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Exchange rate volatility and its impact on risk management with internal models in commercial banks

Abstract

Financial markets create a business environment of a commercial bank. Price movement of an asset is an important attribute of a financial market and is defined with its size. Central banks adjust price movements with monetary policy based on market activity. The same holds for foreign exchange markets where central bank affects market activity with its exchange rate. Due to the capital decree legislated by Bank of Slovenia, Slovenian commercial banks can apply internal models for capital requirements calculation for currency risk and selected market risks (general position risk in line with debt and equity instruments, price change risk for commodities) as an alternative or in combination with standardized methodology. If banks use internal models for capital charge calculations all features of financial markets should be embedded in the internal model in order to assure proper accuracy of the model. The goal of this paper is to identify reasons for a decay factor application in internal models in small financial markets, and to show a proper back testing procedure in case of application of a decay factor. A proper back testing procedure shall be found using linear programming.

Keywords: monetary policy, central bank, financial markets, risk management, decay factor, internal models, back testing.
JEL Classification: F31, G12, G15, G21, G32.

Introduction

In this paper we shall understand a commercial bank as a financial investor. When calculating capital charge for foreign exchange risk, commercial banks can use standardized principle or internal model. Internal risk management model is an effective approach for managing foreign exchange risk as it assures prudential capital charge based on precise risk measurement. When commercial banks use internal models for risk management and consequentially calculating capital charge, value at risk (VaR) as a risk measure[1] should be applied. Commercial banks commonly use two different principles for VaR calculation. The first principle is historical simulation and the second one is delta normal approach. Historical simulation principle of VaR calculation is favourably used in practice because of its independence of distribution. The delta normal principle is based on as assumption of multinormal distribution of returns, which results in underestimating the risk in case distributions differ from normal distribution. If necessary, commercial banks can apply time weighting of asset returns in order to obtain better risk estimation to which they are being exposed to.

The goal of this paper is to identify the reasons for application of time weights in internal model and to explain the modification of backtesting approach in case when time weights are being applied. Due to the directives 2000/12/EC, CAD 93/6/EEC and CAD3 directives, commercial banks can apply internal models for capital requirements calculation for currency risk and selected market risks (general position risk in line with debt and equity instruments, price change risk for commodities) as an alternative or in combination with standardized methodology.

When applying for internal VaR model, a commercial bank should, in line with directives, use time series of data, which is no shorter than one trading year. Time series of data therefore can be longer, but it should never be shorter. Application of time weighting and therefore a decay factor effectively shortens time series of data. The condition in directives refers to an effective length of time series of data. Effective observation period is calculated with average time lag of the individual observations, which cannot be less than six months (Basel Committee on Banking Supervision, 1996).

1. Theoretical background

Holton (1998) proposed a solution to reweighting historical scenarios which are able to adjust any moments (standard deviation, kurtosis, correlation) or a variety of other parameters. Hull and White (1998 and 1998a) published a crude reweighting methodology which is able to match only one or two moments. Holton (1999) introduced the methodology of weighted scenarios that can be used to enhance either Monte Carlo VaR or historical VaR. The technique solves the problem of "phantom drift" that arises when historical VaR is based upon data from a period that experienced a net increase or decrease in a risk factor's value.

Richardson, Boudoukh, Whitelaw (1998) presented a hibrid approach which combines two most popular methods of VaR estimation: RiskMetrics and historical simulation. It estimates the VaR of a portfolio by applying exponentially declining weights to

[1] More about VaR can be found in Jorion (2001).

past returns and then finding the appropriate percentile of this time-weighted empirical distribution. Empirical tests show a significant improvement in the precision of VaR forecasts using the hybrid approach relative to RiskMetrics and Historical Simulation. It is especially appropriate for calculating the VaR of fat-tailed and highly skewed data with rapidly changing moments.

A good review of various papers that proposed reweighting schemes can be found in Dowd (2005).

