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Estimation of the degree of market imperfections: theory and application in currency futures markets

Abstract

This study aims to address two new issues related to market imperfections and currency futures pricing. First, this study presents a measurement model for valuing the degree of market imperfections between the currency futures market and its underlying spot market, and investigates the applicability of the model to estimate the degree of market imperfections for the CME Japanese yen, the CME British pound, the CME Swiss franc, and the CME Australian dollar futures markets. Furthermore, this study also proposes a theoretical hypothesis and an empirical test regarding the relationship between the degree of market imperfections and the currency futures pricing. The empirical results indicate that the measurement model of the degree of market imperfections appears to provide a reasonable and practice-oriented measure for real market imperfections. Additionally, this study finds that larger market imperfections are relatively more mispriced by the cost of carry model based on the perfect-market assumptions, suggesting that there exists a significant positive relationship between the market imperfections and the mispricing of currency futures.

Keywords: pricing of currency futures, a measurement model of degree of market imperfections, EWMA.
JEL Classification: C00, G13, G15.

Introduction

Until now, the cost of carry model has been the most widely used model for pricing currency futures. This model was developed under the assumption of perfect markets and no-arbitrage arguments[1]. In perfect markets, if the "carrying cost" is in a state of disequilibrium (that is, the actual price of currency futures deviates from its theoretical value predicted by the cost of carry model), then the riskless and profitable arbitrage would ensure that the difference between the currency futures price and the spot price converges the carrying cost.

However, market conditions in the real world frequently violate the assumption of perfect markets. In real markets there exist market imperfections. First, arbitrage trades involve transaction costs, including commissions, spread between the bank's ask and bid prices, and taxes. Second, the ability to borrow and lend at the same risk-free interest rate is questioned. In fact, differential borrowing and lending rates exist in each market. Moreover, when investors are engaged in currency arbitrage activities, they usually face four rates – two in each country. Third, futures contracts are not perfectly divisible. The indivisibility of the futures contract prevents investors from purchasing (or selling) exact amounts of futures. Fourth, in practice, it is difficult to instantaneously buy or sell the underlying spot in precise weights. Given delayed spot market execution of arbitrage trades, errors are likely to occur between estimated and actual arbitrage profits. Fifth, traders cannot process quickly available information. Finally, traders may have asymmetric information. Copeland (1976) proposed that information is made available to various market participants sequentially. Kao and Ma (1992) found that short-term price dependence in currency futures market may be caused by trades possessing a heterogeneous information set.

Market imperfections differ among markets. Compared to immature markets, mature markets have relatively high level of information efficiency. Moreover, financial market frictions such as transaction costs, regulatory barriers, liquidity risk, and capital constraints are less likely in mature markets. Therefore, the extent of market imperfections is lower for mature markets than for immature ones. However, a key question is how to precisely estimate the extent of market imperfections for different markets. Following the works of Hsu and Wang (2004) and Wang and Hsu (2006), who developed a model of the degree of market imperfections for the stock index futures, this study aims to address two new issues related to market imperfections and currency futures pricing. First, this study uses an argument regarding the incompleteness of arbitrage mechanisms to develop a measurement model for valuing the degree of market imperfections between the currency futures market and its underlying spot market (hereafter the degree of market imperfections). Meanwhile, by using the CME (Chicago Mercantile Exchange) Japanese yen futures, the CME British pound futures, the CME Swiss franc futures, and the CME Australian dollar futures, this study also tests the applicability of the model to predict the degree of market imperfections for real markets.

[1] The cost of carry model with continuous compounding is the same as the interest rate parity model with continuous compounding of interest rates. For a discussion of this point, see Jabbour (1994).

Next, this study proposes a theoretical hypothesis regarding the relationship between the degree of market imperfections and the currency futures pricing. Meanwhile, using the four currency futures, this study implements an empirical test of this theoretical hypothesis.

The paper is organized as follows. Section 1 develops a model for valuing the degree of market imperfections. Section 2 provides a theoretical hypothesis regarding the relationship between the degree of market imperfections and the currency futures pricing. Empirical procedures and data are presented in Section 3. Section 4 reports the empirical results. Finally, the last section presents conclusions.

