# “Optimal omega-ratio portfolio performance constrained by tracking error”

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# OPTIMAL OMEGA-RATIO PORTFOLIO PERFORMANCE CONSTRAINED BY TRACKING ERROR

## Abstract

The mean-variance framework coupled with the Sharpe ratio identifies optimal portfolios under the passive investment style. Optimal portfolio identification under active investment approaches, where performance is measured relative to a benchmark, is less well-known. Active portfolios subject to tracking error (TE) constraints lie on distorted elliptical frontiers in return/risk space. Identifying optimal active portfolios, however defined, have only recently begun to be explored. The $\Omega$ – ratio considers both down and upside portfolio potential. Recent work has established a technique to determine optimal $\Omega$ – ratio portfolios under the passive investment approach. The authors apply the identification of optimal $\Omega$ – ratio portfolios to the active arena (i.e., to portfolios constrained by a TE) and find that while passive managers should always invest in maximum $\Omega$ – ratio portfolios, active managers should first establish market conditions (which determine the sign of the main axis slope of the constant TE frontier). Maximum Sharpe ratio portfolios should be engaged when this slope is > 0 and maximum $\Omega$ – ratios when < 0.

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## INTRODUCTION

Investment styles follow one of two broad approaches: active and passive. Active fund managers trade frequently and engage energetically with the market. Successful active managers identify high-performing assets and time trades to extract maximal performance, buying when prices are low and selling when they are high. Skill in this space is usually measured relative to a benchmark, usually a market index or an assembly of similar securities with constraints on portfolio weights, asset quality, and acceptable risk. Passive managers select and purchase desired securities and hold these for investment horizons, spanning periods of economic booms and busts. Such managers' proficiency is measured on an absolute basis; they minimize transaction fees and aver that "good" securities outperform in the long run.

Both styles have pros and cons, and the ebb and flow of economic activity often dictate investor style selection: passive usually in stable markets and active in volatile ones. Events such as the 2020 COVID-19 pandemic, which severely shocked global markets, emphasize the importance of agile, active investing. Managers capable and eager to quickly dispose of airline, oil, or tourism-related stocks avoided the worst of the downturn and significantly outperformed less nimble investments.

Modern portfolio theory led to the design and application of the widely-used efficient frontier, which plots – in return-risk space – the locus of portfolios whose arrangement of constituent security weights generates maximal returns at each specified risk level. Sharpe identified the optimal portfolio on this frontier: one whose excess return (usually over the risk-free rate) per unit of risk taken to achieve that return was maximized. This framework of asset selection is ideally suited to the passive investment style. Identifying an optimal portfolio using this construction implies that markets are relatively static and that buying and holding the optimal portfolio will eventually lead to the desired risk/return characteristics (Rudd, 1980).

Active investment strategies require more complex structures. Portfolios whose performance and risk are measured relative to a benchmark follow a different locus of possibilities in return-risk space (Strub & Baumann, 2018). Jorion (2003) demonstrated that such portfolios occupy a distorted ellipse in this space – rather than the efficient frontier's hyperbola for absolute risk and return. This ellipse's dimensions and orientation are governed by many factors, including the variance-covariance matrix of underlying security returns, benchmark weights in the permissible universe of investable assets, constituent portfolio weights relative to the benchmark, and the size of the TE.[1] The greater the deviation from benchmark weights, the higher the possibility for outperforming (or underperforming) that benchmark (and the higher the TE). To limit excessive risk-taking, active managers are often limited by mandates not to exceed a prescribed TE. There are profound differences in how portfolio risk and return evolves and is measured under active and passive investment styles. In common use for passive portfolios, standard performance metrics require complex reformulation and behave in unfamiliar ways in an active space.

The Ω – ratio, a performance metric that makes no distributional assumptions about asset returns, is popular amongst passive investors, but determining the asset allocation to generate an optimal Ω – ratio portfolio eluded the researchers for years. The Ω – ratio's definition imbues it with non-convex properties, which do not yield to standard optimization techniques. Recently, Kapsos, Zymler, Christofides, and Rustem (2011) accomplished this feat using linear programming, but their approach has not been applied to active portfolios, i.e., those constrained by TEs. The authors identify maximum Ω – ratio portfolios on the constant TE frontier under different market conditions and compare these portfolios' performance over time to universal (unconstrained) Ω – ratio portfolios.

The remainder of this article proceeds as follows. A literature review, which provides information regarding the development of the relevant investment strategies and the metrics, which govern performance measurement, follows in section 1. Section 2 describes the data chosen and explains the mathematics governing TE constrained portfolio performance. The Ω – ratio framework, the identification of the optimal Ω – ratio portfolio in passive space, and how this structure may be applied in active space are also discussed in this section. Section 3 presents and discusses the results of the analysis and possible ramifications. The last section concludes and provides some recommendations.

# 1. LITERATURE REVIEW

Modern portfolio theory (MPT) is a well-established and widely implemented paradigm, which asserts that investors select portfolios based upon their level of risk aversion. Set out by Markowitz (1952), the framework gives rise to a set of efficient portfolios – those characterized by the maximum possible return at any given risk level – which trace out a boundary in return-risk space known as the efficient frontier. The literature is replete with improvements and adaptations, augmentations and variations of MPT. Markowitz, Schirripa, and Tecotzky (1999), for example, showed how – by pooling assets – investors could collectively provide constituent members higher expected returns for given risks than individuals could generate alone. These results were confirmed and extend-

1 Defined as the standard deviation of the difference between portfolio and benchmark returns (themselves governed by the difference between portfolio and benchmark weights).


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ed by Kwan (2003), and more recent innovations are provided by Canakgoz and Beasley (2008) and Calvo, Ivorra, and Liern (2012).

Sharpe (1966) identified an optimal portfolio on the efficient frontier, the highest risk-adjusted portfolio return, measured as the quotient of portfolio return above the risk-free rate and the portfolio's risk (defined by its volatility), $SR = \left(\mu_P - r_f\right) / \sigma_P$, where *SR* is the Sharpe ratio, $\mu_P$ is the portfolio annual return, $r_f$ is the annualized risk-free rate, and $\sigma_P$ is the annualized portfolio volatility (or risk). This portfolio represents the single intersection point of the capital market line (CML) hinged at the risk-free rate on the return axis and the frontier, i.e., where the CML is tangent to the frontier. Despite many assumptions embedded in the determination of this optimized portfolio (e.g., Muralidhar, 2015), it remains a popular metric (Sharpe, 1994; Guastaroba & Speranza, 2012; Lo, 2012; Qi, Rekkas, & Wong, 2018).

MPT and the tangent portfolio reflect the passive portfolio management style in which assets are bought and held for "long" investment horizons, usually several months or years. This style enjoys the benefits of low trading costs and a through-the-cycle view of market performance resting on the assumption that superior assets will outperform the broader market even though influenced by it. The active management style (strategic stock selection and timing), while more expensive because of trading expenses, is dominated by fund managers who purchase and sell securities when prevailing conditions signal danger or opportunity. This style has been eclipsed in recent times by index (passive) investing, but this has given rise to systemic problems (Anadu, Kruttli, McCabe, Osambela, & Shin, 2018), and the alleged inferior performance of the active investment style has been challenged by Cremers, Fulkerson, and Riley (2019) whose research found evidence that 'conventional wisdom' had been unfairly critical of the value of active management which continues to outperform the passive style (Berk & van Binsbergen, 2015; Pederson, 2018; Mutunge & Haugland, 2018; Dolvin, Fulkerson, & Krukover, 2018).

Active fund performance is assessed relative to a benchmark, commonly a market index or a selection of securities constrained by investor preferences. Superior active funds should outperform the returns generated by the benchmark and simultaneously not exceed a prescribed, relative risk measure: the TE defined as the standard deviation of the differences between portfolio and fund returns (Filippi, Guastaroba, & Speranza, 2016).

Using a mean/variance (Markowitz) framework, Roll (1992) established a description of portfolio optimization relative to a benchmark for a given TE. These optimal, but constrained, portfolios trace out a frontier much like the efficient frontier (but shifted to the right – i.e., lower potential returns with higher risk), in return/risk space. Bertrand, Prigent, and Sobotka (2001) and Larsen and Resnick (2001) reconsidered the problem of mean-variance maximization under TE constraints, but Jorion (2003) was the first to mathematically formulate the constant TE frontier, a distorted ellipse in return/risk space comprising TE-constrained portfolio return/risk coordinates. Stowe (2014) provides a recent, comprehensive treatise on the relevant mathematics governing TE constrained portfolios.

Maxwell, Daly, Thomson, and van Vuuren (2018) and Maxwell and van Vuuren (2019) adapted and extended Jorion's (2003) approach to TE constrained portfolio optimization by establishing a technique which identified the tangent portfolio on the constant TE frontier. Analogous to the tangent portfolio on the efficient frontier, this portfolio (where the analogous CML – also hinged at the risk-free rate on the return axis – is tangent to the constant TE frontier) represents the maximal risk-adjusted return portfolio constrained by a given TE. Daly, Maxwell, and van Vuuren (2018) explored $\alpha$, $\beta$ and investor utility behavior for TE constrained portfolios, and Evans and van Vuuren (2019) investigated several portfolio performance metrics on the constant TE frontier. Gunning and van Vuuren (2019) surveyed the mechanisms which drive constrained portfolio performance by examining the influence of macroeconomic conditions on the shape of the TE frontier (certain market conditions alter the slope of the main axis of the constant TE frontier ellipse (often from $> 0$ to $< 0$), which profoundly influences TE constrained portfolio performance).