Hsueh, Shyng, Lin (2002) compared the accuracy of various approaches to historical simulation (alternative weighting schemes for historical exchange rate data). Hsieh and Lin (2003) employed alternative exponentially weighted moving average estimator which is based on the maximum likelihood estimator of the variance of generalized error distribution in conjunction with historical simulation when computing Value-at-Risk of exchange rate. They suggest that incorporating generalized error distribution into the historical simulation method is a substantial improvement in exchange rate.

Foreign exchange risk management process in a commercial bank should include all risk factors to which a bank is being exposed to. Each risk factor is managed by a central bank and is a function of macroeconomic variables. Stability in foreign exchange markets is in general one of the basic goals of monetary policy. Bank of Slovenia since ERM2 entrance maintains stable nominal exchange rate with monetary policy. Its interest rate policy is subordinated to assure stability of nominal exchange rate. Monetary policy goal of exchange rate stability and potential structural adaptability of instrument set of central bank require adaptability of commercial banks in trading on foreign exchange markets. Central bank with a monetary and exchange rate policy determines business environment of a commercial bank. This should be considered within internal model for foreign exchange risk management by a commercial bank and has a special role in the stress testing programme which is a part of an internal risk management model. Risk factors define business environment of a commercial bank and should be captured in a risk measuring process and in a stress testing program of a bank.

For capital requirements calculation purpose commercial banks can use standardized methodology or internal models. When using an internal model a commercial bank has to take into consideration macroeconomic environment as its business surrounding. Moreover, the exchange rate against home currency is determined by a central bank. Exchange rate can be more or less variable depending on goals that central bank has. If a commercial bank manages exchange rate risk with an internal model it has to observe the variability of an exchange rate. Foreign exchange markets can be distinguished according to their daily trading volume. Using this criterion foreign exchange markets can be large or small size foreign exchange markets. On small financial markets we can expect larger price movements and therefore yield clustering more frequently. Higher variability can be shown with a leptokurtosis of a yield probability distribution function. Business environment of a commercial bank defines corporate and retail customers with their demand and a central bank with its monetary and exchange rate policy. The latest is of a great importance.

Yield independence is one among key assumptions of a temporally independently and identically distributed or IDD model[1].

For empirical analysis EUR/SIT, EUR/HRK, EUR/CSD, EUR/CZK and EUR/PLN currency pairs were selected. Durbin-Watson test shall be used in order to show evidence of first order autocorrelation. In this research Reuters exchange rates from June 28, 2004 to December 30, 2005 were considered. Each currency has its own holidays. These holidays are included in the time series and as there are no market data available for these dates, the lengths of all time series are different. Daily exchange rate values were used to calculate continuously compounded daily yields, which are used in the analysis. When we write about exchange rates in this article we will refer to daily exchange rate yields.

1.1. Autocorrelation test and stationarity test. In EUR/SIT exchange rate time series from ERM2 entrance onward evidence of a negative first order autocorrelation has been detected using Durbin-Watson test. The value of d statistic is 3,023. Autocorrelation diagnostic for EUR/HRK exchange rate shows there is no first order autocorrelation present as Durbin-Watson d statistic is 1,989. First order autocorrelation can be detected on EUR/CSD time series as the value of d statistic is 2,491. There was no first order autocorrelation detected either for EUR/CZK ar for EUR/PLN time series. The value of d statistic for EUR/CZK is 2,007, and the value of d statistic for EUR/PLN is 1,9633. Both results show there is no evidence of first order autocorrelation. Therefore a null hypothesis for Durbin-Watson test cannot be rejected. Autocorrelation diagnostic for EUR/USD exchange rate time series shows IDD assumption holds as Durbin-Watson d statistic for EUR/USD is 2,005.

[1] More about IDD model can be found in Campbell, Lo, MacKinlay (1997).

