -2.378E-05 & 2.181E-06 & 2.896E-06 \end{bmatrix}. \tag{13}$$

As $\hat{\mu}_{1,\text{EW}}$ and $\hat{\mu}_{1,\text{US}}$ are negative, both EW and US experience a decrease in the mortality level parameter A_1. The positive sign of $\hat{\mu}_{2,\text{EW}}$ and $\hat{\mu}_{2,\text{US}}$ indicates that mortality rates decrease less for the older than for the young. From the $\hat{V}$ matrix it is clear that the level factor $A_1(t)$ and the tilt factor $A_2(t)$ have higher volatility for EW than US. The correlation matrix in Table 1 reveals high negative correlation between A_1 and A_2 for both EW and US, indicating that when mortality rates improve, they improve more for the young than for the old. Furthermore, we observe positive correlations between EW and US as expected.

Table 1. Correlation matrix on first differences of *A* (*t*)

	$A_{1,EW}$	$A_{1,US}$	$A_{2,EW}$	$A_{2,US}$
$A_{1,EW}$	1.00	0.28	-0.79	-0.05
$A_{1,US}$		1.00	-0.21	-0.55
$A_{2,EW}$			1.00	0.19
$A_{2,US}$				1.00

3. Extracting the market price of mortality risk from Vita III

The stochastic mortality rate model developed in section 2 can be used to evaluate SSLIP under the real-world probability measure, P. However, to estimate the value of SSLIP, a Q-probability measure needs to be developed. More precisely, the Q-dynamics of the two-region-two-factor model needs to be derived so that expected payoffs of SSLIP under the Q-measure can be discounted at the risk-free rate of return to find the value of SSLIP. The time t value of SSLIP is given by

$$\Pi_{t,V} = \sum_{j=A,B,C} \Pi_{t,j} = P(t,T)E^{Q}\left[V_T\right], \tag{14}$$

with the Q-dynamics of the underlying stochastic $A(t)$ process given as

$$A(t) = A(t-1) + \mu + C\left[Z(t) + \lambda\right], \tag{15}$$

where $\lambda = \left[\lambda_{1,\text{EW}}, \lambda_{1,\text{US}}, \lambda_{2,\text{EW}}, \lambda_{2,\text{US}}\right]'$. Rearranging, we have

$$A(t) = A(t-1) + \tilde{\mu} + CZ(t), \tag{16}$$

where

$$\tilde{\mu} = \mu + C\lambda. \tag{17}$$

Note that equation (16) is equal to $C\lambda$ added to equation (11).

The difference between the P- and Q-probability measures depends on the market price of risk, λ. As λ is not specified within our model, it must be extracted from the market price of assets with the same underlying source of uncertainty, i.e., $A(t)$.

3.1. Method applied to find λ. To estimate the market price of risk λ, a payoff model in accordance with Vita III is built based on the stochastic mortality rate model in section 2. Vita III is evaluated yearly, and if trigger levels are reached before maturity, Swiss Re receives payouts at the end of that year. We simplify and say that all payouts happen at maturity, but still evaluate every year separately. Then we have the following payoff structure and loss for Vita III series j:

$$L_j^Q = \min[\sum_{t=t_0}^{T_j} \frac{\max[0,(q_t^Q + q_{t-1}^Q)/2 - q_0 AP_j]}{q_0(EP_j - AP_j)}, 100\ \%], \quad (18)$$

where t_0 is 2007, T_j is the maturity of series j (2010 or 2011), AP_j is the attachment point, EP_j is the exhaustion point, q_t^Q is the mortality rate index under the Q-measure at t, and

$$q_0 = (q_{2004} + q_{2005})/2.$$

Simulations with different values of λ are conducted until the discounted expected risk-adjusted payoff is in accordance with the actual price the bonds were sold at. When this is true, we have an estimate of λ. Or, in mathematical terms, λ and the corresponding Q-measure are found when equation (19) holds

$$\prod\nolimits_{t_0,j} = P(t_0,T)E^Q\left[1 - L_j^Q\right]fv_j + \\ + \sum_{t=t_1}^{T} P(t_0,t)(r_f + \delta_j)fv_j, \quad (19)$$

where t_1 is the time of the first interest payment. For Vita III, the value of bond series j at $t = t_0$ is the same as the face value fv_j ($\Pi_{t_0,j} = fv_j$). Thus, investors are only compensated for possible losses by being paid a spread δ_j, over LIBOR/EURIBOR (see Appendix A). L_j is the percentage loss on the bond series j (see equation (18)). By using constant interest rates[1], equation (19) may be rewritten as

$$EL_j^Q = E^Q\left[L_j\right] = \delta_j \sum_{t=t_1}^{T} (1 + r_f)^{T-t}. \quad (20)$$

EL_j^Q is the simulated risk-adjusted payoff. We vary λ until equation (20) holds. Or, by defining the annual loss rate under Q as

$$d_j = \frac{E^Q\left[L_j\right]}{\sum_{t=t_1}^{T}(1 + r_f)^{T-t}}, \quad (21)$$

we try to achieve

$$d_j = \delta_j \quad (22)$$

when running simulations for different values of λ[2].