The $\Omega$ – ratio is a portfolio performance measure that captures both portfolios down and upside potential while remaining consistent with utility maximization (Keating & Shadwick, 2002). Although now widely used, an optimal Ω – ratio portfolio long eluded practitioners because it is a non-convex function, which does not lend itself to standard optimization techniques. Kane, Bartholomew-Biggs, Cross, and Dewar (2005) explored this problem empirically using simulated returns from a portfolio comprising three assets. Several local solutions were found by changing asset weights to maximize the Ω value and assuming no short-selling. Extending this work to ten (real) assets and employing a global optimization technique (not disclosed), Kane, Bartholomew-Biggs, Cross, and Dewar (2005) identified maximal Ω portfolios and compared their performance with portfolios produced using MPT (i.e., tangent portfolios). The results showed that the allocation of weights for portfolios' constituent assets was considerably different from those based on risk minimization. Passow (2004) and Gilli, Schumann, Di Tollo, and Cabej (2008) made laudable attempts to resolve the problem of Ω portfolio optimality. However, their solutions were heuristic (which did not guarantee the accurate identification of the global optimum), and their threshold accepting methods were numerically unstable, requiring complicated fine-tuning of the underlying parameters.

The optimization of the Ω – ratio subject to portfolio constraints has been considered in other portfolio optimization research. Examples include the consideration of transaction costs (Beasley, Meade, & Chang, 2003), a maximum number of permissible portfolio constituents (Guastaroba & Speranza, 2012), and portfolio weight lower and upper bounds (Gnagi & Strub, 2020).

Kapsos, Zymler, Christofides, and Rustem (2011), using the Ω – ratio quasi-concave property (which permits its transformation into a linear program), overcame the non-convex function problem, and established an exact optimization formulation. This solution is a direct analog to the mean-variance framework and its associated Sharpe ratio maximization. Kapsos, Zymler, Christofides, and Rustem (2011) work applies to passive investment approaches. To date, no attempts have been made to explore Ω optimal portfolios, which are also subject to TE constraints (i.e., active style).

In this article, several strands of related research are combined. The authors augment the work described above relating to TE constrained portfolios by adapting Kapsos, Zymler, Christofides, and Rustem (2011) optimal Ω – ratio solutions to accommodate TE constrained portfolios. The authors explore some properties of optimal TE constrained Ω – ratio portfolio performance and contributed to the literature by presenting, for the first time, these results obtained and the implications of these results on the theory and practice of portfolio optimization.

# 2. DATA AND METHODOLOGY

## 2.1. Data

The data for both benchmark and portfolios comprised 15 stocks (from six market sectors to ensure a degree of diversification) selected from a major emerging economy's (South Africa) stock exchange – the Johannesburg Stock Exchange's (JSE) All-Share Index 40 (ALSI40), which comprises 40 of the largest companies listed on the entire JSE, ranked by market capitalization. The behavior of the ALSI40 represents a reasonable reflection of the overall South African stock market because, although it contains only 10% of JSE-listed companies, it includes over 80% of the JSE market capitalization. The 15 stocks selected from the ALSI40 for this work – in turn – represent about 87% of the ALSI40 by market capitalization (roughly 70% of the entire South African stock market) and 79% of the total ALSI40 liquidity determined as the average daily volume as a percentage of total volume for each stock, over ten years from January 2010 to January 2020 (Courtney Capital, 2020). Thus, these stocks are highly liquid, frequently traded by active managers, and all are dual-listed on international stock exchanges.

The authors also analyzed portfolio performance sourced from similarly liquid, high market capitalization (as a percentage of total capitalization) stocks from diverse sectors in the US, UK, and European indices over the same period but found similar results. This is not an unexpected result: diversified, highly liquid portfolios perform similarly regardless of market milieu because return outliers are reduced, and portfolio return distributions are close to normal (Janabi, 2009).



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Monthly returns spanning 20 years from Jan-00 to Jan-20 were used, thus covering an era characterized by different market conditions: the years of expansionary conditions, which preceded the 2007–2009 credit crisis, credit crisis, and post-credit crisis turmoil. The currently ongoing economic ramifications of the COVID-19 pandemic (May 2020) should contribute to an interesting case study.

The benchmark comprised equal proportions of these stocks and was rebalanced monthly. Different combinations of benchmark constituent weights (other than equal weights) were instituted and tested but did not change the outcomes reported here. The principal driving factor is the degree of deviation from the benchmark weights of the relevant portfolios.

For the analysis that follows, five years of monthly returns were used to generate portfolio returns, volatilities, and correlations. The calculations were rolled forward one month at a time (maintaining a five-year period to generate the relevant parameters) to explore the behavior of TE constrained optimal Ω – ratio portfolios, the impact of a sign-changing constant TE frontier main axis slope, and the observed differences between security weights for different optimal portfolios constrained by TE. Although all rolling five-year periods were examined for this work (from Jan-00 to Jan-20), the authors selected (and present) the analytical results, which most strongly demonstrate these impacts mentioned earlier. The two five-year periods identified were:

1. Oct-00 – Oct-05 characterized by

a. relatively low volatility,
b. high returns – despite including the 9-11 US terrorist attack,
c. risk-free rate of 7.0%, and
d. main axis slope of the constant TE frontier > 0 and

2. Oct-09 – Oct-14 characterized by

a. high volatility,
b. lower returns – in the aftermath of the 2007/8 global financial crisis,
c. risk-free rate of 5.8%, and
d. main axis slope of the constant TE frontier < 0.

Portfolio behavior subject to TEs from 1% to 12% (in 1% increments) was explored. Descriptive statistics of these securities are set out in Table 1. While the risk-free rates of 7% and 5.8% may seem excessive post the credit crisis of 2008–2009, these high rates are relevant for many emerging economies. The magnitude of the risk-free rate on the analysis that follows does not influence the conclusions.

**Table 1.** Descriptive statistics for the period Oct-00 to Oct-05 and Oct-09 to Oct-14

Source: Bloomberg and authors' calculations.

| Statistics | Energy | | | Materials | | | Retail | | | IT | | Consumer | | Financial | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| **2000–2005** | | | | | | | | | | | | | | | |
| $\bar{\mu}$ (%) | 30.0 | 15.3 | 9.3 | 3.9 | 22.5 | 13.8 | 16.0 | 43.7 | 18.9 | 11.5 | 10.4 | 30.2 | 29.6 | 14.0 | 20.1 |
| $\mu_{max}$ (%) | 24.0 | 18.0 | 24.8 | 18.2 | 37.6 | 49.3 | 19.9 | 51.6 | 18.2 | 33.4 | 40.2 | 29.0 | 27.3 | 44.1 | 15.2 |
| $\mu_{min}$ (%) | –13.9 | –18.2 | –18.1 | –20.9 | –23.3 | –21.9 | –13.1 | –23.3 | –13.9 | –28.6 | –44.7 | –11.9 | –25.0 | –47.1 | –11.7 |
| $\bar{\sigma}$ (%) | 32.6 | 21.3 | 33.6 | 29.8 | 36.9 | 57.0 | 26.9 | 42.5 | 22.6 | 41.1 | 51.0 | 26.0 | 29.3 | 51.7 | 19.9 |
| s | 0.22 | 0.04 | 0.56 | –0.16 | 0.75 | 0.67 | 0.50 | 1.13 | 0.04 | 0.13 | –0.61 | 0.77 | 0.09 | –0.33 | 0.30 |
| k | –0.77 | 1.13 | 0.05 | –0.23 | 1.34 | 0.06 | –0.30 | 3.23 | –0.03 | 0.60 | 2.09 | 1.22 | 2.18 | 1.89 | –0.02 |
| **2009–2014** | | | | | | | | | | | | | | | |
| $\bar{\mu}$ (%) | 14.2 | –1.1 | –2.0 | 9.0 | –0.4 | –26.5 | 20.8 | 41.8 | 17.1 | 14.8 | 39.8 | 37.2 | 12.2 | 37.5 | 7.4 |
| $\mu_{max}$ (%) | 16.8 | 14.0 | 15.9 | 17.9 | 19.4 | 23.2 | 17.3 | 25.9 | 11.4 | 15.8 | 24.1 | 15.5 | 19.6 | 21.9 | 13.1 |
| $\mu_{min}$ (%) | –10.5 | –14.3 | –21.1 | –11.8 | –16.8 | –28.3 | –17.6 | –14.1 | –15.3 | –9.0 | –14.6 | –19.0 | –15.6 | –11.4 | –9.6 |
| $\bar{\sigma}$ (%) | 17.3 | 20.3 | 26.0 | 24.6 | 28.2 | 36.0 | 23.6 | 24.5 | 18.4 | 18.1 | 28.1 | 24.4 | 25.5 | 28.8 | 17.2 |
| s | 0.52 | –0.07 | –0.17 | 0.27 | 0.22 | –0.35 | –0.38 | 0.54 | –0.57 | 0.45 | 0.05 | –0.69 | 0.40 | 0.48 | 0.31 |
| k | 0.72 | 0.06 | 0.18 | –0.17 | –0.69 | 0.50 | 0.31 | 1.90 | 0.56 | 0.38 | –0.13 | 0.89 | –0.12 | –0.52 | –0.04 |

*Note:* Key: $\bar{\mu}$ – mean annual return, $\mu_{max}$ – max monthly return, $\mu_{min}$ – min monthly return, $\bar{\sigma}$ – mean annualized volatility, *s* – skewness, and *k* – kurtosis.

## 2.2. Methodology

Active investment positions differ from benchmark positions according to the risk appetite of investors. Low TEs mean active weights must be small, while for higher TEs, active weights are larger (permitting a wider range of weights relative to the benchmark). The underlying variables, matrices, and matrix notation are defined further for a sample of $N$ constituent securities:

- $q$ $1\times N$ vector of benchmark weights;
- $x$ $1\times N$ vector of deviations from the benchmark;
- $q_P = (q + x)$ $1\times N$ vector of portfolio weights;
- $E$ $1\times N$ vector of expected returns;
- σ $1\times N$ vector of benchmark component volatilities;
- ρ $N\times N$ benchmark correlation matrix;
- $V$ $N\times N$ covariance matrix of asset returns;
- 1 $1\times N$ vector of 1s; and
- $r_f$ risk-free rate.