1. A model for valuing the degree of market imperfections

This study uses the following assumptions to derive a measurement model for valuing the degree of imperfections between the currency futures market and its underlying spot market: (1) The domestic risk-free interest rate (r) and foreign risk-free interest rate (r_f) are constant. (2) The spot exchange rate, S, follows a geometric Wiener process:

$$dS = (u - r_f)S\, dt + \sigma S\, dZ,$$

or

$$\frac{dS}{S} = (\mu - r_f)dt + \sigma dZ\ , \tag{1}$$

where dZ is a Wiener process. The parameters u and σ denote the instantaneous constant expected rate of return in S and the instantaneous constant volatility of S, respectively. This stochastic process for an exchange rate is the same as that for a stock paying a dividend yield equal to the foreign risk-free interest rate (see Biger and Hull, 1983; and Hull 2006).

Let the currency futures price, F, be a twice continuous and differentiable function of spot exchange rate, S, and once continuous and differentiable function of time, t. Using Ito's Lemma, we have:

$$dF = \frac{\partial F}{\partial S} dS + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 =$$

$$= [\frac{1}{2}\sigma^2 S^2 F_{ss} + (u - r_f)SF_s + F_t]\, dt + (\sigma SF_s)\, dZ. \tag{2}$$

Let u_f and σ_f denote the instantaneous expected return on futures, and the instantaneous standard deviation of return on futures, respectively. Then, equation (2) can be rewritten as follows:

$$\frac{dF}{F} = u_f\, dt + \sigma_f\, dZ, \tag{3}$$

where

$$u_f = (\frac{1}{2}\sigma^2 S^2 F_{ss} + (u - r_f)SF_s + F_t) / F, \tag{4}$$

and

$$\sigma_f = (\sigma SF_s) / F. \tag{5}$$

Now, consider a portfolio P that consists of one unit of the spot position and x units of currency futures position. The change in the value of the hedged portfolio, dP, is then given by:

$$dP = [\, x\,(dF) + (dS)] + Sr_f dt. \tag{6}$$

It is assumed that no initial cash outflow is required for the futures contracts. Therefore, current value of the portfolio is equal to the current spot price, that is, $P = S$. The rate of return of the hedged portfolio is then given by

$$\frac{dP}{P} = [\frac{x}{S}(dF) + (\frac{dS}{S})] + r_f dt$$

$$= [\frac{xF}{S}(\frac{dF}{F}) + (\frac{dS}{S})] + r_f dt. \tag{7}$$

Define $w_f = \frac{xF}{S}$. Then equation (7) becomes:

$$\frac{dP}{P} = [w_f(\frac{dF}{F}) + (\frac{dS}{S})] + r_f dt. \tag{8}$$

Substituting equations (1) and (3) into equation (8) yields:

$$\frac{dP}{P} = (w_f u_f + u)dt + (w_f\sigma_f + \sigma)dZ. \tag{9}$$

If $w_f = -\frac{\sigma}{\sigma_f}$, then $w_f\sigma_f + \sigma = 0$. In this case, the return of P is certain and the portfolio is riskless. However, notice that the portfolio is not permanently riskless. Theoretically, it remains riskless only for an instantaneous point of time. As S and t change, w_f also changes. To keep the portfolio riskless, it is necessary to "continuously" rebalance w_f until the expiration of the futures contract. This cannot actually be done in real capital markets. The reason is that there exist market imperfections in real foreign exchange and currency futures markets. For example, currency arbitrage involves transaction costs, and futures contracts are not perfectly divisible. Thus, under the assumptions of perfect markets, it is possible to continuously choose w_f so that $w_f\sigma_f + \sigma = 0$. Under the no-arbitrage argument, the portfolio must earn the risk-free rate at any instant. However, in imperfect markets, because arbitrage mechanism can not be complete and currency arbitrage is exposed to large risk, the portfolio cannot be riskless at any instant. This means that the portfolio must earn some expected rate of return (which can be greater than, smaller than, or equal to the risk-free rate), rather than the risk-free rate at any instant.