There are three securities from Vita III we can use to estimate λ, Vita III – series 1, 2, and 7 (see Appendix A). Series 3 is identical to series 1, except for the currency, while series 4, 5, and 6 include a guarantor and, therefore, have a much lower δ[3].

3.2. Assumptions made to find λ. The two-region-two-factor model allows for two regions, hence, a total of four λ s: $\lambda_{1,\mathrm{EW}}$, $\lambda_{1,\mathrm{US}}$, $\lambda_{2,\mathrm{EW}}$, and $\lambda_{2,\mathrm{US}}$. From equations (16) and (10) we can see that λ_1 represents the market price of risk of a shift in the mortality curve, and λ_2 the market price of risk of a tilt in the mortality curve.

If only one of the bond series of Vita III is used, there are infinitely many possible combinations of $\lambda_{1,\mathrm{EW}}$, $\lambda_{1,\mathrm{US}}$, $\lambda_{2,\mathrm{EW}}$, and $\lambda_{2,\mathrm{US}}$ that fulfill equation (20). Four variables cannot be uniquely specified with only one equation or by applying only one price from the Vita III series. As a compromise we set $\lambda_{2,\mathrm{EW}} = \lambda_{2,\mathrm{US}} = 0$. It is then, in principle, possible to solve for three unique sets of $\lambda_{1,\mathrm{EW}}$ and $\lambda_{1,\mathrm{US}}$ by combining the three series of Vita III. Unfortunately, none of the combinations could fulfill equation (20) for both series with one unique set of $\lambda_{1,\mathrm{EW}}$ and $\lambda_{1,\mathrm{US}}$. This is addressed further in section 3.3. Therefore, we also assume that

$$\lambda_{1,\mathrm{EW}} = \lambda_{1,\mathrm{US}} = \lambda_1 \quad (23)$$

for all further analysis.

[1] To simplify we do not distinguish between LIBOR/EURIBOR and the risk-free rate of return, r_f. Issues regarding interest rates are not the main focus of this paper, and as a simplification we set LIBOR/EURIBOR and r_f at all maturities to 4.5% annually compounded for all further calculations.

[2] The elements in the variance-covariance matrix in equation (13) are small in magnitude. Throughout the paper we run 500,000 simulations, which are sufficient to obtain estimates with low standard errors and also lead to acceptable computation times.

[3] If we knew how much Swiss Re paid for the guarantee, we could have used these series as well, but this information has not been available to us.

Another important restriction used in this paper is that λ is time independent. Without this restriction there is too little data to estimate λ. This is a common assumption in financial economics (for further discussion, see Cairns et al. (2006, footnote 22)).

As explained in section 2, we have chosen to only have two regions in the mortality rate model. Vita III is linked to indices in five countries: the US, EW, Germany, Japan, and Canada. In section 2.1, the US and EW are chosen as the two regions because they account for 80% (62.5% and 17.5%, respectively) of the Vita III index. The volatility of EW mortality rates is larger than the volatility of the US mortality rates. One of the reasons for this is larger fluctuations in mortality rates around World War II, specifically in 1939 and 1945[1]. Germany and Japan were also hit hard during and after World War II. For EW, the volatility is even greater when looking at data as far back as 1900, due to World War I and the Spanish flu, both before 1933[2]. We like SSLIP to work as an insurance against catastrophes. Because of these facts, we keep the weight of US at 62.5% and set the weight of EW (with the highest volatility) to 37.5%.

To find λ_1 we need to calculate an age-weighted mortality rate index corresponding to Vita III. In addition to weighting for gender and country, the Vita III mortality rate index is weighted for age. A weighted average of mortality rates for 5-year age groups for EW and 10-year age groups for the US is found. We use the same weights, but exclude ages below 31 and above 60 years old as our model is not calibrated for these ages[3]. To get each age group assigned to one specific mortality rate, we split the US age groups to match the EW age groups, and then find the middle age value. The mortality rates for these middle values are weighted to find the index mortality rate.

The target equation, equation (20), shows that the EL_j^Q we seek and the corresponding λ_1 depend on r_f. However, λ_1 is not very sensitive to r_f. For instance for Vita III – series 1, EL^Q varies from 4.53% to 4.96% as r_f varies from 2% to 8%.

[1] When removing 1939 and 1945 from the dataset, the variance of first differences of A_1 and A_2 for 1933-2003 decreases by 81% and 85% for EW, and only by 2% and 3% for US.

[2] When including 1900-1932 in the dataset, the variance of first differences of A_1 and A_2 increases by 77% and 40%, respectively.

[3] For EW, 31-60 year-old males account for 86.50% of the total male population in the portfolio. For the US, 25-64 year-old males account for 95.50% of the total male population, and we assume 31-60 year old males to account for 86.33%.