Net short sales are allowed, so the total active weights $(q_i + x_i)$ may be > 0 for individual securities. No assets outside the benchmark's set may be included using Roll's methodology – although, in principle, this is, of course, possible. Using matrix notation, expected returns and variances are:

- $\mu_B = qE'$ expected benchmark return;
- $\sigma_B = \sqrt{qVq'}$ volatility (risk) of benchmark return;
- $\mu_\varepsilon = xE'$ expected excess return; and
- $\sigma_\varepsilon = \sqrt{xVx'}$ TE.

The active portfolio expected return and variance is given by $\mu_P = (q+x)E' = \mu_B + \mu_\varepsilon$ and $\sigma_P = \sqrt{(q+x)V(q+x)'}$. The portfolio must be fully invested, so $(q+x)1' = 1$.

The following definitions are also required: $a = EV^{-1}E', b = EV^{-1}1', c = 1V^{-1}1', d = a - b^2/c$ and $\Delta_1 = \mu_B - b/c$ where $b/c = \mu_{MV}$ and $\Delta_2 = \sigma_B^2 - 1/c$ with $1/c = \sigma_{MV}^2$ where $MV$ is the minimum variance portfolio (Merton, 1972).

It is useful to recall the relevant mathematics, which generates various frontiers.

#### 2.2.1. *Mean variance frontier*

Minimize $q_P V q_P'$ subject to $q_P 1' = 1$ and $q_P E' = G$ where $G$ is the target return. The vector of portfolio weights is $q_P = (a - bG/d)q_{MV} + ((bG - b^2/c)/d)q_{TG}$, where $q_{MV}$ is the vector of asset weights for the minimum variance portfolio given by $q_{MV} = V^{-1}1/c$, and $q_{TG}$ is the vector of asset weights for the tangent portfolio (with $r_f = 0$), i.e., $q_{TG} = V^{-1}E/b$. The weights of the tangent portfolio's components, $q_{TP}$, with $r_f \neq 0$, are:

$$q_{TP}' = \frac{V^{-1}(E - r_f \cdot 1)'}{1 \cdot V^{-1}(E - r_f \cdot 1)'}$$

#### 2.2.2. *TE frontier*

Maximize $xE'$ subject to $x1' = 0$ and $xVx' = \sigma_\varepsilon^2$. The solution for the vector of deviations from the benchmark is

$$x' = \pm\sqrt{\frac{\sigma_\varepsilon^2}{d}}V^{-1}\left(E - \frac{b}{c}1\right)'.$$

The solution to this optimization problem generates the TE frontier, a portfolio's maximal return at a given risk level, and subject to a TE constraint.

#### 2.2.3. *Constant TE frontier*

Maximize $xE'$ subject to

$$x1' = 0, \quad xVx' = \sigma_\varepsilon^2$$
$$\text{and} \quad (q+x)V(q+x)' = \sigma_P^2.$$

The vector of deviation weights from the benchmark is $x' = -(1/(\lambda_2 + \lambda_3))V^{-1}(E' + \lambda_1 + \lambda_3 Vq')$ where $\lambda_1 = -\frac{\lambda_3 + b}{c}$, $\lambda_2 = \pm(-2)\sqrt{\frac{d\Delta_2 - \Delta_1^2}{4\sigma_\varepsilon^2\Delta_2 - y^2}} - \lambda_3$ and $\lambda_3 = -\frac{\Delta_1}{\Delta_2} \pm \frac{y}{\Delta_2}\sqrt{\frac{d\Delta_2 - \Delta_1^2}{4\sigma_\varepsilon^2\Delta_2 - y^2}}$. The solution


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for the weights which generate the tangent portfolio (to the constant TE frontier) was shown by Maxwell, Daly, Thomson, and van Vuuren (2018) to involve solving for $\sigma_P$ using:

$$\frac{(r_f-\mu_B)}{\sigma_P^2}+\frac{\dfrac{(\Delta_1^2-d\Delta_2)\cdot(\sigma_P^2-\sigma_B^2-\sigma_\varepsilon^2)}{\sqrt{(\Delta_1^2-d\Delta_2)\left[(\sigma_P^2-\sigma_B^2-\sigma_\varepsilon^2)^2-4\Delta_2\sigma_\varepsilon^2\right]}}+\Delta_1}{\Delta_2}-$$

$$-\frac{\sqrt{(\Delta_1^2-d\Delta_2)\left[(\sigma_P^2-\sigma_B^2-\sigma_\varepsilon^2)^2-4\Delta_2\sigma_\varepsilon^2\right]}+\Delta_1\cdot(\sigma_P^2-\sigma_B^2-\sigma_\varepsilon^2)}{2\Delta_2\sigma_P^2}$$

then establishing $\mu_P$ on the efficient segment of the constant TE frontier and then backing out the relevant weights.

#### 2.2.4. Constant TE frontier main axis slope, $S_{MA}$

The main axis slope, $S_{MA}$ is calculated using

$$S_{MA}=\frac{\Delta_1}{\sigma_B-\sigma_{MV}}=\frac{\mu_B-b/c}{\sigma_B-\sigma_{MV}}=\frac{\mu_B-\mu_{MV}}{\sigma_B-\sigma_{MV}},$$

where $\Delta_1$ determines the sign of $S_{MA}$ because the denominator is always > 0 since $\sigma_B - \sigma_{MV} > 0$ always. A necessary and sufficient condition for $S_{MA} < 0$ is $\mu_B < \mu_{MV}$. Note that the $S_{MA}$ is independent of TE since none of its components depend explicitly thereon. For the first time, the authors measure and evaluate the sign and magnitude of the SMA and explore how these (and constituents of the $S_{MA}$) change over time as market conditions evolve plus their influence on TE constrained portfolio performance.

#### 2.2.5. Optimal Ω portfolios

Let us consider a market with $N$ stocks. The current time is $t = 0$, and the end of the investment horizon is $t = T$. A portfolio is completely characterized by a vector of weights $w\in R^N$, such that $\sum_{i=1}^N w_i=100\%$. The element $w_i$ denotes the percentage of total wealth invested in the $i$-th stock at time $t = 0$. Let $\tilde{r}_i$ indicate the random return of asset $i$ and boldface the vector of return variables $\tilde{r} \in R^N$. The random return of a portfolio of assets is defined as $\tilde{r}_P = w^T\tilde{r}$.

Let $F(r_i)$ and $f(r_i)$ denote the cumulative density function and the probability density function. For an asset $i$, Keating and Shadwick (2002) define the Ω – ratio as:

$$\Omega(\tilde{r}_i)=\frac{\int_\tau^{+\infty}\left[1-F(r_i)\right]dr_i}{\int_{-\infty}^\tau F(r_i)\,dr_i}. \tag{1}$$

Integration by parts and some algebraic transformation, the Ω – ratio may be written:

$$\Omega(\tilde{r}_i)=\frac{\int_\tau^{+\infty}(\tilde{r}_i-\tau)f(r_i)\,dr_i}{\int_{-\infty}^\tau(\tau-\tilde{r}_i)f(r_i)\,dr_i}=$$

$$=\frac{\mathbb{E}\left[(\tilde{r}_i-\tau)^+\right]}{\mathbb{E}\left[(\tau-\tilde{r}_i)^+\right]}=\frac{\mathbb{E}(\tilde{r}_i)-\tau}{\mathbb{E}\left[(\tau-\tilde{r}_i)^+\right]}+1.$$

Therefore, the portfolio Ù – ratio is:

$$\Omega(\tilde{r}_p)=\frac{w^T\mathbb{E}(\tilde{r})-\tau}{\mathbb{E}\left[(\tau-w^T\tilde{r})^+\right]}+1. \tag{2}$$

In this paper, portfolio optimization problems are investigated that aim to maximize the Ω – ratio subjected to additional constraints on portfolio weights. The Ω maximization problem can be written as:

$$\max_{\omega\in\mathbb{R}^n}\frac{w^T\mathbb{E}[\tilde{r}]-\tau}{\mathbb{E}\left[(\tau-w^T\tilde{r})^+\right]}, \tag{3}$$

s.t. $w^T1=100\%$ and $\underline{w}\le w\le\overline{w}$.

The objective is to determine the allocation that gives the optimal weights ($w \in R^N$) that result in the portfolio with the maximum Ω – ratio. The constraints above relate to the budget constraint and the upper and lower bound on any individual investment.

The discrete analog for (2) is

$$\Omega = \frac{w^T \overline{r} - \tau}{\sum_j \left[ \tau - w^T r_j \right]^+ p_j}. \tag{4}$$

The optimization problem is

$$\max_w \frac{w^T \overline{r} - \tau}{\sum_j \left[ \tau - w^T r_j \right]^+ p_j}, \tag{5}$$

s.t. $\sum w_i = 1$, and $\underline{w} \le w \le \overline{w}$ and where $p_j = 1/N$.

Using the portfolio weights derived from (5), $\Omega s$ may be calculated, and their component numerators and denominators graphed. Figure 1 presents interesting similarities with MPT's mean-variance framework and Sharpe ratio optimization. Points above the frontier are unattainable, and investors may choose better solutions for all coordinates below. Kapsos, Zymler, Christofides, and Rustem (2011) named this locus of points the Ω frontier and found that it is concave, non-decreasing feature arose from the optimization problem's (5) convexity property. For each point on the frontier, the Ω – ratio is determined by the gradient of the line passing through it and the origin, so an optimal solution (maximum Ω – ratio) is that point at which the line which passes through the origin has the highest slope (i.e., tangent to the Ω frontier).

# 3. RESULTS AND DISCUSSION

The aim is to generate constant TE frontiers for varying levels of TE (1% to 12% in 1% intervals) and then, for each constant TE frontier, establish the risk and return (and corresponding portfolio constituent weights) for each of the following maximal portfolios: return, Sharpe, and Ω. There are two "maximum Ω – ratio portfolios"; one which simultaneously satisfies the relevant TE constraint and maximizes the return at each TE-constrained risk level and the other unconstrained by TE, i.e., a universal maximum Ω portfolio. The former is identified by first selecting the (known) asset weights, which generate the upper hemisphere of portfolios on the constant TE frontier (i.e., from minimum to maximum variance portfolios on the ellipse). These weights are then used to generate portfolio returns over the chosen period of interest (five years of monthly returns, rolled forward one month at a time, since Jan-00) and the associated Ω – ratio calculated for each set of 60 (5y) returns. By construction, these portfolios lie on the constant TE frontier.

