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Life office management perspectives by actuarial risk indexes

Abstract

The study focuses on the quantitative risk analysis of a pension scheme referred to a portfolio of beneficiaries entering in the retirement state at the same time. The analysis starts from the retirement time of the contractors and concerns the dynamic behavior of the financial periodic portfolio fund cut down year by year by the payments due to the survivals. The fund arises from the payment stream consisting in constant payments due at the beginning of each year in case of life of the pensioner in the deferment period. It gushes that the two forces operating in opposite directions from the retirement age on, in and out of the portfolio, are the increasing effect due to the interest maturing on the accumulated fund and in the outflow represented by the benefit payments due to the survival. The two processes are compared in a scenario in which the financial risk and the demographic risk are considered. Posing the time of valuation coinciding with the contract entry time, we consider deterministically unknown the dynamic of the future behavior of both the interest rates maturing on the fund and of the mortality. The mortality trend betterment in very long periods, as in the pension cases happens, leads to the need of a careful consideration of the systematic deviations of the number of deaths from the expected values. As known, if the risk arising from the accidental deviations of mortality can be hedged by pooling strategies, such that it is possible to neglect this risk source in the case of sufficiently large portfolios, the risk arising from the longevity phenomenon cannot be avoided without making dangerous mistakes consisting in underestimation of future obligations. The scenario in which the study is framed consists in stochastic hypotheses on the evolution in time of the interest rates of return on investment of the fund and in a more complex description of the mortality trend. The aim of the paper is, in particular, to study the effects of the change in the mortality description on the fund portfolio values. The survival forecasting made at the time of the contract issue, even if considered with a certain degree of projection, isn't likely to be the same we can forecast, for example, at the time of the retirement age. The choice of the "right" mortality table means the choice of the "right" projection level to attribute to the mortality trend. Risk filters and opportune indexes are considered and illustrated.

Keywords: life insurance, portfolio net value, survival laws, conditional moments.
JEL Classification: G22, J11, G13.

Introduction

General life insurance contracts, mostly issued for saving purposes, and the global section of pension annuities, are characterized by long or very long durations. In these cases it is opportune to carefully define the details concerning the technical basis in order to correctly compute the premium amounts at the time of issue; the dynamic relationship between the two contractors is based on the specific financial and demographic assumptions on the future background moving evolution.

1. Risk affecting valuations in case of life

The assumptions above introduced have to be outlined knowing the complexity of their display and the market ductility to changes, events surely likely to happen in long periods, both from the financial side and from the demographic one.

The financial risk originates in the financial markets in which the insurer or the pension plan invests. The two fundamental processes involving the management matching purposes are the financial incomes earned by the portfolio fund period by period and the market prices of the financial instruments purchased by the insurer or pension

plan to front the future obligations (cf. Olivieri Pitacco, 2003). Choosing the stochastic approach, the processes follow two different paths due to the peculiarity of each financial market section. If the first one can be described by means of a stochastic model expressing the instantaneous global rate of return on assets in the shape of a short rate, the second must reflect the investment strategy followed to fulfil the obligations towards the policyholders or the pensioners; in this case a stochastic term structure of interest rates based on no arbitrage assumptions can be suitable to the aim.

On the other hand, the demographic risk lies in the uncertainty in the number of survivors at each defined contract term; this can be realized by the uncertainty in the number of premiums going in during the deferment period or of benefits going out during the retirement interval. The demographic risk is due to the accidental deviations of the number of survivors from the expected value at each term and, more important, to the systematic deviations caused by the betterment in mortality. This last gives origin to the longevity risk.

At portfolio valuation level, the first demographic risk component, the mortality risk, can be considered sufficiently hedged in portfolios large enough to achieve pooling effects. On the contrary, to cover the longevity risk component no pooling strategies can be adopted, having to hedge systematic deviations acting on each contract in the same sense.

Only careful assumptions about future evolution of mortality can be at the base of a correct policy aimed to future obligation meeting management, thus implying efficiency in the survival phenomenon monitoring. In what follows we will specifically focus on the demographic systematic side of the risk environment.

