“Z-score vs minimum variance preselection methods for constructing small portfolios”

AUTHORS	Francesco Cesarone https://orcid.org/0000-0003-2326-4204 Fabiomassimo Mango https://orcid.org/0000-0002-3092-6731 Gabriele Sabato

ARTICLE INFO	Francesco Cesarone, Fabiomassimo Mango and Gabriele Sabato (2020). Z-score vs minimum variance preselection methods for constructing small portfolios. *Investment Management and Financial Innovations*, *17*(1), 64-76. doi:10.21511/imfi.17(1).2020.06
DOI	http://dx.doi.org/10.21511/imfi.17(1).2020.06
RELEASED ON	Friday, 14 February 2020
RECEIVED ON	Wednesday, 25 September 2019
ACCEPTED ON	Tuesday, 24 December 2019

JOURNAL	"Investment Management and Financial Innovations"
ISSN PRINT	1810-4967
ISSN ONLINE	1812-9358
PUBLISHER	LLC “Consulting Publishing Company “Business Perspectives”
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

BUSINESS PERSPECTIVES

LLC "CPC "Business Perspectives"
Hryhorii Skovoroda lane, 10,
Sumy, 40022, Ukraine

www.businessperspectives.org

Francesco Cesarone (Italy), **Fabiomassimo Mango** (Italy), **Gabriele Sabato** (UK)

Z-SCORE VS MINIMUM VARIANCE PRESELECTION METHODS FOR CONSTRUCTING SMALL PORTFOLIOS

Abstract

Several contributions in the literature argue that a significant in-sample risk reduction can be obtained by investing in a relatively small number of assets in an investment universe. Furthermore, selecting small portfolios seems to yield good out-of-sample performances in practice. This analysis provides further evidence that an appropriate preselection of the assets in a market can lead to an improvement in portfolio performance. For preselection, this paper investigates the effectiveness of a minimum variance approach and that of an innovative index (the new Altman Z-score) based on the creditworthiness of the companies. Different classes of portfolio models are examined on real-world data by applying both the minimum variance and the Z-score preselection methods. Preliminary results indicate that the new Altman Z-score preselection provides encouraging out-of-sample performances with respect to those obtained with the minimum variance approach.

Received on: 25th of September, 2019
Accepted on: 24th of December, 2019

Keywords asset allocation, risk diversification, risk parity, portfolio optimization, credit scoring

JEL Classification C61, C63, G11

Francesco Cesarone, Assistant Professor, Department of Business Studies, Roma Tre University, Italy.

Fabiomassimo Mango, Associate Professor of Banking and Finance, Department of Management, Sapienza University of Rome, Italy.

Gabriele Sabato, Ph.D. in Banking and Finance, Sapienza University of Rome; Co-founder and CEO of Wiserfunding Limited, UK.

INTRODUCTION

The issue of constructing small portfolios is a well-known problem in the financial industry, particularly in the case of small investor who should stem costs due to the complexity of management. However, also big investors could take advantage of this practice if small portfolios can achieve better performance than large portfolios. This analysis provides further evidence that an appropriate preselection of the assets in a market can lead to a significant improvement in portfolio performance. More precisely, this paper investigates the effectiveness of the Z-score index for preselecting the assets of an investment universe compared with that achieved by the minimum variance approach. The Z-score is a predictive index of creditworthiness expressed as a numerical score, which essentially measures the default probability of a company. It is used here to classify the quality of a company and its out-of-sample performance in terms of the market price. Different classes of portfolio models are examined on real-world data by applying both the minimum variance and the Z-score preselection methods. Preliminary results show that the new Altman Z-score method produces encouraging out-of-sample performances with respect to those obtained with the minimum variance approach.

The structure of this paper is as follows. Section 1 presents a survey of the literature on the main research topics covered in this work. Section 2 provides details on the research methodology. More precisely, subsection 2.1 describes the portfolio selection strategies analyzed.

http://dx.doi.org/10.21511/imfi.17(1).2020.06

Subsection 2.2 is devoted to discussing the new Altman Z-score model, while, subsection 2.3 explains the preselection procedures applied to an investment universe, and describes the method used to evaluate the performance. The computational results based on real-world data are presented in section 3, where the main empirical findings are also discussed. Finally, the last section contains some concluding remarks.

1. LITERATURE REVIEW

The first empirical evidence showing that small portfolios tend to achieve a drastic elimination of the diversifiable risk in a market is probably due to a work by Evans and Archer (1968) who discovered that the average standard deviation decreases quickly when the portfolio size increases. They concluded that no more than about ten assets are needed to almost completely eliminate the non-systematic risk in the portfolio return. From then on, several contributions in the literature show that investing in a small number of assets from an investment universe is sufficient to obtain a significant in-sample risk reduction in terms of variance and of some other popular risk measures, and good out-of-sample performances in practice (see, e.g., Statman, 1987; Newbould & Poon, 1993; Tang, 2004; Cesarone, Scozzari, & Tardella, 2013, 2016, 2018, and references therein).

After the global financial crisis started in 2008, the weakness of some classical portfolio selection approaches based on risk-gain analysis (Markowitz, 1952, 1959) has given rise to a new research stream that is based on capital (DeMiguel, Garlappi, & Uppal, 2009; Tu & Zhou, 2011; Pflug, Pichler, & Wozabal, 2012) and risk diversification (see Cesarone & Tardella, 2017; Cesarone & Colucci, 2018; Cesarone, Scozzari, & Tardella, 2019; Lhabitant, 2017; Roncalli, 2014, and references therein). Furthermore, in the last few decades, several scholars have proposed portfolio selection models based on stochastic dominance criteria (see, e.g., Fábián, Mitra, Roman, & Zverovich, 2011; Roman, Mitra, & Zverovich, 2013; Bruni, Cesarone, Scozzari, & Tardella, 2017; Valle, Roman, & Mitra, 2017, and references therein).

This study considers several of these approaches for portfolio selection purposes and investigates the effectiveness of the Z-score index for preselecting the assets of an investment universe compared with that achieved by the minimum variance approach.

The original Z-score index was introduced by Altman (1968) for evaluating the default probability of a company. However, following several findings that show the relation between market prices and credit ratings (Hand, Holthausen, & Leftwich, 1992; Hsueh & Liu, 1992; Kliger & Sarig, 2000; Gonzalez, Haas, Persson, Toledo, Violi, Wieland, & Zins, 2004; Hull, Predescu, & White, 2004; Norden & Weber, 2004; Micu, Remolona, & Wooldridge, 2006; Grothe, 2013), a new version of the Altman credit-scoring model (Altman, 2002; Altman & Hotchkiss, 2006; Altman, 2013) is used here to classify the quality of a company and its out-of-sample performance in terms of market price.

