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International Bank Capital Regulation for Market Risk

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Abstract

Banking is one of the most regulated industries. The central role of banks in the financial intermediation system necessitates international effort to adopt common bank capital standards. Bank capital adequacy has been a subject of interest, especially during economic downturn when bank failures further destabilize the financial system. This paper discusses the inadequacy of regulation for market risk based on the statistical properties of normal market data. An attempt is made to investigate the properties of "crash" market data via wavelet multiresolution analysis. The idea is to contribute to the development of better risk modelling techniques that will lead to more efficient bank capital regulation.

1. Introduction

Modigliani and Miller (1958) proposition stated that in a frictionless world of full information and complete markets, capital structure can't affect the value of the firm. Research on 'optimal' capital structure is aligned with the departures from this frictionless world such as taxes, costs of financial distress, transaction costs and asymmetric information problems. 'Optimal' leverage has particular relevance to financial institutions especially from a regulatory perspective. Berger et al. (95) define market capital 'requirement' as the capital structure that maximizes the value of the bank. Narrowing the gap between market-generated capital 'requirements' and regulatory ones has been the objective of adequate bank capital guidelines.

Banking is one of the most regulated industries. Bank capital earned the status of most importance in this regard because of its role in banks' soundness and risk-taking incentives. The central role of banks in the financial intermediation system necessitates international effort to adopt common bank capital standards. Bank capital adequacy has been a subject of interest, especially during economic downturn when bank failures further destabilize the financial system. The social costs of bank failure have induced governments to provide financial support and other forms of protection. The safety net for banks (federal deposit insurance, unconditional payment guarantees, access to the discount window, etc.) might cause the lowest equity-to-asset ratios among all industries. The historical evolution of bank capital ratios is consistent with this hypothesis.

Benston, G. (2000), analyzes nine reasons that validate government regulation of banks and concludes that deposit insurance is the only public-policy justifiable rationale for regulation.

2. Bank Capital Regulations: Basel Accord

The main objectives of a good capital adequacy system should be set at reduction of the real costs associated with bank failures, while at the same time allowing banks efficient functioning. The existence of deposit insurance should not create moral hazard and inefficient resource allocation. According to Sharpe (1978), in the world of perfect information and absence of moral hazard, bank soundness can be achieved equally well either with risk-based insurance premium or risk-related capital standards. According to Flannery (1991), in the real world, observations of bank risks are masked by noise distorting the equivalency of these alternative pricing mechanisms. Therefore, the design of deposit insurance and capital standards should be pooled together towards a common goal.

In the last century commercial banks have been financed with less capital relative to debt. Lower capital might lead to more bank failures, jeopardizing the financial stability and the viability of the monetary system. The principal concern is systemic risk. Public information on banks' condition is imperfect. Bhattacharya and Thakor (1993) believe that when some banks fail, de-

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structive "panic" runs on the bank system may develop. Guttentag and Herring (1987) propose that interbank markets transmit problems from one bank to other banks since monitoring is difficult. Mayer (1965) offers a strategy to prevent economic loss to depositors in the event of bank failure, while Cohen (1970) raises awareness of potential bank failures among weakly capitalized banks. Shay (1974) argues that losses from bank failure exceed the costs incurred in capital regulation. The total social costs of bank failure as well as the opportunity cost of overcapitalization are discussed in Santomero and Watson (1977).

These systemic problems can impose heavy social costs. A set of regulations is needed to protect deposit insurance funds from the effect of excessive risk taking by bank managers. The collapse of the savings and loan system in the United States during the late 1980 would cost the American taxpayers billions of dollars as stated by the General Accounting Office (1990). Black et al. (1978) and Acharya and Dreyfus (1989) prove that the government should price risk through deposit insurance premiums and capital standards. Buser et al. (1981) assert that the indirect means of persuading banks to raise capital ratios, such as cease-and-desist orders, total withdrawal of insurance coverage, prohibition of dividend payments, etc., are blunt and uncertain. Such actions may also create additional moral hazard, which depends on how far the capital ratio is from the closure point according to Herring and Vankudre (1987) and Davies and McManus (1991).

Thus regulatory capital requirements differ from market-based capital "requirements". Capital regulation is motivated by fear of negative consequences of bank failure. Obvious solution will be high equity capital that makes the probability of default insignificant. But Berger et al. (1995) argue that equity beyond the market "requirement" reduces the value of the bank and increases its cost of financing.

Many countries introduced formalized capital requirements in the last 20 years. The US and the UK were leading the way by adoption of minimum capital requirements in 1981. Many central bankers had expressed concern over the erosion of capital levels in commercial banks but there was no consensus on the definition of bank capital and its required levels. National differences in the level of capital provided some countries with competitive advantage in the marketplace. A bilateral agreement on capital standards, reached by the Federal Reserve System and Bank of England in 1987 threatened to create a zone of exclusion to the international banks of other countries according to Kapstein (1994). Central bankers realized that an international agreement on capital adequacy had become necessary. Common minimum capital requirements were implemented in the G-10 countries with the introduction of the Basel Capital Accord and put into practice by around 100 countries world-wide later.

