“Management of tangible assets using a modified market value price formation model”

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MANAGEMENT OF TANGIBLE ASSETS USING A MODIFIED MARKET VALUE PRICE FORMATION MODEL

Abstract

The paper deals with the economic measurements of the market value of enterprise assets, which are of great importance for their effective management. The use of more accurate economic measurements is an integral part of an optimal strategy to manage business assets. Therefore, reduction of evaluation results uncertainty is a necessary condition for effective management. To achieve mentioned goals, the paper aims to determine the mathematical base for the assets valuation methodology of value/depreciation that change over time, which can be applied to its dynamic objective quantitative analysis. The basic hypothesis suggests that all tangible assets, characterized by removable depreciation, are inclined to a negative periodic depreciation during short inter-service periods when remedial repair works are carried out to eliminate depreciation.

The methodical approaches concerning a mathematical description of assets value/depreciation dynamics are considered. It is shown that both traditional, progressive and regressive value/depreciation dynamics models change over time. They do not correspond to the actual state since they do not take into account increased objects value and negative periodic depreciation. To evaluate value/depreciation change over time more precisely, a new kind of mathematical model is proposed, which equations take into account the opposite signs of periodic depreciation during operational service periods and non-operational inter-service periods. It is proved that the actual indicators of fair market value and periodic depreciation of enterprise assets can be determined with higher reliability based on a new mathematical model.

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INTRODUCTION

It is crucial to determine the actual maket value of enterprise assets for their proper management. This may be done by economic measurements, using applied econometric independent evaluation methods. One of three traditional evaluation methodological approaches, namely the costs approach, is widely used to measure the market value of the assets. Its classical appraisal methodology involves taking into account the indicators of assets depreciation. These depreciation indicators are usually obtained by the appraiser based on object field inspection and information about the previous period of economic life. In this case, the appraiser takes into account both the actual physical condition of the property and its actual age, operating conditions and the corresponding stage of the life cycle. In particular, the appraiser is based on the object value, which is considered the most adequate method for a present situation. Well-known evaluation models, which change over time, are limited to positive values of the depreciation only, which is not always true.

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For example, negative depreciation is common for the intellectual property, along with the traditional generally accepted positive depreciation during certain periods. Different periodic depreciation signs can be manifested in certain periods of goodwill, trademarks and brands' existence, and these signs may change many times (Pozdnyakov, 2019, p. 91; Pozdnyakov & Lapishko, 2018, p. 728; Pozdnyakov & Lapishko, 2019, p. 393). It's stated that goodwill is not a depreciable asset since its value depends on how a business runs. Listed above intangible assets have the infinite and intense capability to either appreciate or depreciate (even self-destruct), depending on various facts and circumstances and on the capacity and ability of the person who owns them to exploit their underlying value (Narayan, 2021, p. 2). After all, some types of assets in certain periods show their value increase, but currently this indisputable fact is not adequately reflected in the evaluation theory. Moreover, for certain classes of tangible objects, different general patterns of value over time changes are observed, which indicate the negative value of depreciation for some periods. This type of long-term value over time growth model dependence is inherent not only for some intangible assets, but for all objects that are characterized by removable types of depreciation. These are, in particular, numerous objects belonging to the class of machines and equipment (during periods of repair, preventive maintenance, replacement of worn-out units) and real estate in the form of buildings and constructions (during periods of repairs, outbuildings, renovations and reconstructions). There are not enough methods for assessment of assets value and depreciation, which changes over time, in the evaluation theory.

The bottleneck of assests value and depreciation patterns, which change over time, and the relationship between certain characteristics of depreciation and individual price-forming factors requires immediate solution and is undoubtedly relevant. The models relevance of value and depreciation, which change over time, is that the accuracy of depreciation estimation directly affects the uncertainty of valuation results. Therefore, regarding the property of different market segments and different conditions of its use, the characteristics of such dependencies may differ significantly. Depreciation characteristics are not stable over time and may change following price-forming factors and market conditions. This requires in-depth studies of value and depreciation patterns, which change over time, and development of relevant models. Based on the revealed depreciation indicators, a more adequate, exact and relevant mathematical model of these dependencies is offered. A more adequate mathematical description of depreciation dynamics is undoubtedly an urgent task, as it reduces the uncertainty of assets valuation, and thus provides more reliable data for their effective management.