2. The mortality model choice

Several models capture in varied ways the behavior of the improvement in the mortality trends; a wide production of projected survival models witnesses to it. An interesting and thorough treatment of the subject can be found Pitacco, 2004; in the paper the survival dynamic models are focused both on their mathematical details and on their longevity description capacity, both in the deterministic and in the stochastic approach. In that paper a wide literature on the matter is reported.

The fundamental observation giving weight to this aspect in actuarial valuations is that tables incorrectly representing the future evolution of the mortality trend, inevitably lead to the underestimation of future costs and consequently to asset liability mismatching risk.

The use of the "right" projected mortality table surely fronts this risk but the question is how to manage the choice? The paper exactly deals with the impact of the choice of different mortality tables in the framework of the portfolio fund by means of an opportune risk index. The aim of the study is to catch useful management information concerning the risk index quantification and behavior in a general plan with pension annuity structure.

The layout of the paper is the following. In section 3 we expose the portfolio fund model. In section 4 we introduce the risk index for measuring the impact of the uncertainty in the mortality table chosen for valuations. Section 5 studies a specific case of a pension annuity portfolio, developing the analysis of the risk index trend in a specific contractual hypotheses with different demographic scenarios, equipped by illustrations and comparisons. The last section provides some remarks with decisional aims concerning the demographic model choice.

3. The portfolio fund model

We consider a pension annuity on an individual aged x at issue time. From this time on, a sequence of premiums P, paid at the beginning of each contract term in case of the beneficiary's life, takes place up to his retirement age. After this point, the sequence of constant benefits R begins at constant time intervals, at the beginning of each contract term if the beneficiary is living. The premiums are deposited in a fund earning interest term by term (cf. Olivieri et al, 2003).

In a portfolio perspective, we consider a cohort of N^0 persons of the same age x, entering in the pension plan above described and reaching the retirement state at the same time n, that is at the age $x + n$.

During the premium payment phase, the portfolio gets richer thanks to the payment of P made at the beginning of each period, say k (k=1,2,…,n-1) by the N^k survivals at time k belonging to the initial group of insureds. These deposits earn a random financial interest rate i^*_k in the time period $(k\text{-}1,k)$.

From time n on, the survivals receive the constant benefit R at the beginning of each contract term, until the contractor is alive.

Indicating by Z_k the portfolio fund consistency at time k, we can write:

$$Z_k = Z_{k-1}\left(1 + i^*_k\right) + N^k P$$

with $k = 1, 2, \dots, n-1$

$$Z_k = Z_{k-1}\left(1 + i^*_k\right) - N^k R$$ (1)

with $k = n, n+1, n+2, \dots, w - x$.

The first formula in (1) refers to the premium deposit period, the *accumulation phase*, while the second one refers to the benefit period, callable *annuitization phase*.

In particular we have:

portfolio fund amount during the accumulation phase:

$$Z_h = \sum_{i=0}^{h-1} N^i P \prod_{j=i+1}^{h}\left(1 + i^*_j\right) + N^h P,$$ (2)

$h = 1, 2, \dots, n-1,$

portfolio fund amount during the annuitization phase:

$$Z_n = Z_{n-1}\left(1 + i^*_n\right) - N^n R$$

and

$$Z_{n+m} = Z_{n-1}\prod_{l=n}^{n+m}\left(1 + i^*_l\right) - \left[\sum_{p=n}^{n+m-1} N^p R \prod_{q=p+1}^{n+m}\left(1 + i^*_q\right) + N^{n+m}R\right],$$ (3)

$m = 1, 2, \dots w\text{-}(x+n).$

4. The risk index

In what follows we consider the random variables *"curtate future lifetime of the i-th contractor"* belonging to the pension plan, independent and identically distributed and independent on the financial process investing the portfolio fund.