The Basel Accord was published in July 1988, aiming to become a basis for national regulations. The main objectives were to strengthen the soundness and stability of the international banking system by boosting banks' capital positions and reducing competitive inequalities. The arrangement was intended to adapt capital requirements to differences in risk profiles, take into account off-balance sheet exposure in assessing capital adequacy and lower the disincentives to holding liquid, low risk capital. Miles (1995) argues that there is a significant difference between the optimal capital adequacy scheme and the BIS risk-based capital. The theoretical model suggests that it is not the default risk itself but rather banks' returns in high default states, which are the key determinants for the capital requirements.

Under the 1988 Basel Capital Accord there are two basic ways of improving capital ratios: by either increasing the measures of regulatory capital or decreasing the measures of total risk. Several studies suggest that capital requirements persuade weakly capitalized banks to increase their capital ratios faster. At the same time the question whether higher capital requirements give banks greater incentives to increase asset risk remains open. Evidence suggests that in the short run banks tended to increase capital through retained earnings. Banks' response to capital requirements also varies in relation to the financial situation of the bank and the business cycle in the most cost effective way. Banks are likely to reduce lending when raising capital is too expensive. When assessing the effect of the Basel Accord it is important to determine whether the adoption of fixed capital requirements led banks to maintain higher capital ratios and whether the increase in ratios was attained by increasing capital or reducing lending.

The 1988 regulatory framework deferred low-risk transactions as they consumed too much capital while at the same time the stated capital ratios may not reflect the actual risk being taken. The concern that higher capital standards will induce banks to take higher risk might not be valid. Furlong and Keeley (1989) analyze theoretically the relationship between capital regulation and bank asset risk. Value-maximizing banks meet higher capital requirements by raising additional capital and do not tend to increase asset portfolio risk. Theoretical proof is provided that more stringent capital regulation will reduce the risk exposure of the deposit insurance system. In contrast, Gennotte and Pyle (1991) demonstrate that deposit guarantees lead to inefficient investment and higher capital requirements lead to increased bank asset risk. Results suggest that increases in bank capital requirements are not a substitute for monitoring and control of asset risk by the regulators. Flannery (1989) shows that capital requirements encourage higher risk taking. Diamond and Rajan (2000) suppose that binding capital requirements even make banks riskier in the long run. Koehn and Santomero (1980), Keeton (1988) and Kim and Santomero (1988) demonstrate that in a utility maximization framework an increase in the capital requirements may lead to either higher or lower portfolio risk. Furlong and Keeley (1989) and Keeley and Furlong (1990) prove that value maximizing banks with publicly traded stock will always reduce portfolio risk in response to a higher equity requirement. Gennotte and Pyle (1991) argue that if bank investments are subject to decreasing returns then the bank may increase its portfolio risk in response to higher capital requirements.

The empirical evidence suggests that higher equity is related to lower overall bank risk and lower future probability of default according to Lane et al. (1986), Avery and Berger (1991) and Cole and Gunther (1995). Several empirical studies have analyzed the correspondence of Basel Accord's risk-weighted assets (RWA) with actual risk. Avery and Berger (1991) and Bradley et al. (1991) found that RWA were weakly and positively related to the probability of failure. Cordell and King (1995) found few problems with the relative risk weights and also conclude that the accounting measures of capital may overstate the actual value of capital. Koehn and Santomero (1980, 1988) and Rochet (1992) prove that inadequately assigned risk weights lead to higher risk of banks' assets. Measures of risk exposure may be subject to manipulation by bank management. Merton (1995) provides an example of how banks may be able to restructure their transactions to reduce capital requirements without reducing their actual risk exposures. Securitization and other financial innovations provide opportunities for reduction of capital requirements with little or no drop in overall risk, called regulatory capital arbitrage. Regulatory capital requirements may have played a role in the expansion of securitization. Jones (2000) analyzes techniques for capital arbitrage and the difficulties to intercept these activities under the current capital framework. Arbitrage undermines capital ratio measures and erodes capital standards. This requires alignment of regulatory measures of risk with banks' true economic risks. Capital requirements may also decrease monitoring incentives and affect the quality of bank portfolios according to Besanko and Kanatas (1993) and Boot and Greenbaum (1993).

It was intended that the fixed capital requirements would limit risk taking relative to capital. Conversely capital requirements equally valid for a broad class of assets may encourage a switch to the riskier assets in the class, increasing the overall risk of the bank's portfolio. The academic literature in this area is inconclusive. Available data do not allow accurate measurements for banks' risk-taking.

