A Simple Model of Patent Races with an Application to Strategic Subsidies in R&D

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Abstract

This work presents a model of patent races in which we study the incentives to win the race taking into consideration four different factors: (i) the importance of the innovation, (ii) appropriability of the patent, (iii) the strength of competition, and (iv) the structure of the profit functions. We study all cases and find that the processes of creative destruction and monopoly persistence are separated by very simple conditions. Our results are congruent with the recognized stylized facts (potential entrants stimulate progress both through their own investment and by provoking incumbents to invest more) and offer some new results. For instance if the cost of the innovation is small enough, there are equilibria in which the incumbent remains the monopolist. We also present an application of the model to strategic subsidies to R&D.

Key words: Patent Races, Strategic subsidies, R&D.

1. Introduction

An important topic in industrial organization is the strategic interaction between an incumbent firm and a potential entrant. Of special interest is the case in which the entrant is able to enter the market because of an investment in the appropriate technology. This problem has received attention, at least, since Schumpeter (1942). Early formal treatments include Arrow (1962), with a model in which only one part (either the entrant or the incumbent) has the possibility of investing to get a drastic innovation (an innovation that makes the inventor the only firm in the market, since competing with the old technology gives no profits). This author finds that the incentive for the entrant to invest is greater than for the incumbent. This result is known as the replacement effect and formalizes the Schumpeterian “process of creative destruction”.

Since then, other works show ways to complement the above findings. It has been shown, for instance, that the result is very sensitive to the way in which the innovation follows the investment. If the investment is made in the form of a bid in an auction to get a patent, then the incumbent will be willing to pay more (Gilbert and Newbery, 1982). However, if both firms invest and the one who finds the invention first is the only one allowed to use the technology (a standard patent race), then the entrant is shown to have an incentive to invest more under some conditions (Reinganum, 1983). This difference in results has been attributed to the underlying uncertainty. In the case of the auction with no uncertainty, the firm who enters the highest bid gets the innovation and the patent. However, in Reingamun (1983) a higher investment implies only a higher probability of winning the race.

Latter on, Katz and Shapiro (1987) develop a more general version of the auction model to find that it is not always the case that the firm with the larger incentive to preempt (presumably the incumbent) wins the auction, since the other firm may have larger stand-alone incentives. More recent works on patent races study, among others, the topics of optimal patent length, learning and sequential innovation. However a complete taxonomy of the cases when the entrant or the incumbent has the bigger incentive to invest in innovation is still missing.

Here we present a simple model of patent races in which we fully characterize the equilibria (who has the larger incentive to win the race) taking into consideration four different factors: (i) the importance of the innovation (drastic or not), (ii) who gets the patent (only one firm or both),

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(iii) the strength of competition (Cournot vs. Bertrand), and (iv) the relative size of the profits in different situations. We find 31 different cases that are easily analyzed. Furthermore, the cases of creative destruction and of monopoly persistence are separated by very simple conditions. As far as we know, no other model offers this characterization in a simpler way. For instance, the main result in Reingamun (1983) of creative destruction corresponds to our case *D1C* (Drastic innovation, only one firm gets the patent, and Cournot competition). By continuity arguments Reingamun (1983) argues that the result is also valid for a non-drastic innovation that is close to being drastic, but offers no conditions on how close it has to be. In our model, Proposition 1 establishes necessary and sufficient conditions for the prevalence of the process of creative destruction with non-drastic innovations. Furthermore, the proposition includes conditions for the cases of Bertrand competition and for the cases in which both firms can get a patent on a better technology.

Our propositions are congruent with the recognized stylized facts (potential entrants stimulate progress both through their own investment and by provoking incumbents to invest more) and offer some new results. For instance, since our model is deterministic in the sense that the innovation comes after the investment, this kind of uncertainty is not a necessary condition to get the result. It is also shown that, although it is true that in the presence of drastic innovations the entrant is willing to invest more and will replace the incumbent, if the cost of the innovation is known to be small enough, there are equilibria in which the incumbent remains the monopolist.

Our model is static and (almost) deterministic. Both the incumbent and the entrant have to decide whether to invest a quantity r or not. This investment is the cost of developing the innovation, which follows deterministically after the investment. If only one firms invests, it gets the patent; however if both invest, we consider two cases, in one, the patent is obtained randomly (the one who gets first to the patents office), in the other, both obtain the patent (the innovations obey different processes). Then we examine the structure of the equilibria of this game as r changes, thus giving meaning to the idea of who has the greater incentive to innovate by looking at relevant equilibria with greater values of r (for a very big value of r no one invests, but this is not a relevant case).

Cabral (1994) develops a static version with uncertainty of the model in Reinganum (1983) which is rich enough to capture its basic features. However, this version is still too complex to provide the characterization we present in this paper and relies in some *ad-hoc* assumptions on the profit functions.

Finally, we present an application of the model to strategic subsidies to R&D. The literature on this issue includes Spencer, Barbara and Brander (1983), Bagwell and Staiger (1992 and 1994), Brander and Spencer (1985) and Ecton and Grossman (1986). Following the models in these works, we will assume an economy of two firms, each belonging to a different country, competing in a third market, and find again a taxonomy of the cases in which subsidies are a profitable option for one country. In case there is a not-too-high limit to the size of subsidies, we find that, if the innovation is drastic, then both countries are willing to spend the same amount in subsidies if this implies that their firm gets the patent. This implies two equilibria (in pure strategies), in each of them a different country gives the subsidy while the other does not. This result holds for both Cournot and Bertrand competition. If the innovation is non-drastic and only one patent is possible, then under Cournot competition only the country of the incumbent subsidizes. However, if competition is *a la* Bertrand, there are again two equilibria, with either country being the only one to subsidize.