Let u_p and σ_p denote the instantaneous expected rate of return of the hedged portfolio, and the coefficient of dZ in equation (9), respectively. Thus, we have

$$w_f u_f + u = u_p, \tag{10}$$

$$w_f \sigma_f + \sigma = \sigma_p. \tag{11}$$

From (10) and (11), we obtain the following equilibrium condition in imperfect markets for the currency futures:

$$\frac{u_f}{\sigma_f} = \frac{u - u_p}{\sigma - \sigma_p}. \tag{12}$$

Substituting equations (4) and (5) into equation (12), we obtain the partial differential equation (PDE) for currency futures prices in imperfect markets:

$$\frac{1}{2}\sigma^2 S^2 \left(1-\frac{\sigma_p}{\sigma}\right) F_{ss} + \left[(u_p - r_f) - (u - r_f)\frac{\sigma_p}{\sigma}\right] SF_s + + F_t \left(1-\frac{\sigma_p}{\sigma}\right) = 0. \tag{13}$$

Further simplifying, we have:

$$\frac{1}{2}\sigma^2 S^2 F_{ss} + u_\alpha S F_s + F_t = 0, \tag{14}$$

where $u_\alpha = [(u_p - r_f) - (u - r_f)\frac{\sigma_p}{\sigma}]/(1-\frac{\sigma_p}{\sigma})$.

Using the partial differential equation technique and the no-arbitrage argument, the second order PDE can also be derived for currency futures prices in perfect markets, as follows:

$$\frac{1}{2}\sigma^2 S^2 F_{ss} + (r - r_f) S F_s + F_t = 0. \tag{15}$$

We compare PDE (14) and PDE (15). When the markets are perfect and the arbitrage mechanism is complete, the portfolio consisting of currency futures contracts and the underlying asset becomes a perfectly hedged portfolio and $\sigma_p = 0$. In this case, $\frac{\sigma_p}{\sigma}$ also equals zero. As $\frac{\sigma_p}{\sigma} \rightarrow 0$, then $(1-\frac{\sigma_p}{\sigma}) \rightarrow 1$ and $[(u_p - r_f) - (u - r_f)\frac{\sigma_p}{\sigma}] \rightarrow (r - r_f)$ in PDE (14). PDE (14) thus is identical to PDE (15). In other words, the PDE for currency futures prices in imperfect markets is identical to the PDE for the cost of carry model in perfect markets. Conversely, when the markets are imperfect and the arbitrage mechanism cannot work completely, the hedged portfolio cannot continuously be riskless and $\sigma_p > 0$. In this case, $\frac{\sigma_p}{\sigma}$ is also greater than zero and the two PDEs are not identical. Since $\frac{\sigma_p}{\sigma}$ reflects the total effects of all market imperfections between the currency futures market and its underlying spot market when implementing arbitrage activities, $\frac{\sigma_p}{\sigma}$ is defined as the degree of market imperfections and it can be used to measure the extent of market imperfections between the currency futures market and its underlying spot market.

The next question is how to measure the degree of market imperfections. The major problem in applying $\frac{\sigma_p}{\sigma}$ to measure the extent of market imperfections in practice is that direct estimation of σ_p is complex. To estimate σ_p, a hedged portfolio needs to be formed, rebalanced continuously, and then the variance of the hedged portfolio must be calculated. To minimize the work, an equivalent measurement for the degree of market imperfections is provided. In imperfect markets, because arbitrage mechanism cannot be complete, the volatility of the hedged portfolio contains a systematic risk, which is not diversifiable. Thus, we can only find a w_f that minimizes the volatility of the hedged portfolio. That is, w_f is chosen such that:

$$Var(w_f R_f + R_s) = \sigma^2_{\min},$$

or

$$w_f^2 \sigma_f^2 + \sigma^2 + 2 w_f \sigma_{fs} = \sigma^2_{\min}, \tag{16}$$

where R_f and R_s denote the instantaneous return on currency futures and the instantaneous spot return, respectively; σ_{fs} represents the instantaneous covariance between the currency futures return and the spot return in the hedged portfolio; and $\sigma^2_{\min}$ is the minimum variance of the return of the hedged portfolio.

The value of w_f that minimizes (16) can be found as follows:

$$\frac{\partial\left[w_f^2 \sigma_f^2 + \sigma^2 + 2 w_f \sigma_{fs}\right]}{\partial w_f} = 2 w_f \sigma_f^2 + 2\sigma_{fs} = 0. \tag{17}$$

The solution is:

$$w_f = \frac{-\sigma_{fs}}{\sigma_f^2} = -\rho\frac{\sigma}{\sigma_f}, \tag{18}$$

where ρ is the instantaneous correlation coefficient between the currency futures and the spot returns in the hedged portfolio.