The denominator of the Basel Accord risk-based capital ratio reflects value imperfectly. Researchers have suggested different ways to improve it. The need for new kind of 'risk accounting' focused on exposures was on the agenda. The Basel Committee has decided to revise the Accord for a number of reasons but most important it has been rapid development of new risk management techniques leaving the Accord looking outdated. Many leading banks have argued that their internal risk evaluation is more accurate than the Accord framework. The reason already discussed is the effect of financial innovations circumventing capital rules. In effect the Accord has become progressively less binding. In July 1999, a consultative paper set out a number of options for reforming the Accord.

On January 16, 2001, the Basel Committee on Banking supervision issued a proposal for a New Basel Capital Accord that will replace the current 1988 Accord. The idea behind it is to align capital requirements with underlying risk. It is also aimed to provide banks with different options for capital adequacy evaluation. The Committee considered the suggestions from supervisors around the world, expecting that all significant banks around the globe will adhere to the New Accord. The New Basel Capital Accord is more extensive and complex, reflecting the advancement and innovations in finance and the need for a more risk sensitive framework. The focus is not only on refined measurement for minimum capital requirements but also on supervisory examination of the internal assessment processes as well as on market discipline involving disclosure and sound banking practices.

In the proposals for minimum capital requirements, the old one-size-fits-all framework will be replaced by a variety of options. This gives the banks the incentives to continuously improve their risk management. A standardized or an Internal Ratings-Based (IRB) approach for credit risk is available for use depending on banks' complexity. If finalized by the end of 2001, the New Accord will be implemented in 2004 thus allowing adjustment of national regulatory agencies and tuning of banks' internal systems. The proposal promotes strong risk management practices with a sensitive-to-risk framework. It is believed that it will contribute to financial stability globally.

The simplicity of the 1988 Basel Accord allowed its implementation in different countries achieving security and international competition in the banking system. It is remarkable that a multilateral standard could be developed and accepted in such a short time. A major shortfall was the lack of differentiation in the quality of the borrower, encouraging increased risk taking and regulatory capital arbitrage through securitization or credit derivatives.

3. Modelling Market Risk for Regulatory Purpose

The Accord has been changed to account for financial innovations and some of the risks it did not consider initially. Several amendments were added to the original accord. The Basel Committee addresses the risk related to interest rate instruments and equities on the trading books as well as foreign exchange and commodities risk throughout the bank in the Amendment to the Capital Accord of January 1996. It is a part of 2001 Basel Accord. These capital requirements for market risk are intended to apply on a worldwide basis.

This 1996 amendment requires banks to set aside capital to cover the risk of losses arising from movements in market prices. The main novelty of this amendment is that it allows banks to use their internal models to determine the required charge for market risk as an alternative to the standard approach. The standard approach defines the risk charges associated with each position and specifies how these charges are to be aggregated into an overall market risk capital charge. A risk factor representing market movements in equity prices must be formulated for each of the markets where the bank holds significant positions at a minimum. In a more detailed approach various sectors would be represented, while the most sophisticated model will contain the volatility of individual assets.

Two broad classes of methodologies for measuring market risk are allowed. Either the standardized one proposed in the Accord or ones based on banks' internal risk management models. The internal model's approach allows a bank to use its own model to estimate the Value-at-Risk (VaR) in its trading account. The use of the latter is conditional upon approval of the bank's supervisory authority. Along with the general criteria and qualitative standards, specification of market risk factors is put forward.

Although banks are given options to model market risk, the minimum standards are set for "Value-at-Risk" to be computed on a daily basis and a 99% one-tailed confidence interval to be used. The quantitative standards also prescribe a 10 trading days "holding period", a year historical observation period and every three months data sets update. VaR covers so-called normal market behavior and the impact of extreme price changes can be determined by using stress testing. The accuracy of the VaR estimates is of crucial importance because of the cost associated with holding both too much and too little capital.