The presence of first order autocorrelation is one reason why a bank could consider an application of a decay factor in its internal risk management model. Besides this feature of exchange rate behavior banks should also consider clustering of returns. Assume there is no clustering of high and/or low returns. Then there is no need for application of a decay factor. In case of evidence of yield clustering, bank will face time subperiods of high exchange rate variance, and time subperiods with low exchange rate variance. We shall test clustering of exchange rate yields with a concept of time series stationarity. If a time series is stationary, then its mean, variance and autocovariance should be time invariant. According to the financial theory, asset prices follow random walk and are therefore non-stationary. But the first differences of a random walk time series are stationary. The value of an asset should be equal to its price on the previous day plus a random shock. If there is no random shock a time series has no unit root problem and is therefore stationary (Gujarat, 1991).

The research shows selected time series of exchange rates exhibit a first order autocorrelation. These time series are therefore also non-stationary as they do not fulfil a requirement of time invariant autocovariance. Stochastic process is stationary if there is no autocovariance in time and if also its mean and variance are time invariant. In order to see if mean and variance of these exchange rate time series are time invariant, we shall split the observed time horizon in two time subperiods and test the assumption of difference between means and the assumption of equal variances between so defined time series.

For the research purpose time horizon shall be split in two equal parts. The first part shall include the first half of exchange rate time series, and the second one shall include the second half of time series.

Table 1. Independent samples test of equal means and equal variances

		Levene's test for equality of variances		t-test for equality of means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. error difference	95% confidence Interval of the difference	
									Lower	Upper
EUR/SIT	Equal variances assumed	2,492	,115	,263	392	,793	,000003	,000012	-,000021	,00002695
	Equal variances not assumed			,263	327,687	,793	,000003	,000012	-,000021	,00002696
EUR/CSD	Equal variances assumed	4,492	,035	1,345	392	,180	,000315	,000234	-,000146	,00077596
	Equal variances not assumed			1,345	368,929	,180	,000315	,000234	-,000146	,00077605

Source: Reuters data and own calculation with SPSS 12.1 for Windows.

Before testing the assumption of a difference between means, analysis of variances should be performed. Testing the equality of two variances in this research will be done with Levene's test. The null hypothesis has a general form of $H_0 : \sigma_1 = \sigma_2$ and adequate alternative hypothesis has a general form of $H_1 : \sigma_1 \neq \sigma_2$. For EUR/SIT it holds

$$F_L = 2{,}492 < F_{(\alpha = 0{,}05,\, m_1 = 1,\, m_2 = 392)} = 3{,}865 .$$

Null hypothesis cannot be rejected. The variances in two time subperiods are not different and therefore no yield clustering exists. For EUR/SIT stationarity was not rejected but it cannot be confirmed either. We cannot be sure if EUR/SIT is stationary as this research captures only two time subperiods and ignores the combinations of all remaining subperiods. There might be volatility clustering present in other time subperiods which were not tested in this research. For stationary time series of asset prices the use of a decay factor in risk management process is not grounded.

For EUR/CSD Levene's test shows significant differences between variances for two time subperiods. Comparing expected yields between two time subperiods for EUR/SIT and for EUR/CSD with equality of means test shows no significant differences. Therefore the assumption of equal means $H_0 : \mu_1 = \mu_2$ cannot be rejected but it cannot be confirmed either. The assumption holds only for selected time subperiods of data. For total confirmation of the assumption of equal means, the set of all samples of rime subperiods should be tested.

For all other currency pairs, EUR/HRK, EUR/CZK, EUR/PLN and EUR/USD, Dickey-Fuller test will be applied in order to test stationarity of exchange rate time series. Let *Y* be an exchange rate time series and let u_t be an error term. In this research Dickey-Fuller test will be applied to the regression in the following form (Gujarati, 1991):

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + u_t$$

Table 2. Regression model coefficients

Model	Unstandardized coefficients		Standardized coefficients	t	Sig.
	B	Std. error	Beta		
1 (Constant)	,001	,001		1,102	,271
t	-3,38E-06	,000	-,049	-1,371	,171
EURUSD (t-1)	-1,004	,050	-,711	-19,909	,000

Note: Dependent variable: dEUR/USDt.
Source: Reuters data and own calculation with SPSS 12.1 for Windows.