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the process of creative destruction and provides other results. Section 4 shows the application to R&D subsidies. Finally, Section 5 concludes. The whole set of equilibria is presented in the appendix.

2. The model

An incumbent monopolist is currently producing with a technology of constant marginal costs equal to $\bar{c}$ and no fixed costs. Both the monopolist and a potential entrant firm may have access to a better technology that allows production at constant marginal cost $\underline{c} < \bar{c}$ (again no fixed costs). The cost of adopting the new technology is r.

Suppose that one firm has cost $\underline{c}$ and the other has cost $\bar{c}$. If the monopolistic price with costs $\underline{c}$ is $p > \bar{c}$ the innovation is said to be drastic, otherwise the innovation is nondrastic. Profits of one firm when both firms are present in the market, have low costs and compete *a la* Cournot is denoted by $\Pi(\underline{c},\underline{c})$. The expressions $\Pi(\underline{c})$ and $\Pi(\bar{c})$ denote the profits of a firm that acts as a monopolist with low and high costs, respectively.

Firms may choose whether to invest the amount *r* or not to invest. For every value of *r* we compute the set of equilibria, and then study the changes on this set as the value of *r* changes. We consider two interpretations for the equilibria. First, for a given value of *r*, a situation in which the incumbent chooses not to invest in equilibrium while the entrant chooses to invest can be interpreted as a situation in which the entrant has a higher incentive to invest. This interpretation requires to look at all values of *r*. In the second interpretation, only the high values of *r* are considered relevant to the question of who has the higher incentive to invest. This interpretation is discussed in Section 3, when, in a state previous to the game, firms can influence the value of the variable *r*.

This model is somewhat different from the standard literature, in which the success of the investment is random and the firm investing more in equilibrium is said to have a higher incentive to invest. However, these two ways of studying the willingness to pay for an innovation provide a rationale for the process of creative destruction. Our new perspective is what makes the model simpler and more tractable, while being sufficiently rich in the set of phenomena that is able to explain.

Next we show the games played by both firms in different scenarios. The incumbent *I* is the row player and the entrant *E* is the column player. The sets of equilibria for different values of *r* and for different relations in the profits are described in Appendix I. In the next section, propositions 1 to 4 summarize the interesting features of these equilibria.

2.1. Drastic innovation (D)

2.1.1 Both firms can get the patent (2).

Cournot (C). If both firms invest, they both compete with low costs. If only one firm invests, it will dominate the market as a monopolist. If no one invests, the incumbent remains a monopolist with high cost.

	E invests	*E* does not invest
I invests	$\Pi(\underline{c},\underline{c})-r$, $\Pi(\underline{c},\underline{c})-r$	$\Pi(\underline{c})-r$, 0
I does not invest	0, $\Pi(\underline{c})-r$	$\Pi(\bar{c})$, 0

Bertrand (B). If both firms invest, Bertrand competition makes price equal to marginal costs and firms lose the investment. If only one firm invests, it will enjoy monopolistic profits as before. The new game is:

	E invests	*E* does not invest
I invests	$-r$, $-r$	$\Pi(\underline{c})-r$, 0
I does not invest	0, $\Pi(\underline{c})-r$	$\Pi(\bar{c})$, 0

2.1.2. Only one firm can get the patent (1).

If both firms invest, only one of them can get the patent. The probability of getting the patent is 1/2 for either firm. See that in this case it never occurs that both firms compete in the market. The game is:

	E invests	*E* does not invest
I invests	$\frac{1}{2}\Pi(\underline{c})-r$, $\frac{1}{2}\Pi(\underline{c})-r$	$\Pi(\underline{c})-r$, 0
I does not invest	0, $\Pi(\underline{c})-r$	$\Pi(\bar{c})$, 0

2.2. Nondrastic innovation (N)

2.2.1. Both firms can get the patent (2).

Cournot (C). In this case, when the entrant invests, it shares the market with the incumbent, however, if only the incumbent invests, it will enjoy monopolistic profits. Denote by $\Pi\left(\underline{c},\bar{c}\right)$ the profits of the firm who has the low costs and shares the market (competing *a la* Cournot) with a firm with high costs. Similarly, let $\Pi\left(\bar{c},\underline{c}\right)$ be the profits of the firm with high costs that shares the market with a low cost firm (again, in Cournot competition). The game is as follows.

	E invests	*E* does not invest
I invests	$\Pi\left(\underline{c},\underline{c}\right)-r$, $\Pi\left(\underline{c},\underline{c}\right)-r$	$\Pi\left(\underline{c}\right)-r$, 0
I does not invest	$\Pi\left(\bar{c},\underline{c}\right)$, $\Pi\left(\underline{c},\bar{c}\right)-r$	$\Pi\left(\bar{c}\right)$, 0

Bertrand (B). Bertrand competition will make firms lose their investments if they both invest. If the incumbent is the only one that invests, it will enjoy monopolistic profits. If it is the entrant is the only one to invest, then it will set a price equal to the high cost its profits will be $\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right)$ where $q\left(\bar{c}\right)$ is the demanded quantity at price $\bar{c}$. The game is as follows:

	E invests	*E* does not invest
I invests	$-r$, $-r$	$\Pi\left(\underline{c}\right)-r$, 0
I does not invest	0, $\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right)$	$\Pi\left(\bar{c}\right)$, 0

2.2.2. Only one firm can get the patent (1).

Cournot (C). The difference with respect to the case where both can get the patent is that, if both invest, the situation is either *NI* or *IN* with equal probabilities. The game is as follows.