While VaR models have proven to be useful management tools, they have showed certain limitations – excessive dependency on history or unrealistic statistical assumptions. An improvement of VaR was proposed by the Extreme Value Theory, which achieves more accurate modelling of the tail distribution of returns. In particular EVT provides foundation for the estimation of VaR for very low probability or ”extreme” events. VaR and EVT methodologies assume fat tails, based on distribution estimates at fixed time scale. De Vries (1994), Lux (1996) and Pagan (1996) consent that the distribution of returns is leptokurtic with fat tails and the tails follow approximately a power law P(return□x) ~C/xm , the tail index m estimated in the range 2-4. Extrapolating this distribution to infinite values leads to infinite fourth and higher moments. This holds for return distributions of a few minutes to about three weeks. Gopikrishnan et al. (1998) and Plerou et al. (1999) confirm that, at a larger scale, distributions slowly converge to Gaussian with finite moments. Alternative descriptions of fat-tail distributions with finite moments are still far from sufficient for modelling market moves as “one-point” statistics according to Campbell et al. (1997) and Lo and MacKinlay (1999). Even correlations and volatilities (two-point statistics) are still limited in information content. The assumption of independent returns implies that the degree of fatness of the tails decreases as the holding horizon extends. Johansen and Sornette (2000) argue that returns exhibit strong correlation exactly at the time of extreme events. Therefore, using fixed time scale is not adapted to the real dynamics of price moves. Loworder statistics with adjustment to the time scales of the market might be more efficient description. Dacorogna et al. (1996) propose to expand periods of high volatility and contract those of low volatility. Muzy et al. (2001) and Breymann et al. (2000) show that return volatility display long-term correlations from large to small time scales. Fixed time scales are not adequate for capturing the perception of risk and return. A better insight into the dynamics of financial markets can be achieved with time-adaptive framework. This could be accomplished with the introduction of wavelet analysis of the financial time series.

Johansen and Sornette (1998) differentiate between price variation with fat tail distribution and crashes as separate phenomena. Market in normal times exhibits self-organization with fat tail distributions, while criticality describes crashes preceded by increased susceptibility and precursory signals similar to critical instability.

Statistical financial models do not deliver in crashes because the statistical properties of data are different from the statistical properties in stable times. Therefore, a model based on normal times data may not be of much use in times of crisis. Morris and Shin (1999), Danielsson and Zigrand (2001), and Danielsson et al. (2001) suggest that most statistical modelling is based on misunderstanding of the properties of risk. Danielsson (2001) proposes a risk model based on the aggregate actions of market participants executing the same trading strategies during crisis and changing the distribution of risk. As a result risk modelling is not only ineffective in lowering systemic risk but may exacerbate the crisis. The role of regulation is complex. If the regulations restrict the banks’ scope for pursuing individually optimal strategies, causing banks to act in a similar manner during crisis, it may lead to an escalation of the crisis according to Danielsson and Zigrand (2001). They also conclude that unregulated financial institutions, such as hedge funds, are essential for the prevention of systemic crisis.

The analysis focuses on market risk models. Market risk modelling is easier than the other risk factors because of the relative abundance of accurate market risk data and simpler and more established methodology. Danielsson (2001) identifies a number of shortcomings with regulatory VaR. Moreover, existing risk models break down in times of crisis because the stochastic process of market prices is ruled by the actions of market participants. The fundamental criticism of risk modelling remains and also applies to credit, liquidity and operational risk. Therefore, considering the weaknesses in market risk modelling, it is unfortunate that Basel II proposals basically extend the VaR regulatory methodology to credit and operational risk. Assuming that regulations are here to stay, the important question is whether it is possible to create a regulatory mechanism that is successful in reducing systemic risk. Unfortunately, we do not know enough about market crashes to design an effective supervisory system. Much research is being done, both theoretical and applied. The proposition is that when we know more about market crashes we would be able to regulate more effectively.

4. Market Crash Patterns

The objective of this research is to extract important information on the dynamics of the stock markets by studying large crashes. The stock market behavior before a crash is related to the transient behavior preceding a final equilibrium state according to Johansen (1998). A normally functioning financial market exposes properties common to complex system. Analogy of the financial crashes to critical points in statistical mechanics has been recently researched. Criticality implies scale invariance. Johansen et al. (2000) claim that oscillations appear in the price of the asset just before the critical date.

Several attempts were made to explain the origin of the October 1987 crash with the properties of trading and the structure of markets. It was hypothesized that stock market crashes are caused by the slow buildup of long-range correlations leading to a collapse in a critical instant. An unprecedented market increase took place during 1987 and even before. In mathematical terms “critical” points are defined as the explosion to infinity in a complex dynamical system. A possible link between stock market crashes and critical points is worth researching. Anderson et al. (1988) confirm that the theory of critical phenomena has been already applied to other economic models. A dynamic model of the stock market possibly will exhibit well-defined critical points. Mathematical properties of a critical point are independent of the specific model; therefore, predictions should be relatively robust to model misspecification. Johansen and Sornette (1998) apply statistical analysis to market indices and demonstrate that the largest crashes might be outliers, probably triggered by amplifying factors. Further on, Johansen et al. (2000) identify patterns of near-critical behavior years before crashes. Precursors of market crashes are associated with dynamical critical points by Sornette et al. (1996). On the October 1987 crash precursory patterns are identified as well as characteristic oscillations of relaxation and aftershock signatures in analogy of critical phase transitions in physics.