As $w_f = -\rho \frac{\sigma}{\sigma_f}$, equation (11) can be rewritten as follows

$$\sigma_p = (1-\rho)\sigma , \quad (19)$$

or

$$\frac{\sigma_p}{\sigma} = (1-\rho) . \quad (20)$$

Equation (20) can be used as a measurement model of the degree of market imperfections. When the markets are perfect and the arbitrage mechanism works completely, the riskless arbitrage ensures that the difference between the currency futures and the spot prices equals the carrying cost. This means that ρ should be one, and that $\frac{\sigma_p}{\sigma}$ tends to zero in equation (20). In contrast, in imperfect markets, the relationship between the currency futures price and the spot price sometimes violates the carrying cost. ρ can not approach one (or ρ is less than one). In this case, $\frac{\sigma_p}{\sigma} > 0$ in equation (20). The more the market imperfections (that is, the smaller the correlation coefficient (ρ)) are, the larger the degree of market imperfections is.

Finally, from equation (20), the estimation of $\frac{\sigma_p}{\sigma}$ is very simple. Empirically, historical data can be used to estimate the parameter ρ in equation (20).

2. Relationship between the degree of market imperfections and the currency futures pricing

When the markets are perfect and the arbitrage mechanism works completely, the actual price of currency futures is equal or very close to the theoretical price estimated by the cost of carry model. In contrast, in imperfect markets, there exist market imperfections, the relationship between the currency futures price and the spot price sometimes violates the carrying cost.

Wang and Hsu (2006) proved that the absolute discrepancy between the actual price of stock index futures and the theoretical price estimated by the cost of carry model increases with the degree of market imperfections. Based on their work, this study also proposes a hypothesis regarding the relationship between the degree of market imperfections and the currency futures pricing.

Theoretical hypothesis: The larger the degree of market imperfections is, the larger is the absolute discrepancy between the actual price of currency futures and the theoretical price estimated by the cost of carry model.

This hypothesis can be further proved as follows:

Let $F_{\cos t,t}$ denote the theoretical futures price estimated by the cost of carry model on day *t*. $F_{\cos t,t}$ can be expressed as follows:

$$F_{\cos t,t} = S_t e^{(r-r_f)(T-t)} , \quad (21)$$

where S_t represents the current spot exchange rate; and *T-t* denotes the time to maturity of futures.

Let AF_t represent the actual price of currency futures on day *t*. If the currency futures price (AF_t), spot price (S_t), foreign risk-free interest rate (r_f), and time to maturity of futures (*T-t*) are known, a u_p should be found so that AF_t can be expressed as follows:

$$AF_t = S_t e^{(\mu_p - r_f)(T-t)} . \quad (22)$$

Using (21) and (22), we have:

$$\frac{AF_t}{F_{\text{cost,t}}} = e^{(u_p - r)(T-t)} . \quad (23)$$

Taking a natural logarithm on both sides of equation (23) and rearranging it, we get:

$$LnAF_t - lnF_{cost,t} = (u_p - r)(T - t). \quad (24)$$

Additionally, from equations (10) and (11), we have:

$$u_p = u - u_f \left(\frac{\sigma - \sigma_p}{\sigma_f} \right) = u - u_f \left(\frac{1 - \sigma_p / \sigma}{\sigma_f / \sigma} \right) . \quad (25)$$

Letting $\sigma_p / \sigma = y$ and $\sigma_f / \sigma = b$, equation (25) can be simplified as:

$$u_p = u - u_f \left(\frac{1-y}{b} \right) . \quad (26)$$

The partial derivative $\frac{\partial u_p}{\partial y}$ is:

$$\frac{\partial u_p}{\partial y} = \frac{u_f}{b} , \quad (27)$$

where $b > 0$. The meaning of equation (27) is briefly discussed as follows:

a) if $u_f > 0$, then $\frac{\partial u_p}{\partial y} > 0$. u_p is a strictly increasing function of $\frac{\sigma_p}{\sigma}$;

b) if $u_f < 0$, then $\frac{\partial u_p}{\partial y} < 0$. u_p is a strictly decreasing function of $\frac{\sigma_p}{\sigma}$.

From the above inferences (a) and (b), and equation (24), it can be proved that the absolute discrepancy between the actual price of currency futures and the theoretical price estimated by the cost of carry model increases with the degree of market imperfections.