1. THEORETICAL BASIS

In the value concept, all depreciable objects tend to diminish their value over time, due to forces such as obsolescence, depreciation, and inadequacy to market requirements. Absolute depreciation can be determined by a difference between the asset primary value and its present value. Consequently, periodical depreciation can be determined by a difference between the appraised value of assets at the end of a single period and its value at the beginning of that period (Pozdnyakov, 2019, p. 91). Generally, value and depreciation models, which change over time, can be obtained by series of periodic appraisal estimates of object market value. The change in value between such estimates becomes a measure of the depreciation, attributable to the period between estimates. Annual estimates make possible long-time monitoring of any tangible or intangible evaluation object. Moreover, the period can be reduced to semi-annual, quarterly, monthly, etc., to obtain a more detailed result. Considering the profitable real estate objects, an alternative way can be to identify the dynamics of the present value changes, based on the future income stream that assets are capable to generate (Gribovskiy, 2001, p. 64). Alekseev (2011, p. 1) gives an overview of calculating methods of accumulated depreciation, taking into account the change of qualitative characteristics of evaluation objects. There are two main types of depreciation: removable and irremovable (non-removable, unremovable, or irreparable). Veyg (2009, p. 49) concluded that a difference between removable and irremovable depreciation depends on the physical/technical ability and economic feasibility to eliminate depreciation.

Typical for buildings and constructions is the nonlinear model of value change over time, characterized by a slow decline in value at the beginning and accelerated decline at the end of the economic life. As a result, its market value decreases slowly at the beginning of the economic life. At the end of its life, due to the gradual loss of usefulness and defects accumulation, there are even more functional and physical signs of depreciation. Changes in the general economic situation and external conditions cause an increase of the economic depreciation rate. Over a long time gap, the discrepancy between the actual and the most effective use of that object will increase, and this factor supports external (economic) depreciation growth (Friedman & Ordway, 1995, p. 318). Progressive depreciation can be defined as a loss of an assets value at an increasing rate. There are three main models of depreciation: progressive depreciation, conditionally linear depreciation, and regressive depreciation (Gribovskiy, 2001, p. 72). Linear straight-line depreciation is an inadequate oversimplified model, and research has confirmed the feasibility of such practices only for a group of specific objects and expanding the practice of accelerated depreciation model. Authors offered to adopt the preferable method of logistic-curve depreciation that more adequately represents real depreciation and obsolescence of capital assets, as well as evaluates depreciation more detailed (Pankratov & Grabovy, 2017, p. 4).

Progressive depreciation characterizes multi-storey residential real estate buildings (Gribovskiy, 2001, p. 72; Yiu, 2007, p. 6; Sirmans et al., 2005, p. 33), and digressive depreciation is relevant for equipment and, in particular, for cars (Olsen, 2016, p. 3) and some non-residential and commercial real estate buildings (Bokhari & Geltner, 2016, p. 4). At the same time, depreciation is often viewed from the accounting point of view, without paying enough attention to their correspondence and compliance with real changes in assets market value. It is still impossible to formulate one single criterion for the choice of depreciation model. There are no clear recommendations on how to choose the best method for depreciation relevant for businesses of various legal forms and different groups of fixed assets. The chosen method should help accelerating the upgrade of fixed assets, determining the real value of profit, and analyzing all the factors associated with the operation of the fixed assets (Shmygol et al., 2019, p. 77). In a broader context, the main tasks of depreciation policy are boost of investment activities and accelerated renewal of fixed assets (Trachova, 2017, p. 411; Trachova, 2018, p. 150). The choice of depreciation model is considered from the perspective at the enterprise level (Lypetsky, 2010, p.183; Pedan, 2009, p. 224), or at a state level (Pankratov & Grabovy, 2017, p. 4; Shmygol et al., 2019, p. 77).

Almost a hundred years ago, researchers began to work on retirement characteristics of group properties at Iowa State College. Robley Winfrey and Edwin Bernard Kurtz published articles (Winfrey & Kurtz, 1931; Winfrey, 1935; Kurtz, 1937), which opened a promising area of research. Today this classical technique of mathematical modeling is the commonplace in the evaluation methodology, called Iowa curves. The most fully mathematical models of value dependencies, which change over time, have been considered by Robley Winfrey (1967, 1969, 1970) and his followers (Henderson, 1968; Marston et al., 1982). The groups of properties were analyzed by reflecting variations through the use of one composite retirement survivor curve for the entire property group (Russo, 1978).