Then (cf. Coppola et al, 2002):

$$DMRM = Var_T\left[E[Z_k]\right] \qquad (4)$$

is a measure of the demographic model risk on the portfolio value in k. By conditioning on T we mean the variability due to the randomness in the choice of the survival function used to determine the survival probabilities.

5. Numerical applications

5.1. Preliminary remarks. We consider a pension scheme referred to a cohort of c = 1000 beneficiaries aged x = 45 at time t = 0 and entering in the retirement state 20 years later, that is at the age 65.

The cash flow structure consists in the sequence of constant premiums P, payable at the beginning of each year up to t = 20 in case of the beneficiary's life (*accumulation phase*) and in the sequence of constant benefits R = 100 payable at the beginning of each year after t = 20 (*annuitization phase*), again in case of the beneficiary's life.

The problem outlined in the preceding sections involves the uncertainty in the choice of the mortality model used for valuations.

The goal of this section is exactly the measuring of the impact of this choice on the portfolio fund values. This quantification will be made by the measurement tool, namely the *table risk index DMRM*, introduced in section 4, applied on the portfolio fund values treated in section 3, on the basis of a stochastic hypotheses on the evolution of the interest rates earned by the fund. The study will be developed considering two moments in which the demographic scenarios can change, t = 0, the contract issue time, and t = n = 20, the accumulation period ending time coinciding with the time of entering in the retirement period.

The aim is to investigate the *DMRM* trend in different demographic environments, in order to identify the less risky demographic statement for the pension or life annuity plan and get some information useful for management purposes.

5.2. Financial hypotheses. In these numerical applications we adopt the Vasicek model for describing the instantaneous global rate of return on the assets linked to the portfolio under consideration, without making specific assumptions

on an investment strategy (Olivieri et al., 2003). The short interest rate r_t is given by:

$$dr_t = \alpha(\gamma - r_t)dt + \sigma dW_t, \qquad (5)$$

where $\{W_t\}$ is a standard Wiener process and α, γ and σ are positive constants. As well-known, formula (5) could give negative values and this is not in contrast with our idea of considering a short rate reflecting the global investment strategy connected to the portfolio. For this numerical exemplification, we assign the following values to the model parameters, as in Olivieri et al., 2003: $r_0 = 5\%, \alpha = 0.1, \gamma = 4\%, \sigma = 1\%$.

5.3. Mortality hypotheses. The products we are referring to are clearly affected by the longevity risk, that is by the risk deriving from the stochastic improvement in the mortality trend characterizing all the industrialized countries. In order to capture the effects of this kind of risk, we choose three different mortality tables with different degree of projection. In particular, we model the mortality by considering the following "odds":

$$\frac{q_x}{p_x} = GH^x,$$

which is the third term of the Heligman-Pollard (H-P) law, describing the old-age pattern of mortality (Heligman et al., 1980); G expresses the level of senescent senescent mortality and H the rate of increase of senescent mortality itself. Giving the following values to these parameters (estimated as in Olivieri et al., 2003) :

Table 1. Parameters for the Heligman-Pollard law

	Medium	Maximum
G	0,000002	0,0000001
H	1,13451	1,17215

we get two mortality tables with medium and maximum degrees of projection respectively.

In the application developed in what follows, we consider a third mortality table referring to the Lee Carter (L-C) model, offering a good description of the survival phenomenon being able to correct itself year by year, capturing the changes in the trend due to the longevity phenomenon. The projection level of the Lee Carter survival probabilities is included between the HP medium and the HP maximum probabilities.

5.4. Scenario simulations. Now, referring to the financial and the demographic stochastic environments described above, we evaluate the periodic portfolio funds arising from the two flows in and out of the portfolio under consideration, the first consisting in the

increasing effect due to the interest maturing on the accumulated fund and the second in the outflow represented by the benefit payments due in case of life of the pensioners. These portfolio fund values are considered stochastic both from the point of view of the stochastic term structure of interest rates and for the choice of the mortality table. We assign defined probabilities to the use of each one of the three mortality hypotheses introduced above. In particular, we pose the probabilities 0.20, 0.30, 0.50 to choose the table HP medium, HP maximum and LC, respectively. The aim is to study the table risk behavior in order to collect information about the table risk trend. The application is expanded on three different scenarios. Hence, we furnish a measure of this risk by formula (4) to the end of identifying the less risky scenario.