2. METHODS

2.1. Portfolio selection models

This subsection gives a brief review of the portfolio selection models used for this analysis. Specifically, three different classes of models for selecting a portfolio are considered:

1) risk minimization;
2) capital or risk diversification;
3) second-order stochastic dominance.

Hereafter, the linear return of the k-th asset at time t is denoted by

$$r_{t,k} = \frac{p_{t,k} - p_{t-1,k}}{p_{t-1,k}},$$

where $p_{t,k}$ represents its price at time t.

The portfolio return at time t is

$$R_t(x) = \sum_{i=1}^{n} x_i r_{i,t},$$

where x_i is the percentage of capital invested in the asset i, and n indicates the number of tradable assets belonging to an investment universe.

2.1.1. Minimum risk portfolios

This subsection describes two portfolio selection models focused on the minimization of portfolio risk, that is measured using both symmetric and asymmetric risk measure.

As for symmetric risk measures, the first portfolio selection model considered aims at minimizing variance, namely a special case of the Mean-Variance model (Markowitz, 1952, 1959). In the case of long-only portfolios, it can be formulated as follows:

$$\begin{cases} \min\limits_{x} \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \\ s.t. \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \;\; i = 1,\ldots,n \end{cases}, \tag{1}$$

where σ_{ij} is the covariance of the returns of asset i and asset j.

As for asymmetric risk measures, the second portfolio selection model analyzed consists in minimizing the Conditional Value-at-Risk at a specified confidence level ε ($CVaR_\varepsilon$), i.e., the mean of losses in the worst 100epsilon% of the cases (Acerbi & Tasche, 2002), where losses are defined as negative outcomes. A formal definition of $CVaR$ is as follows:

$$CVaR_\varepsilon(x) = -\frac{1}{\varepsilon}\int_0^\varepsilon Q_{R_{p(x)}}(\alpha)\,d\alpha, \tag{2}$$

where $Q_{R_{p(x)}}(\alpha)$ is the α-quantile function of the portfolio return $R_{p(x)}$. Thanks to its theoretical and computational properties, $CVaR$, also called expected shortfall or average Value-at-Risk, has become widespread for risk management and asset allocation purposes. From a theoretical point of view, $CVaR_\varepsilon$ satisfies the properties of monotonicity, sub-additivity, homogeneity, and translational invariance, i.e., the axioms of a coherent risk measure (Artzner, Delbaen, Eber, & Heath, 1999). Furthermore, Ogryczak and Ruszczynski (2002) show that the mean-$CVaR$ model is consistent with second-order stochastic dominance. From a computational point of view, the mean-$CVaR$ portfolio can be efficiently solved by means of linear programming (Rockafellar & Uryasev, 2000). The long-only portfolio that minimizes $CVaR_\varepsilon$ can be found by solving the following problem:

$$\begin{cases} \min\limits_{x} CVaR_\in(x) \\ s.t. \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \;\; i = 1,\ldots,n \end{cases}. \tag{3}$$

In these experiments, the confidence level ε is fixed equal to 10%.

2.1.2. Capital and risk diversification strategies

The concept of diversification can be qualitatively related to the portfolio risk reduction due to the process of compensation caused by the co-movement among assets that leads to a potential attenuation of the exposure to risk determined by individual asset shocks. However, the question of which measure of diversification is most appropriate is still open (see, e.g., Meucci, 2009; Lhabitant, 2017).

The oldest and most intuitive way to force diversification in a portfolio is to equally share the capital among all securities in an investment universe (Tu & Zhou, 2011). Formally, the Equally Weighted (EW) portfolio is defined as $x_{EW} = 1/n$. This strategy does not entail the use of any past or future information, nor needs the resolution of complex models. From a theoretical point of view, Pflug et al. (2012) prove that when increasing the uncertainty of the market, represented by the degree of ambiguity on the distribution of the asset returns, the optimal investment strategy tends to be the EW one. Furthermore, from a practical point of view, DeMiguel et al. (2009) empirically investigate its out-of-sample performance, which seems to be generally better than that obtained from different classical and recent portfolio selection models.

Two recent portfolio selection approaches focused on risk diversification are described below and tested in the empirical analysis.

The Risk Parity (RP) strategy, introduced by Maillard, Roncalli, and Teiletche (2010), requires

http://dx.doi.org/10.21511/imfi.17(1).2020.06

that each asset equally contributes to the total risk of the portfolio, which is measured by volatility. The standard approach used for decomposing the portfolio volatility is the Euler allocation, namely

$$\sigma(x)=\sum_{i=1}^{n} RC_i(x),$$

where

$$RC_i(x)=x_i\frac{\delta\sigma(x)}{\delta x_i}=\frac{1}{\sigma(x)}\sum_{k=1}^{n}\sigma_{ik}x_i x_k$$

is the contribution of the i-th asset. Thus, the RP portfolio can be obtained by imposing the following conditions:

$$RC_i(x)=RC_j(x)\Leftrightarrow\sum_{k=1}^{n}\sigma_{ik}x_i x_k=$$

$$=\sum_{k=1}^{n}\sigma_{jk}x_j x_k \quad \forall i,j.$$

Hence, a direct method for finding an RP portfolio is to solve the following system of linear and quadratic equations and inequalities:

$$\begin{cases}\sum_{k=1}^{n}\sigma_{ik}x_i x_k=\lambda \quad i=1,\ldots,n\\ \sum_{i=1}^{n}x_i=1\\ x_i\geq 0 \quad i=1,\ldots,n\end{cases} \tag{4}$$

that has a unique solution, due to the positive semi-definiteness of the covariance matrix Σ (Cesarone et al., 2019).

An alternative approach to diversifying the risk, introduced by Choueifaty and Coignard (2008), consists in maximizing the so-called diversification ratio:

$$DR(x)=\frac{\sum_{i}^{n}x_i\sigma_i}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j}}, \tag{5}$$

where σ_i is the volatility of asset i. Note that, thanks to the subadditivity property of volatility, $DR(x)\geq 1$. As described by Choueifaty, Froidure, and Reynier (2013), the Most Diversified (MD) portfolio, namely the optimal portfolio that maximizes the diversification ratio (5), can be found by solving the following (convex) quadratic programming problem:

$$\begin{cases}\min_{y}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}y_i y_j\\ s.t.\sum_{i=1}^{n}y_i\sigma_i=1\\ y_i\geq 0 \; i=1,\ldots,n\end{cases}. \tag{6}$$

Clearly, the normalized portfolio weights are

$$x_i^{MD}=\frac{y_i^*}{\sum_{k}^{n}y_k^*}$$

with $i=1,\ldots,n$, where y^* is the optimal solution of Problem (6).