3.3. Estimates of λ_1 and discussion. In theory, λ_1 should be equal for all securities on the same underlying (and the same time to maturity), which means series 2 and 7 should yield the same λ_1. Table 2 shows d_j, annual loss rate under Q as defined in equation (21), after simulations with different values for λ_1. The correct λ_1 is found when $d_j = \delta_j$ (δ_j equals the number of basis points series j was issued at).

Table 2. The tables below show d_j as basis points (bps) when running simulations for different values of λ_1 for three different series of Vita III. We search for the λ_1 that yields $d_j = \delta_j$.

Series 1 – 4 years, class B, $\delta_1 = 110$ bps

λ_1	1.260	1.269	*1.270*	1.271	1.280
d_1 [bps]	105	109	*110*	111	114

Series 2 – 5 years, class B, $\delta_2 = 112$ bps

λ_1	1.090	1.103	*1.104*	1.105	1.100
d_2 [bps]	105	111	*112*	113	115

Series 7 – 5 years, class A, $\delta_7 = 80$ bps

λ_1	1.460	1.471	*1.472*	1.473	1.480
d_7 [bps]	76	79	*81*	81	83

As Table 2 shows, the three values for λ_1 do not match exactly. Also, in the previous subsection we find that there are no unique solutions to a set of $\lambda_{1,\text{EW}}$ and $\lambda_{1,\text{US}}$. There are at least two possible reasons:

1. The stochastic mortality rate model is misspecified.
2. Some of the assumptions we make for pricing do not hold.

There are a number of stochastic mortality rate models, and they are all influenced by the data used to calibrate them. Including jumps could be reasonable when pricing CAT bonds like Vita III. However, we have chosen not to include jumps in the analysis.

The other reason may be violations of assumptions of the pricing theory applied. One of the assumptions is that there exists a liquid and frictionless market, where assets on the same underlying are traded. Obviously, this assumption is violated. Different λ_1 estimates from the different series of Vita should in theory give arbitrage possibilities. In the present market, though, with few securities and low liquidity, the theoretical arbitrage possibilities may be difficult to exploit. As a result, different investors with different appetite for risk may buy the different series, and no one is able or willing to exploit the theoretical arbitrage possibility.

Other assumptions, like, e.g., constant λ over time, may also not hold, and thus, explain why the λ_1 estimates differ between series 1 and 2, which have different maturities.

The rating and spread paid on series 7 (the most secure series) may not reflect the underlying mortality risk, but rather the credit quality of the collateral and the swap counterparty. Thus, the λ_1 results from series 7 (see Table 2) are likely to be overestimated. Estimates of λ_1 from series 1 and 2 can also be affected by these non-mortality risks, although not by as much as for series 7. We, therefore, place less emphasis on the λ_1 corresponding to series 7.

4. Analysis of the securitization structure

We now have an estimate of the market price of mortality risk and can, hence, analyze and price SSLIP. When we refer to *the price of* SSLIP we mean the risk premium (expected rate of return) investors demand on the three classes. The prices of classes A and B are presented as basis points (bps) above r_f (δ_A and δ_B), and for C the price is presented as the total required rate of return ($r_f + \delta_C$). For classes A and B one can also find the yield to maturity, YTM. This is the rate of return investors earn if there is no loss and investors receive face values at maturity, and is, therefore, greater than the risk premium.

To analyze and find the prices of the different classes of SSLIP we run simulations. As the only stochastic underlying is mortality rates, we run simulations with different development of mortality rates for ages from 31 to 60. For each simulation we use λ to derive the risk-adjusted mortality rates under Q. In that way we use the same random numbers for all calculations under both P and Q. Given a set of specified input parameters, we calculate the terminal value of the SPV, V_T, under both P and Q for each simulation. Furthermore, for each simulation we use V_T to calculate the payoffs on classes A, B, and C, according to the claim functions given in equation (3). By averaging over the simulated payoffs, we obtain estimates for the expected values $E^P\left[\Phi_j(V_T)\right]$ and $E^Q\left[\Phi_j(V_T)\right]$ for all classes.

4.1. Base case parameters. We first present results from a simulation with *base case parameters*. Then we analyze how prices vary when some of these parameters are changed. Our base case parameters have values as shown in Table 3.

Table 3. The parameters used when simulating the base case.

Parameter	Value
Region	US
Gender	Males
Agreed benefit, fv_p	\$200,000
Number of contracts, m_0	100,000
Market price of risk of tilt factor, λ_2	0
Market price of risk of shift factor, λ_1	1.10
Safety margin, s	55%
Face value class A, fv_A	55%
Face value class B, fv_B	15%
Years in term policy, n	10 years
Age at t_0, x	35 years old
Risk-free interest rate, r_f	4.5%

The first five parameters in Table 3 are considered constant throughout the paper while the last seven are changed when doing sensitivity analysis. The US is chosen as base case. Mortality rates are different for males and females. Analyse of different mortality rates are done by varying age, not gender. According to Standard & Poor's (2007), 65% of Vita III policyholders are males, and we, therefore, use mortality rates for males in the base case. *Agreed benefit* is proportional to the value of the SPV at T ($fv_p \sim V_T$) and does, thus, not affect the results. *The number of contracts* in the securitization only affects the basis risk, and is, therefore, only changed when analyzing this risk. For all other cases, basis risk is ignored, and hence, the number of contracts does not affect the prices of the classes. The *market price of risk of the tilt factor*, λ_2, is set to 0 as explained in section 3.2.