However, in all these works only the service period of the object's economic life was considered, and short inter-service periods, aimed at ilimination of depreciation, were not taken into account. Thus, a hypothesis is stated that for proper estimation methodologies not only service periods must be taken into account. The whole economical life covers also inter-service periods, which strongly affect general depreciation characteristics. The main appraisal technology to obtain functional dependencies of depreciation, which change over time, is based on the dynamics of object market value during economical life.

In this regard, it is crucial to elaborate a detailed theoretical basis for a more accurate determination of depreciation. This requires determining a mathematical base for assets value/depreciation functions, which change over time, that can be applied to its dynamic's objective quantitative analysis. The main objective is to reduce uncertainty in results when viewing the economic measurements of assets value. Accurate measure-

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ment results are integral for developing a management strategy regarding business assets. To solve this problem, equations must be derived to formulate a **more reliable** mathematical model that takes into account the opposite signs of periodic depreciation during operational service periods and non-operational inter-service periods for tangible assets, which are characterized by removable depreciation. It seems appropriate to test this model, applying the hypothesis formulated above – with the assumption that these kind of tangible assets may show negative periodic depreciation during short-term inter-service periods when remedial repair works are carried out to eliminate the depreciation.

2. RESULTS

To determine the quantitative indicators of depreciation, an econometric methodology was used (Pozdnyakov, 2019, p. 91). The basis for these assessments is investigated changes in assets market value during the relevant periods. To define the indexes of assets depreciation, two kinds of depreciation are used: periodical (in our case annual) depreciation and total (complete) accumulated depreciation (from asset creation date to evaluation date). Aiming to analyze exposure of changes in assets value over time, the absolute annual depreciation index $A_{aad\ i}(t)$ was taken for each i year of life period. It is mathematically determined as an absolute annual increase of assets value, which is taken with a reversed sign, and measured by monetary dimension:

$$A_{aad\ i}(t) = -A_{aavi\ i}(t), \tag{1}$$

where $A_{aavi\ i}(t)$ is absolute annual increase of appraised assets value, which is determined by the formula

$$A_{aavi\ i}(t) = V_i - V_{i-1}, \tag{2}$$

where V_i is an index of the appraised asset value during i year (for example, on the evaluation date); V_{i-1} is an index of the appraised asset value during previous (i – 1) year.

Equation (1), taking into account formula (2), can be written as

$$A_{aad\ i}(t) = V_{i-1} - V_i. \tag{3}$$

Annual (or periodical) absolute depreciation is considered as a statistical analogue of the first derivative of asset value change, which change over time. The one-year period is selected due to practical considerations. In general, this period can be changed, depending on the concrete research task. Therefore, for a more detailed analysis of investigated parameters, periodical depreciation for shorter periods can be used – for example, for a quarter, for a month, etc., up to daily periodical absolute depreciation. It is easy to prove that the shorter the time Δt for periodical depreciation $A_{aavi\ i}(t)$ during each i period is, the closer numeral values to $A_{aavi\ i}$ function is:

$$\lim_{\Delta t \to 0} A_{aad\ i}(t) = \frac{\partial V_i(t)}{\partial t}. \tag{4}$$

Therefore, an annual depreciation can be examined as a measuring index of $V_i(t)$ asset value change during time t dynamics. This characteristic of periodical depreciation allows considering it as an optimal and most comfortable instrument for the evaluation tasks that are related to the asset value change during time t.

Periodical depreciation and complete accumulated depreciation are interconnected in such a way that the index of periodical depreciation $A_{aavi\ i}(t)$ simultaneously is also an indirect measuring index $A_{cad\ i}(t)$ of complete accumulated depreciation. Parameter $A_{cad\ i}(t)$ shows a quantitative estimation of complete depreciation, accumulated in a period from zero to t time, which means that it reflects the change in the asset value from the moment when an object was created/purchased and registered up to the assessment date. As the index of the complete accumulated depreciation during i period, absolute complete change of appraised asset value is taken, on the interval from the creation date to the assessment date:

$$A_{cad\ i}(t) = -A_{acvi\ i}(t), \tag{5}$$

where $A_{acvi\ i}(t)$ is an absolute increase of accumulated asset value during i year, which is determined by the formula

$$A_{acvi\ i}(t) = V_i(t) - V_0, \tag{6}$$

where V_i (t) is an an index of the asset value during i year; V_0 is an index of the initial assets value druing zero year, at the assets creation or registration date.