2.1.3. Portfolio selection based on SSD

Second-order Stochastic Dominance (SSD) is a rational principle of decision making under uncertainty, widely studied and investigated in the literature (see, e.g., Bruni et al., 2017; Valle et al., 2017, and references therein). This subsection discusses the portfolio optimization method for Enhanced Indexation (EI), provided by Fábián et al. (2011), Roman et al. (2013) who select a portfolio whose return distribution SSD dominates that of a given benchmark. For finding an SSD efficient portfolio w.r.t. a specific benchmark R_B, in the case of T equally likely scenarios, the authors propose the following multi-objective optimization problem:

$$\begin{cases}\max_{x}\min_{1\leq t\leq T}\left(Tail_{\frac{t}{T}}\left(R_p(x)\right)-Tail_{\frac{t}{T}}\left(R_B\right)\right)\\ s.t.\sum_{i=1}^{n}x_i=1\\ x_i\geq 0 \; i=1,\ldots,n\end{cases}, \tag{7}$$

where $Tail_{t/T}(R_p)$ is the unconditional expectation of the worst $(t/T)100\%$ outcomes of R_p. This problem can be expressed as an LP problem, using the $CVaR$ reformulation of Rockafellar and Uryasev (2000, 2002). However, due to the high number of variables and constraints (more than T^2), Problem (7) is solved by implementing cutting planes techniques, as explained in Roman et al. (2013).

The complete list of portfolio models analyzed in this study is reported in Table 1.

Table 1. List of portfolio strategies

Model	Abbreviation
Minimum risk strategy	
Minimum variance portfolio	MinV
Minimum conditional value-at-risk portfolio with ε = 0,10	MinCVaR
Capital diversification strategy	
Equally weighted portfolio	EW
Risk diversification strategy	
Risk parity portfolio	RP
Most diversified portfolio	MD
Portfolio selection based on Second-order Stochastic Dominance	
SSD portfolio	SSD

2.2. *Z*-score: from default to price prediction

The Z-score index was introduced by Altman (1968) to predict the default probability of a firm. This index was originally built as multiple linear regression of five explanatory variables represented by common business ratios. Given its high accuracy and effectiveness in predicting a firm bankruptcy (see, e.g., Altman, Haldeman, & Narayanan, 1977; Altman, 2002; Altman & Hotchkiss, 2006; Altman, 2013), the Altman Z-score model has become one of the state-of-the-art approaches for assessing the credit risk of a company. This version of the Altman credit-scoring model, also called SME Z-score, is calibrated by the country and industrial sector to maximize its prediction power. The SME Z-score index is obtained by multiple linear regression with forty explanatory variables, which can be divided into three main groups:

- *financial* variables such as those belonging to the following accounting ratio categories: leverage, liquidity, profitability, coverage, activity;
- *corporate governance and managerial* variables such as size and age of the company, industry sector, age/experience of managers, location, market position, number of board members, etc.;
- *macroeconomic* variables such as industry default rate, GDP growth rate, consumer confidence index, consumer price index, unemployment rate, interest rate, etc.

As mentioned in the introduction, following several findings that highlight the connection between market prices and credit ratings, in this paper, a variant of the SME Z-score model is applied on a set of large corporates belonging to the Eurostoxx market, assessing the effectiveness of this approach to classify the quality of a listed company and its future performance in terms of market price. In this variant of the SME Z-score model, the book value of a company is substituted by its market value.

2.3. Preselection process and methodology

This paper aims to study and compare the ability of the new Altman Z-score index and that of the minimum variance criterion for preselecting assets. The two preselection strategies are performed as follows:

1) on the date where the assets are preselected, the current monthly values of the new Altman Z-score of all assets in the investment universe are collected, and ten assets with the highest scores are chosen (Z-score preselection);

2) on the same date, the Minimum Variance (MinV) portfolio (1) is computed on in-sample data of 1 year (250 financial days), and ten preselected assets are those with the highest weights in such MinV portfolio (minimum variance preselection).

The empirical analysis is based on a rolling time windows approach. As already mentioned, an in-sample time window of 1 year is used. The portfolio performance is then assessed in the following month (20 financial days, called out-of-sample window). Next, the in-sample window is shifted by one month, thus

http://dx.doi.org/10.21511/imfi.17(1).2020.06

covering the previous out-of-sample period; again, the optimal portfolio w.r.t. the new in-sample window is computed, and this procedure is repeated up to the end of the data. Thus, for each monthly portfolio rebalancing, ten assets are preselected through both the Z-score preselection and minimum variance preselection strategies. Then, all the portfolio selection approaches, listed in Table 1, are applied on these ten preselected assets.

The out-of-sample performance of each portfolio strategy is examined using as a benchmark the Equally Weighted (EW) portfolio throughout the investment universe (Bench). More specifically, the following performance measures (where the constant risk-free rate of return is set equal to 0) are considered: mean (Mean); volatility (Vol), Sharpe ratio (Sharpe) (Sharpe, 1966, 1994), maximum drawdown (MDD) (see, e.g., Chekhlov, Uryasev, & Zabarankin, 2005, and references therein), Ulcer index (Ulcer) (MacCann, 1989), Sortino ratio (Sortino & Satchell, 2001) (Sortino), Rachev ratio (with a confidence level equal to 5% and 10%, named Rachev5 and Rachev10, respectively) (Rachev, Biglova, Ortobelli, & Stoyanov, 2004), Jensen's Alpha (JensenA) (Jensen, 1968), and Information ratio (Info) (Goodwin, 1998).

3. RESULTS AND DISCUSSION

This section provides the empirical results obtained by all the strategies listed in Table 1 with and without preselection on the Eurostoxx market. Specifically, a subset of this investment universe, containing 31 assets, is considered, where companies belonging to the banking, insurance, and financial sector are excluded. The reasons for this choice are closely linked to the time availability of the new Altman Z-score index, which starts from February 2009, and to the elements on which the Altman Z-score model is based. Indeed, this model aims to assess the possible bankruptcy of non-financial companies, which can be traded, or not, in a market. As described in subsection 2.2, the SME Z-score model uses several categories of budget indicators to forecast the default probability of a company, but these variables are explanatory only for a specific sector. Indeed, financial companies are based on completely different rules and dynamics w.r.t. non-financial ones. For instance, some budget indicators are representative of the degree of solvency only for non-financial companies, and therefore cannot be used for the same purpose in the case where the debt is part of the production process. In fact, the banks admit the debt as an element of production as they systematically collect resources for credit activities, mainly aimed at commercial banks and at investments in the securities market. Furthermore, at least in theory, financial companies can borrow indefinitely. Indeed, except for specific regulatory constraints, they can cover all (or almost all) costs of production factors if they are able to generate a significant and positive spread between the lending and borrowing rates. On the other hand, non-financial companies should not directly allow debts to produce goods and/or services, but they should use debts only to meet the needs of the circulating and fixed capital. In addition, they can borrow up to a specific threshold, beyond which the cost of the debt is too expensive for any profitable use. Also, in the case of insurance companies, the new Altman Z-score index cannot be evaluated by the model described in subsection 2.2. Indeed, they have an inverted economic cycle w.r.t. the financial and non-financial companies: revenues occur before production costs due to the collection of insurance premiums.