In section 3.3, Table 2, we get three different values for λ_1. Series 2 is more similar to SSLIP than series 1 and 7 and we have, therefore, set $\lambda_1 = 1.104$, c.f., Table 2. For Vita III, class B has lower trigger levels than class A, and is, therefore, more similar to SSLIP in that less extreme events affect the payoff. Also, the λ_1 estimate from series 7 might be biased because of strong influence from other risk factors than mortality (see discussion in section 3.3). Because SSLIP has a maturity of ten years in the base case, the five-year bonds of Vita III are more similar than the four-year bonds. Hence, series 2 is used in the base case. Although we find λ_1 to the nearest thousandth in section 3.3, we round to the nearest hundredth here and use 1.10.

The safety margin, s, is set to 55%. With s = 55%, $\Pr^P(V_T > 0) \approx 1$. The total cost for the insurer to transfer risk to investors is not affected by s (see section 4.3), as opposed to the δ s. 55% yields sensible prices of the three classes. By sensible prices we mean that prices and rating roughly correspond to other previously traded assets.

The face values of classes A and B are set to 55% and 15% of $E^P[V_T]$, respectively. Although the total cost for insurers is independent of these numbers, they have great influence on the δ s. After trying many different combinations of fv_A and fv_B, we found these values to yield sensible prices and ratings for all three classes.

10-year term insurance policies are chosen for the base case. In ten years mortality rates might change a lot, and insurers are, hence, exposed to a significant risk. Also, 10-year term contracts are typical for the US life insurance market.

35 years old is chosen as age for the base case. People with families are more likely to buy life insurance products than other people and 35 year old males are good representatives for this group.

The *risk-free interest rate*, r_f, is as before set to 4.5% annually compounded for the base case[1].

4.2. Base case results. Results from simulating the base case are reported in Table 4.

Table 4. Results from 500,000 simulations with base case parameters

The price of class A, δ_A	8 bps
Expected loss, class A, EL_A^P	0.0005%
Moody's rating, class A	Aaa
YTM, class A, y_A	4.58%
The price of class B, δ_B	220 bps
Expected loss, class B, EL_B^P	0.1204%
Moody's rating class B	Aa3
YTM, class B, y_B	6.72%
The price of class C, $\delta_C + r_f$	20.41%
Weighted average prices, wa_{prices}	3.16%
Insurer's minimum markup, im_{min}	14.34%
Number of SPV defaults	0
StdDev($E^P[V_T]$)	10.69%

The prices are found as explained above using equation (6). Another interesting measure is the expected loss for classes A and B,

$$EL_j^P = 1 - \frac{E^P[\Phi_j(V_T)]}{fv_j} \quad \text{for } j = \text{A, B.} \tag{24}$$

Investors are concerned with expected loss, and it is possible to make some assumptions about what kinds of investors are willing to buy the classes, and at what price level.

According to Moody's Investors Service (2000), the expected loss rate has become the primary measure of credit quality, particularly for structured finance securities. Numerical simulations are often conducted to estimate expected loss rates. Different levels of expected loss rates correspond to different rating levels that give an impression of relative credit quality of the different classes. The cumulative expected loss rates estimated in this paper are mapped with Moody's Idealized Loss Rate Table, which is shown in Appendix B. YTM is found using equation (7)[2].

The weighted average of prices, wa_{prices}, is the total risk premium investors demand for the SPV as a spread above r_f, and is found by solving for wa_{prices} in

$$e^{(wa_{prices}+r_f)(T-t_0)} = \frac{E^P[V_T]}{\Pi_{t_0,V}}. \tag{25}$$

Insurer's minimum markup, im_{min}, is the cost for the insurer to transfer the risk to the capital market, as percent of received fair premiums. Hence,

$$im_{min} = \frac{sm_0Z_{x:n} - \Pi_{t_0,V}}{m_0Z_{x:n}} = s - \frac{\Pi_{t_0,V}}{m_0Z_{x:n}}, \tag{26}$$

and is one of the measures we find the most interesting.

Number of SPV defaults is for how many of the simulations we observe $V_T < 0$ and StdDev($E^P[V_T]$) is the standard deviation of the expected values of V_T under P.

4.3. Sensitivity analysis. To analyze how the results depend on the different parameters, simulations are performed when varying one parameter at a time. Some parameters affect the underlying mortality rates and, hence, the cost for the insurer to transfer the risk to the investors. Other parameters do not affect this value, but changes the prices investors are willing to pay for the different claims. In all tables presented in

[1] All interest rates and prices are presented as annually compounded. Equation (6) gives the prices as continuously compounded, so annually compounded rates have been computed from these before presented here.

