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ARTICLE INFO	Enrico De Giorgi and Thorsten Hens (2009). Prospect theory and mean-variance analysis: does it make a difference in wealth management?. *Investment Management and Financial Innovations*, *6*(1)
RELEASED ON	Monday, 16 March 2009
JOURNAL	"Investment Management and Financial Innovations"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

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Prospect theory and mean-variance analysis: does it make a difference in wealth management?

Abstract

We show that prospect theory is a valuable paradigm for wealth management. It describes well how investors perceive risk and with appropriate modeling it can be made consistent with rational decision making. Moreover, it can be represented in a simple reward-risk diagram so that the main ideas are easily communicated to clients. Finally, we show on data from a large set of private clients that there are considerable monetary gains from introducing prospect theory instead of mean-variance analysis into the client advisory process.

Keywords: behavioral finance, prospect theory, risk profile, mean-variance analysis.
JEL Classification: D03, D14, D81, G11.

Introduction

Behavioral Finance researchers have amassed evidence that the prospect theory of Kahneman and Tversky (1979) provides a better description of investors' choices than the mean-variance model of Markowitz (1952). For recent surveys of this evidence see, for example, Camerer (1995), De Bondt (1998) and Barberis and Thaler (2003). Prospect theory has been applied to explain low participation in equity markets (Benartzi and Thaler, 1995; Barberis, Huang and Thaler, 2006), the disposition effect (Shefrin and Statman, 1985)[1], insufficient diversification (Barberis, Huang and Thaler, 2006), high trading activities (Gomes, 2005) and investors' preferences for positively skewed payoff distributions (Barberis and Huang, 2008).

Despite the growing consensus among researchers that prospect theory is superior to mean-variance for describing individual preferences, the mean-variance model of Markowitz (1952) remains the industry standard in wealth management. The adherents of mean-variance analysis do not adapt prospect theory for (at least) the following reasons: (I) prospect theory is associated with irrational decisions while mean-variance analysis is believed to lead to rational decisions; (II) prospect theory is more complicated than mean-variance analysis which imposes burdens both on the computational skills and on the communication of the analysis to clients; (III) under standard simplifying assumptions like normally distributed returns, prospect theory and mean-variance analysis almost coincide hence believers in normality see no point to adopt prospect theory.

The goal of this paper is to address all the above arguments in favor of mean-variance analysis and to show that none of them are well hold. We also go further and show that the superiority of prospect theory compared to mean-variance for describing individual preferences translates into a significant additional monetary value to real-world investors. Hence we argue that prospect theory can be introduced as a worth-while innovation in wealth management.

We introduce the main results of this paper in three parts. Firstly, it may well be that prospect theory leads to rational decisions while mean-variance analysis does not. Indeed, mean-variance preferences might lead to violations of monotonicity, i.e., mean-variance investors might display a preference for smaller payoffs when returns are not normally distributed. This is the case with a large number of asset classes, from stocks to alternative investments and structured products.

Secondly, prospect theory can be formulated in a simple reward-risk way similar to mean-variance analysis, i.e., the prospect theory analysis can be displayed in a simple reward-risk diagram which can be interpreted in the same way as the mean-variance diagram, i.e., higher reward implies higher risk and optimal portfolios are those which maximize reward given a risk constraint. Moreover, since prospect theory describes how investors perceive risk, the interaction between clients and financial advisors is facilitated by prospect theory, as the risk-reward diagram is more meaningful to the client.

Finally, we do an empirical exercise to show that mean-variance portfolios are inefficient for real investors, who are in fact best described by prospect theory. This is due to the fact that the observed distributions of returns strongly deviate from normality even for standard asset classes. Thus mean-variance analysis and prospect theory do not coincide in real-world applications. We show that the added value delivered to clients when using prospect theory in-

We are grateful to Mila Winter and Dieter Niggeler for their research assistance.
[1] The disposition effect is the observation that investors tend to hold loser positions too long and sell winner positions too earlier. Recently, Hens and Vlceck (2005) and Barberis and Xiong (2008 a) show that prospect theory does not explain the disposition effect. Barberis and Xiong (2008 b) introduce a model where investors get utility from realized gains and losses and use this model to explain the disposition effect.

stead of mean-variance is high enough to justify the effort of integrating prospect theory into the wealth management process[1]. Also, clients might be willing to pay an additional fee for the service offered, given the superiority of prospect theory asset allocations from their perspective.

The remainder of the paper is structured as follows. Section 1 introduces prospect theory. Section 2 presents an empirical analysis where we compare prospect theory with the mean-variance analysis. The last section concludes.

1. Risk from a Behavioral Finance perspective

While the Modern Portfolio Theory of Markowitz (1952) evolved as a "top-down process" which was influenced by the limited mathematical abilities of the 1950s (see Markowitz, 1991), Behavioral Finance has been developed as a "bottom-up process" by the findings of innumerable controlled laboratory experiments. In the Behavioral Finance based risk theory, the prospect theory of Kahneman and Tversky (1979), aversion to losses is more important than aversion to volatility, which was postulated by Markowitz (1952) as the only measure of risk[2]. Moreover, it is observed that investors are risk averse when comparing two gains, and risk seeking when they can choose between a sure loss and a gamble which gives them the chance to break-even. Finally, prospect theory departs from mean-variance analysis since the former allows investors to overweight small probabilities in their decisions. Since this latter aspect is however also a departure from rational choice, as formalized by expected utility, we will not consider it here. Hence, the recommendations based on prospect theory we consider are consistent with rational choice. Under this assumption prospect theory is described by a value function similar to the risk utility of von Neumann and Morgenstern.