	E invests	*E* does not invest
I invests	$\frac{1}{2}\Pi\left(\underline{c}\right)+\frac{1}{2}\Pi\left(\bar{c},\underline{c}\right)-r$, $\frac{1}{2}\Pi\left(\underline{c},\bar{c}\right)-r$	$\Pi\left(\underline{c}\right)-r$, 0
I does not invest	$\Pi\left(\bar{c},\underline{c}\right)$, $\Pi\left(\underline{c},\bar{c}\right)-r$	$\Pi\left(\bar{c}\right)$, 0

Bertrand (B). The game is as follows.

	E invests	*E* does not invest
I invests	$\frac{1}{2}\Pi\left(\underline{c}\right)-r$, $\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right)-r$	$\Pi\left(\underline{c}\right)-r$, 0
I does not invest	0, $\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right)-r$	$\Pi\left(\bar{c}\right)$, 0

3. Equilibrium patterns

Pure strategies equilibria will be denoted by *II*, *IN*, *NI*, *NN*, where *I* stands for *Invests* and *N* stands for *Does not invest*, and where first the strategy of the incumbent is listed, and second the strategy of the entrant *ME* will denote a unique mixed strategy equilibrium and *3E* will refer to a situation of multiple equilibria (two in pure strategies, *IN* and *NI*, and one in mixed strategies).

We will refer to the changes in the equilibrium structure as *r* increases as the equilibrium pattern. All equilibrium patterns except three start in *II*. For convenience, those three will be identified with the equilibrium patterns with the same continuation after *II*. All patterns end up in *NN*. We will only write the transition equilibrium structures, which are defined as the equilibrium patterns without the first (*II*) and last (*NN*) equilibria that correspond to the situations in which the cost of the investment is very cheap or very expensive, respectively. (Equilibria are computed in Appendix I.)

Equilibrium patterns:

One transition	Two transitions	Three transitions
NI	*3E→NI*	*NI→3E→IN*
IN	*3E→IN*	*NI→3E→NI*
ME	*IN→ME*	*IN→ME→NI*
	ME→NI	*IN→3E→NI*
		IN→3E→IN

There are 31 cases in the model and a total of 12 equilibrium patterns out of 64 possibilities (4 with one transition, 4×3 = 12 with two, and 4×3×4 = 48 with three). Some interesting patterns do not occur: A mixed strategy structure (not multiple equilibria) never precedes *IN*, and never occurs after *NI*, a multiple equilibria are never the last structure (before *NN*), and *IN* and *NI* never precede each other immediately.

3.1. Two interpretations of the equilibrium patterns

The equilibrium patterns have a straightforward interpretation. They show the set of equilibria for different values of *r*. With this interpretation, the process of creative destruction (*NI*) may occur for some values of *r*, but not for others, in most of the cases. Proposition 2 shows which are these cases.

Another, and perhaps more interesting, interpretation is to pay attention to the patterns in which the entrant is the only firm investing for high values of *r*, thus showing a higher willingness to pay for the innovation. Suppose that the amount *r* is the price established in an auction in which the incumbent and the entrant bid for a patent (as in Arrow, 1962). A firm only invests if it wins the auction, otherwise, it does not invest. In this case, the game represents a reduced form of the bigger game when we consider high values of *r*. Proposition 1 shows the cases of creative destruction for this interpretation.

Other cases compatible with the second interpretation may arise if we preserve the structure that, previous to the game, firms can choose an action r_i ($i \in \{I, E\}$, where *I* stands for incumbent and *E* stands for entrant), and that this action is costless, but implies that $r = \max\{r_I, r_E\}$ if both invest, and $r = r_i$ if only Firm *i* invests. For instance, firms can advertise a quality associated with the innovation, and suffer a great lose in terms of reputation if they do not fulfill the expectations in case they decide to invest. In this case, Firm *i* will choose r_i to be in highest rank compatible with a favorable equilibrium.

3.2. The process of creative destruction

The next propositions summarize the results regarding the process of creative destruction. All the proofs follow after a careful and tedious inspection of the 31 equilibrium structures displayed in the appendix.

Proposition 1. The only equilibrium patterns (more properly, transitions) ending in *IN* occur in the cases of nondrastic innovation with condition $\Pi(\underline{c}, \overline{c}) \leq \Pi(\underline{c}) - \Pi(\overline{c})$ in the Cournot cases and $(\overline{c} - \underline{c})q(\overline{c}) \leq \Pi(\underline{c}) - \Pi(\overline{c})$ in Bertrand competition.

The proposition states that, for high values of *r*, the process of creative destruction is prevalent except if there is little to be gained by the entrant. These are the cases of a nondrastic innovation that come along with a condition on profits that favors the monopolist. See also that this condition on monopolistic profits is weaker for the cases of Bertrand competition, because this is again a condition that gives small gains to the entrant. This proposition is congruent with the findings in the literature. For instance, Reinganum (1983) shows that, in the Cournot framework, and with only one firm able to patent the technology, the process of creative destruction appears if the innovation is drastic and, by continuity, if the innovation is not drastic, but close to being drastic. This author also shows that if the innovation reduces the cost only a little, then the process of creative destruction does not take place. In our model, Proposition 1 gives the precise conditions that makes this process possible, not only in the case considered in Reinganum (1983), but also in the cases of Bertrand competition and for a different patent regime.

The next proposition shows new results. It states that, for almost all cases, there is a range of values of *r* for which the result *IN* can be sustained in equilibrium or as the realization of a mixed strategies equilibrium. Of course this proposition is not in contradiction with the previous one. The former was about the persistence of creative destruction as *r* increases, while this one is about the possibility that this process does not arise for appropriate costs of the innovation.