[2] Because YTM does not take into account the possibilities of losses, it is greater than or equal to the price added the risk-free rate, $y \geq \delta + r_f$.

this subsection, the slanted characters are used for the base case results.

4.3.1. The market price of risk of a shift in the mortality curve, λ_1. The market price of risk of a shift in the mortality curve, λ_1, affects the mortality rates under the Q-measure, and hence, all valuations of SSLIP. From Table 5 we see that the higher the market price of risk is, the higher compensation do investors require to take on mortality risk. Even if λ_1 is as high as 1.50, the insurer only needs a markup of 20.09% to transfer the risk to the capital market.

Table 5. Sensitivity analysis of the market price of risk of a shift in the mortality curve, λ_1

λ_1	wa_{prices}	im_{min}
0.70	1.83%	8.94%
0.90	2.46%	11.62%
1.10	*3.16%*	*14.34%*
1.30	3.94%	17.17%
1.50	4.81%	20.09%

4.3.2. Safety margin, s. When varying the safety margin, s, im_{min} is constant. This makes sense, because the underlying risk and the premiums do not vary with im_{min}, and hence, should im_{min} neither do so. Table 6 shows the results when varying s. The only actual requirement is that s is big enough to obtain $Pr^P(V_T < 0) \approx 0$. This is true for all trials with s except when s = 20%. It is, thus, no problem to use s = 30% as the safety margin.

In theory (assuming a liquid and efficient market), the attractiveness of the securities is not affected by these values. However, we want to create securities that are rated as investment grade and thus, $StdDev(E^P[V_T])$ cannot be too high. We find that when $StdDev(E^P[V_T]) \approx 10\%$ and hence, $wa_{prices} \approx 3\%$ we achieve the ratings we seek on the structure. If one wants to create classes with Aaa ratings for both classes A and B, and at the same time one wishes to avoid too high prices for class C, one needs to set s even higher than what is done in the base case.

Table 6. Sensitivity analysis of the safety margin, s

s	Defaults	wa_{prices}	StdDev($E^P[V_T]$)
20%	474	13.96%	29.63%
30%	0	6.94%	19.69%
40%	0	4.68%	14.75%
50%	0	3.54%	11.76%
55%	*0*	*3.16%*	*10.69%*
60%	0	2.85%	9.78%
70%	0	2.39%	8.40%
80%	0	2.06%	7.35%
90%	0	1.80%	6.52%

Note: Defaults is the number of times V_T < 0 out of the 500,000 simulations.

4.3.3. Face value class A and B, fv_A *and* fv_B. The face values of classes A and B, fv_A and fv_B, are expressed as percentages of $E^P[V_T]$. Class C receives what is left, and therefore, has no face value. How the investors split the SPV at maturity does not affect the value of the SPV at maturity, V_T. Hence, wa_{prices} and im_{min} are constant when the face values are varied. For the capital market to be interested in buying the different classes, the classes should be as transparent as possible, i.e., the payoff profiles should resemble other securities in the market.

Figure 4 shows the prices and Moody's ratings of the three classes for different values of fv_A and fv_B. The rightmost bar shows the base case, with fv_A = 55% and fv_B = 15%. The prices and corresponding ratings for the classes are attractive and also compare somewhat to similar securities traded, for example, the Queensgate securities presented in Lane (2006).

Fig. 4. The prices and Moody's ratings of the three classes for different values of fv_A and fv_B, expressed as percentages of $E^P[V_T]$. The rightmost bar shows the base case

4.4.3. Years in term, n. Term insurance policies are usually sold with maturity of 1, 5, 10, 15, 20, or 30 year(s). When modelling the two-region-two-factor model from section 2.1 we look at mortality rates ten years ahead, and hence, are not able to analyze maturities longer than ten years. We do, however, look at results for 1-year and 5-year policies. Table 7 shows the results when varying years in term. Results show that when reducing n, im_{min} decreases. This makes sense, because one is now exposed to the underlying mortality risk for only one or five years and hence the cost of transferring risk is less. The StdDev($E^P[V_T]$) also decreases with decreasing maturity because s is kept constant. The value for s is set to fit $n = 10$ and could be lower when $n = 5$ to get the same StdDev($E^P[V_T]$) as for the base case.

Table 7. Sensitivity analysis of years in term, n

n	im_{min}	StdDev($E^P[V_T]$)
10	14.34%	10.69%
5	7.49%	9.12%
1	2.41%	6.70%

4.3.5. Age, x. Age is important because it directly influences the only underlying, mortality rates. Mortality rates increase as age increases. The same absolute increase in mortality rates, therefore, results in a smaller relative increase for high ages compared to lower ages. Table 8 shows results when varying age, x. It is clear that as x increases, wa_{prices}, im_{min}, and StdDev($E^P[V_T]$) decrease. They do not vary much though, and, apparently, SSLIP works for all ages presented here. The small differences also make it unproblematic to combine several age groups without changing the properties of SSLIP much. Note that the variation of StdDev($E^P[V_T]$) indicates that a lower safety margin, s, may be applied when securitizing policies for higher ages.