Financial markets are exposed to external factors; therefore precursors of crashes of the order of few years should be taken with caution. No fundamental theory supports precursory, universal log-periodic oscillations on large time-scale. On the other side, no precise definition links crashes to critical points in statistical mechanics. The crash corresponding to discontinuity in the derivative of a market index is not defined in terms of the amount of decline needed. What is to be considered only a ‘correction’ and what is a signal of a real crash? Johansen and Sornette (1998) study drawdowns (cumulative loss from the last local maximum to the next local minimum) and find that the largest events seem to be outside the exponential fit, concluding that the three largest crashes are outliers. This implies that they are probably triggered by additional amplifying factors. In such case, a specific signature similar to precursors before instabilities could exist. Drozdz et al. (1999) provide examples where the log-periodic oscillations accompany fast increase of the market index and log-periodicity on various time scales exhibits self-similarity. The stock market log-periodicity reveals much richer structure than just lowest order Fourier expansion. Sornette and Johansen (1997) propose that large market crashes exhibit an analogy to critical points in statistical physics with log-periodic corrections to scaling. They propose a more general formula with additional degrees of freedom to better capture the behavior away from the critical point.

Sornette et al. (1996) argue that scale invariance and self-similarity are the dominant concepts in the October 1987 crash, which could be the result of worldwide cooperative phenomena, with signatures in an analogy with critical phase transition in physics. Vandewalle et al. (1998) analyze the evolution of several financial indices before the crash of October 1987, assuming that the crash is similar to a phase transition and particularly to a specific heat jump. It is not an attempt to forecast the crash event but rather to establish an analogy between the crash and the jump quantity at a critical point. Critical exponents are estimated, considering the background first. Vandewalle et al. (1999) detect clustering of huge volatility of stock markets and panic-correlation appearing before crashes. Fluctuations and correlations play an important role in thermodynamic phase transitions.

Statistical evidence proves that the largest stock market crashes are outliers. Johansen and Sornette (1998) differentiate between price variation with fat tail distribution and crashes as separate phenomena. Market in normal times exhibits self-organization with fat tail distributions, while criticality describes crashes preceded by increased susceptibility and pre-cursory signals similar to critical instability. Sornette and Johansen (1998) assert that large market crashes are analogous to critical points in statistical physics. All the subparts of the dynamical system respond cooperatively and the response to a small external perturbation becomes infinite. The hierarchical model of traders with "crowd" behavior illustrates the concept of criticality, where a large proportion of the actors decide simultaneously to sell their stocks. Vandewalle et al. (1998) analyze stock market indices before crashes and uncover a logarithmic dependence, superimposed on a well-defined oscillation pattern. The "log-periodic" oscillations are interpreted as precursors to predict the crash time, the point where these oscillations accumulate. Sornette at al. (1996) analyze the behavior of S&P500 before October 1987 market crash and identify precursory patterns similar to dynamical critical points. The evolution of the index, as a function of time, is represented by a power law. Feigenbaum & Freund (1997) analyze the daily S&P500 data prior to the October 1997 correction and reveal the typical log-periodic pattern.

The analogy between market crashes and critical points is probed at an experimental level with the application of self-similar analysis of time series in Gluzman and Yukalov (1997, 1998). A resummation technique of theoretical physics, based on the algebraic self-similar renormalization is used to model market dynamics around crisis. Renormalization of asymptotic series can be used to forecast future values from historical data. The algebraic self-similar renormalization is developed and illustrated by several examples from the stock market index time series.

Research in the area of stock market crashes' similarity with critical points has been conducted mainly by physicist. No substantial theoretical back up exists and the empirical evidence is still not sufficient. Laloux et al. (1999) review the methods inspired by the physics of critical phenomena and consider the numerical solution for the log-periodic oscillations rather dubious, since a seven parameters fit to a noisy data is required. Further research with an alternative methodology is essential to reinforcement of the log-periodic oscillations "hypothesis" before crashes. The application of wavelets would enable us to perform time scale analysis and to achieve de-noising of the data for better detection of log-periodic oscillations. Wavelets allow for multiresolution of signals and yield temporal and frequency analysis with contracted and dilated versions of a chosen prototype function.

Physicists derive the final model from the characteristics of the data, dealing with systems with a large number of degrees of freedom. It explains why economics and finance enjoy increasing popularity among physicists starting with Mandelbrot and Taylor (1962). Working without an a priori assumed model is characteristic of the modern approach to economics. The model is to be inferred from the data. Wavelet transforms offer generic analysis to assist in this approach. Greenblatt (1998) claims that Matching Pursuit, Basis Pursuit and Method of Frames techniques based on wavelet decomposition appear to be promising in this respect. Another advantage of a wavelet transform is its ability to carry out in non-stationary environment. Flandrin (1989) proposes the use of the wavelet transform to analyze the behavior of fractional Brownian motion, a highly non-stationary random process.