The choice of 5y monthly sample period was based on the observation that fund managers generally use between three and five years of monthly return data for portfolio construction and performance metric estimation (Marhfor, 2016). The authors opted for the longer-range to include more return data and embrace a greater fraction of the business cycle (the average business cycle frequency in South Africa is about seven years

Source: Authors' calculations.

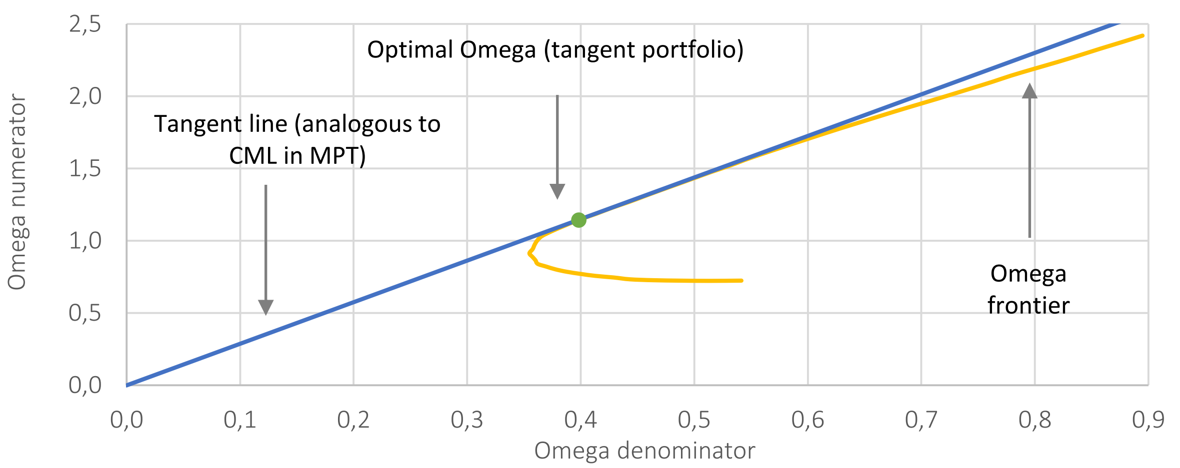


**Figure 1.** Ω frontier, analogous capital market line, and location of the optimal Ω portfolio



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(Thomson & van Vuuren, 2016), so the 5y sample period encapsulates almost one full cycle). Shorter periods encompass too few data to provide robust metrics and longer periods use too many return data because markets change considerably over longer periods, styles alter, market dynamics adapt to different trends.

The results of the strategy are impacted by choice of rolling window length (and the date at which the strategy is installed and implemented), but this work aims to illustrate portfolio performance during two different sets of market conditions: one which results in a positive main axis on the constant frontier and one which results in a negative main axis. Although the period choice is significant, the selection of different market milieus was deliberate in evaluating and categorizing behavior in these times.

The Ω – ratio – measured using as threshold the benchmark return – for each portfolio is then plotted on the same x-axis (risk) as the constant TE frontier (the solid black line in Figure 2 for *TE* = 6%). The unconstrained (universal) Ω – ratio, using the same threshold, is also shown in Figure 2, along with the efficient frontier and the capital market line (CML) for the constant TE frontier (Maxwell, Daly, Thomson, & Van Vuuren, 2018).

In Figure 2, the period selected was Oct-00 to Oct-05 using monthly returns. This period preceded the credit crisis of 2007–2009 when markets enjoyed buoyant returns and reduced volatility, giving rise to a constant TE ellipse with a positive main axis (Gunning & van Vuuren, 2019). The maximum Ω – ratio portfolio – constrained by TE – lies between (in terms of risk and return) the maximum Sharpe ratio and maximum return portfolios. In contrast, the universal (unconstrained) Ω – ratio portfolio lies outside the constant TE frontier with higher risk and higher return than all other constant TE frontier portfolios (in this example where *TE* = 6% and $r_f$ = 7.0%).

In Figure 3, the period selected was Oct-09 to Oct-14, i.e., post the worst of the turbulent market volatility instituted by the credit crisis of 2007–2009. Portfolio annual returns are substantially lower than those observed in the period preceding the credit crisis and annual risk is higher: the configuration resulting in a negative main axis for the constant TE ellipse. The maximum TE-constrained Ω – ratio portfolio again lies between (in terms of risk and return) the maximum Sharpe ratio and max-

Source: Bloomberg and authors' calculations.

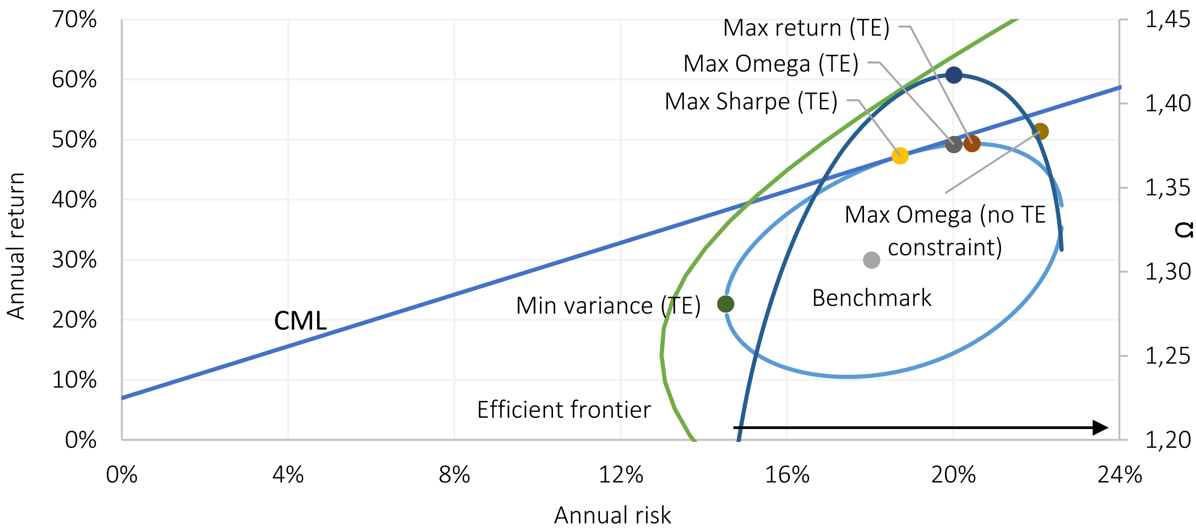



**Figure 2.** Orientation of relevant components in Oct-05. *TE* = 6% and $r_f$ = 7.0%. The Ω – ratio as a function of risk is shown as a solid black line, tied to the right-hand axis (the maximum Ω – ratio on this curve is indicated). All other elements are linked to the left-hand axis

Source: Bloomberg and authors' calculations.

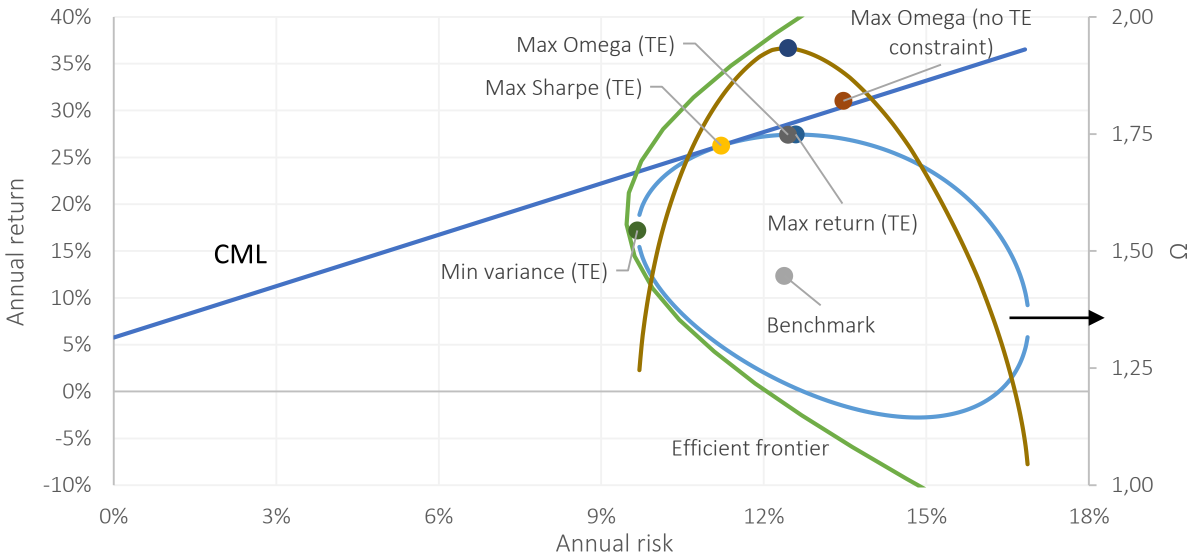



**Figure 3.** Orientation of relevant components in Oct-14. *TE* = 6% and $r_f$ = 5.8%

imum return portfolios. In contrast, the universal (unconstrained) Ω – ratio portfolio again lies outside the constant TE frontier with higher risk and higher return than all other constant TE frontier portfolios (in this example where $TE = 6\%$ and $r_f = 7.0\%$).

Figure 4(a) shows the Ω frontiers for portfolios with returns selected from the two periods (Oct-00 – Oct-05 and Oct-09 – Oct-14). Figure 4(b) plots Ω – ratio at each corresponding threshold. Ω – ratios are higher in the latter period because the high volatility here leads to a higher dispersion of portfolio returns. The overall increase in quantity and magnitude of returns > 0% in this period, combined with the greater dispersion, elevates Ω – ratios $\forall \tau > 0\%$.