Table 8. Sensitivity analysis of age, x

x	wa_{prices}	im_{min}	StdDev($E^P[V_T]$)
31	3.18%	14.51%	12.61%
35	3.16%	14.34%	10.69%
40	3.13%	14.17%	8.77%
45	3.09%	13.99%	7.69%
50	3.03%	13.79%	7.71%

4.3.6. Risk-free interest rate, r_f. The risk-free interest rate affects what returns the SPV gets on invested capital. Because the focus of this thesis is mortality risk, and not interest rate risk, we assume interest rates to be constant. We still run simulations to analyze scenarios with higher or lower interest rates. Results are shown in Table 9.

Table 9. Sensitivity analysis of the risk-free interest rate, r_f

r_f	EL_A^P	EL_B^P	δ_A	δ_B	$\delta_C + r_f$	im_{min}
2.0%	0.0006%	0.1477%	10	245	18.35%	14.86%
4.5%	0.0005%	0.1204%	8	220	20.41%	14.34%
7.0%	0.0003%	0.1066%	6	197	22.48%	13.85%

The consequence to V_T of 100 unexpected deaths is greater if it happens in year one relative to year ten. This is due to lost return on the collateral when benefits are paid out early. Interest rates strongly influence how important the development in mortality rates in the first years is relative to the last years. Higher interest rates make the first years more important, while lower interest rates make the last years more important. The mortality rate model applied is random walk, and hence, big deviations from expected mortality rates are most likely in the last years. As a consequence, lower interest rates make V_T relatively more dependent on the last years' volatile mortality rates, and hence, make V_T more volatile.

Increased volatility of V_T under P caused by lower interest rates, results in higher expected loss on classes A and B. The adjustment between Q and P mortality rates increases every year. With relatively more weight on the last years due to lower interest rates, the payoffs under Q are reduced compared to under P. Hence, when the difference between $E^Q[V_T]$ and $E^P[V_T]$ increases, the risk premium δ_j increases, cf. equation (6). Most importantly, the sensitivity analysis shows that im_{min} increases with lower interest rates, but is not very sensitive.

4.4. Basis risk. To analyze basis risk is not straightforward. Basis risk is defined as the remaining risk associated with hedging a position in SSLIP with mortality index linked securities. Or, put differently, the risk of deviation between realized mortality in SSLIP and officially published population mortality rates. There are two reasons for this risk. One is that the policyholders do not represent the population, and hence, are subject to different mortality rates. We do not analyze this type of basis risk, and continue to assume it can be ignored.

The second source of basis risk is simply statistical sample risk[1]. When the number of policyholders in the sample portfolio is increased, the realized sample mortality rate is likely to vary less from the population mortality rate. Hence, this risk is diversifiable, and investors should not demand a risk premium for this risk. However, basis risk results in increased volatility of V_T. Because of the non-linear payoff structure of the different classes, the increased volatility affects the expected payoff for all classes under both P and Q. So, even if investors do not demand a risk premium, basis risk affects YTM on A and B.

To analyze this risk we use the binomial distribution to simulate sample mortality under P. It is not straightforward to simulate sample mortality under the risk adjusted Q-measure. Thus, we are not able to value the claims directly. We are, however, able to look at $StdDev(E^P[V_T])$, EL_A^P, and EL_B^P, and from that we can draw some conclusions of how the values and YTM of the different classes are affected.

[1] Hereafter, when we refer to basis risk, we mean only this second, statistical type of basis risk. Dahl (2004) refers to this type of risk as "unsystematic mortality risk".

The maturity of the policies, n, is important when considering basis risk. As n increases, basis risk becomes less important, because the variation tends to even out over the years. When including basis risk in the model, some results change, while others are unchanged. $E^P[V_T]$ is not affected by basis risk, but $StdDev(E^P[V_T])$ increases, as shown in Figure 5. Although basis risk is small for m = 200,000 when n = 10 years, it is still present.