3. Empirical procedures and data

3.1. Empirical procedures. To empirically test the theoretical hypothesis, the absolute pricing error (*AE*) is regressed on the degree of market imperfections ($\frac{\sigma_p}{\sigma}$). That is:

$$AE_t = \alpha_0 + \alpha_1 \frac{\sigma_{p,t}}{\sigma_t} + \varepsilon_t , \quad (28)$$

where $\mathrm{AE_t} = \left|\mathrm{AF_t} - \mathrm{F_{cost,t}}\right|$. Theoretical hypothesis is supported if the estimated coefficient α_1 is significantly positive.

The theoretical futures price on day $t, F_{\cos t,t}$, can be calculated from (21). Additionally, the input parameters required to produce a value of $\frac{\sigma_p}{\sigma}$ (that is, 1-ρ) are the variance and covariance (σ_{sf}) of the spot and currency futures returns. A popular approach to estimating the time-varying variance-covariance matrix of returns is the exponentially weighted moving average (EWMA), which is as follows:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)R_{s,t-1}^2 , \quad (29)$$

$$\sigma_{f,t}^2 = \lambda\sigma_{f,t-1}^2 + (1-\lambda)R_{f,t-1}^2 , \quad (30)$$

$$\sigma_{\mathrm{fs,t}} = \lambda\sigma_{fs,t-1} + (1-\lambda)R_{f,t-1}R_{s,t-1} , \quad (31)$$

where $R_{s,t-1}$ and $R_{f,t-1}$ are the spot and currency futures returns on day *t*-1, respectively; and λ is the decay factor. In the widely used Riskmetrics value-at-risk package of J.P. Morgan, λ is set to 0.94 for daily data (see J.P. Morgan. 1996).

3.2. Data. This study investigates the applicability of the model and implements an empirical test of the theoretical hypothesis for four currency futures contracts traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange: the Japanese yen, the British pound, the Swiss franc, and the Australian dollar. For the four currency futures, the nearest maturity contracts all have significant trading volume. To reduce thin trading problems, this study only considers the near-month contracts. The daily closing prices of the four currency futures contracts are collected from Datastream. The daily spot prices for each currency also are collected accordingly. The sample period is from January 2, 1996 to August 31, 2005.

The empirical tests also require risk-free interest rates. The US 91-day Treasury bill rates are used as the domestic risk-free interest rates. In the cases of Australia, the United Kingdom, and the Switzerland, the risk-free interest rates are based on the three-month interbank rates. The risk-free interest rates for Japan are based on the three-month Gensaki rates. Interest rate data are obtained from Datastream.

4. Empirical results

First, this study investigates the applicability of the model. Table 1 shows the frequency distribution of the degrees of market imperfections for the four currency futures markets. Because the futures prices and spot prices generally change in the same direction, in practice, the estimated ρ should lie between 0 and 1 and the boundary of $\frac{\sigma_p}{\sigma}$ (that is, 1- ρ) should be [0, 1]. Table 1 illustrates that all of the degrees of market imperfections fall into the 0.05-0.90 interval. Thus, the $\frac{\sigma_p}{\sigma}$ measures listed in Table 1 are consistent with the real market situation.

The frequencies in Table 1 indicate that most of the degrees of market imperfections fall into the 0.05-0.06, 0.06-0.07, 0.07-0.08, and 0.08-0.09 intervals for the Japanese yen and the British pound markets. Meanwhile, for the Australian dollar market, most of them fall in the 0.07-0.08, 0.08-0.09, 0.09-0.10, and 0.10-0.11 intervals. Table 1 also indicates that 1446 trading days (59.98%) have degrees of market imperfections that are less than 0.10 for the Japanese yen market. Meanwhile, 1399 of the degrees of market imperfections (58.02%) for the British pound market, and 1188 of the degrees of market imperfections (49.28%) for the Swiss franc market are below 0.10. However, for the Australian dollar market, only 915 trading days (37.95%) have degrees of market imperfections that are less than 0.10. Overall, Table 1 indicates that the Japanese yen market had the lowest degree of market imperfections, followed closely by the British pound market, the Swiss franc market, and the Australian dollar market.