Constituent deviations from the benchmark (i.e., $x_i$) in the optimal, universal, unconstrained Ω

Source: Bloomberg and authors' calculations.

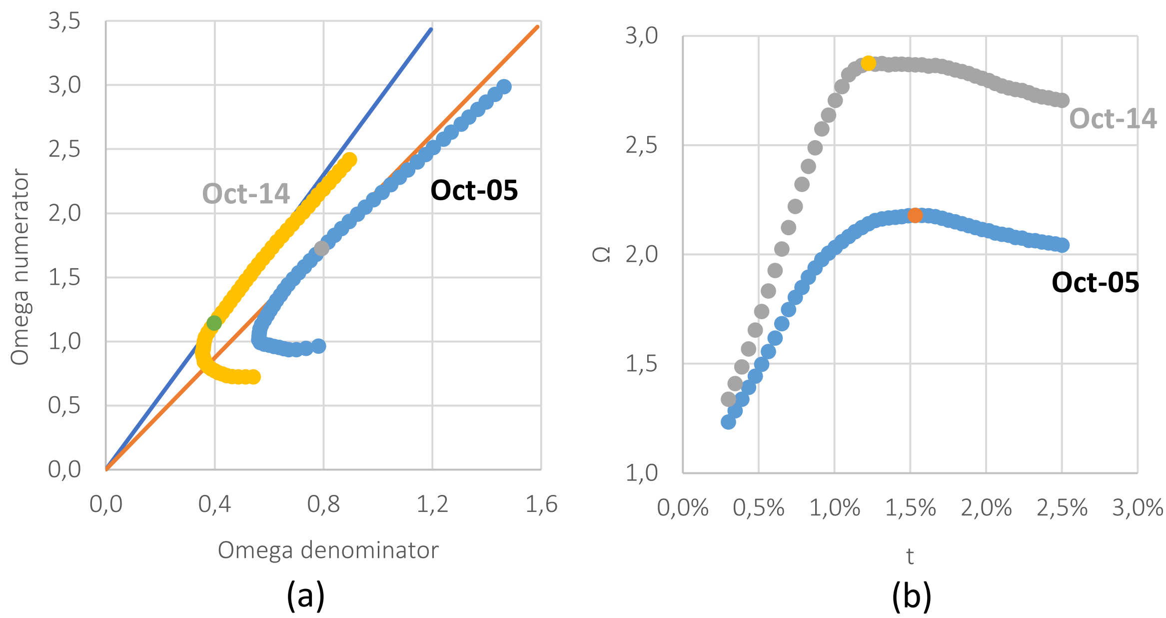



**Figure 4(a)-(b).** Ω frontiers and (b) maximum Ω(τ) for Oct-00 – Oct-05 and Oct-09 – Oct-14



http://dx.doi.org/10.21511/imfi.17(3).2020.20

Source: Bloomberg and authors' calculations.

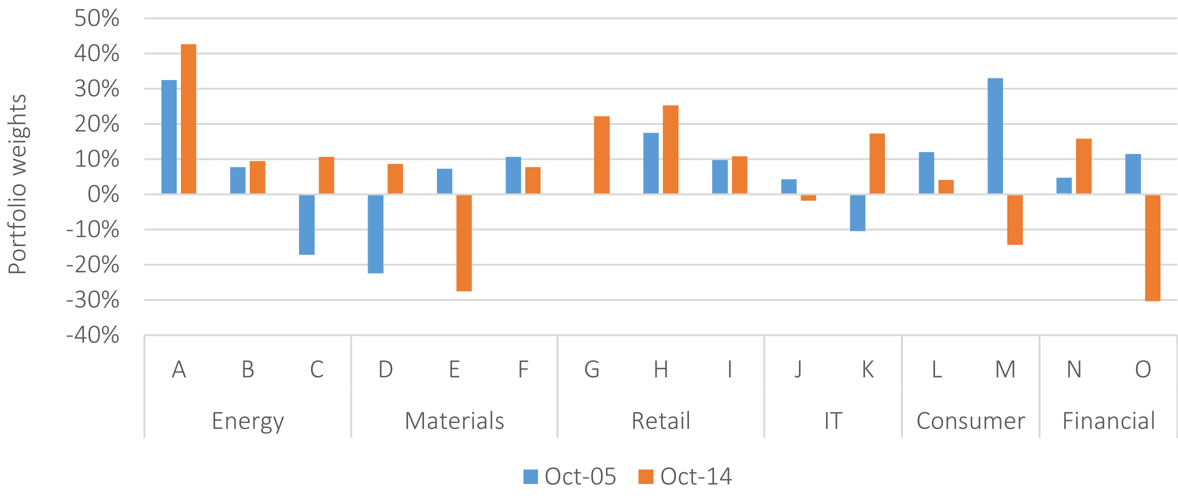



**Figure 5.** Weights in optimal, unconstrained Ω portfolios for Oct-00 – Oct-05 and Oct-09 – Oct-14

portfolios for the two periods are provided in Figure 5, grouped by the market sector.

In the former period, the optimal unconstrained Ω portfolio strongly overweighs the assets A and M while strongly underweighting C and D. Both A and M witnessed considerable growth in the low risk, high return pre-crisis period, skewing their return distributions to the right. Simultaneously, C and D both experienced large losses during this time, skewing both return distributions to the left.

Similar observations were noted for assets A (strong growth post the credit crisis), and E and O (large losses with widely dispersed returns) in the latter period.

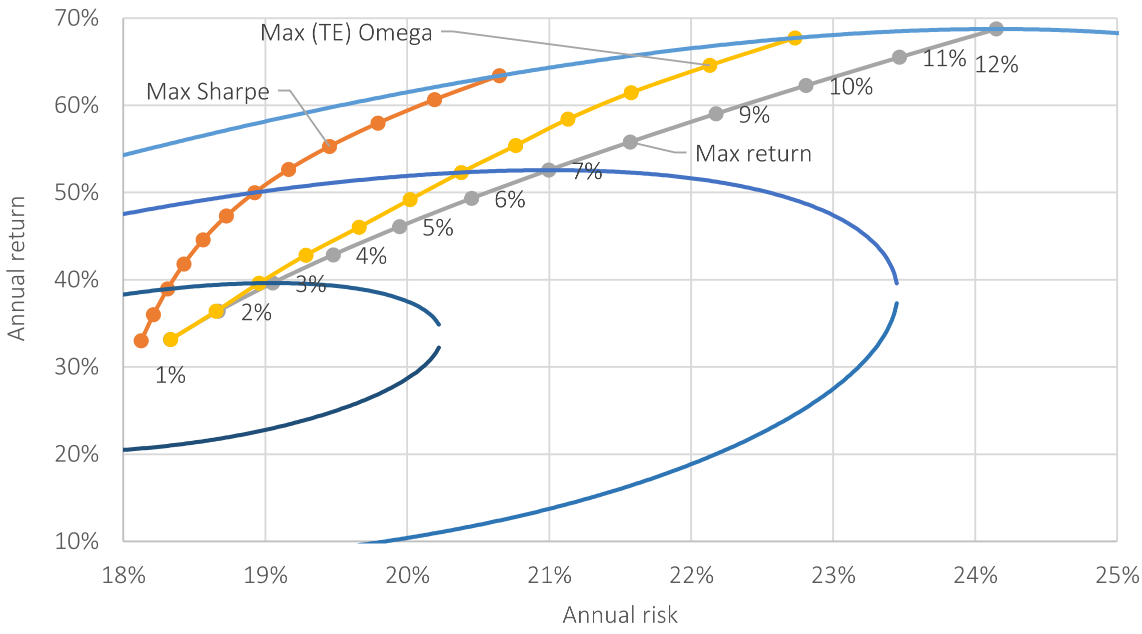
The remainder of the assets' performance was unremarkable; their return distributions were characterized by low skewness and low excess kurtosis. Thus, deviations from the benchmark weights are small.

The maximum Sharpe ratio's behavior, maximum constrained Ω, and maximum return portfolios as a TE function are shown in Figure 6(a) for Oct-00 – Oct-05. The locus of the return/risk coordinates all increase monotonically as TE increases, and the relative configuration is preserved for all TEs (both the risk and return of the maximum Sharpe ratio portfolio less than that of the maximum Ω, and, in turn, less than that of the maximum return portfolio). Figure 6(b) shows the associated Sharpe ratios for all portfolios as TE's function again monotonically increasing with the maximum Sharpe ratio portfolio exhibiting, as expected, the highest Sharpe ratio for all TEs. Constant TE frontiers of 3%, 7%, and 12% are displayed for scale, and the Sharpe ratio for the constrained optimal Ω portfolio is shown as a dotted line in Figure 6(b) for comparison. The vertical scales in Figures 6(a) and (b) are the same as those for Figures 7(a) and (b) for direct comparison.

Figure 7 duplicates the analysis presented in Figure 6 but for the period Oct-09 – Oct-14. When the constant TE frontier's main axis is negative, returns for all maximal portfolios increase monotonically with increasing TE, while the risk for these portfolios decreases then increases again as TE increases.

For $TE < 5\%$, the constrained maximum Ω – ratio portfolio does not lie on the efficient constrained portfolio set – it lies to the right of the maximum return portfolio, i.e., it has higher risk and lower return (recall that the efficient set spans the upper hemisphere of the ellipse from the minimum variance portfolio on the left to the maximum return portfolio on the right. Portfolios outside this region are inefficient). Because the same level of return is possible for this maximum constrained Ω portfolio, it is inefficient. This may not be true because the Ω – ratio makes no assumptions of return distribution normality. Instead, it uses the empirical distribution and may still be

Source: Bloomberg and authors' calculations.