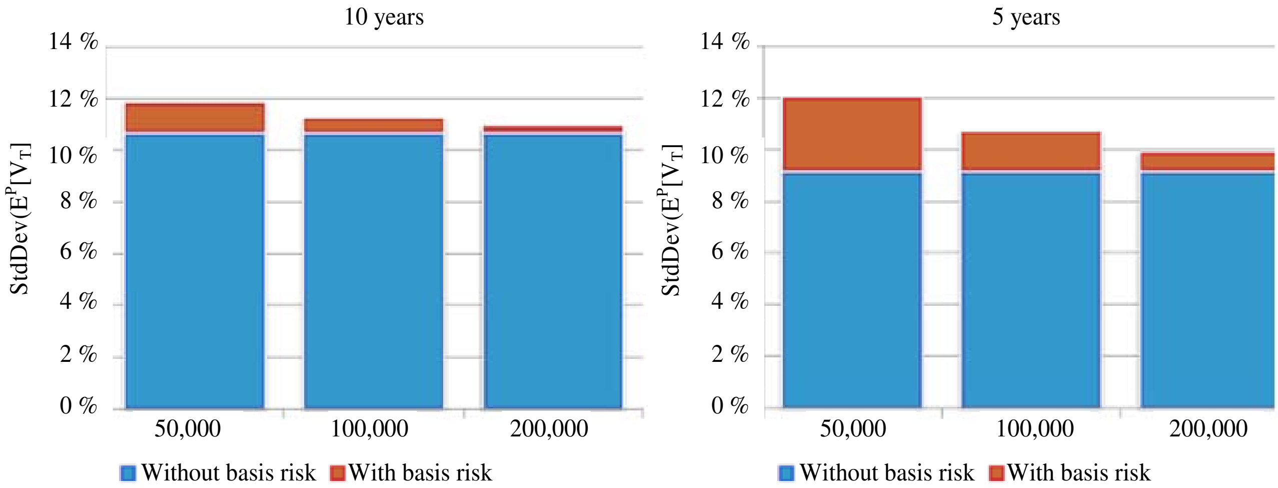


Fig. 5. These charts show that when including basis risk, the standard deviation of the expected value of the *SPV* at maturity, $StdDev(E^P[V_T])$, decreases as number of contracts, m, increases and maturity of policies, n, increases. Basis risk is, thus, more important for 5-year bonds than for 10-year bonds

When $StdDev(E^P[V_T])$ increases, expected loss under the P-measure of the two bond classes A and B, EL_A^P and EL_B^P, also increase. Thus, ratings might worsen, and the YTM demanded by investors, y_A and y_B, increase. For class C, it is opposite and the initial value of class C, $\Pi_{t_0,C}$, increases due to its call option characteristics[1].

Figure 6 shows the mortality risk and the basis risk for class A and B for n = 10 years with different number of policies included in the SPV. It is clear that basis risk affects the expected loss of the bond classes. One should include at least 100,000 policies in the SPV to reduce the basis risk. 200,000 are of course even better. If this is difficult for one insurer to achieve alone, several insurers can cooperate. However, the expected loss and corresponding rating do not change much. For 10 years, only class B with 50,000 policies changes rating one class. The others are unchanged. Note that one should focus on expected loss, and that whether the rating changes or does not depend on the choice of base case parameters.

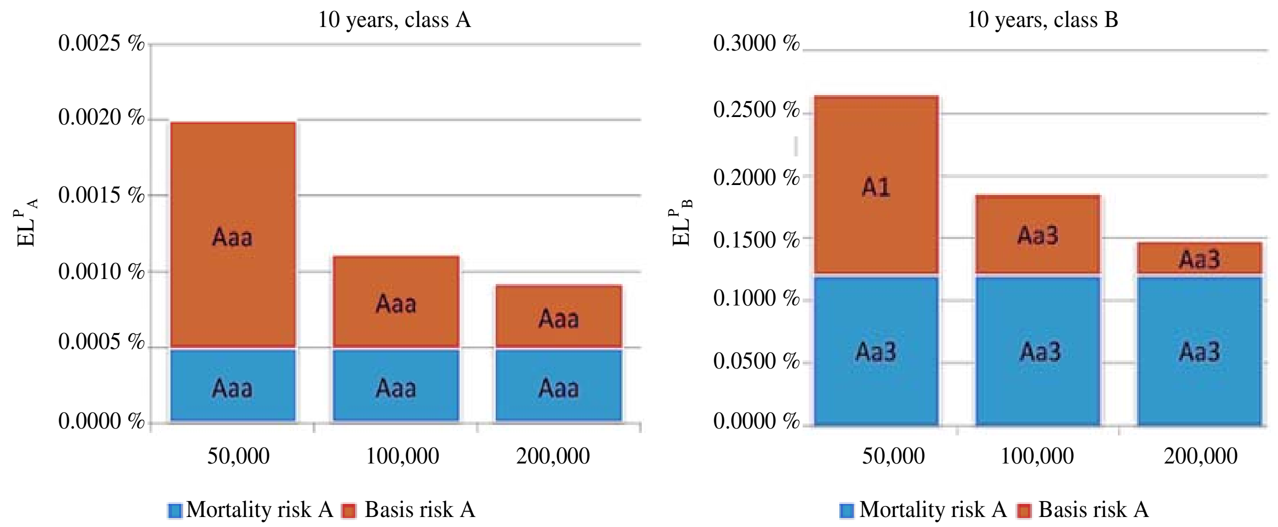


Fig. 6. These charts show how basis risk affects the expected loss and corresponding ratings for classes A and B for n = 10 when including a different number of policies, m, in the *SPV*

[1] As is known from option pricing theory, call options increase in value with increasing volatility. Thus, the value of class C, $\Pi_{t_0,C}$, increases when $StdDev(E^P[V_T])$, which is the volatility of the underlying of class C, increases.