5. Wavelet Analyses of Market Crash Patterns

Wavelets have been developed interdisciplinary. Yves Mayer (mathematician), Jean Morlet (geophysicist) and Alex Grossman (theoretical physicist) with common interest in time-frequency localization and multiresolution analysis, developed the foundations of wavelet analysis. The innovations of Daubechies (1992) and Mallat (1998) are applicable to signal processing. Ogden (1997) explores the analogy between signal processing and statistical analysis using wavelets. Wavelets are used to determine periodicities, cyclicities and intermittencies of financial time series. According to Los (2002), spectrograms and scalograms of the financial markets might be helpful in detection and prevention of financial crisis.

As mentioned before, wavelets are most useful in function estimation when the underlying function exhibits sharp spikes and jumps, a situation typical in market crashes. Wavelets can be used not only to estimate the localizations and sizes of these jumps but also the actual recovery of the function. This is the primary focus of the general change-point problem according to Page

(1954, 1955). Relatively little work has been done applying wavelets to the statistical change-point problem and it is only natural to apply wavelet methods to the problem. In terms of financial time series, wavelets might be more robust in analyzing market indices around crashes compared to the numerical analysis with several free parameters estimated by physicists.

Wavelets offer the means to automatically adapt the localization window in time and in frequency. Moreover, while standard methods of statistical function estimation rely upon certain assumptions about the smoothness of the function estimated with wavelets, such assumptions are relaxed considerably. These characteristics make wavelets suitable for analysis of stock market indices returns before market crashes. Struzik (2001) uses wavelets to detect heavy oscillations and characteristic patterns before the biggest crashes of S&P500. Capobianco (1999) analyzes Nikkei daily stock index volatility using wavelets as a smoothing operator. This threshold de-noising procedure improves the volatility prediction power. The results support the proposed methodology for this research. In addition, Cizeau (1997) determines the Hurst exponent of S&P500 to be 0.9 in line with already existing findings on fractional Brownian motion of Mandelbrot (1967) and Mandelbrot and van Ness (1968).

Mallat (1989) develops the discrete wavelet transform that decomposes a time series in terms of trend and details. The process can be iterated with different bases delivering various degrees of resolution. Coifman and Wickerhauser (1992) suggest a method for adaptively selecting the best basis.

5.1. Properties of Stock Market Crash Data

Several measurement approaches will be used to characterize the data around stock market crashes. The goal is to reveal the statistical differences in "normal" and "distressed" market data, as an argument against existing regulatory risk models based on normal market data.

Wavelet MRA allows the identification of the Hausdorff fractal Dimension of financial time series (Kyaw, Los and Zong, 2002; Lipka and Los, 2002). The homogeneous (monofractal) Hölder-Hurst exponent (*H*) of daily market index data will be calculated to determine the global self-similarity of such series. We'll begin the analysis eight years prior to crashes, as research reports recognition of patterns up to that length, and extended to three years after. Subsamples of daily prices before and after the crash, as well as groups of shorter lengths will be tested for the stability of the Hurst exponent. We'll accept a provisional definition of a stock market crash of "consecutive draw-downs of 15% and more," as these price variations are outside the range of a simple exponential fit. It was found that a power law truncated by an exponential provides a reasonable fit at short to medium time scales.

The research will then be extended to high frequency data for series. We intend to analyze high-frequency data from two months prior and two months after the crash. Based on these data, we'll compute multifractal singularity spectra for various stock markets.

Regular time series can always be fitted by a polynomial dynamic process. The degrees of irregularity are measured by the Lipschitz exponent α_L. The α_L measures the degree of randomness of a time series and is based on Taylor series expansion error. It will be used to assess the degree of irregularity of each singularity separately. For each stock market return series, a singularity spectrum of this α_L. will be computed for the time of a "crash" (15% and more draw - down). Additional analysis and comparison of different types of singularities will be also conducted for the purpose of identifying patterns preceding, a "critical maximum," if such patterns exist. Another reason for researching the details in the irregularities of each singularity is to identify any clues of magnitude/impact relationship of market shocks as a further goal of this research.

5.2. Data

The data for the research consists of daily returns of world stock market indices that experienced sudden drop or continuous draw-downs larger than 15%. The sample is divided between “end of a bubble” and “unexpected sudden events” time series. The finance.yahoo.com data source is used. The idea is not only to verify the results obtained with numerical methods but also extend to additional data sets and hypothesis testing such as lack of log-periodic oscillation precursory patterns preceding drawdowns due to unexpected sudden events. Wavelet analysis has not been previously used to detect precursory patterns of market crashes. The extension to high frequency data analysis increases the power of discovering turbulence structure. So far, hypothetical models of precursory stock market pricing patterns have only been tested on daily data

Financial markets are exposed to external factors; therefore precursors of crashes of the order of few years should be taken with caution. No fundamental theory supports precursory, universal log-periodic oscillations on large time-scale. On the other side, no precise definition links crashes to critical points in statistical mechanics. The crash corresponding to discontinuity in the derivative of a market index is not defined in terms of the amount of decline needed. What is to be considered only a ‘correction’ and what is a signal of a real crash?