Table 3. Regression model summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,710[a]	,504	,502	,00553

Note: [a]. Predictors: (constant), EUR/USD (t-1), t.
Source: Reuters data and own calculation with SPSS 12.1 for Windows.

As computed $t = \tau = -19{,}909$ for the EUR/USD indicates null hypothesis $H_0 : \delta = 0$ can be rejected and alternative hypothesis $H_1 : \delta \neq 0$ therefore applies. For all remaining currency pairs in the financial analysis the value of τ statistics corresponds to inequality $\tau \leq -19{,}513$. $\tau = -19{,}513$ holds for EUR/PLN currency pair. All time series of exchange rates in research were shown to be stationary. Consequently, homoscedasticity has been shown along with the stationarity and therefore no volatility clustering has been detected based on selected time horizon.

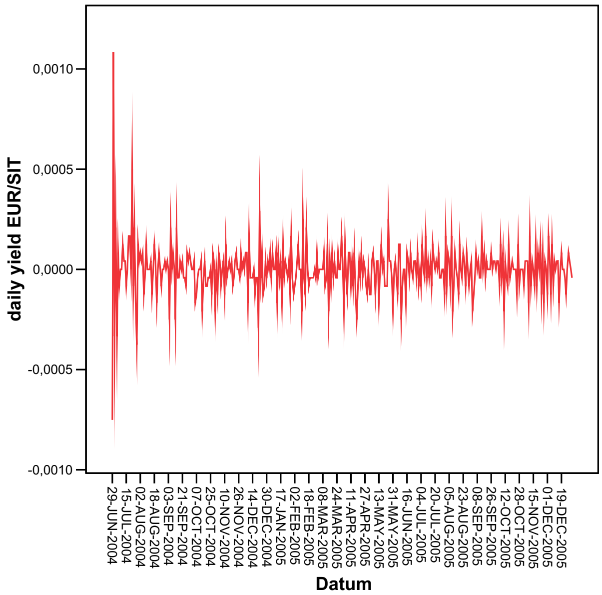
In case of yield clustering detected a decay factor should be applied in order to improve the risk exposure assessment accuracy. The value of applied decay factor should be in a negative correlation with the detected kurtosis of a yield probability distribution function. The existence of kurtosis proves yields of an asset are not stationary. The higher the kurtosis of a yield distribution function is, the lower should be the value of a decay factor. In case there is no excess kurtosis then the kurtosis κ of a yield distribution function is $\kappa = 3$ and a corresponding decay factor λ should be $\lambda = 1$. Then the following equation therefore applies:

$$\lim_{\kappa \to \infty} f(\lambda) = 0.$$

Table 4. Descriptive statistic for selected exchange rate yields

Curreney pairs	N	Std.	Kurtosis	
	Statistic	Statistic	Statistic	Std. error
EUR/PLN	393	,00536	,454	,246
EUR/CZK	394	,00292	,975	,245
EUR/SIT	394	,00012	21,000	,245
EUR/HRK	394	,00204	,814	,245
EUR/CSD	394	,00233	4,280	,245
EUR/USD	396	,00553	,302	,245
Valid N (listwise)	393			

Source: Reuters data and own calculation with SPSS 12.1 for Windows.



Source: own calculation.

Fig. 1. EUR/SIT exchange rate movements from ERM2 entrance onward

1.2. Modelling and back testing. Let us assume a commercial bank is using an internal model for managing foreign exchange rate risk. When exchange rate yield is more variable and when clustering of variability can be observed, a commercial bank has to properly model these specifics. Clustering can be at latest shown in back testing results as clustering of excessions can prove an existence of yield variability clustering. Proper risk management with an internal model would indicate the need to use time weightening and implement the use of a decay factor. Of course there are several levels of foreign exchange market liquidity to observe and several levels of leptokurtosis can be assigned. The higher the leptokurtosis of a yield probability distribution of an exchange rate as an instrument, the lower decay factor should be assigned.