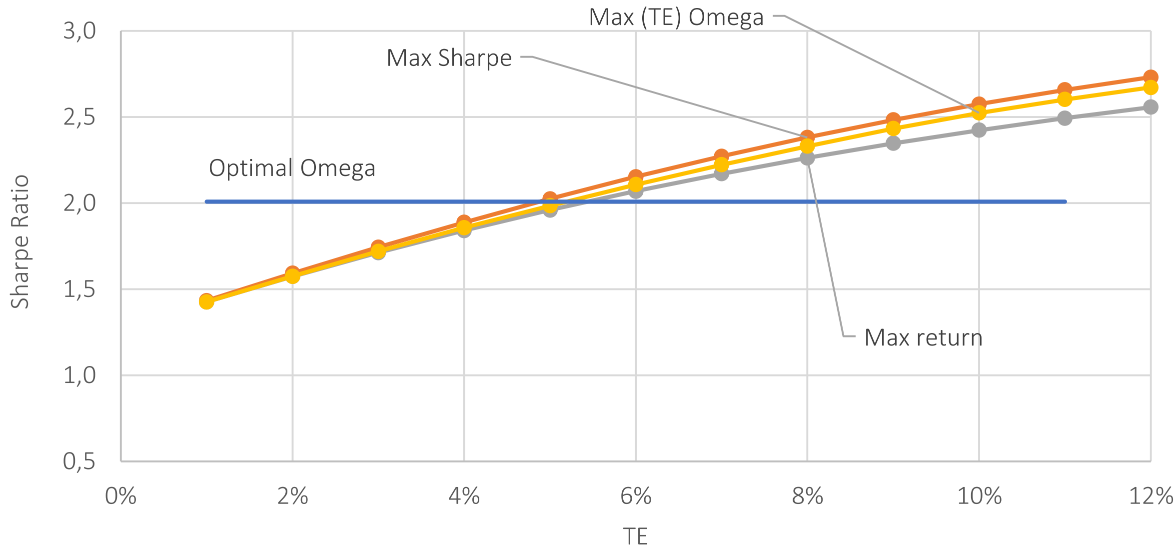



(a)

**Figure 6(a).** Return/risk profiles for relevant portfolios as a function of TE (percentages indicate TE values)

efficient because both the max Sharpe and max return portfolios assume a normal distribution of returns.

Figure 8 compares weight deviations from the three portfolios' benchmark over (a) the Oct-00 – Oct-05 period and (b) the Oct-09 – Oct-14 period.




(b)

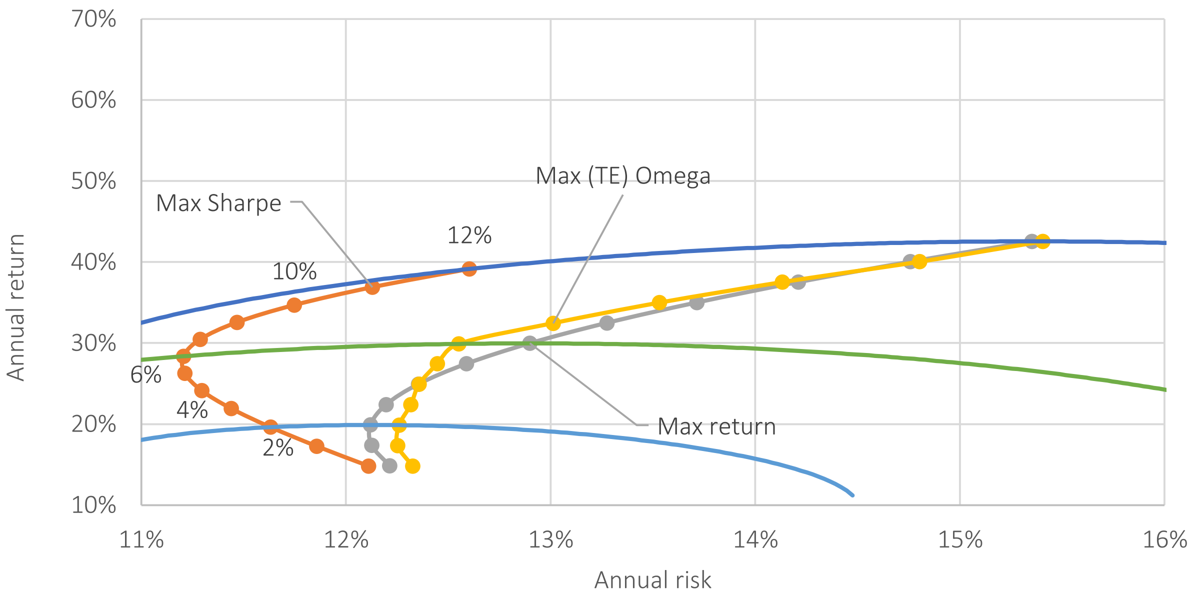
*Note:* Constant TE frontiers at 3%, 7%, and 12% are also shown for comparison. The optimal Ω – ratio's Sharpe ratio is indicated as a dashed line.

**Figure 6(b).** Sharpe ratios versus TE for Oct-00 – Oct-05



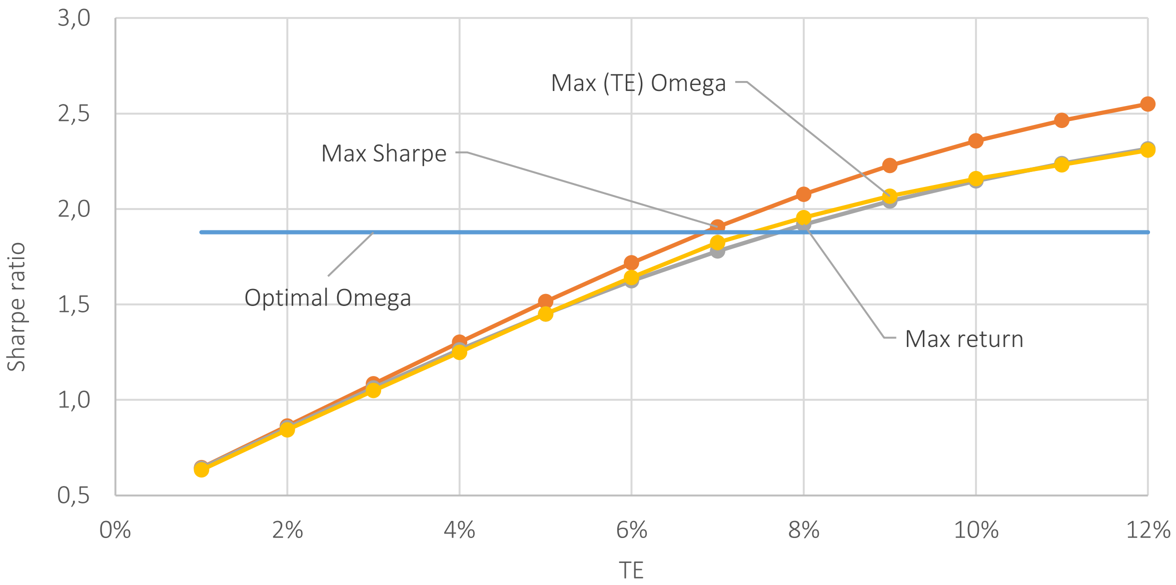
http://dx.doi.org/10.21511/imfi.17(3).2020.20

Source: Bloomberg and authors' calculations.




(a)

**Figure 7(a).** Return/risk profiles for relevant portfolios as a function of TE




(b)

*Note:* Constant TE frontiers at 3%, 7% and 12% are also shown for comparison.

**Figure 7(b).** Sharpe ratios versus TE for Oct-14

The reasons for large over or underweighting remain – for either period – as discussed previously (for Figure 5). In Figure 8(a) for Oct-00 – Oct-05, the constant TE frontier's main-axis slope is > 0 (Figure 2) while for Figure 8(b) for Oct-09 – Oct-14 the main-axis slope is < 0 (Figure 3). Deviations of asset weights from the benchmark vary considerably over the two periods. The relative weights differ in magnitude, the signs (overweight/underweight) are also often different. The size of constituent asset deviation from the benchmark weights (> 0% or < 0%) is also greater when the main axis slope is > 0, but although the relative weights of the constituents often have different signs (like Oct-05), the mag-

Source: Bloomberg and authors' calculations.