Proposition 2. There exists a set of values of *r* such that the equilibrium structure includes *IN*, *3E* or *ME* for all cases except (i) drastic innovation, both may patent, Cournot competition and $\Pi(\underline{c})-\Pi(\bar{c})\leq\Pi(\underline{c},\underline{c})$, (ii) drastic innovation, one patent and $\frac{1}{2}\Pi(\underline{c})\leq\Pi(\bar{c})$, (iii) non drastic innovation, both may patent and $\Pi(\underline{c})-\Pi(\bar{c})\leq\Pi(\underline{c},\underline{c})$, and (iv) non drastic innovation, one patent, Cournot and $\max\left\{\Pi(\underline{c})-\Pi(\bar{c}),\frac{1}{2}\Pi(\bar{c})-\frac{1}{2}\Pi(\bar{c},\underline{c})\right\}\leq\max\left\{\frac{1}{2}\Pi(\underline{c},\bar{c}),\Pi(\underline{c},\underline{c})\right\}$.

The cases in which the outcome *IN* cannot occur are the ones favorable to the entrant: not a fierce competition and profits not so great for the monopolist. Among the cases, it is interesting to observe that, contrary to results in other models (e.g., Reinganum, 1983), there may be no creative destruction when the innovation is drastic and firms compete *a la* Cournot if the cost of innovation is not too big and if $\frac{1}{2}\Pi(\underline{c})\leq\Pi(\bar{c})$. Finally, the next proposition shows that the cases when the outcome *IN* occurs but the outcome *NI* does not are very few.

Proposition 3. The cases in which the only equilibrium is *IN* (other than those of Proposition 1) are as follows:

(i) Nondrastic innovation, only one firm gets the patent, Cournot competition and $\frac{1}{2}\Pi(\underline{c},\bar{c})\leq\Pi(\underline{c})-\Pi(\bar{c})\leq\Pi(\underline{c},\bar{c})$, except if $\Pi(\underline{c})-\Pi(\bar{c},\underline{c})\leq\Pi(\underline{c},\bar{c})$.

(ii) Nondrastic innovation, only one firm gets the patent, Bertrand competition and $\frac{1}{2}(\bar{c}-\underline{c})q(\bar{c})\leq\Pi(\underline{c})-\Pi(\bar{c})\leq(\bar{c}-\underline{c})q(\bar{c})$.

3.3. The incumbent invests more because of the entrant

See that, in the absence of an entrant, the condition for the incumbent to invest is $r<\Pi(\underline{c})-\Pi(\bar{c})$. Thus, the phenomenon in which the incumbent invests because of the entrant are those equilibria with the monopolist investing as a possible result and in which $\Pi(\underline{c})-\Pi(\bar{c})\leq r$. The cases when this happens are summarized in the next proposition.

Proposition 4. The cases in which the equilibria *II*, *IN* or *ME* occur for $\Pi(\underline{c})-\Pi(\bar{c})\leq r$ are:

(i) Drastic innovation, both firms can get the patent, Cournot competition and $\Pi(\underline{c})-\Pi(\bar{c})\leq\Pi(\underline{c},\bar{c})$.

(ii) Drastic innovation, only one firm gets the patent, Cournot competition and $\frac{1}{2}\Pi(\underline{c})\leq\Pi(\bar{c})$.

(iii) Nondrastic innovation, both firms can get the patent, Cournot competition and $\Pi(\underline{c})-\Pi(\bar{c})<\Pi(\underline{c},\underline{c})-\Pi(\bar{c},\underline{c})\leq\Pi(\bar{c},\underline{c})$.

(iv) Nondrastic innovation, only one firm gets the patent, Cournot competition and $\max\left\{\Pi(\underline{c})-\Pi(\bar{c}),\frac{1}{2}\Pi(\underline{c},\bar{c})\right\} \leq \max\left\{\frac{1}{2}\Pi(\underline{c})+\frac{1}{2}\Pi(\underline{c},\underline{c}),\Pi(\underline{c},\bar{c})\right\}$ except

$$\Pi(\underline{c})-\Pi(\bar{c})\leq \frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c})\leq \frac{1}{2}\Pi(\underline{c},\bar{c})\leq \Pi(\underline{c},\bar{c}).$$

(v) Nondrastic innovation, only one firm gets the patent, Bertrand competition and $\max\left\{\Pi(\underline{c})-\Pi(\bar{c}),\frac{1}{2}(\bar{c}-\underline{c})q(\bar{c})\right\} \leq \max\left\{\frac{1}{2}\Pi(\underline{c}),(\bar{c}-\underline{c})q(\bar{c})\right\}$.

See that a necessary condition for this result is that $\Pi(\underline{c})-\Pi(\bar{c})$ cannot be too big with respect to oligopolistic profits $\Pi(\underline{c},\underline{c})$, $\Pi(\underline{c},\bar{c})$ or $(\bar{c}-\underline{c})q(\bar{c})$. *I.e.*: $\Pi(\underline{c})-\Pi(\bar{c})$ cannot be too high or the monopolist would have invested if left alone, and $\Pi(\underline{c},\underline{c})$, $\Pi(\underline{c},\bar{c})$ or $(\bar{c}-\underline{c})q(\bar{c})$, depending on the case, have to be high enough to take the entrant's threat seriously.

4. Strategic subsidies to R&D

One question that has been addressed in the literature is the possibility of strategic subsidies in R&D. The idea is that one country may choose to subsidize technological investments to induce an oligopolistic equilibrium that is more favorable to its national firms. From the viewpoint of the country this subsidy may be optimal.