Since the new Altman Z-score index is available only for non-financial and non-insurance companies, the empirical analysis is performed with the following datasets:

- Eurostoxx, containing 31 assets of the Euro Stoxx 50 Market Index (Europe) from February 1, 2009 to January 31, 2019 (daily frequency, source: Bloomberg);
- the new Altman Z-score, assessed on the same 31 assets from February 2009 to January 2019 (monthly frequency, source: Wiserfunding Limited).

Table 2 reports some details of 31 assets belonging to the analyzed investment universe.

All models have been implemented in Matlab 8.5 on a workstation with Intel Core CPU (i7-6700, 3.4 GHz, 16 Gb RAM) under MS Windows 10.

Figures 1 and 2 show the ten preselected assets of the investment universe described in Table 2 for each rebalancing date using the Z-score and the

Table 2. List of 31 assets belonging to the investment universe considered

No.	Company name	Ticker symbol	ISIN number	Ticker Bloomberg
1	DAIMLER AG	DAI	DE0007100000	DAI GY Equity
2	TOTAL S.A.	FP	FR0000120271	FP FP Equity
3	BAYERISCHE MOTOREN WERKE AKTIENGESELLSCHAFT	BMW	DE0005190003	BMW GY Equity
4	SIEMENS AG	SIE	DE0007236101	SIE GY Equity
5	ENI S.P.A.	ENI	IT0003132476	ENI IM Equity
6	ENEL SPA	ENEL	IT0003128367	ENEL IM Equity
7	BASF SE	BAS	DE000BASF111	BAS GY Equity
8	KONINKLIJKE AHOLD DELHAIZE N.V.	AD	NL0011794037	AD NA Equity
9	TELEFONICA SA	TEF	ES0178430E18	TEF SQ Equity
10	LVMH MOET HENNESSY – LOUIS VUITTON SE	MC	FR0000121014	MC FP Equity
11	VINCI	DG	FR0000125486	DG FP Equity
12	ORANGE	ORA	FR0000133308	ORA FP Equity
13	BAYER AG	BAYN	DE000BAY0017	BAYN GY Equity
14	SANOFI	SAN	FR0000120578	SAN FP Equity
15	IBERDROLA, S.A.	IBE	ES0144580Y14	IBE SQ Equity
16	FRESENIUS SE & CO. KGAA	FRE	DE0005785604	FRE GY Equity
17	L'OREAL SA	OR	FR0000120321	OR FP Equity
18	SCHNEIDER ELECTRIC SE	SU	FR0000121972	SU FP Equity
19	DANONE S.A.	BN	FR0000120644	BN FP Equity
20	SAP SE	SAP	DE0007164600	SAP GY Equity
21	NOKIA OYJ	NOKIA	FI0009000681	NOKIA FH Equity
22	SAFRAN S.A.	SAF	FR0000073272	SAF FP Equity
23	ADIDAS AG	ADS	DE000A1EWWW0	ADS GY Equity
24	L'AIR LIQUIDE	AI	FR0000120073	AI FP Equity
25	KONINKLIJKE PHILIPS N.V.	PHIA	NL0000009538	PHIA NA Equity
26	KERING	KER	FR0000121485	KER FP Equity
27	VIVENDI	VIV	FR0000127771	VIV FP Equity
28	ASML HOLDING N.V.	ASML	NL0010273215	ASML NA Equity
29	ESSILORLUXOTTICA	EL	FR0000121667	EL FP Equity
30	UNILEVER NV	UNA	NL0000009355	UNAT NA Equity
31	CRH PUBLIC LIMITED COMPANY	CRG	IE0001827041	CRH ID Equity

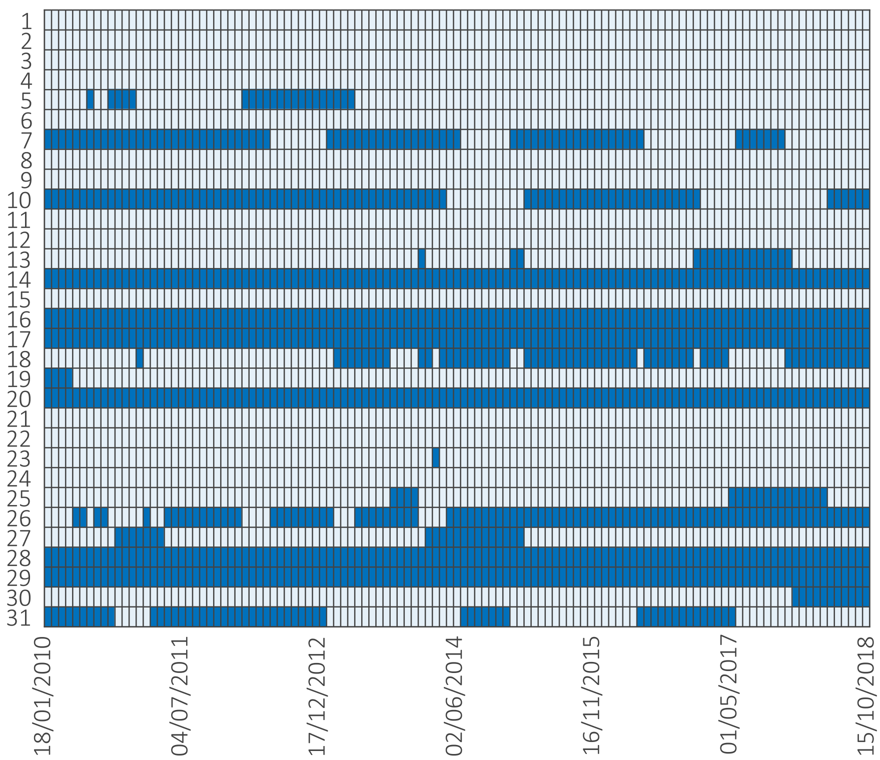

Figure 1. Ten preselected assets for each rebalancing date using Z-score preselection method