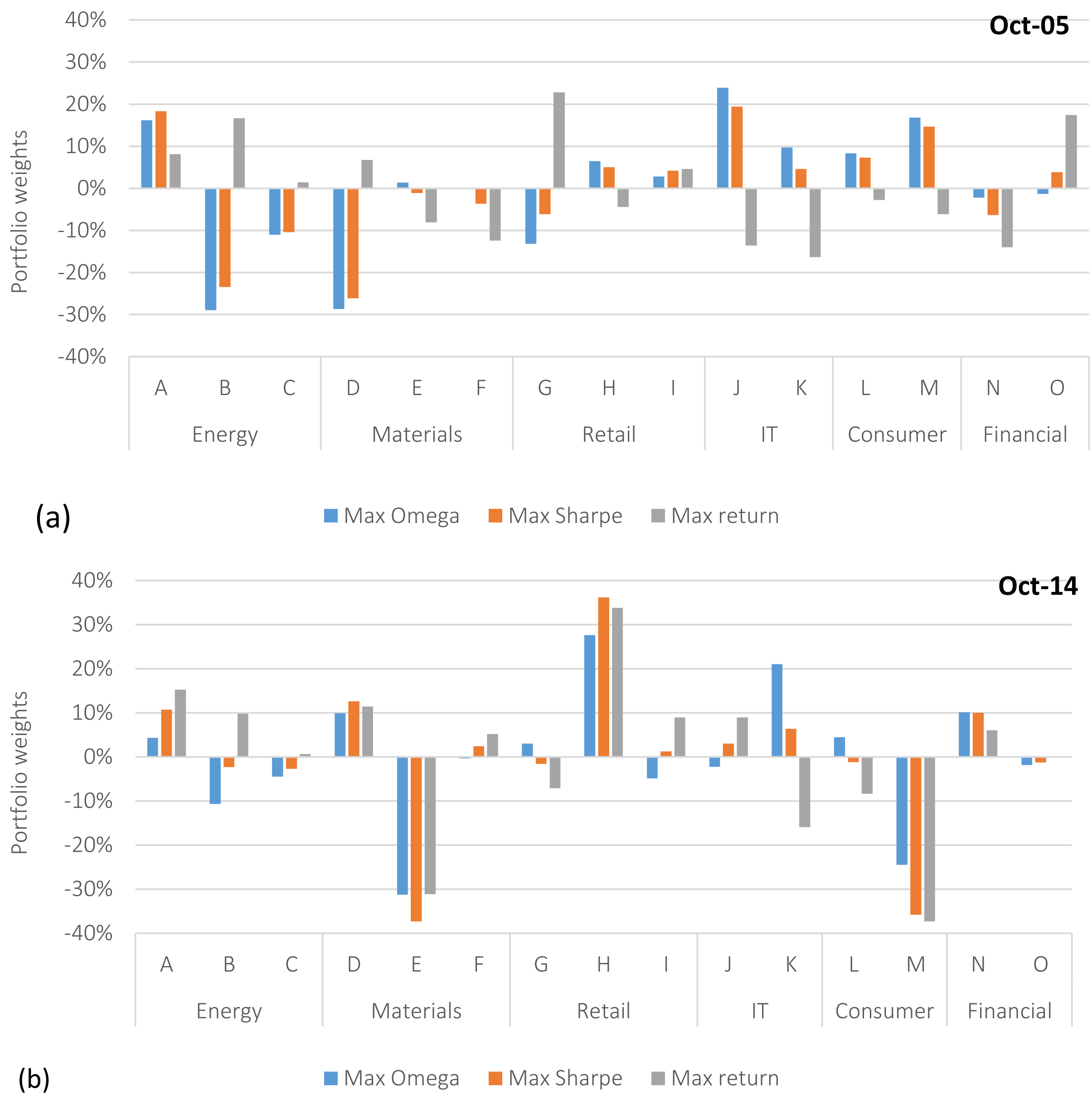



**Figure 8.** Benchmark weight deviations for relevant portfolios in (a) Oct-00 – Oct-05 and (b) Oct-09 – Oct-14

nitude of the differences are negligible. These observations are explained by the fact that when the main-axis slope is > 0, the range of risks spanned by the efficient portfolio set is greater than when the main-axis slope is < 0 (earlier period's approximately 14.5% to 20.5% (6% risk range) compared with later period's 9.5% to 12.5% (3% risk range) in this example).

Figure 9 presents asset K's weight deviations from the benchmark over the two periods as TE's function. Several interesting features are apparent. The profiles are broadly similar regardless of the main axis slope: all increase or decrease monotonically as TE increases. This reflects the stability of benchmark deviations as TE changes – these are gradual, not abrupt. The benchmark weight deviations for the maximum Sharpe ratio and maximum return portfolios are almost identical over the two periods, while those for the maximum Ω portfolio are notably higher in the second period.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Source: Bloomberg and authors' calculations.

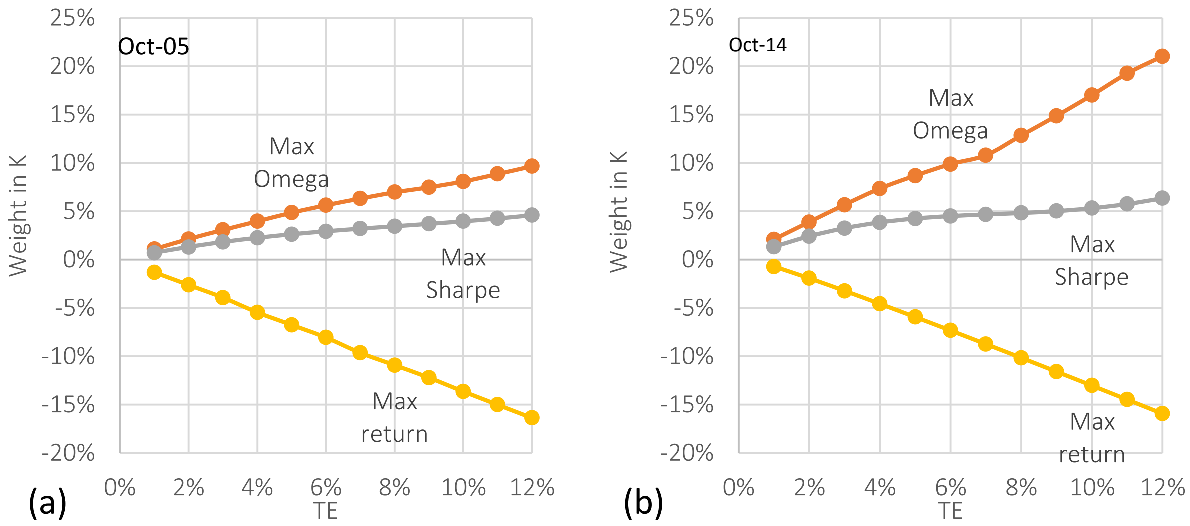


**Figure 9.** Asset K's deviation in weight from the benchmark for the relevant portfolios in (a) Oct-00 – Oct-05 and (b) Oct-09 – Oct-14

Asset K is a large conglomerate, which enjoyed unprecedented success after the credit crisis. Several fortuitous mergers and acquisitions buoyed the company's profitability, it's weighting in the overall market index swelling from 1% to 10% over this period. K's return distribution becomes increasingly skewed to the right as time passes. The weights in the maximum Ω – ratio portfolio increase accordingly, as the Ω – ratio balloons. This is untrue the maximum Sharpe portfolios because this approach assumes that returns are normally distributed. As K's volatility increases over time (due to large positive returns), the Sharpe ratio is penalized (as a risk-adjusted return portfolio), so weights do not change much despite a strongly-performing asset. The maximum return portfolio underweights K relative to the benchmark because its performance is strongly tied to the benchmark (itself representing the broad market, being well-diversified, and having equally weighted components). This leads to strong positive correlations with other asset returns, which perform favorably over the periods but not spectacularly. Adding asset K increases the risk relative to the benchmark beyond that of the specified PD, so the only option is to underweight this asset at the others' expense. This may generate the highest return portfolio, but it is only the maximum Ω – ratio portfolio that fully exploits a strongly outperforming asset.

## CONCLUSION AND RECOMMENDATIONS

Identifying and characterizing portfolios' behavior with a maximum Ω – ratio and constrained by *TEs* has been implemented and investigated here for the first time. Such portfolios differ from universal – unconstrained Ω – ratio portfolios, both in risk and return characteristics and constituent asset weights (and, therefore, they differ in their respective weight deviations from the benchmark). Unconstrained Ω portfolios distribute weights among components depending on both positive and negative return configurations. Portfolios that limit the magnitude of negative returns and encourage positive returns have the highest Ω – ratio and are selected for optimality. Portfolios constrained by TE, however, must allocate component asset weights differently. Because the relative risk level of these portfolios must equal the TE, constrained Ω portfolios penalize assets whose risk profile prevents reaching relative risk equal to the TE (while favoring assets which generate portfolios with more positive returns than negative ones). This arises from the complex interplay of not only component volatilities, but also correlations between components. Individual assets whose returns are

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strongly correlated with those of the benchmark (or "market" if the benchmark represents broad market exposure) – while appearing favorable due to high positive returns – may be penalized because their inclusion leads to relative risk different from the TE.

When the constant TE frontier's main axis is < 0 (a feature that arises only in conditions of high market turbulence, usually short-lived, lasting only a few months), the range of possible return/risk combinations for optimal portfolios (Sharpe, Ω – ratio or return) is considerably reduced. Portfolios under these conditions exhibit similar risks and returns and have similar component weights. When the constant TE frontier's main axis slope is > 0 (a longer-lasting and far more prevalent feature of the constant TE frontier, arising from "normal" market conditions), the range of possible return-risk combinations is considerably greater. Variation in component asset weights is also higher when the constant TE main axis slope is > 0.

Passive asset managers relying on absolute rather than relative performance should always allocate asset weights using the universal maximized Ω – ratio portfolio. Such portfolios have been shown to outperform other vaunted "optimal" alternatives. Active managers who require portfolios to outperform a prescribed benchmark while maintaining a prescribed level of risk relative to it subvert the mechanisms employed by optimal Ω – ratio portfolio construction, reducing – or eliminating – its effectiveness. In these cases, market conditions – which dictate the sign of the constant TE frontier's main axis slope – should also be considered. When the constant TE frontier's main axis is > 0, strategic active asset managers should allocate asset weights using a maximum Sharpe ratio framework. Tactical (shorter-term) active asset managers should use a maximum Ω – ratio approach to determine asset weights.

Possible future work could include an explicit, long-term, empirical investigation of the veracity of these conclusions. Current (2020) highly volatile market conditions, due to the fallout induced by the COVID-19 pandemic, could serve as an interesting case study to gauge and calibrate these effects.

Future work could also explore the dual impacts of trading commissions and trading costs. Investigating restrictions on short-selling in available portfolios may also be of interest. However, improved trading platforms and computation speeds continue to erode such limitations, so such work's impact may diminish in the future.

## AUTHOR CONTRIBUTIONS

Conceptualization: Wade Gunning.
Data curation: Wade Gunning.
Formal analysis: Wade Gunning.
Investigation: Wade Gunning.
Methodology: Wade Gunning.
Project administration: Gary van Vuuren.
Software: Wade Gunning.
Supervision: Gary van Vuuren.
Validation: Wade Gunning, Gary van Vuuren.
Writing – original draft: Wade Gunning.
Writing – review & editing: Gary van Vuuren.