Back testing shows the accuracy of an internal model with a number of excessions. It uses VaR as a criteria number along with actual and hypothetical portfolio loss. Clustering of loss excessions according to VaR could imply high autocorrelation in risk. If the internal model in a commercial bank is used for foreign exchange risk management, the autocorrelation refers to an exchange rate autocorrelation. An optimal back testing result is an even distribution of excessions in different volatility regimes, which shows that the VaR model is responsive to a variety of market conditions. Market conditions are a business environment of a commercial bank and should therefore also been captured within VaR internal model.

The clustering of loss excessions therefore supports selection of a decay factor. If VaR is not responsive to the increased revenue volatility, this indicates poor parameterization and lower decay factor should be selected.

1.3. Model. Here we shall define what an efficient time series is to be used within internal models when a decay factor has been applied. When a decay factor (smaller than 1) was applied, time series of data should be extended as a condition for internal model use requires an efficient time series of data, which should be no shorter than 250 daily data.

Let us assume that a commercial bank is using an internal model for managing foreign exchange rate risk and no time weightening has been applied. In this case a commercial bank has to use 250 daily risk factor values. Let μ be a weighted average and an effective number of data in time series. Let N be a number of all data in observed time series, and let n_i be a successive number of a data in a time series, where i stands for its place in the range queue. When there is no time weightening in general holds the following equation:

$$\mu = \frac{\sum_{i=1}^{N} n_i}{N} = \frac{\frac{N \cdot (N+1)}{2}}{N} = \frac{N+1}{2} = \frac{250+1}{2} = 125{,}5.$$

Effective number of data in time series should not be shorter than 125,5. This corresponds to the requirement that effective observation period cannot be less than six months (Basel Committee on Banking Supervision, 1996).

The idea of an effective time series is identical to the average weighted maturity concept of a security. If we assume that unconditional returns are not IDD, then it can be assumed that the data on returns from the nearer past are better representative of future risk than other. As possible solution to the problem Boudoukh, Richardson and Whitelaw (1998) suggested generalized historical simulation method known as BRW model. BRW model assigns different weights to returns, depending on time of their origin. Last historical return r_t has assigned weight $a_1 = 1$, the return before that r_{t-1} has assigned weight a_2, where $a_2 = a_1\lambda$ and analogically. λ represents exponential decay factor with values on the interval between 0 and 1. The largest weights are assigned to the yields from nearer past.

If a commercial bank applies time weightening with a_i weightening scheme, then the general equation holds:

$$\mu = \frac{\sum_{i=1}^{N} a_i \cdot n_i}{N} = \frac{a_1 + 2a_2 + \ldots + 250a_{250}}{250}.$$

Let λ be a decay factor. Then the following applies:

$$\mu = \frac{\sum_{i=1}^{N} a_i \cdot n_i}{N} = \frac{a_1 + 2a_2 + \ldots + 250a_{250}}{250} =$$

$$= \frac{\overbrace{a_1}^{a_1=1} + 2 \cdot \left(\overbrace{a_1\lambda}^{1\cdot\lambda=\lambda} \right) + 3 \cdot \left(\overbrace{a_2\lambda}^{a_2=a_1\lambda \Rightarrow a_2\lambda=a_1\lambda^2} \right) \ldots + 250a_{250}}{250} =$$

$$= \frac{1 + 2\cdot\lambda + \ldots + 250a_{250}}{250} \geq 125{,}5.$$

If the last equation does not hold, a commercial bank has to extend its time series of data. Time series of data should be extended that the following condition would be met:

$$\mu = \frac{\sum_{i=1}^{N} a_i \cdot n_i}{N} = \frac{a_1 + 2a_2 + \ldots + 250a_{250} + \ldots + Na_N}{N} \geq 125{,}5 \cdot$$

The bank would seek for a minimal N so that the last inequality would hold. This is a solution of a minimization problem for which a bank will seek a solution. The length of time series can be longer therefore μ can be $\mu \geq 125{,}5$. When a bank applies a decay factor, the value of a decay factor determines relative importance of a daily trading result in a time series of trading results on a daily basis. In case there is no time weightening applied, the sum of all weights is determined with equality $\sum_{i=1}^{N} a_i = 250$. As a commercial bank can use a longer time series of data and as 250 trading days is a minimum length of a time series, equality