Scenario A

♦ Premium calculation assumptions:

HP medium,

Fixed interest rate 3%.

In Figure 1 the trend of the portfolio fund from the retirement age 65 on, is illustrated when three different mortality assumptions are made; they are:

♦ HP medium

♦ HP maximum

♦ LC

The figure shows three different behaviors becoming more and more distinct when the age increases.

In Figure 2 the risk index is represented, when assigning the probabilities defined in the subsection 5.4. The table risk increases when the age increases, pointing out a slight stationarity in the age interval about 90.
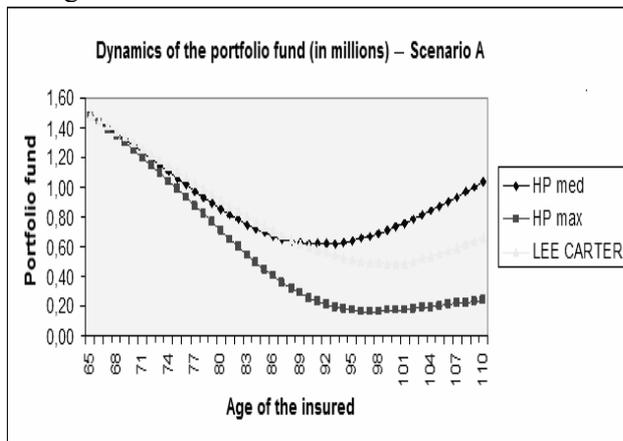

Fig. 1. Portfolio funds – Scenario A

Scenario B

♦ Premium calculation assumptions:

Three different assumptions (HP medium, HP maximum, LC),
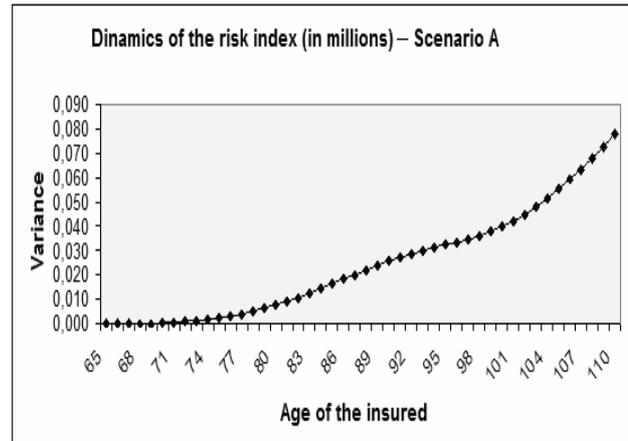
Fixed interest rate 3%.

Fig. 2 Risk index, annuitization phase – Scenario A

In Figure 3 the portfolio fund trend from the retirement age on is again represented, when using in this period the same mortality assumptions made in the premium calculation.

For this reason, scenario B appears to have a sort of homogeneity quality in the demographic description.

The portfolio funds, stressing the performances showed in scenario A, are greater than the corresponding values relating to this scenario. The behavior of the table risk index appears much more interesting and is reported in figure 4, considering the same probabilities of the table choice used in scenario A. In this case we observe an age interval in which the table risk strongly decreases.
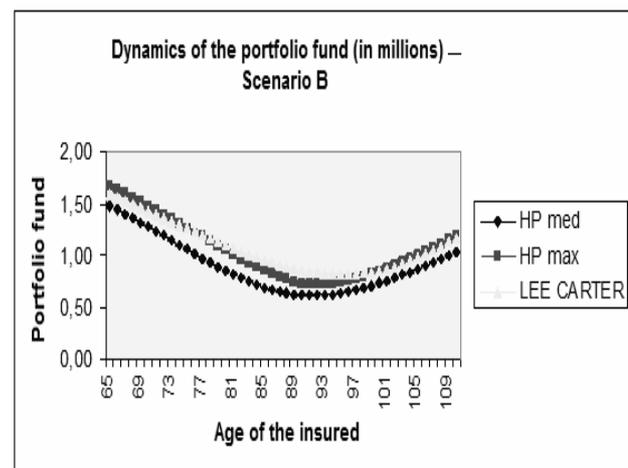