Similarly to Equation 3, Equation 5, and taking into account Equation 6, the next modification can be written as:

$$A_{cad\,i}(t) = V_0 - V_i. \tag{7}$$

Mathematically defined depreciation indexes above are the most concrete and complete descriptions that can be applied for the objective quantitative analysis of V_i (t) asset value/depreciation change during time t. In general, it is characterized by positive depreciation during long-time service periods and negative depreciation during short-time inter-service periods.

Iowa survivor curves allow determining the current value of assets, stating that its value at any time is defined by the present value of the investment operating returns over its expected future residual useful economic life. The expected economic life is unknown on the assessment date – so, the general economic life for a specific separate object is undetermined. Based on a statistical analysis, the average economic life for a group of identical assets can be determined, but for one specific object, this indicator is difficult to find due to depreciation elimination (repairs, reconstructions, modernization etc.).

Therefore, for each specific object, the expected residual life and the total economic life usually are difficult to determine. To overcome this uncertainty, the principle of financial equivalence was formulated. It states that the financial equivalent of future operating returns over the expected residual life can be calculated on the assumption that all options for future operating return curves can be defined based on estimating the value of the cost of creating/acquiring new property object. Robley Winfrey (1967, 1969, 1970) states that the most convenient is the assumption that the sums of a homogeneous annuity operating returns series are equal in the amount for the variable annual future operating returns over the total life expectancy. Using the above concept of equivalence, for each age of the asset, the author has proposed an indicator called the "condition percent factor", by which the acquired/created new property value, or the cost of a new asset, can be converted to the property present value index at the present age. Robley Winfrey (1969, p. 26) defines the concept of "condition percent factor" as "the ratio of the present value of the depreciable property relative to its depreciable value when new". The analytical expression for this factor has the following form:

$$c = \frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1}, \tag{8}$$

where c is the condition percent factor, n is the probable object economic total life in years, x is the current age of the object in years, r is the annual rate of net operating returns.

On the other side, the author defines condition percent factor c as the ratio between the property present value V_i on the current age x, and its initial value V_0 of the new property object:

$$c = \frac{V_i}{V_0}. \tag{9}$$

From (9) we can easily obtain a formula for calculating the present property value V_i at the valuation date, at its current age x:

$$V_i = V_0 \cdot c. \tag{10}$$

Thus, the model of asset value over time change is mathematically defined, and expressions (8) and (10) are necessary and sufficient. Based on this model, it is easy to determine the characteristics of accumulated depreciation, which is refered to as the total absolute depreciation $A_{cad\,i}(t)$ by analytical expression (7), and the accumulated depreciation factor $K_i(t)$:

$$K_i = 1 - \frac{V_i}{V_0}. \tag{11}$$

By substitution (10) into (7), after elementary transformations, the following form was obtained

$$A_{cad\,i}(t) = V_0(1-c). \tag{12}$$

Accordingly, by using the expressions (10), (11), we arrive at

$$K_i = 1 - c. \tag{13}$$

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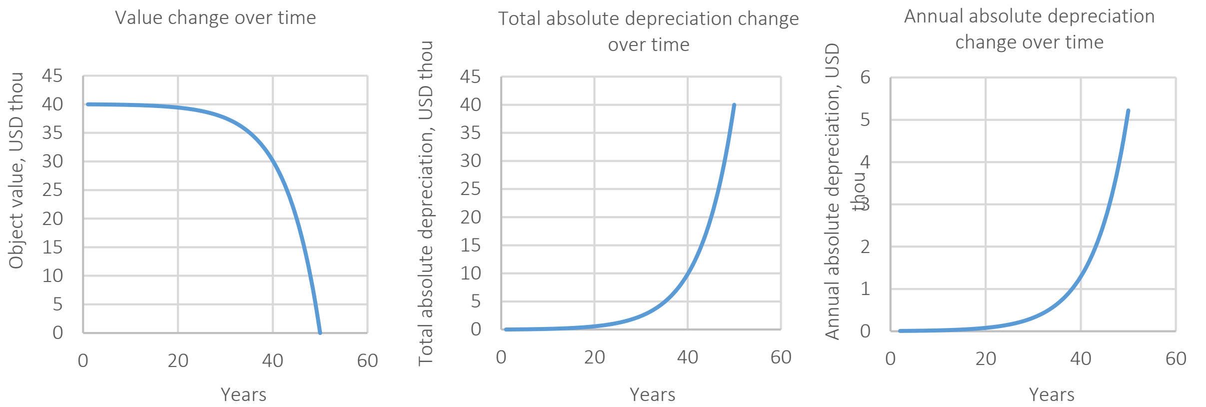

Figure 1. Graphs of the value change over time (left); total absolute accumulated (middle) and the annual (periodic) absolute depreciation (right). Traditional progressive depreciation model, based on (8)

In the same way the absolute annual depreciation $A_{aad\,i}(t)$ for the i year is defined as absolute annual increment of the object value. It has the monetary unit's dimension, according to analytical expression (3), as a difference between estimated object value V_{i-1} during previous to x year $(i-1)$, and estimated object value V_i during x year, at the evaluation date.