$\sum_{i=1}^{N} a_i = 250$ can be generalized to $\sum_{i=1}^{N} a_i \geq 250$. The last inequality is very important. The sum of all weights should be at least 250, what makes sense. But this is only one condition in a constraint set when the required length of time series is being determined. The other constraint refers to an effective observation period. When determining required length of a time series, the larger value of all constraint will be selected as all constraints should be met. Therefore the value of N corresponds to the solution of the following optimization problem:

Min N subject to $\sum_{i=1}^{N} a_i \geq 250$

$$\mu = \frac{\sum_{i=1}^{N} a_i \cdot n_i}{N} =$$

$$= \frac{a_1 + 2a_2 + \ldots + 250\, a_{250} + \ldots + Na_N}{N} =$$

$$= \frac{a_1 \cdot \left(1 + 2\lambda + 3\lambda^2 + \ldots + N\lambda^{N-1}\right)}{N} \geq 125{,}5$$

$$a_i \geq 0 \forall i \in \{1, 2, \ldots, N\}$$

$$N \geq 0 \ .$$

Suppose $\lambda = 0{,}999$. If a commercial bank considers only $N = 250$ data in exchange rate time series, we get:

$$\sum_{i=1}^{N} a_i \geq 250,$$

$$\frac{1}{250}\sum_{i=1}^{N} a_i \geq 1,$$

$$\frac{1}{250}\sum_{i=1}^{250} a_i = \frac{1}{250}\left(1 + \lambda + \lambda^2 + \ldots + \lambda^{250-1}\right) = 0{,}885187 \cdot$$

Calculation result shows that the length of exchange rate time series data is too short and should therefore be extended. Searching the solution only for the condition $\sum_{i=1}^{N} a_i \geq 250$ would give a solution of $N = 288$. The calculation shows that average effective length of time series with $N = 288$ data is 119,6049. Therefore the first condition is fulfilled, but the second one is not fulfilled. We shall now search the minimal length of time series so that the second condition in predefined optimization problem would be fulfilled. The following therefore applies:

$$\min\left(\frac{a_1 + 2a_2 + \ldots + 250a_{250} + \ldots + Na_N}{N} > 125{,}5\right),$$

$$\min\left(\frac{a_1 + 2a_2 + \ldots + 250a_{250} + \ldots + Na_N}{N} > 125{,}5\right) = 306 \cdot$$

We have calculated that the solution of optimization problem min N subject to

$$\sum_{i=1}^{N} a_i \geq 250$$

$$\mu = \frac{\sum_{i=1}^{N} a_i \cdot n_i}{N} =$$

$$= \frac{a_1 + 2a_2 + \ldots + 250a_{250} + \ldots + Na_N}{N} =$$

$$= \frac{a_1 \cdot \left(1 + 2\lambda + 3\lambda^2 + \ldots + N\lambda^{N-1}\right)}{N} \geq 125{,}5$$

$$a_i \geq 0 \forall i \in \{1, 2, \ldots, N\}$$

$$N \geq 0$$

when $\lambda = 0{,}999$ is $N = 306$.

Conclusion

The goal of this paper was to search for reasons when and why a commercial bank should apply a decay factor in internal model for foreign exchange risk management. We showed the autocorrelation or stationarity is the reason to implement a decay factor in an internal model for foreign exchange risk management purpose in a commercial bank. Autocorrelation has been shown for EUR/SIT and EUR/CSD currency pairs. EUR/SIT and EUR/CSD currency pairs were shown to have a unit root due to existing first order autocorrelation. Presence of a volatility clustering or mean variability in time cannot be rejected as the research was not based on the set of all samples. All remaining currency pairs in the research were shown to be stationary. If a time series is non-stationary, the reason for a decay factor implementation is supported. When a decay factor is applied, the length of a time series should be extended. The length of a time series can be calculated with an optimization mathematical model, which has first been explained and then applied with a selected decay factor.

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