6. Measurement Methodologies

6.1. Global Self-similarity Test

A random process $x(t)$ is said to be self-similar with parameter if for any $a > 0$ it obeys the scaling relation

$$x(t) = a^{-H} x(at) . \quad (1)$$

H is called the Hurst exponent of the series. Fractional Brownian Motions are series with $0 < H < 1$ (Geometric Brownian motion has H =0.5).

The risk (= volatility = energy = power) spectrum $P(\omega)$ can be used to determine the self-affinity of a time series

$$P(\omega) \propto \omega^{-\gamma} \quad (2)$$

with $\gamma=2H+1$, so that H can be identified. Such a risk spectrum can be accurately measured by a scalegram or averaged scalogram, based on wavelet MRS.

Fleming *et al*. (2001) extract the set of detail coefficients $\{ d_{m,k} \}$ using a wavelet, $\psi(t)$ with N vanishing moments. To determine the Hurst exponent of the series

$$Var(d_{m,k}) \propto 2^{-m\gamma} . \quad (3)$$

If $0 < \gamma < 2N$, one can plot the $\log_2 Var(d_{m,k})$ versus the level of decomposition m to produce a straight line of slope $-\gamma$ for the given time series. Deviations in these log-variance plots have been shown to reveal coherent risk structures (risk-concentrating features) present at a given scale with an increase in the variance.

6.2. Lipschitz Irregularity Analysis

An efficient implementation of the discrete Wavelet Transform allows decomposition of time series in terms of approximation provided by the scaling function and details provided by the wavelets. The second order moment (variance) of the wavelet resonance coefficients allows computation of the average Lipschitz exponent α_L. Pointwise irregularity of time series can also be analyzed with wavelets and measured by local $\alpha_L s$. We will use the modulus maxima and maxima lines to detect and measure the complete multifractal singularity spectra. From Hwang, Mallat theorem (Los, 2003), we know that a financial time series can be singular at a time point, if there are wavelet maxima lines that converge towards this time point at the finest scale. Gábor's Gaussian Wavelet Transform will be used to detect these maxima lines for the particular series of interest. The decay rate of the maxima along the maxima lines indicates the order of the various isolated singularities. The display of the modulus maxima of the Wavelet Transform as a function of scale in a log-log plot allows us to compute the Lipschitz α_L : the slope d of the plot will be $d=\alpha_L+0.5$. The degree of irregularity of each singularity can thus be assessed separately and locally.

Two caveats are important for the data analysis. First, by the nature of wavelet analysis one should expect maxima line to converge at the end points (= abrupt beginning and abrupt end) of the time series. These are analytic artifacts and the data set should therefore be extended, so that the singularity of interest should be somewhere in the middle of the scalogram. With historic data we can easily extend the sample to avoid such artifacts. But such "midsample limitation" imposes restrictions on contemporary data.

Second, one must analyze the time series just before the crash without the masking impact of the large jump (maxima lines of preceding singularities might merge). For this purpose I'll attempt to analyze series of original data extended by the mirror image of itself. The singularity at the flex point will depend on the acceleration of the price increase (decrease). For historic data, such flex points will be chosen at the local maxima (minima). According to Johansen *et al*. (2000) the intervals between consecutive maxima follow geometric contraction and tend to zero at time point t_C .

Applying this technique, we would also achieve analysis of the reverse signal that may reveal additional information. "Synthetic" series with flex points different than the local maxima or minima will be also researched for patterns of differences with max/min points. The results will be useful for the analysis of contemporary data in pattern recognition experiments.

6.3. Computing Multifractal Spectra

The complete singularity spectrum of multifractals will be computed from the local modulus maxima of the Wavelet Transform with a Gaussian wavelet basis, following Mallat's (1998) procedure. First, the Wavelet Transform will be computed for all translations and all dilations. The largest dyadic scale $a=2^j$ depends on the number of available sample points and the distances between the singularities. After finding the modulus maxima for each scale a as:

$$\sup_a \left|W(\tau,a)\right| \sim a^{(\alpha_L+0.5)}, \tag{4}$$

maxima lines are drawn in the scalograms (software is availbale) that allow the computation of Gibbs' partition function

$$Z(q,a)=\sum_\tau \sup_a \left|W(\tau,a)\right|^q \sim a^{\tau(q)}. \tag{5}$$

The decay scaling exponent is computed as the slope of $\log_2 Z(q,a) \approx \tau(q)\log_2 a + C(q)$. The multifractal spectrum $D(\alpha_L)$ can be found as the inverse Legendre transform of the scaling exponent $\tau(q)$.