A graphical interpretation of the present property value $V_i(t)$ and depreciation indicators $A_{cad\,i}(t)$, $A_{aad\,i}(t)$ dependencies, for a non-residential real estate object with characteristics V_0= 40,000 USD, $r = 0.15$, is presented above. The probable term n of the object expected economic life for the buildings of capital group IV is 50 years. The graph shape of the accumulated depreciation $K_i(t)$ dependence here and in the following diagrams is similar to the graph of total absolute accumulated depreciation $A_{cad\,i}(t)$ – with the difference that on the ordinate axis is postponed its dimensionless numerical values, variated from 0 to 1.

Figure 1 shows diagrams that correspond to the model of progressive depreciation, which is increasing over time. Using a well-known traditional model of asset value, the dependencies described by smooth continuous curves that do not take into account inter-service periods of depreciation elimination were obtained. The change patterns of real estate object value is described by a sharply nonlinear convex upward graph with the negative first and the positive second derivatives. According to the literature review, this kind of dependencies is considered typical for real estate objects, especially for buildings and constructions. A relatively slow value decreases at the beginning of the economic life, which is constantly increasing over time, reaching maximum values at the end of its life, is observed.

This model is not relevant for machines, mechanisms, equipment, high-tech information technology devices, and especially for electronics and computer technologies. For these asset classes opposite patterns, with high depreciation rates in the initial years of the service period and their gradual decrease, are appropriate. In contrast to progressive depreciation, the model of regressive depreciation reflects the inverse nature of value change over time. This model is characterized by a rapid depreciation at the beginning of the operating period, which gradually slows down. The change function for this model is described by a monotonically decreasing concave graph with the negative first and second derivatives. At the same time, the functional depreciation is the dominant in the integrated accumulated depreciation, because new models with better functional characteristics appear. In general, the two most common nonlinear depreciation models may be viewed: progressive and regressive ones. For progressive depreciation, modified Iowa type survivor curves were developed, according to which the total depreciation is defined as

$$K_i = \frac{(1+r)^n - (1+r)^{n-x}}{(1+r)^n - 1}, \tag{14}$$

where $K_i(t)$ is accumulated depreciation factor, determined by (11), others is the same as in (8). In terms of its economic content, the accumulated depreciation factor $K_i(t)$ traditionally is considered as a dimensionless coefficient of accumulated depreciation, which in classical evaluation theory changes from 0 to 1 during the ecomomic life. The annual rate of net operating returns r must correspond to the discount rate, accepted for discount period of the object expected rest of economic life ($n - x$). This formula needs to be adjusted paying attention to the decrease of equipment efficiency, as well as the salvage value, as the cost of equipment disposal. Therefore, the method gives good results when evaluating the equipment, operating conditions of which are close to average ones. The method does not take into account short inter-service periods of depreciation elimination, which are considered as individual events in a specific single object lifetime (Kozlov, 2010, 2012, 2016).

A graphical interpretation of the current value $V_i(t)$, and depreciation $A_{cad\,i}(t)$, $A_{aad\,i}(t)$ dependencies, using the model of regressive depreciation, is presented below. Here another example of evaluation practice is used, namely, the object of the "machinery and equipment" class drilling machine 2H125 with characteristics $V_0 = 2139$ USD, $r = 0.15$, $n = 28.57$ years (Gokhberg & Shcherban, 2007). Hereinafter the value of $n = 29$ years is used. The expected duration of the economic life n is chosen based on the normative amortization rate.

Two classical models are considered above, none of which takes into account depreciation elimination. In fact, such an elimination drastically change the patterns of value/depreciation change over time, increasing the value of objects in a short time and creating periods of negative periodic depreciation. None of these models includes a negative periodic depreciation during inter-service periods when the cost of facilities increases.