Table 1. Frequency distribution of the degrees of market imperfections between the currency futures market and its underlying spot market

Degree of market imperfections	Japanese yen	British pound	Swiss franc	Australian dollar
0.05~0.06	145 (6.01%)	145 (6.01%)	53 (2.20%)	12 (0.50%)
0.06~0.07	471 (19.54%)	345 (14.31%)	187 (7.76%)	141 (5.85%)
0.07~0.08	390 (16.18%)	380 (15.76%)	370 (15.35%)	213 (8.83%)
0.08~0.09	262 (10.87%)	324 (13.44%)	299 (12.40%)	280 (11.61%)
0.09~0.10	178 (7.38%)	205 (8.50%)	279 (11.57%)	269 (11.16%)
0.10~0.11	146 (6.06%)	207 (8.59%)	191 (7.92%)	260 (10.78%)
0.11~0.12	146 (6.06%)	154 (6.39%)	147 (6.10%)	171 (7.09%)
0.12~0.13	86 (3.56%)	106 (4.40%)	149 (6.18%)	137 (5.68%)
0.13~0.14	82 (3.40%)	92 (3.82%)	101 (4.19%)	121 (5.02%)
0.14~0.15	75 (3.11%)	62 (2.57%)	91 (3.77%)	98 (4.07%)
0.15~0.20	233 (9.66%)	184 (7.63%)	319 (13.23%)	379 (15.72%)
0.20~0.30	149 (6.18%)	131 (5.44%)	157 (6.51%)	187 (7.76%)
0.30~0.40	30 (1.24%)	34 (1.41%)	27 (1.12%)	66 (2.74%)
0.40~0.50	17 (0.71%)	13 (0.54%)	3 (0.12%)	39 (1.62%)
0.50~0.60	1 (0.04%)	8 (0.33%)	17 (0.71%)	23 (0.95%)
0.60~0.70	0 (0.00%)	1 (0.04%)	19 (0.79%)	6 (0.25%)
0.70~0.80	0 (0.00%)	12 (0.50%)	2 (0.08%)	9 (0.37%)
0.80~0.90	0 (0.00%)	8 (0.33%)	0 (0.00%)	0 (0.00%)
	2411 (100.00%)	2411 (100.00%)	2411 (100.00%)	2411 (100.00%)

The right-hand-side panel of Table 2 further summarizes statistics on degrees of market imperfections. The average degree of market imperfections is smallest for the Japanese yen market. Additionally, compared to the other markets, the Australian dollar market displays high average degree of market imperfections. This result is similar to that shown in Table 1.

The columns labeled absolute percentage error in Table 2 summarize the statistics on absolute percentage error, and the t-test and Wilcoxon Sign-Rank test statistics based on mean absolute percentage error (MAPE) and mean $\frac{\sigma_p}{\sigma}$ are reported in Table 3. Table 2 shows that the Japanese yen market had the smallest average degree of market imperfections (0.1100), followed closely by the British pound market (0.1160), the Swiss franc market (0.1254), and the Australian dollar market (0.1421). Meanwhile, Table 2 also indicates that the Japanese yen market had the lowest MAPE value (0.103%), followed by the British pound market with MAPE of 0.112%, the Swiss franc market with MAPE of 0.142%, and the Australian dollar market with the highest MAPE value (0.173%). Thus, overall, Table 2 indicates that the rankings of the average degree of market imperfections precisely match those of the mean absolute percentage error for the four markets. Additionally, Both t-test and Wilcoxon Sign-Rank test statistics in Table 3 further indicate that the MAPE and the mean $\frac{\sigma_p}{\sigma}$ for the Japanese yen market are significantly smaller than for the other markets at the 1% level. The MAPE and the mean $\frac{\sigma_p}{\sigma}$ are also significantly larger for the Australian dollar market than for the other markets. To summarize, the results from Tables 2 and 3 demonstrate that higher degrees of market imperfections are associated with higher absolute percentage errors. Moreover, the measurement model of the degree of market imperfections appears to provide a reasonable measure of the degree of market imperfections for real currency futures markets.

Table 2. Summary statistics on absolute percentage error and degree of market imperfections

	Absolute percentage error				*Degree of market imperfections*			
	Mean (%)	*Std* (%)	*Max* (%)	*Min* (%)	*Mean*	*Std.*	*Max*	*Min*
Japanese yen	0.103	0.118	1.230	0.000	0.1100	0.0623	0.5048	0.0522
British pound	0.112	0.143	4.457	0.000	0.1160	0.0878	0.8569	0.0546
Swiss franc	0.142	0.193	1.820	0.000	0.1254	0.0825	0.7493	0.0518
Australian dollar	0.173	0.211	3.778	0.000	0.1421	0.0936	0.7506	0.0583

Note: the absolute percentage error is computed as $\left|AF_t - F_{\cos t,t}\right| / AF_t$.