http://dx.doi.org/10.21511/imfi.17(1).2020.06

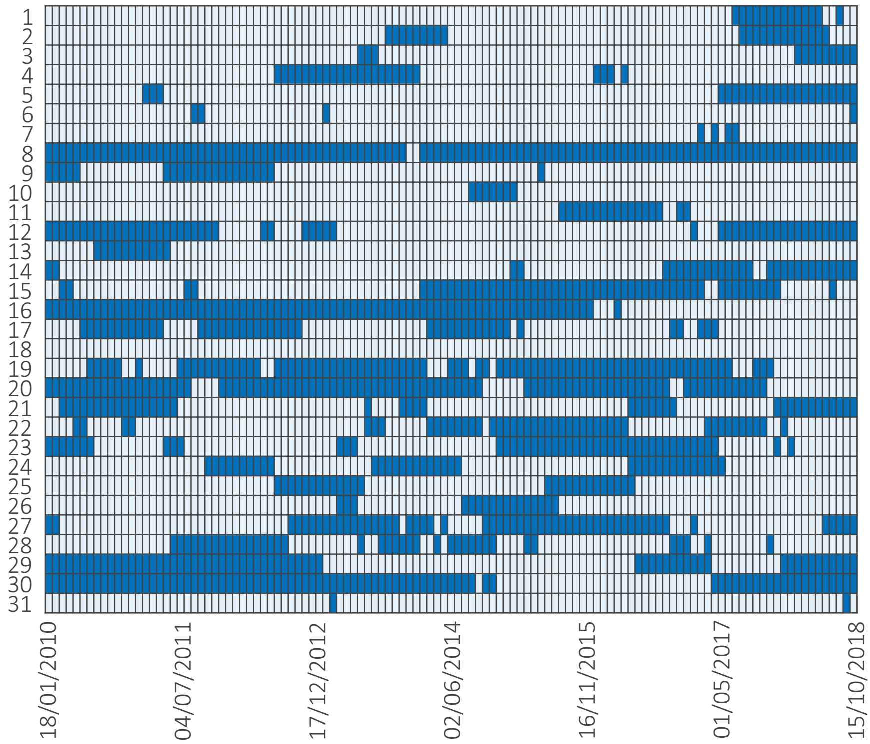

Figure 2. Ten preselected assets for each rebalancing date using the minimum variance preselection method

minimum variance preselection methods, respectively. According to the rolling time windows approach discussed above, Figures 1 and 2 are heatmaps with 31 rows (a row for each asset) and 117 columns (a column for each rebalancing date), where the preselected assets are marked in blue.

3.1. Out-of-sample performance results without preselection

Computational results are presented here for all the portfolio models listed in Table 1 without using any preselection procedure. Table 3 reports the out-of-sample performance for each portfolio strategy, where the rank of the performance is shown in different colors. For each column, the colors span from deep-green to deep-red, where deep-green depicts the best performance, while deep-red the worst one. Such a visualization style allows for easier revelation of (possible) persistent behavioral pattern of a portfolio approach (corresponding to a row). Note that the best performances are generally obtained from SSD and MD portfolios. This behavior is also confirmed by the trend of the cumulative out-of-sample portfolio returns reported in Figure 3. Note that there is a clear dominance of the SSD portfolio, followed by the MD portfolio.

3.2. Out-of-sample performance results using minimum variance and Z-score preselection

This subsection provides the empirical results for all the portfolio strategies (see Table 1) applied to a subset of ten assets, which are obtained by means of the minimum variance and the Z-score preselection procedures described in subsection 2.3. As already mentioned, Figures 1 and 2 show, in the rolling time windows scheme of evaluation, ten companies preselected by the Z-score and minimum variance methods, respectively.

Table 4 reports the out-of-sample performance for each portfolio model when the minimum variance preselection is used. Again, the rank of the performance is indicated with different colors, as in subsection 3.1. Note that the minimum variance preselection tends to be ineffective compared to the results obtained without assets preselection. This is also highlighted in Figure 4, where the cumulative out-of-sample portfolio returns are

Table 3. Out-of-sample results without preselection

Approach	Mean	Vol	Sharpe	MDD	Ulcer	Sortino	Rachev5	Rachev10	JensenA	Info
MinVaR	4.34E-04	8.76E-03	4.95E-02	–0.144	0.043	7.09E-02	0.986	1.008	1.94E-04	1.29E-02
MinCVaR	4.48E-04	8.95E-03	5.01E-02	–0.156	0.046	7.15E-02	0.984	1.005	2.05E-04	1.50E-02
EW	3.58E-04	1.10E-02	3.24E-02	–0.230	0.071	4.65E-02	0.991	1.009	0.00E+00	–
RP	3.82E-04	1.04E-02	3.66E-02	–0.209	0.061	5.26E-02	0.994	1.005	4.50E-05	2.31E-02
MD	5.71E-04	9.78E-03	5.84E-02	–0.185	0.046	8.52E-02	1.019	1.039	2.83E-04	4.64E-02
SSD	7.49E-04	9.88E-03	7.58E-02	–0.244	0.059	1.13E-01	1.056	1.082	4.96E-04	5.73E-02
Bench	3.58E-04	1.10E-02	3.24E-02	–0.230	0.071	4.65E-02	0.991	1.009	–	–