Based on these simplified models, it is possible to receive a more adequate description of value/depreciation change over time, which reflects short-term inter-service periods when object value increases. All the above mathematical models, which are used in independent valuation theory, involve the use of only positive values of periodic depreciation because during the whole economic life assets mostly lose their value during service periods. There are several assets kinds that demonstrate negative depreciation only during short terms when elimination of depreciation is carried out. These types of assets include almost all buildings and constructions, that may be repaired and reconstructed several times during their economic life, as well as a wide range of equipment, machines and mechanisms, automobile vehicles, railway, aircraft, river, pipeline transport, vessels, electrical power lines, etc. In general, these are tangible assets, characterized by removable depreciation. Following the above stated hypothesis, during the short-term inter-service periods, comparing to service periods, a qualitatively

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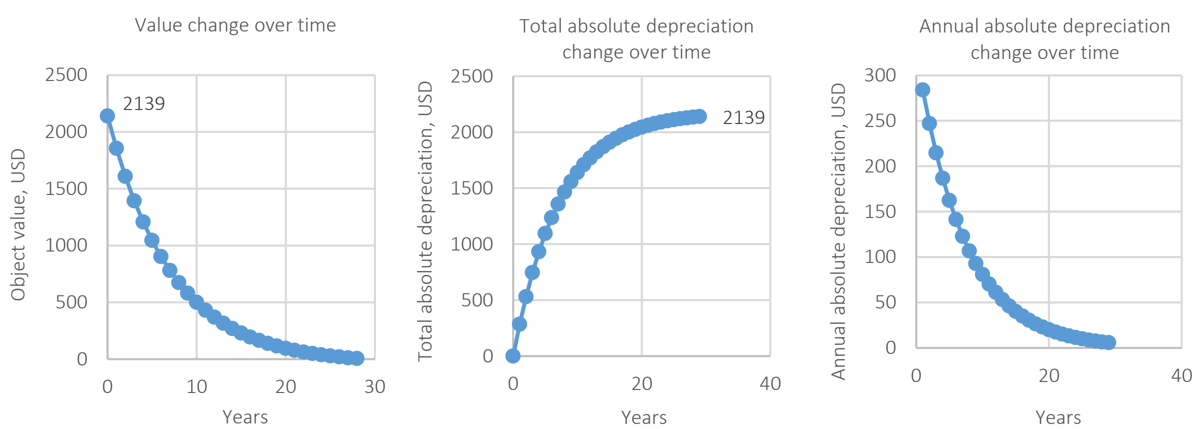

Figure 2. Graphs of the value change over time (left); total absolute accumulated depreciation (middle) and the annual (periodic) absolute depreciation (right). Traditional regressive depreciation model, based on (14)

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different depreciation is observed. When object value increases drastically, it is showing a strong negative periodic depreciation. In fact, long service periods with positive periodic depreciation, which are widely analyzed in the literature, usually differ from the short inter-service periods when periodic depreciation takes negative values.

Let's consider the possibility of creating a model that would be devoid of the identified higher shortcomings. First, it should be taken into account that during inter-service periods assets value increase and its corresponding total depreciation decrease. Following the basic valuation principles of contribution, marginal productivity and changes, the object value during these periods increases, showing a negative periodic depreciation. The principle of contribution, or marginal productivity, implies increase of object value due to any additional factor that improves its functional characteristics from the consumer's point of view. Additional factors, such as equipment repairs and modernization, clearly increase its value. The percentage of this increase may be less or more corresponding to the costs of such works, depending on the efficiency of investments, i.e., how much the value of the object increases. It is assumed that the funds associated with the depreciation elimination increase the object value strictly to the same degree. Thus, it is stated that if during the start x_{rb} and the end x_{rf} of repair works the costs for these works amounted to V_r, then during this period object value has increased on V_r.

Based on the analysis, the improved model of value/depreciation change over time is formulated. It adequately corresponds to the evaluation conditions. According to the proposed approach, it is suggested to use the improved version instead of the primary model with the above-mentioned shortcomings. The improved version takes into account the alternating nature of periodic depreciation. Thus, for a long-term service period, the function of value over time changes for the machinery and equipment (regressive depreciation) can be analytically described by the expression (15) is shown below.

$$V_i = \begin{cases} V_0 \cdot \left(1 - \dfrac{(1+r)^n - (1+r)^{n-x}}{(1+r)^n - 1}\right), \text{ when } x_{rb} \ge x > x_0; \\ \dfrac{(x - x_{rb}) \cdot (V_{rf} - V_{rb})}{(x_{rf} - x_{rb})} + V_{rb}, \text{ when } x_{rf} \ge x > x_{rb}; \ , \\ V_0 \cdot \left(1 - \dfrac{(1+r)^n - (1+r)^{q-x}}{(1+r)^n - 1}\right), \text{ when } x_e \ge x > x_{rf}, \end{cases} \tag{15}$$

Where x_0 – date of the asset creation or acquisition at the primary (initial) cost of V_0; x_{rb} – date of the asset repair and restoration works beginning, when it reaches the value of V_{rb}; x_{rf} – date of the asset repair and restoration works finishing, when it reaches the value of V_{rf}; x_e – date of the end of the asset economic life; q – the number of periods before the end of the economic life of the asset will be reached.