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# REFERENCES


1. Anadu, K., Kruttli, M., McCabe, P., Osambela, E., & Shin, C. H. (2018). *The shift from active to passive investing: potential risks to financial stability?* (Finance and Economics Discussion Series 2018-060). Washington: Board of Governors of the Federal Reserve System. https://doi.org/10.17016/FEDS.2018.060
2. Beasley, J. E., Meade, N., & Chang, T. J. (2003). An evolutionary heuristic for the index tracking problem. *European Journal of Operational Research, 148*(3), 621-643. https://doi.org/10.1016/S0377-2217(02)00425-3
3. Berk, J., & van Binsbergen, J. (2015). Measuring skill in the mutual fund industry. *Journal of Financial Economics, 118*(1), 1-20. https://doi.org/10.1016/j.jfineco.2015.05.002
4. Bertrand, P., Prigent, J. L., & Sobotka, R. (2001). Optimisation de portefeuille sous contrainte de variance de la tracking-error. *Banque & Marchés, 54*(1), 19-28. Retrieved from http://test.greqam.fr/sites/default/files/_dt/greqam/00a36.pdf
5. Calvo, C., Ivorra, C., & Liern, V. (2012). On the computation of the efficient frontier of the portfolio selection problem. *Journal of Applied Mathematics, 2012*(1), 1-25. https://doi.org/10.1155/2012/105616
6. Canakgoz, N. A., & Beasley, J. E. (2008). Mixed-integer programming approaches for index tracking and enhanced indexation. *European Journal of Operational Research, 196*(1), 384-399. https://doi.org/10.1016/j.ejor.2008.03.01
7. Courtney Capital. (2020). *JSE top 40 shares*. Retrieved from https://www.courtneycapital.co.za/trade/jse-top-40-shares/
8. Cremers, K. J. M., Fulkerson, A., & Riley, T. B. (2019). Challenging the conventional wisdom on active management: a review of the past 20 years of academic literature on actively managed mutual funds. *Financial Analysts Journal, 75*(4), 8-35. https://doi.org/10.1080/0015198X.2019.1628555
9. Daly, M., Maxwell, M., & van Vuuren, G. (2018). Feasible portfolios under tracking error, β, α and utility constraints. *Investment Management and Financial Innovations, 15*(1), 141-153. http://dx.doi.org/10.21511/imfi.15(1).2018.13
10. Dolvin, S., Fulkerson, J., & Krukover, A. (2018). Do "good guys" finish last? The relationship between Morningstar sustainability ratings and mutual fund performance. *Journal of Investing, 28*(2), 77-91. https://doi.org/10.3905/joi.2019.28.2.077
11. Evans, C., & van Vuuren, G. (2019). Investment strategy performance under tracking error constraints. *Investment Management and Financial Innovations, 16*(1), 239-257. http://dx.doi.org/10.21511/imfi.16(1).2019.19
12. Filippi, C., Guastaroba, G., & Speranza, M. (2016). A heuristic framework for the bi-objective enhanced index tracking problem. *Omega, 65*(C), 122-137. https://doi.org/10.1016/j.omega.2016.01.004
13. Gilli, M., Schumann, E., Di Tollo, G., & Cabej, G. (2008). *Constructing long/short portfolios with Ω -ratio*(Swiss Finance Institute Research Paper No. 08-34). Swiss Finance Institute. Retrieved from https://ideas.repec.org/p/chf/rpseri/rp0834.html
14. Gnagi, M., & Strub, O. (2020). Tracking and outperforming large stock market indices. *Omega, 90*(C), 119-129. https://doi.org/10.1016/j.omega.2018.11.008
15. Guastaroba, G., & Speranza, M. G. (2012). Kernel Search: An application to the index tracking problem. *European Journal of Operational Research, 217*(1), 54-68. https://doi.org/10.1016/j.ejor.2011.09.004
16. Guastaroba, G., Mansini, R., Ogryczak, W., & Speranza, M. G. (2016). Linear programming models based on Omega ratio for the enhanced index tracking problem. *European Journal of Operational Research, 251*(3), 938-956. https://doi.org/10.1016/j.ejor.2015.11.037
17. Gunning, W., & van Vuuren, G. (2019). Exploring the drivers of tracking error constrained portfolio performance. *Cogent Economics, 7*(1), 1-15. https://doi.org/10.1080/23322039.2019.1684181
18. Janabi, M. (2009). Commodity price risk management: Valuation of large trading portfolios under adverse and illiquid market settings. *Journal of Derivatives and Hedge Funds, 15*(2), 15-50. https://doi.org/10.1057/jdhf.2008.27
19. Jorion, P. (2003). Portfolio optimization with tracking-error constraints. *Financial Analysts Journal, 59*(5), 70-82. Retrieved from https://www.researchgate.net/profile/Philippe_Jorion/publication/228183529_Portfolio_Optimization_with_Tracking-Error_Constraints/links/02bfe50f02c016cc41000000/Portfolio-Optimization-with-Tracking-Error-Constraints.pdf
20. Kane, S. J., Bartholomew-Biggs, M. C., Cross, M., & Dewar, M. (2005). Optimizing Omega. *Journal of Global Optimization, 45*(1), 153-167. Retrieved from https://link.springer.com/article/10.1007/s10898-008-9396-5
21. Kapsos, M., Zymler, S., Christofides, N., & Rustem, B. (2011). Optimizing the Omega ratio using linear programming. *Journal of Computational Finance, 17*(4), 49-57. https://doi.org/10.21314/JCF.2014.283
22. Keating, C., & Shadwick, W. F. (2002). A universal performance measure. *Journal of Performance Measurement, 6*(3), 59-84. Retrieved from https://www.actuaries.org.uk/system/files/documents/pdf/keating.pdf
23. Kwan, C. C. (2003). Improving the efficient frontier. *The Journal*

*of Portfolio Management, 29*(2), 69-79. https://doi.org/10.3905/jpm.2003.319874

24. Larsen, G. A., & Resnick, B. G. (2001). Parameter estimation techniques, optimization frequency, and portfolio return enhancement. *Journal of Portfolio Management, 27*(4), 27-34. https://doi.org/10.3905/jpm.2001.319810
25. Lo, A. (2012). The statistics of Sharpe ratios. *Financial Analysts Journal, 58*(4), 36-52. Retrieved from https://alo.mit.edu/wp-content/uploads/2017/06/The-Statistics-of-Sharpe-Ratios.pdf
26. Marhfor, A. (2016). Portfolio performance measurement: review of literature and avenues of future research. *American Journal of Industrial and Business Management, 6*(4), 432-438. http://dx.doi.org/10.4236/ajibm.2016.64039
27. Markowitz, H. (1952). Portfolio selection. *The Journal of Finance, 7*(1), 77-91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x
28. Markowitz, H., Schirripa, F., & Tecotzky, N. (1999). A more efficient frontier. *The Journal of Portfolio Management, 25*(5), 99-108.
29. Mausser, H., Saunders, D., & Seco, L. (2006). Optimising Omega. *Risk,* 88-92. Retrieved from https://www.researchgate.net/publication/230786885_Optimizing_Omega
30. Maxwell, M., & van Vuuren, G. (2019). Active investment strategies under tracking error constraints. *International Advances in Economic Research, 25*(3), 309-322. Retrieved from https://link.springer.com/article/10.1007/s11294-019-09746-3
31. Maxwell, M., Daly, M., Thomson, D., & van Vuuren, G. (2018). Optimizing tracking error-constrained portfolios. *Applied Economics, 50*(54), 5846-5858. https://doi.org/10.1080/00036846.2018.1488069
32. Merton, R. (1972). An analytic derivation of the efficient portfolio frontier. *The Journal of Financial and Quantitative Analysis, 7*(4), 1851-1872. https://doi.org/10.2307/2329621
33. Muralidhar, A. (2015). The Sharpe ratio revisited: what it really tells us. *Journal of Performance Measurement, 19*(3), 6-12. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2692859
34. Mutunge, P., & Haugland, D. (2018). Minimizing the tracking error of cardinality constrained portfolios. *Computers and Operations Research, 90,* 33-41. Retrieved from https://isiarticles.com/bundles/Article/pre/pdf/107838.pdf
35. Passow, A. (2004). *Omega portfolio construction with Johnson distributions* (FAME Research Paper Series RP120). International Center for Financial Asset Management and Engineering. Retrieved from https://ideas.repec.org/p/fam/rpseri/rp120.html
36. Pedersen, L. (2018). Sharpening the arithmetic of active management. *Financial Analysts Journal, 74*(1), 21-36. https://doi.org/10.2469/faj.v74.n1.4
37. Qi, J., Rekkas, M., & Wong, A. (2018). Highly accurate inference on the Sharpe ratio for autocorrelated return data. *Journal of Statistical and Econometric Methods, 7*(1), 21-50. Retrieved from http://www.scienpress.com/Upload/JSEM%2fVol%207_1_2.pdf
38. Roll, R. (1992). A mean/variance analysis of tracking error. *The Journal of Portfolio Management, 18*(4), 13-22. Retrieved from https://jpm.pm-research.com/content/18/4/13
39. Rudd, A. (1980). Optimal selection of passive portfolios. *Financial Management, 9*(1), 57-66. Retrieved from https://www.jstor.org/stable/3665314
40. Sharpe, W. F. (1966). Mutual fund performance. *Journal of Business, 39*(1), 119-138. http://dx.doi.org/10.1086/294846
41. Sharpe, W. F. (1994). The Sharpe ratio. *The Journal of Portfolio Management, 21*(1), 49-58. https://doi.org/10.3905/jpm.1994.409501
42. Stowe, D. L. (2014). *Tracking error volatility optimization and utility improvements* (Working paper). Retrieved from http://swfa2015.uno.edu/F_Volatility_&_Risk_Exposure/paper_221.pdf.
43. Strub, O., & Baumann, P. (2018). Optimal construction and rebalancing of index-tracking portfolios. *European Journal of Operational Research, 264*(1), 370-387. http://doi.org/10.1016/j.ejor.2017.06.055
44. Thomson, D., & van Vuuren, G. (2016). Forecasting the South African business cycle using Fourier Analysis. *International Business and Economics Research Journal, 15*(4), 175-192. https://doi.org/10.19030/IBER.V15I4.9755



http://dx.doi.org/10.21511/imfi.17(3).2020.20