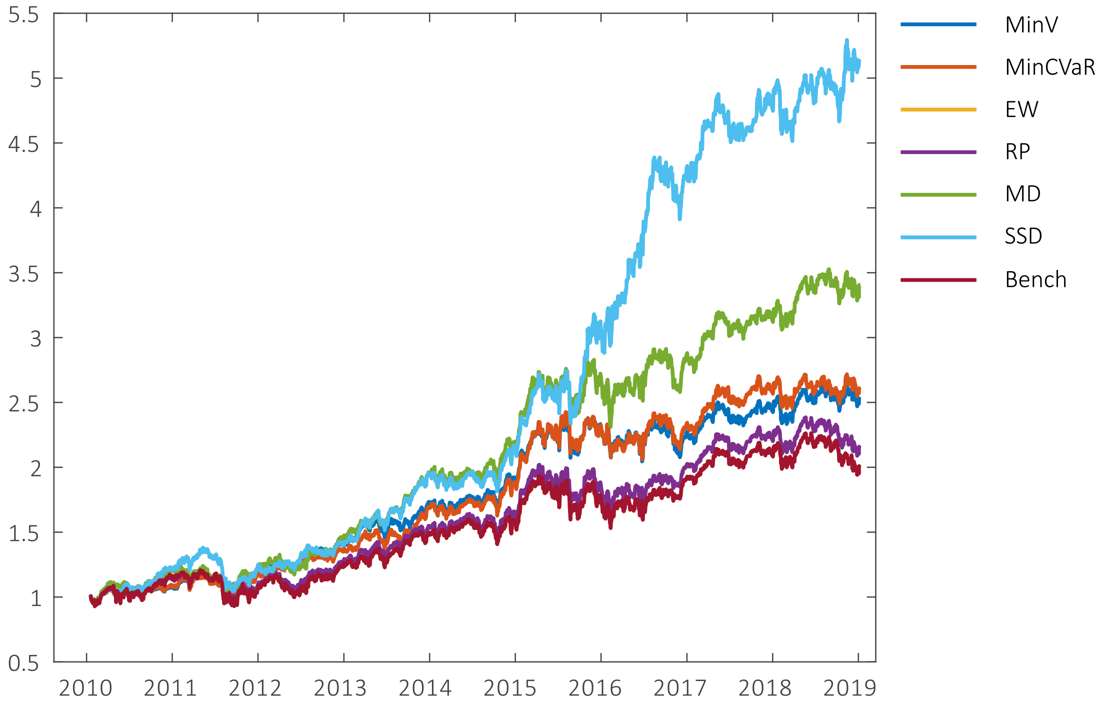

Figure 3. Out-of-sample compounded return for all models without preselection

shown for all the portfolio strategies analyzed.

Conversely, using the new Altman Z-score preselection, general improvement in the performance of all the models analyzed can be observed, except for the SSD model. This phenomenon is easily verifiable by comparing Table 5 and Figure 5 with Table 3 and Figure 3, respectively. Furthermore, observe that the empirical tests have also been performed considering a Z-score preselection made on its average over the in-sample period (1 year), but the results tend to remain unchanged w.r.t. the direct use of the monthly Z-score values (see subsection 2.3).

Table 4. Out-of-sample results using the minimum variance preselection

Approach	Mean	Vol	Sharpe	MDD	Ulcer	Sortino	Rachev5	Rachev10	JensenA	Info
MinVaR	4.27E-04	8.77E-03	4.87E-02	–0.144	0.043	6.97E-02	0.988	1.009	1.88E-04	1.18E-02
MinCVaR	4.30E-04	8.96E-03	4.80E-02	–0.153	0.047	6.85E-02	0.995	1.011	1.88E-04	1.18E-02
EW	3.87E-04	9.34E-03	4.14E-02	–0.179	0.048	5.98E-02	1.008	1.029	1.03E-04	7.43E-03
RP	4.08E-04	9.10E-03	4.49E-02	–0.166	0.044	6.49E-02	1.009	1.029	1.34E-04	1.22E-02
MD	4.74E-04	9.13E-03	5.19E-02	–0.173	0.046	7.54E-02	1.030	1.043	2.14E-04	2.19E-02
SSD	6.66E-04	1.00E-02	6.66E-02	–0.207	0.067	9.95E-02	1.065	1.086	4.22E-04	4.15E-02
Bench	3.58E-04	1.10E-02	3.24E-02	–0.230	0.071	4.65E-02	0.991	1.009	–	

http://dx.doi.org/10.21511/imfi.17(1).2020.06

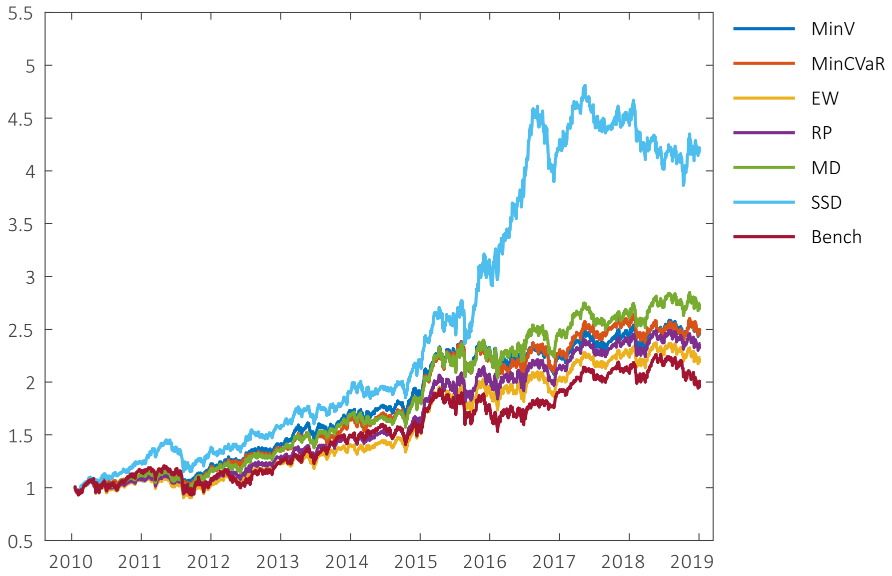

Figure 4. Out-of-sample compounded return for all models using minimum variance preselection

Table 5. Out-of-sample results using Z-score preselection

Approach	Mean	Vol	Sharpe	MDD	Ulcer	Sortino	Rachev5	Rachev10	JensenA	Info
MinVaR	5.85E-04	9.92E-03	5.90E-02	–0.174	0.048	8.55E-02	0.993	1.021	3.16E-04	3.77E-02
MinCVaR	5.93E-04	1.01E-02	5.89E-02	–0.199	0.055	8.50E-02	0.984	1.008	3.23E-04	3.77E-02
EW	4.87E-04	1.11E-02	4.37E-02	–0.211	0.062	6.33E-02	0.999	1.020	1.42E-04	3.92E-02
RP	5.21E-04	1.07E-02	4.88E-02	–0.203	0.056	7.07E-02	1.001	1.017	1.94E-04	4.58E-02
MD	6.49E-04	1.06E-02	6.12E-02	–0.183	0.047	8.96E-02	1.017	1.037	3.41E-04	5.85E-02
SSD	5.31E-04	1.06E-02	4.98E-02	–0.177	0.061	7.16E-02	0.979	1.010	2.53E-04	2.56E-02
Bench	3.58E-04	1.10E-02	3.24E-02	–0.230	0.071	4.65E-02	0.991	1.009	–	–