It must be noted that in the described above model (15) object value during the period of repair and restoration works realization is determined by the linear function of its increase from the value of V_{rb} at the beginning of this period to V_{rf} at its end.

3. DISCUSSION

Let's consider the proposed model on a specific example. The task conditions are supplemented by the following data. Let's assume that the operation of the same object during the 8 and 9 years of its economy is overhauled at a total cost of V_r = 614 USD, with the distribution of repair costs over periods under the linear function. Such a long period of repair was chosen for a more suitable visual representation of the value/depreciation change over time. In fact, repair often lasts several months in a year. If a more detailed reflection is required, a smaller accounting period, such as a month or a week, can be used. Figure 3 shows a graphical interpretation of the obtained dependencies.

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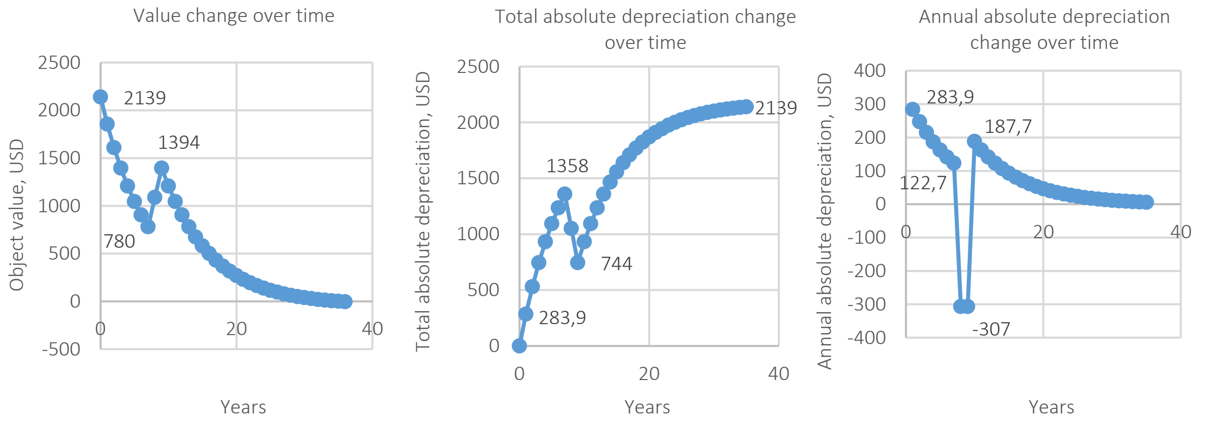

Figure 3. Graphs of the value change over time (left); total absolute accumulated depreciation (middle) and the annual (periodic) absolute depreciation (right). Proposed regressive depreciation model, based on (16)

According to the proposed model (16), at the time interval from the asset's creation date x_0 or acquisition date at initial cost V_0, its change of value over time is similar to the primary model, defined by expressions (11) and (14). As it can be observed, due to the regressive nature of depreciation at the end of the first year, the object value decreased from V_0 = 2139 USD to 1855.1 USD, which led to a positive total absolute depreciation $A_{cad\,i}(t)$ = 283.9 USD and the same index of annual absolute depreciation $A_{aad\,i}(t)$. At the end of the first year, both of these indicators are identical. However, during the next years the situation is opposite: when object value is reduced on a concave graph with the negative first and second derivatives, the indicators of total absolute depreciation increase on a convex graph with the positive first and second derivatives. The indicators of annual absolute depreciation decrease according to the concave function with the negative first and second derivatives, similar to the regularity of value change over time. This is explained by the fact that the indicator of annual depreciation is characterized by the total absolute depreciation. If the first and second derivatives are positive, then annual absolute depreciation will be negative, respectively. From this point of view, the annual absolute depreciation is similar to the first derivative of the total absolute depreciation, but with increments determined by the selected period. As the duration period decreases, its value approaches the first derivative, and at infinitesimal increments, it is equal to it. In some cases, linear nature of the total absolute depreciation change the annual depreciation is constant. In case of its progressive nature with the positive first and negative second derivatives, as described by the concave graph, the value of annual absolute depreciation is characterized by regularity of a convex curve with the negative first and a positive second derivative:

$$\left|\lim_{\Delta t \to 0} A_{aad\,i}(t)\right| = \frac{\partial A_{cad\,i}(t)}{\partial t}. \qquad (16)$$