In a typical model of patent races in an oligopolistic context, there are situations in which a firm chooses not to invest in equilibrium. However, if this firm was able to move first, the equilibrium may change and be more favorable for this firm. Being a first move by the country of the firm, the subsidy may actually work as a commitment device that makes the outcome in which the firm invests more likely.

A sample of this literature includes Spencer, Barbara and Brander (1983), Bagwell and Staiger (1992 and 1994), Brander and Spencer (1985) and Ecton and Grossman (1986). Following the models in these works, we will assume an economy of two firms, each belonging to a different country, competing in a third market. This way of modeling reduces surplus comparisons to comparisons of profits minus subsidies. Our treatment of the patent race makes the analysis particularly simple while offering new insights into this part of the literature.

Take the model of patent races as developed in Section 2. Say that the incumbent monopolist belongs to Country *I* and the potential entrant – to Country *E*. Now, before decisions to invest take place, each country has the opportunity to pay in advance for part of the cost *r* of the innovation. If the subsidy is high enough, the firm has a dominant strategy in investing, which is equivalent to making the decision before the other firm. Being a transfer of surplus between two agents of the same country, the subsidy makes sense if, in equilibrium, the firm makes a higher profit (after substracting the cost of innovation *r*) than in the equilibrium with no subsidies. We will refer to this situation as subsidies being profitable. In this setting one can study different questions namely: Which are the cases in which subsidies increase the surplus of the country? Which is the minimum size of the subsidy that does the job? Which country needs a smaller subsidy to favor its firm?

Take, for instance the case D1 (drastic innovation, one patent), with $\frac{1}{2}\Pi(\underline{c})<r<\Pi(\underline{c},)-\Pi(\bar{c})$. Suppose now that, previous to the game, only Country *I* can make a strategy subsidy s_I. If the equilibrium with $s_I = 0$ is *NI*, then the choice of $s_I > r-\frac{1}{2}\Pi(\underline{c})>0$ has the consequence of changing the equilibrium to *IN*, as investing is now a dominant strategy for the incumbent, whose profits change form 0 to $\frac{1}{2}\Pi(\underline{c})-r+s_I$. Total profits for Country *I* increase as $\frac{1}{2}\Pi(\underline{c})-r>0$. Since s_I is just a monetary transfer within the country, any amount $s_I > r-\frac{1}{2}\Pi(\underline{c})$ is the best action as a subsidy.

Similarly, if the equilibrium with $s_I = 0$ is *IN*, any subsidy $s_I \geq 0$ leaves the equilibrium and total profits unchanged. Similar results are obtained if only Country *E* can pay the subsidy.

Suppose now that both countries can choose to pay a subsidy, and consider the case where the equilibrium with no subsidies is *IN*. There are two pure strategy equilibria in this case. In the first, Country *I* makes the subsidy $s_I > r-\frac{1}{2}\Pi(\underline{c})$ while Country *E* chooses $s_E = 0$. In the second equilibrium, we have the reverse situation, with $s_I = 0$ and $s_E > r-\frac{1}{2}\Pi(\underline{c})$. To check that these strategies are indeed equilibria, notice that, given $s_i > r-\frac{1}{2}\Pi(\underline{c})$, Country *j* will change its profits from zero to $\frac{1}{2}\Pi(\underline{c})-r<0$ in case it changes its subsidy from $s_j = 0$ to $s_j > r-\frac{1}{2}\Pi(\underline{c})$. The argument is similar for any other case. Thus we can conclude that only one country will make the strategic subsidy.

A different result is obtained if there is a limit to the size of the subsidy that a given country can pay to its firm. In this setting, consider again the previous example, then $\left(s_i = 0, s_j > r-\frac{1}{2}\Pi(\underline{c})\right)$ is an equilibrium strategy for the countries only if $r-\frac{1}{2}\Pi(\underline{c}) < \bar{s}_j$, where $\bar{s}_j$ is the maximum subsidy that Country *j* can pay.

It will be, then, of interest, to know which is the smallest subsidy that makes the *I* the dominant strategy for each country. Denote these subsidies by s_I^* and s_E^*, respectively for the country of the incumbent firm and the country of the entrant. The next question of interest would be to compare the relative sizes of subsidies s_I^* and s_E^*. If $s_i^* > s_j^*$, this would be an indication of a situation more favorable to Country *i*. In the example above (drastic innovation, one patent), we have $s_I^* = s_E^* = r-\frac{1}{2}\Pi(\underline{c})$.

The following propositions show the values of s_I^* and s_E^* in all the cases. As before, the proofs are straightforward inspections of the cases displayed in Appendix I. Proposition 5 shows that, for the cases of drastic innovation, and when subsidies can induce a change in the equilibrium to improve the net outcome of the country, then $s_I^* = s_E^*$. Proposition 6 shows that, if the innovation is non-drastic, and if there can be only one patent, then the minimum subsidy for the incumbent is higher than for the entrant $\left(s_I^* > s_E^*\right)$ if the competition is *a la* Cournot. However, in Bertrand competition, both minimum subsidies are the same $\left(s_I^* = s_E^*\right)$. For all other cases (non-drastic innovation and two patents) there are not neat conditions to separate the cases in which subsidies are an interesting option, or the cases in which one country needs to pay less than the other (see Appendix II).

Proposition 5. In the model with subsidies described above, if the innovation is drastic, then $s_I^* = s_E^*$. Furthermore, the values are as follows:

(i) One patent, Cournot: $s_I^* = s_E^* = r - \frac{1}{2}\Pi(\underline{c},\underline{c})$ in all cases.