7. Models of Market Crashes

Attempts have been made to explain the origin of the October 1987 crash with the properties of trading and of the structure of markets. It has been hypothesized that stock market crashes are caused by the slow buildup of long-range correlations leading to a collapse in a critical instant. An unprecedented market volatility increase took place during 1987 and even before. In mathematical terms "critical" points are defined as the explosion to infinity of the moments of a complex dynamical system. A possible link between stock market crashes and critical points is thus worth researching. Anderson et al. (1988) and Los (2003) show that the theory of critical fluid mechanics phenomena has been already applied to other financial-economic models. A dynamic model of the stock market is expected to exhibit well-defined critical points.

A crash occurs when a synchronization of the individual actions takes place. An increasing synchronization, or correlation, is observed in physics when a phase transition, especially a critical point, is approached. The idea of critical points has been generalized to self-organized critical points in open equilibrium systems.

The modern financial-economics' approach is to infer the model from the data. Wavelet MRA offers a very generic analysis so that complex models can be fitted to the decomposition components.

8. Conclusions

The evident need of bank capital regulations led to the introduction of the Basel Accord of 1988, which addressed credit risk only. The financial innovations and deregulation of the banking industry created the need for new regulations. After a number of amendments to the original Accord, a proposal for Basel II Accord is on the way. It will also include capital charges for market, liquidity and operation risks. The new proposal has already incurred criticism from the banking community as well as serious arguments against its effectiveness raised by academic researchers. Concerns that regulation could even exacerbate an impending financial crisis are grounded in the binding nature of capital requirements.

In order to improve the effectiveness of regulation we ought to access its impact on institutions functioning in extreme situations. The focus of this research is on market risk. Market risk modelling has been extensively researched, but the VaR methodology based on normal market conditions fails in crashes. Better knowledge of market crashes will enable better regulation.

The existing research on precursors of market crashes uses numerical analysis based on physical phenomena and raises doubts for the lack of enough empirical evidence or theoretical underpinning. Wavelet analysis offers robust methodology to confirm the log-periodic oscillation findings or uncover other precursory patterns.

Research in the area of stock market crashes' similarity with critical points in fluid or gas dynamics has been conducted mainly by physicists. No substantial theory, or modelling exists yet and the empirical evidence is still insufficient. Laloux et al. (1999) review the methods inspired by the physics of critical phenomena and consider the numerical model solution for the log-periodic oscillations rather dubious, since a seven degrees of freedom fit to noisy data is required. Therefore, further research with an alternative methodology is essential to corroborate or falsify the hypothesis of log-periodic oscillations preceding stock market.

Wavelets allow for multiresolution analysis (MRA) of market return time series and yield temporal and frequency analysis with contracted and dilated versions of a chosen prototype function. The application of wavelet MRA enables us, first, to perform time-scale or time-frequency analysis and, second, to achieve de-noising of the data for better detection of log-periodic oscillations. Wavelet MRA is most useful, when the time series exhibits sharp spikes and jumps, a situation typical for stock market crashes. Wavelets can be used not only to estimate the localizations and sizes of these singularities, but also the actual recovery of the market pricing process.

Relatively little work has been done applying wavelets to the statistical change-point problem and it is only natural to apply wavelet methods. In terms of financial time series, wavelets might be more robust in analyzing market indices around crashes compared to the numerical analysis with several free parameters by classical physicists.

The evident need of bank capital regulations led to the introduction of the Basel Accord of 1988, which addressed default or credit risk only. The financial innovations and deregulation of the banking industry created the need for new regulations. After a number of amendments to the original Accord, a proposal for Basel II Accord is on the way. It will also include capital charges for market, liquidity and operation risks. In order to improve the effectiveness of such regulation of financial institutions, we ought to access its impact on their functioning in extreme situations. The focus of this research is on stock market risk. Market risk modelling has been extensively researched, but the VaR methodology based on normal market conditions fails during crashes. Better knowledge of market crashes will enable better regulation.

The above described methodology is likely to enable us to answer at least some of the following questions and determine the direction for future research on this important topic:

Do crashes signal phase transitions in stock markets?

Can crashes be predicted through detection of logperiodic oscillations or other patterns?

What should be called a stock market "crash"?

There is always persistent turmoil in capital markets on smaller scales. Little is done to quantify the magnitude of financial crisis. Olsen and Associates, Zurich, have recently constructed a scale based on the Gutenberg-Richter law to measure market shocks. If we are successful in determining the magnitude of the crash and the jump quantity at a critical point it is only logical to extend the research into the magnitude/impact area with implementation of an indicator measuring market shocks.

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