Fig. 3. Portfolio funds – Scenario B

Scenario C

♦ Premium calculation assumptions:
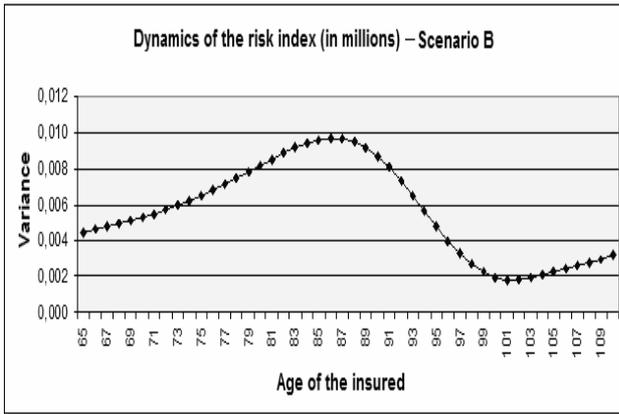
LC,

Fixed interest rate 3%.

Fig. 4. Risk index, annuitization phase – Scenario B

Figure 5 illustrates the portfolio fund performance from the retirement age on when the three different mortality assumptions are made from the contract issue on.

In this case the portfolio funds are quite similar to those of scenario A and the risk index behavior mostly confirms the one represented in scenario A framework.
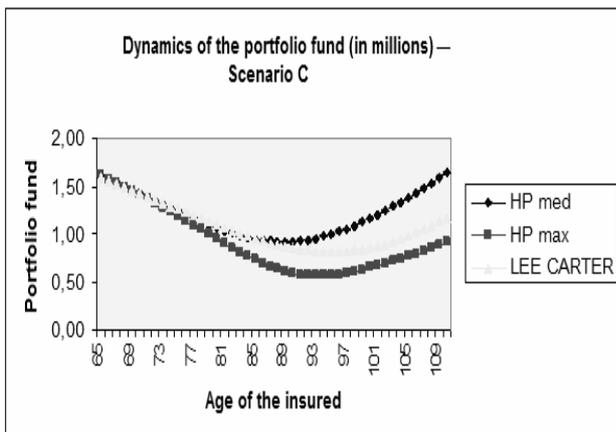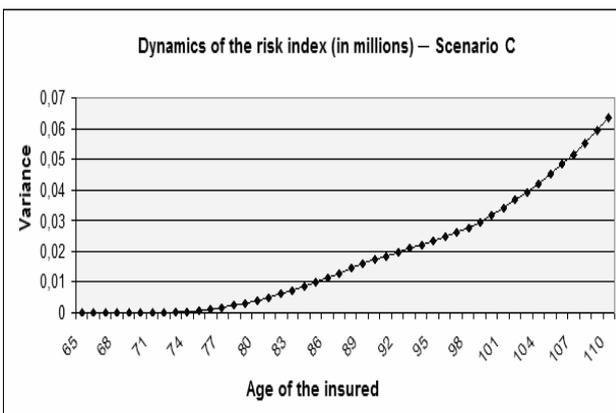