(ii) One patent, Bertrand: $s_I^* = s_E^* = r$ in all cases.

(iii) Two patents, Cournot: $s_I^* = s_E^* = r - \frac{1}{2}\Pi(\underline{c})$ in all cases.

Proposition 6. In the model with subsidies described above, if the innovation is non-drastic, and if there can be only one patent, then $s_I^* > s_E^*$ if the competition is *a la* Cournot, and $s_I^* = s_E^*$ if the competition is *a la* Bertrand. More precisely:

(i) Under Cournot, subsidies are profitable if and only if $\Pi(\underline{c},\underline{c}) < r < \Pi(\underline{c},\bar{c})$. In this case $s_I^* = r - \Pi(\underline{c},\underline{c}) + \Pi(\underline{c},\bar{c})$ and $s_E^* = r - \Pi(\underline{c},\underline{c})$.

(ii) Under Bertrand, $s_I^* = s_E^* = r$ in all cases.

As discussed above, these results imply that, in the game in which countries decide whether to subsidize their own firm, and when $\bar{s}_I$ and $\bar{s}_E$ are not too high, there is only one equilibrium for the case of a non-drastic innovation, one patent, and Cournot competition, which consists of only Country *I* subsidizing its national firm. For the other cases in propositions 5 and 6, there are two possibilities in equilibrium, with either country being the only one subsidizing.

5. Conclusion

We have developed a simple model of patent races that, although static and almost deterministic, is rich enough to explain the stylized facts of creative destruction and of incumbents investing because of the threat of entrants. Furthermore, the model fully characterizes the equilibrium patterns for all the combinations of (i) size of the innovation, (ii) patent structure, (iii) type of competition, and (iv) relative profits. Although the total of cases is 31, the conditions that provide an answer to the interesting questions are quite simple. These results are congruent with those obtained in the literature, and clarify some other aspects like whether or not the incumbent may be the only firm investing for a drastic innovation. We have presented an application to strategic subsidies. It would be of interest to apply this model to the study of other issues in R&D, like licensing, imitation and others.

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Appendix I

Here we compute all the equilibria for all cases, except the non-generic. The non-generic cases arise when conditions on profits are given by equalities. These cases can occur only by chance and their equilibria are just combinations of the equilibria when there are strict inequalities, and do not alter the equilibrium patterns. I.e. equalities in the conditions for profits are uninteresting, non-generic and only complicate the exposition. The equilibria of each game in Section 2 are as follows (we omit the lowest and the highest values of r as they always give *II* and *NN* respectively, except when otherwise noted):

D2C. Drastic, two patents, Cournot.

Since $\Pi(\underline{c},\underline{c}) < \Pi(\underline{c})$, we can easily compute the equilibria for different values of r.

(i) if $\Pi(\underline{c},\underline{c}) < \Pi(\underline{c}) - \Pi(\bar{c})$:

$$\Pi(\underline{c},\underline{c}) < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow 3E$$

$$\Pi(\underline{c}) - \Pi(\bar{c}) < r < \Pi(\underline{c}) \rightarrow NI$$

(ii) if $\Pi(\underline{c}) - \Pi(\bar{c}) < \Pi(\underline{c},\underline{c})$:

$$\Pi(\underline{c},\underline{c}) < r < \Pi(\underline{c}) \rightarrow \text{NI}$$

D2B. Drastic, two patents, Bertrand.

$$0 < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow 3E$$

$$\Pi(\underline{c}) - \Pi(\bar{c}) < r < \Pi(\underline{c}) \rightarrow NI$$

D1. Drastic, one patent.

(i) $\Pi(\bar{c}) < \frac{1}{2}\Pi(\underline{c})$:

$$\frac{1}{2}\Pi(\underline{c}) < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow 3E$$

$$\Pi(\underline{c}) - \Pi(\bar{c}) < r < \Pi(\underline{c}) \rightarrow \text{NI}$$

(ii) $\frac{1}{2}\Pi(\underline{c}) < \Pi(\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c}) < r < \Pi(\underline{c}) \rightarrow \text{NI}$$

N2C. Nondrastic, two patents, Cournot.

(i) If $\Pi(\underline{c},\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c})$:

$$\Pi(\underline{c},\underline{c}) - \Pi(\bar{c},\underline{c}) < r < \Pi(\underline{c},\underline{c}) \rightarrow \text{NI}$$

$$\Pi(\underline{c},\underline{c}) < r < \Pi(\underline{c},\bar{c}) \rightarrow \text{3E}$$

$$\Pi(\underline{c},\bar{c}) < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow \text{IN}$$

(ii) If $\Pi(\underline{c},\underline{c}) < \Pi(\underline{c}) - \Pi(\bar{c}) < \Pi(\underline{c},\bar{c})$:

$$\Pi(\underline{c},\underline{c}) - \Pi(\bar{c},\underline{c}) < r < \Pi(\underline{c},\underline{c}) \rightarrow \text{NI}$$
$$\Pi(\underline{c},\underline{c}) < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow \text{3E}$$
$$\Pi(\underline{c}) - \Pi(\bar{c}) < r < \Pi(\underline{c},\bar{c}) \rightarrow \mathit{IN}$$

(iii) If $\Pi(\underline{c},\underline{c}) - \Pi(\bar{c},\underline{c}) < \Pi(\underline{c}) - \Pi(\bar{c}) < \Pi(\underline{c},\underline{c})$:

$$\Pi(\underline{c},\underline{c}) - \Pi(\bar{c},\underline{c}) < r < \Pi(\underline{c},\bar{c}) \rightarrow \text{NI}$$

(iv) If $\Pi(\underline{c}) - \Pi(\bar{c}) < \Pi(\underline{c},\underline{c}) - \Pi(\bar{c},\underline{c}) < \Pi(\underline{c},\underline{c})$:

$$\Pi(\underline{c},\underline{c}) - \Pi(\bar{c},\underline{c}) < r < \Pi(\underline{c},\bar{c}) \rightarrow \text{NI}$$

N2B. Nondrastic, two patents, Bertrand.