This is quite similar to the relationship between the functions of object value change over time $V_i(x)$ and the total absolute depreciation $A_{cad\,i}(t)$:

$$\left|\lim_{\Delta t \to 0} A_{cad\,i}(t)\right| = \frac{\partial V_i(t)}{\partial t}, \qquad (17)$$

according to (4).

The modulus of depreciation limit in (17), (18) is determined by the corresponding characteristics (or their change rate), and the derivative is determined by the character of these changes (increase or decrease).

When repair and restoration start during x_{rb} = 8th year (end of the seventh/beginning of the eighth year) the asset reaches the value of V_{rb} = 780.5 USD, with depreciation of $A_{cad\,i}(t)$ = 1358.5 USD, $A_{aad\,i}(t)$ = 122.7 USD. At this point, there is a change in the monotonic nature: the curve of object value V_i (t) changes its first derivative sign – instead of the regressive decrease of the value, its linear characteristics increase. Respectively, the nature of the

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$A_{cad\,i}(t)$ changes. A nonlinear growth of the accumulated total absolute depreciation comparing to its linear decrease is observed. The most interesting thing happens with the indicators of annual depreciation $A_{aad\,i}(t)$: they fall sharply, change the sign and remain constant and negative throughout the whole period of repair. The changes of the $A_{cad\,i}(t)$ is explained by the linear nature of object value and total absolute depreciation changes during this period.

During x_p = 10th years when repair is completed (end of the ninth/beginning of the tenth year) the asset reaches the value V_p = 1394.5 USD, with depreciation $A_{cad\,i}(t)$ = 744.5 USD, $A_{aad\,i}(t)$ = –307 USD. At this point, the graphs change their monotonic nature; object value function $V_i(t)$ changes the sign of the first derivative, its linear growth stops and further regressive decrease of value starts, which was observed before the start of repair. The nature of the $A_{cad\,i}(t)$ also changes abruptly, and linear decrease in the accumulated total absolute depreciation is replaced by its nonlinear growth. The graph of annual absolute depreciation $A_{aad\,i}(t)$ returns to its previous form; its indicators change again, increasing sharply, and continue to decrease nonlinearly, according to the pattern typical for the service period of normal operation of assets, remaining positive and asymptotically approaching a zero level.

CONCLUSION

Inter-service periods, which are ignored in traditional models, lead to a real picture over a long period of distortion and provides for drastic changes in the patterns of value and depreciation during the transition from operational to inter-operational periods and vice versa. The study has shown that during periods of depreciation elimination, the market value of asset increases, and is followed by negative annual depreciation, which also reduces the accumulated total absolute depreciation. It is shown that in case of individual activities, such as repair or restoration, the value increases during short-time inter-service periods, and the coefficient of accumulated depreciation is reduced sharply.

It is possible to conclude that mathematical models are unconditionally necessary, since they are used in evaluation practice, substantiation with more conclusive justification. Indicators of assets value and depreciation dynamics, influenced by individual activites during short-term periods of depreciation elimination, are crucial. It is proved that the change of these indicators during such period completely changes the type of functions that describe the object characteristics. As a result, it is recommended to apply generalized indicators of value and depreciation, statistically defined earlier for certain groups of objects, for evaluation procedures. The strong impact of individual activites proves the need of individual approach to each single unit of property or equipment evaluation. Finally, great variability of the factors associated with these works causes a large variation in the characteristics of the same objects. Specialists of production enterprises and appraisers are well aware that during the initial periods of service life all types of equipment are characterized by almost identical value and depreciation indexes. Instead, as the duration of the operating period increases, the variation of these indicators increases sharply.

A promising area of further research is application of more complicated nonlinear functions, which can be used to empirically determine curves of value/depreciation, which change during the life cycle. A practical and theoretical interest for further research is the quantitative analysis of positive and negative periodic depreciation. Of particular interest is estimation of residual life of assets, which depends directly on single unit's value/depreciation.

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