Fig. 5. Portfolio funds – Scenario C

Fig. 6. Risk index, annuitization phase – Scenario C

5.5. Comparison among scenarios. The three situations examined in the applications indicate that the difference among the trends of the risk index in the different scenarios is due to the degree of homogeneity in the demographic profile. The final

aspect emerged in the preceding subsection leads to the comparison reported in Figure 7, in which the three risk index trends are gathered. The figure clearly shows that the scenario presenting the lowest table risk is scenario B, characterized by a strong homogeneity degree in the mortality assumptions made for premium calculation and fund evolution.
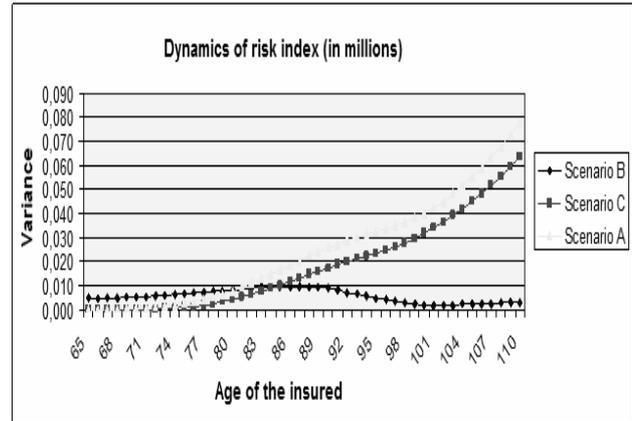

Fig. 7. Comparison among scenarios

5.6. Change of the financial hypotheses. In order to have an idea of the impact of financial hypotheses change on the dynamics of risk indexes in scenarios *A*, *B*, *C*, we consider that during the *annuitization phase period* the fund accrues respectively on the basis of the fixed rate equal to 3% and Vasicek stochastic process, with parameters given in Table 1.

We can say that using a fixed rate equal to 3% produces a squeezing effect on the risk index in each scenario. In Figure 8 we report the comparison of the three risk index behaviors when the rate earned by the portfolio fund is fixed at 3%.
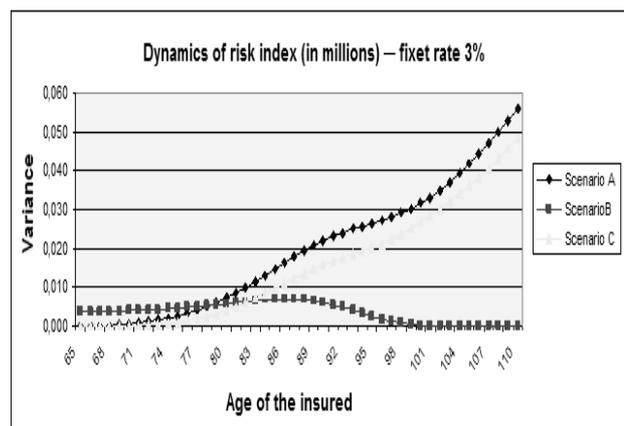

Fig. 8. Comparison among scenarios at fixed rate 3%

Conclusions

The paper deals with the impact of the randomness in the choice of the mortality tables on the portfolio fund values related to a pension scheme or a life annuity plan. In particular we refer to a portfolio of beneficiaries with the same age at entry and retiring

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at the same time as well. The scheme provides for a flow of anticipated constant premiums paid in the accumulation phase and streaming in the fund and for a flow of anticipated constant benefits paid in the annuitization phase and running out of the fund. In this framework, the portfolio fund at time k is defined as the sum effectively existing in the fund at time k, net of the sums going in or of the sums going out at time k. The background hypotheses involve a Vasicek dynamic description for the short rates maturing by the fund and the randomness in the choice of the survival model. The risk index measuring the impact of the mortality table choice is based on variance conditional calculation principle.

The application of the model has been developed pointing out the effects on the table risk index of two statements concerning the demographic assumptions: the first refers to two different demographic assumptions for premium calculation and for the evolution in time of the portfolio fund (scenarios A and C), the second concerns the same demographic assumption for the two processes (scenario B).

The conclusion can be synthesized in the following assertion: the impact of the table risk index on the portfolio fund is strongly compressed when the mortality description is made by means of the same model, that is in frameworks characterized by the homogeneity of theoretical mortality assumptions for the premium calculation and for the dynamics of the portfolio fund. This happens both when a stochastic model describes the evolution in time of the short interest rates earned by the fund and when these rates are assumed to be constant.

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