(i) If $(\bar{c} - \underline{c})q(\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c})$:

$$0 < r < (\bar{c} - \underline{c})q(\bar{c}) \rightarrow \text{3E}$$
$$(\bar{c} - \underline{c})q(\bar{c}) < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow \mathit{IN}$$

(ii) If $\Pi(\underline{c}) - \Pi(\bar{c}) < (\bar{c} - \underline{c})q(\bar{c})$:

$$0 < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow \text{3E}$$
$$\Pi(\underline{c}) - \Pi(\bar{c}) < r < (\bar{c} - \underline{c})q(\bar{c}) \rightarrow \mathit{IN}$$

N1C. Nondrastic, one patent, Cournot.

(i) $\Pi(\underline{c},\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow \text{IN}$$

(ii) $\frac{1}{2}\Pi(\underline{c},\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c}) < \Pi(\underline{c},\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) < r < \Pi(\underline{c}) - \Pi(\bar{c}) \rightarrow \text{IN}$$
$$\Pi(\underline{c}) - \Pi(\bar{c}) < r < \Pi(\underline{c},\bar{c}) \rightarrow \text{ME}$$

(iii) $\Pi(\underline{c}) - \Pi(\bar{c}) < \frac{1}{2}\Pi(\underline{c},\bar{c}) < \Pi(\underline{c},\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) \ < \ r \ < \ \Pi(\underline{c},\bar{c}) \ \rightarrow \ \text{ME}$$

(iv) $\frac{1}{2}\Pi(\underline{c},\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) < \Pi(\underline{c},\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) \ < \ r \ < \ \Pi(\underline{c}) - \Pi(\bar{c}) \ \rightarrow \ \text{IN}$$

$$\Pi(\underline{c}) - \Pi(\bar{c}) \ < \ r \ < \ \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) \ \rightarrow \ \mathit{ME}$$

$$\frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) \ < \ r \ < \ \Pi(\underline{c},\bar{c}) \ \rightarrow \ \mathit{NI}$$

(v) $\Pi(\underline{c}) - \Pi(\bar{c}) < \frac{1}{2}\Pi(\underline{c},\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) < \Pi(\underline{c},\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) \ < \ r \ < \ \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) \ \rightarrow \ \text{ME}$$

$$\frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) \ < \ r \ < \ \Pi(\underline{c},\bar{c}) \ \rightarrow \ \mathit{NI}$$

(vi) $\Pi(\underline{c}) - \Pi(\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) < \frac{1}{2}\Pi(\underline{c},\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) \ < \ r \ < \ \Pi(\underline{c},\bar{c}) \ \rightarrow \ \text{NI}$$

(vii) $\Pi(\underline{c},\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) < \Pi(\underline{c}) - \Pi(\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) \ < \ r \ < \ \Pi(\underline{c}) - \Pi(\bar{c}) \ \rightarrow \ \text{IN}$$

(viii) $\frac{1}{2}\Pi(\underline{c},\bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) < \Pi(\underline{c},\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) \ < \ r \ < \ \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) \ \rightarrow \ \text{IN}$$

$$\frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) \ < \ r \ < \ \Pi(\underline{c},\bar{c}) \ \rightarrow \ \mathit{3E}$$

$$\Pi(\underline{c},\bar{c}) \ < \ r \ < \ \Pi(\underline{c}) - \Pi(\bar{c}) \ \rightarrow \ \mathit{IN}$$

(ix) $\frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c},\underline{c}) < \frac{1}{2}\Pi(\underline{c},\bar{c}) < \Pi(\underline{c},\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) < r < \frac{1}{2}\Pi(\underline{c},\bar{c}) \to \text{NI}$$

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) < r < \Pi(\underline{c},\bar{c}) \to \text{3E}$$

$$\Pi(\underline{c},\bar{c}) < r < \Pi(\underline{c})-\Pi(\bar{c}) \to \textit{IN}$$

(x) $\frac{1}{2}\Pi(\underline{c},\bar{c}) < \frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) < \Pi(\underline{c})-\Pi(\bar{c}) < \Pi(\underline{c},\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) < r < \frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) \to \text{IN}$$

$$\frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) < r < \Pi(\underline{c})-\Pi(\bar{c}) \to \textit{3E}$$

$$\Pi(\underline{c})-\Pi(\bar{c}) < r < \Pi(\underline{c},\bar{c}) \to \textit{NI}$$

(xi) $\frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) < \frac{1}{2}\Pi(\underline{c},\bar{c}) < \Pi(\underline{c})-\Pi(\bar{c}) < \Pi(\underline{c},\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) < r < \frac{1}{2}\Pi(\underline{c},\bar{c}) \to \text{NI}$$

$$\frac{1}{2}\Pi(\underline{c},\bar{c}) < r < \Pi(\underline{c})-\Pi(\bar{c}) \to \textit{3E}$$

$$\Pi(\underline{c})-\Pi(\bar{c}) < r < \Pi(\underline{c},\bar{c}) \to \textit{NI}$$

(xii) $\frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) < \Pi(\underline{c})-\Pi(\bar{c}) < \frac{1}{2}\Pi(\underline{c},\bar{c}) < \Pi(\underline{c},\bar{c})$:

$$\frac{1}{2}\Pi(\underline{c})-\frac{1}{2}\Pi(\bar{c},\underline{c}) < r < \Pi(\underline{c},\bar{c}) \to \text{NI}$$

N1B. Nondrastic, one patent, Bertrand.