Table 3. Results of statistical tests for differences in absolute percentage error and degree of market imperfections

	Absolute percentage error	*Degree of market imperfections*
Australian dollar vs. Japanese yen	14.138*** (13.976***)	14.002*** (14.342***)
Australian dollar vs. British pound	11.679*** (11.253***)	9.986*** (9.352***)
Australian dollar vs. Swiss franc	5.244*** (5.433***)	6.569*** (6.576***)
Swiss franc vs. Japanese yen	8.476*** (8.621***)	7.300*** (7.125***)
Swiss franc vs. British pound	6.153*** (5.849***)	3.834*** (3.645***)
British pound vs. Japanese yen	2.358*** (2.391***)	2.719*** (2.865***)

Notes: numbers in parentheses are the Wilcoxon test statistics. *** denotes significance at the 1% level. If the MAPE for the former futures market is greater than that for the latter futures market in column 1, t-value is positive. If the mean degree of imperfections for the former futures market is greater than that for the latter futures market in column 1, t-value is positive.

Table 4. Regression results on the analysis for testing the relation between the degree of market imperfections and the absolute pricing error

	α_0	α_1	R^2	*DW*	*N*
Japanese yen	7.83E-06*** (14.961)	1.04E-05** (2.526)	0.049	2.020	2411
British pound	0.0012*** (13.706)	0.0051*** (8.358)	0.047	2.014	2411
Swiss franc	0.0008*** (10.321)	0.0020*** (4.108)	0.161	2.049	2411
Australian dollar	0.0007*** (11.297)	0.0030*** (8.597)	0.100	2.021	2411

Notes: numbers in parentheses are the t values. *N* represents the number of observations. ** and *** denote significance at the 5% and 1% levels, respectively.

Table 4 reports the results of regression model (28) for testing the relation between the degree of market imperfections ($\frac{\sigma_p}{\sigma}$) and the absolute pricing error (AE). To control for autocorrelation, the regression coefficients of the equation (28) are estimated with an iterative Cochrane-Orcutt procedure. As expected, for all markets, the estimated coefficient ($\alpha 1$) on the degree of imperfections is significantly positive at the 1% level or the 5% level. For example, for the Australian dollar market, the estimated coefficient ($\alpha 1$) has a value of 0.0030, and the null hypothesis of $\alpha 1 = 0$ is rejected at the 1% level. Thus, the empirical results suggest a significant positive relation between the degree of market imperfections and the absolute pricing error.

Overall, the results from Tables 2 to 4 support the theoretical hypothesis that larger degrees of market imperfections are associated with larger mispricings of the currency futures contracts based on the perfect-market assumptions. This finding is consistent with that of Wang and Hsu (2006) for the stock index futures contracts.

Conclusions

This study aims to address two new issues related to market imperfections and currency futures pricing. First, foreign exchange and currency futures markets are not perfect and market imperfections differ among markets. Following the works of Hsu and Wang (2004) and Wang and Hsu (2006), this study uses an argument regarding the incompleteness of arbitrage mechanisms to develop a measurement model for valuing the degree of market imperfections between the currency futures market and its underlying spot market. Meanwhile, by using the CME Japanese yen, the CME British pound, the CME Swiss franc, and the CME Australian dollar futures contracts, this study tests the applicability of the model to predict the degree of market imperfections for real markets. Moreover, this study also proposes a theoretical hypothesis and an empirical test regarding the relationship between the degree of market imperfections and the currency futures pricing.

The empirical results indicate that the measurement model of the degree of market imperfections appears to provide a useful and practice-oriented measure for real degrees of market imperfections. Additionally, this study finds that larger market imperfections are relatively more mispriced by the cost of carry model based on the perfect-market assumptions, suggesting that the impact of market imperfections on the mispricing of currency futures is enormous, and cannot be neglected. Thus, when investors take a closer look at the applicability of the cost of carry model for pricing both mature and immature currency futures markets, they should note the degree of imperfection for the markets they are participating in.

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