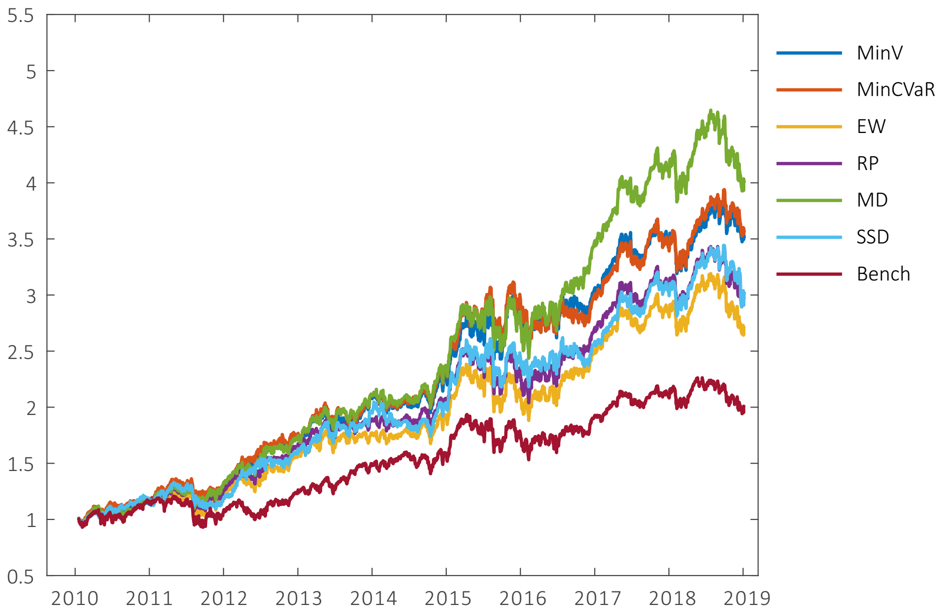

Figure 5. Out-of-sample compounded return for all models using Z-score preselection

CONCLUSION

The qualitative goal of portfolio diversification is to avoid over-concentrating the capital in very few securities. However, an important strand of research has shown that a significant in-sample risk reduction and good out-of-sample performances can be obtained by constructing small portfolios.

This paper examines for the first time the effectiveness of a new credit risk index (the new Altman Z-score) to preselect the assets from an investment universe and compares this with a minimum variance preselection approach. The effects of these two preselection methods on different classes of portfolio models have been investigated using real-world data. The findings demonstrate that the Z-score preselection method tends to generate better out-of-sample performances with respect to those obtained from the minimum variance criterion. Further tests are underway to examine the preselection effectiveness of the Z-score index compared to that achieved by other strategies on different markets, also including financial companies.

REFERENCES

1. Acerbi, C., & Tasche, D. (2002). On the coherence of expected shortfall. *Journal of Banking & Finance, 26*(7), 1487-1503. https://doi.org/10.1016/S0378-4266(02)00283-2
2. Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *The Journal of Finance, 23*(4), 589-609. https://doi.org/10.1111/j.1540-6261.1968.tb00843.x
3. Altman, E. I. (2002). *Revisiting credit scoring models in a Basel 2 environment* (NYU Working Paper No S-CDM-02-06). Retrieved from http://ssrn.com/abstract=1294413
4. Altman, E. I. (2013). Predicting financial distress of companies: revisiting the Z-Score and ZETA models. In *Handbook of Research Methods and Applications in Empirical Finance* (chap 17, pp. 428-456). Edward Elgar Publishing. Retrieved from https://ideas.repec.org/h/elg/eechap/14545_17.html
5. Altman, E. I., Haldeman, R. G., & Narayanan, P. (1977). ZETA™ analysis A new model to identify bankruptcy risk of corporations. *Journal of Banking & Finance, 1*(1), 29-54. https://doi.org/10.1016/0378-4266(77)90017-6
6. Altman, E. I., & Hotchkiss, E. (2006). Corporate Financial Distress and Bankruptcy: *Predict and Avoid Bankruptcy, Analyze and Invest in Distressed Debt.* John Wiley & Sons. Retrieved from https://www.amazon.com/Corporate-Financial-Distress-Bankruptcy-Distressed/dp/0471691895
7. Artzner, P., Delbaen, F., Eber, J., & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance, 9*(3), 203-228. https://doi.org/10.1111/1467-9965.00068
8. Bruni, R., Cesarone, F., Scozzari, A., & Tardella, F. (2017). On exact and approximate stochastic dominance strategies for portfolio selection. *European Journal of Operational Research, 259*(1), 322-329. https://doi.org/10.1016/j.ejor.2016.10.006
9. Cesarone, F., & Colucci, S. (2018). Minimum risk versus capital and risk diversification strategies for portfolio construction. *Journal of the Operational Research Society, 69*(2), 183-200. https://doi.org/10.1057/s41274-017-0216-5
10. Cesarone, F., Moretti, J., & Tardella, F. (2016). Optimally chosen small portfolios are better than large ones. *Economics Bulletin, 36*(4), 1876-1891. Retrieved from https://EconPapers.repec.org/RePEc:ebl:ecbull:eb-16-00671
11. Cesarone, F., Moretti, J., & Tardella, F. (2018). Why small portfolios are preferable and how to choose them. *The Journal of Financial Perspectives, 5*(1), 103-116. Retrieved from https://ssrn.com/abstract=3154353
12. Cesarone, F., Scozzari, A., & Tardella, F. (2013). A new method for mean-variance portfolio optimization with cardinality constraints. *Annals of Operations Research, 205*(1), 213-234. https://doi.org/10.1007/s10479-012-1165-7
13. Cesarone, F., Scozzari, A., & Tardella, F. (2019). An optimization-diversification approach to portfolio selection. *Journal of Global Optimization,* 1-21. https://doi.org/10.1007/s10898-019-00809-7
14. Cesarone, F., & Tardella, F. (2017). Equal risk bounding is better than risk parity for portfolio selection. *Journal of Global Optimization, 68*(2), 439-461. https://doi.org/10.1007/s10898-016-0477-6
15. Chekhlov, A., Uryasev, S., & Zabarankin, M. (2005). Drawdown measure in portfolio optimization. *International Journal of Theoretical and Applied Finance, 8*(1), 13-58. https://doi.org/10.1142/S0219024905002767
16. Choueifaty, Y., & Coignard, Y. (2008). Toward maximum diversification. *The Journal of Portfolio Management, 35*(1), 40-51.
17. Choueifaty, Y., Froidure, T., & Reynier, J. (2013). Properties of the most diversified portfolio.

http://dx.doi.org/10.21511/imfi.17(1).2020.06

Journal of Investment Strategies, 2(2), 49-70. http://dx.doi.org/10.2139/ssrn.1895459

18. DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *Review of Financial Studies, 22*(5), 1915-1953. https://doi.org/10.1093/rfs/hhm075
19. Choueifaty, Y., & Coignard, Y. (2008). Toward Maximum Diversification. *The Journal of Portfolio Management Fall, 35*(1), 40-51. https://doi.org/10.3905/JPM.2008.35.1.40
20. Evans, J. L., & Archer, S. H. (1968). Diversification and the reduction of dispersion: an empirical analysis. *The Journal of Finance, 23*(5), 761-767. https://doi.org/10.1111/j.1540-6261.1968.tb00315.x
21. Fábián, C. I., Mitra, G., Roman, D., & Zverovich, V. (2011). An enhanced model for portfolio choice with SSD criteria: a constructive approach. *Quantitative Finance, 11*(10), 1525-1534. https://doi.org/10.1080/14697680903493607
22. Gonzalez, F., Haas, F., Persson, M., Toledo, L., Violi, R., Wieland, M., & Zins, C. (2004). *Market dynamics associated with credit ratings: a literature review* (ECB occasional paper, 16). Retrieved from https://ssrn.com/abstract=752065
23. Goodwin, T. H. (1998). The information ratio. *Financial Analysts Journal, 54*(4), 34-43. https://doi.org/10.2469/faj.v54.n4.2196
24. Grothe, M. (2013). *Market pricing of credit rating signals* (ECB Working Paper). Retrieved from https://ssrn.com/abstract=2366361
25. Hand, J. R., Holthausen, R. W., & Leftwich, R. W. (1992). The effect of bond rating agency announcements on bond and stock prices. *The Journal of Finance, 47*(2), 733-752. https://doi.org/10.1111/j.1540-6261.1992.tb04407.x
26. Hsueh, L. P., & Liu, Y A. (1992). Market anticipation and the effect of bond rating changes on common stock prices. *Journal of Business Research, 24*(3), 225-239. https://doi.org/10.1016/0148-2963(92)90020-C
27. Hull, J., Predescu, M., & White, A. (2004). The relationship between credit default swap spreads, bond yields, and credit rating announcements. *Journal of Banking & Finance, 28*(11), 2789-2811. https://doi.org/10.1016/j.jbankfin.2004.06.010
28. Jensen, M. C. (1968). The performance of mutual funds in the period 1945–1964. *The Journal of Finance, 23*(2), 389-416. https://doi.org/10.1111/j.1540-6261.1968.tb00815.x
29. Kliger, D., & Sarig, O. (2000). The information value of bond ratings. *The Journal of Finance, 55*(6), 2879-2902. https://doi.org/10.1111/0022-1082.00311
30. Lhabitant, F. S. (2017). *Portfolio Diversification.* Retrieved from https://www.elsevier.com/books/portfolio-diversification/lhabitant/978-1-78548-191-8
31. MacCann, B. B. (1989). *The investor's guide to fidelity funds.* John Wiley & Sons Incorporated.
32. Maillard, S., Roncalli, T., & Teiletche, J. (2010). The Properties of Equally Weighted Risk Contribution Portfolios. *The Journal of Portfolio Management, 36*(4), 60-70. https://doi.org/10.3905/jpm.2010.36.4.060
33. Markowitz, H. (1952). Portfolio selection. *The Journal of Finance, 7*(1), 77-91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x
34. Markowitz, H. (1959). *Portfolio selection: Efficient diversification of investments* (Cowles Foundation for Research in Economics at Yale University, Monograph 16). John Wiley & Sons Inc., New York.
35. Meucci, A. (2009). Managing diversification. *Risk, 22,* 74-79. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1358533
36. Micu, M., Remolona, E. M., & Wooldridge, P. D. (2006). *The price impact of rating announcements: which announcements matter?* https://dx.doi.org/10.2139/ssrn.911598
37. Newbould, G. D., & Poon, P. S. (1993). The minimum number of stocks needed for diversification. *Financial Practice and Education, 3*(2), 85-87. Retrieved from https://www.researchgate.net/publication/293106806_The_minimum_number_of_stocks_needed_for_diversification
38. Norden, L., & Weber, M. (2004). Informational efficiency of credit default swap and stock markets: The impact of credit rating announcements. *Journal of Banking & Finance, 28*(11), 2813-2843. https://doi.org/10.1016/j.jbankfin.2004.06.011
39. Ogryczak, W., & Ruszczynski, A. (2002). Dual stochastic dominance and related mean-risk models. *SIAM Journal on Optimization, 13*(1), 60-78. https://doi.org/10.1137/S1052623400375075
40. Pflug, G., Pichler, A., & Wozabal, D. (2012). The 1/N investment strategy is optimal under high model ambiguity. *Journal of Banking & Finance, 36,* 410-417. https://doi.org/10.1016/j.jbankfin.2011.07.018
41. Rachev, S., Biglova, A., Ortobelli, S., & Stoyanov, S. (2004). Different Approaches to Risk Estimation in Portfolio Theory. *The Journal of Portfolio Management, 31*(1), 103-112. https://doi.org/10.3905/jpm.2004.443328
42. Rockafellar, R., & Uryasev, S. (2000). Optimization of Conditional Value-at-Risk. *Journal of Risk, 2*(3), 21-42.
43. Rockafellar, R. T., & Uryasev, S. (2002). Conditional Value-at-Risk for General Distributions. *Journal of Banking & Finance, 26,* 1443-1471. https://doi.org/10.1016/S0378-4266(02)00271-6
44. Roman, D., Mitra, G., & Zverovich, V. (2013). Enhanced indexation based on second-order stochastic dominance. *European Journal of Operational Research, 228*(1), 273-281. https://doi.org/10.1016/j.ejor.2013.01.035

45. Roncalli, T. (2014). *Introduction to risk parity and budgeting*. Chapman & Hall/CRC Financial Mathematics Series, CRC Press, Boca Raton, FL.

46. Sharpe, W. F. (1966). Mutual fund performance. *Journal of Business, 39*(1), 119-138.

47. Sharpe, W. F. (1994). The sharpe ratio. *The Journal of Portfolio Management, 21*(1), 49-58.

48. Sortino, F., & Satchell, S. (2001). *Managing downside risk in financial markets.* Retrieved from https://www.elsevier.com/books/managing-downside-risk-in-financial-markets/sortino/978-0-7506-4863-9

49. Statman, M. (1987). How many stocks make a diversified portfolio? *Journal of Financial and Quantitative Analysis, 22*(3), 353-363. https://doi.org/10.2307/2330969

50. Tang, G. Y. (2004). How efficient is naive portfolio diversification? An educational note. *Omega, 32*(2), 155-160. https://doi.org/10.1016/j.omega.2003.10.002

51. Tu, J., & Zhou, G. (2011). Markowitz meets Talmud: A combination of sophisticated and naive diversification strategies. *Journal of Financial Economics, 99*(1), 204-215. https://doi.org/10.1016/j.jfineco.2010.08.013

52. Valle, C. A., Roman, D., & Mitra, G. (2017). Novel approaches for portfolio construction using second order stochastic dominance. *Computational Management Science, 14*(2), 257-280. https://doi.org/10.1007/s10287-017-0274-9

http://dx.doi.org/10.21511/imfi.17(1).2020.06