Conclusion

This paper contributes to the literature on mortality rate modelling and on securitization. In the paper we have proposed a possible securitization structure for term insurance policies. The securitization structure has also been calibrated to market data using the Vita III-deal and mortality data for England and Wales and the United States. We find that the structure makes it possible for life insurance companies to reduce their exposure to mortality risk, i.e., changes in mortality rates at a reasonable cost. To make the different classes of the structure transparent for investors, we have constructed classes A and B so as to have ratings Aaa and Aa3. High ratings, combined with decent expected rates of returns that have low correlation with other asset classes are likely to make securitization structures of life insurance policies attractive investments. The only uncertainty included in our model is changes in mortality rates. This simplification makes the structure easy to analyze and hopefully will foster a stronger interest among life insurers and investors towards securitization of life insurance policies.

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Appendix A. Vita Capital III overview

Table 10. Overview of the nine different bonds issued on January 1st, 2007, called Vita III.

Series	1	2	3	4	5	5	6	6	7
Rating	A	A	A	AAA	AAA	AAA	AAA	AAA	AA-
Principal	$90	$50	€30	$100	$100	$50	€55	€55	€100
Class	B	B	B	A	A	B	A	B	A
Attachment	120%	120%	120%	125%	125%	125%	125%	120%	125%
Exhaustion	125%	125%	125%	145%	145%	145%	145%	125%	145%
Maturity	4	5	4	4	5	5	4	4	5
Currency	L	L	E	L	L	L	E	E	E
Margin	110	112	110	21	20	21	21	22	80
Guaranteed				X	X	X	X	X	

Source: Standard & Poor's (2007).

Appendix B. Moody's idealized loss rate table

	Horizon									
Rating	1-Year	2-Year	3-Year	4-Year	5-Year	6-Year	7-Year	8-Year	9-Year	10-Year
Aaa	0,0000%	0,0001%	0,0004%	0,0010%	0,0016%	0,0022%	0,0029%	0,0036%	0,0045%	0,0055%
Aa1	0,0003%	0,0017%	0,0055%	0,0116%	0,0171%	0,0231%	0,0297%	0,0369%	0,0451%	0,0550%
Aa2	0,0007%	0,0044%	0,0143%	0,0259%	0,0374%	0,0490%	0,0611%	0,0743%	0,0902%	0,1100%
Aa3	0,0017%	0,0105%	0,0325%	0,0556%	0,0781%	0,1007%	0,1249%	0,1496%	0,1799%	0,2200%
A1	0,0032%	0,0204%	0,0644%	0,1040%	0,1436%	0,1815%	0,2233%	0,2640%	0,3152%	0,3850%
A2	0,0060%	0,0385%	0,1221%	0,1898%	0,2569%	0,3207%	0,3905%	0,4560%	0,5401%	0,6600%
A3	0,0214%	0,0825%	0,1980%	0,2970%	0,4015%	0,5005%	0,6105%	0,7150%	0,8360%	0,9900%
Baa1	0,0495%	0,1540%	0,3080%	0,4565%	0,6050%	0,7535%	0,9185%	1,0835%	1,2485%	1,4300%
Baa2	0,0935%	0,2585%	0,4565%	0,6600%	0,8690%	1,0835%	1,3255%	1,5675%	1,7820%	1,9800%
Baa3	0,2310%	0,5775%	0,9405%	1,3090%	1,6775%	2,0350%	2,3815%	2,7335%	3,0635%	3,3550%
Ba1	0,4785%	1,1110%	1,7215%	2,3100%	2,9040%	3,4375%	3,8830%	4,3395%	4,7795%	5,1700%
Ba2	0,8580%	1,9085%	2,8490%	3,7400%	4,6255%	5,3735%	5,8850%	6,4130%	6,9575%	7,4250%
Ba3	1,5455%	3,0305%	4,3285%	5,3845%	6,5230%	7,4195%	8,0410%	8,6405%	9,1905%	9,7130%
B1	2,5740%	4,6090%	6,3690%	7,6175%	8,8660%	9,8395%	10,5215%	11,1265%	11,6820%	12,2100%
B2	3,9380%	6,4185%	8,5525%	9,9715%	11,3905%	12,4575%	13,2055%	13,8325%	14,4210%	14,9600%
B3	6,3910%	9,1355%	11,5665%	13,2220%	14,8775%	16,0600%	17,0500%	17,9190%	18,5790%	19,1950%
Caa1	9,5599%	12,7788%	15,7512%	17,8634%	19,9726%	21,4317%	22,7620%	24,0113%	25,1195%	26,2350%
Caa2	14,3000%	17,8750%	21,4500%	24,1340%	26,8125%	28,6000%	30,3875%	32,1750%	33,9625%	35,7500%
Caa3	28,0446%	31,3548%	34,3475%	36,4331%	38,4017%	39,6611%	40,8817%	42,0669%	43,2196%	44,3850%

Note: Cumulative idealized loss rates for Moody's rating classes.
Sourse: Moody’s Investor Service, April 2007, *Special Comment on Default & Loss Rates of Structured Finance Securities*: *1993-2006*, New York.