(i) $\frac{1}{2}(\bar{c}-\underline{c})q(\bar{c}) < \Pi(\underline{c})-\Pi(\bar{c}) < (\bar{c}-\underline{c})q(\bar{c}) < \frac{1}{2}\Pi(\underline{c})$:

$$\frac{1}{2}(\bar{c}-\underline{c})q(\bar{c}) < r < \Pi(\underline{c})-\Pi(\bar{c}) \to \text{IN}$$

$$\Pi(\underline{c})-\Pi(\bar{c}) < r < (\bar{c}-\underline{c})q(\bar{c}) \to \textit{ME}$$

(ii) $\Pi(\underline{c})-\Pi(\bar{c}) < \frac{1}{2}(\bar{c}-\underline{c})q(\bar{c}) < (\bar{c}-\underline{c})q(\bar{c}) < \frac{1}{2}\Pi(\underline{c})$

$$\frac{1}{2}(\bar{c}-\underline{c})q(\bar{c}) < r < (\bar{c}-\underline{c})q(\bar{c}) \to \text{ME}$$

(iii) $\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) < \frac{1}{2}\Pi\left(\underline{c}\right)$:

$\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < r < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) \rightarrow$ IN

(iv) $\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) < \frac{1}{2}\Pi\left(\underline{c}\right) < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right)$:

$\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < r < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) \rightarrow$ IN

$\Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) < r < \frac{1}{2}\Pi\left(\underline{c}\right) \rightarrow$ *ME*

$\frac{1}{2}\Pi\left(\underline{c}\right) < r < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) \rightarrow$ *NI*

(v) $\Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) < \frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < \frac{1}{2}\Pi\left(\underline{c}\right) < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right)$:

$\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < r < \frac{1}{2}\Pi\left(\underline{c}\right) \rightarrow$ ME

$\frac{1}{2}\Pi\left(\underline{c}\right) < r < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) \rightarrow$ *NI*

(vi) $\frac{1}{2}\Pi\left(\underline{c}\right) < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right)$:

$\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < r < \frac{1}{2}\Pi\left(\underline{c}\right) \rightarrow$ IN

$\frac{1}{2}\Pi\left(\underline{c}\right) < r < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) \rightarrow$ *3E*

$\Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) < r < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) \rightarrow$ *NI*

(vii) $\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < \frac{1}{2}\Pi\left(\underline{c}\right) < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right)$:

$\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < r < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) \rightarrow$ IN

(viii) $\frac{1}{2}\Pi\left(\underline{c}\right) < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right)$:

$\frac{1}{2}\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < r < \frac{1}{2}\Pi\left(\underline{c}\right) \rightarrow$ IN

$\frac{1}{2}\Pi\left(\underline{c}\right) < r < \left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) \rightarrow$ *3E*

$\left(\bar{c}-\underline{c}\right)q\left(\bar{c}\right) < r < \Pi\left(\underline{c}\right)-\Pi\left(\bar{c}\right) \rightarrow$ *IN*

Appendix II

In the model described in Section 4, if the innovation is non-drastic, two patents are allowed, and firms compete *a la* Cournot, then:

(i) $s_I^* > s_E^*$ if and only if $\max\left\{\Pi(\underline{c}) - \Pi(\bar{c}), \frac{1}{2}\Pi(\underline{c}, \bar{c})\right\} < r < \Pi(\underline{c}, \bar{c})$ except when $\frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c}, \underline{c}) < \min\left\{\Pi(\underline{c}) - \Pi(\bar{c}), \frac{1}{2}\Pi(\underline{c}, \bar{c})\right\}$ in which case $s_I^* > s_E^*$ if and only if $\frac{1}{2}\Pi(\underline{c}, \bar{c}) < r < \Pi(\underline{c}, \bar{c})$.

(ii) $s_I^* < s_E^*$ if and only if

$$\frac{1}{2}\Pi(\underline{c}, \bar{c}) < \frac{1}{2}\Pi(\underline{c}) - \frac{1}{2}\Pi(\bar{c}, \underline{c}) < r < \min\{\Pi(\underline{c}) - \Pi(\bar{c}), \Pi(\underline{c}, \bar{c})\}$$

(iii) in all other cases, subsidies are not profitable.

Finally, if firms compete *a la* Bertrand:

(i) $s_I^* > s_E^*$ if and only if

$$\Pi(\underline{c}) - \Pi(\bar{c}) < \frac{1}{2}(\bar{c} - \underline{c})q(\bar{c}) < r < \min\left\{\frac{1}{2}\Pi(\underline{c}), (\bar{c} - \underline{c})q(\bar{c})\right\}$$

(ii) $s_I^* < s_E^*$ if and only if $\frac{1}{2}(\bar{c} - \underline{c})q(\bar{c}) < \Pi(\underline{c}) - \Pi(\bar{c})$ and

$$\frac{1}{2}(\bar{c} - \underline{c})q(\bar{c}) < r < \min\left\{\frac{1}{2}\Pi(\underline{c}), \Pi(\underline{c}) - \Pi(\bar{c})\right\} < (\bar{c} - \underline{c})q(\bar{c})$$

(iii) in all other cases, subsidies are not profitable.