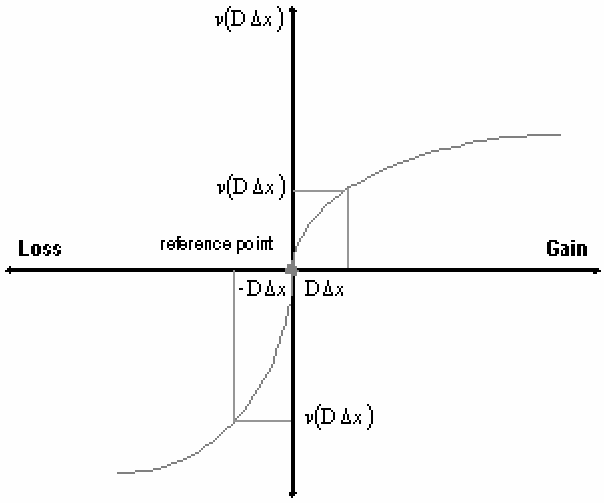
1.1. The prospect theory value function. The prospect theory value function has three important properties:

- It is defined over gains and losses with respect to some natural reference point.
- It is concave in gains and convex for losses.
- The function is steeper for losses than for gains.

These properties of the value function are illustrated graphically in Figure 1, where Δx represents a gain or a loss with respect to some (subjective) reference point and $v(\Delta x)$ is the prospect utility derived from this gain or loss. Tversky and Kahneman (1992) have proposed the following piecewise-power value function:

$$v(\Delta x) = \begin{cases} \Delta x^{\alpha} & \text{for } \Delta x \geq 0 \\ -\beta(-\Delta x)^{\alpha} & \text{for } \Delta x < 0 \end{cases}.$$

Based on experimental evidence they suggest that the median risk and loss aversion of individuals are $\alpha = 0.88$ and $\beta = 2.25$, respectively[3]. This value function has been under fire both theoretically (De Giorgi, Hens and Levy, 2003; Köbberling and Wakker, 2005; Rieger, 2007) and empirically (De Giorgi, Hens and Post, 2007). For example, prospect theory with a piecewise-power value function does not lead to robust asset allocations, i.e., slight differences in investors loss or risk aversion lead to substantially different optimal asset allocations. Consequently, for applications of prospect theory to portfolio selection, other value functions have been used; see, for example, De Giorgi and Hens (2006) and Hens and Bachmann (2008, Chapter 2.4.1).



Note: The x-axis reports gains and losses, while the y-axis reports the corresponding prospect theory value. The origin is the reference point, i.e., the reference point has zero value.

Fig. 1. The prospect theory value function

Given the value function $v(\Delta x)$, the prospect theory decision criterion is described as follows: For any set of scenarios $s = 1,...,S$ occurring with probabili-

[1] The effort for integrating prospect theory mainly refers to the asset allocation side. Indeed, prospect theory requires more advanced optimization techniques. However, the current technology allows to solve the prospect theory asset allocation in few seconds also with large opportunity sets.

[2] Markowitz (1959) suggested semi-variance as measure of risk, which only accounts for negative deviations from the mean.

[3] Experiments also show a high degree of heterogeneity between participants. In our empirical analysis reported in Section 3 we don't use median values of loss and risk aversion, but prospect theory is calibrated to each individual investor.

ties $p_s > 0, s = 1,...,S$ a decision leading to the payoffs $\Delta x_s, s = 1,...,S$ is preferred to one leading to alternative payoffs $\Delta y_s, s = 1,...,S$ if and only if $\sum_{s=1}^{S} p_s v(\Delta x_s) > \sum_{s=1}^{S} p_s v(\Delta y_s)$. Hence, since the value function is increasing, prospect theory is consistent with expected utility theory and thus leads to rational decisions. On the other hand, with general payoffs' distributions mean-variance analysis does not lead to rational decisions since higher payoffs may come along with higher variance, which is the so-called mean-variance paradox. For example the binary lottery delivering a payoff $y > 0$ with a positive probability $p > 0$ while having a zero payoff otherwise will not be preferred by mean-variance investors to the sure payoff of zero if the probability p tends to zero while y tends to infinity and the expected value py is kept constant. In other words, a mean-variance investor may not take a positive payoff even it is without payment (i.e., free), as he perceives risk strictly as variance. Note that this is of high practical relevance since applying the mean-variance criterion to structure products may imply a situation as in the mean-variance paradox. A simple structured product with capital protection has no potential for losses but still it has a positive variance, and may thus be undervalued by mean-variance analysis.

1.2. A reward-risk perspective on prospect theory. A fundamental principle in financial economics that is very useful in the communication with clients is that there is no reward without risk. In the mean-variance framework, the reward-risk trade-off is implemented using the idea that investors who desire to increase the expected return of their investments must accept returns which deviate more strongly from the mean. Actually, the real groundbreaking idea of Markowitz (1952) was the suggestion of a simple reward-risk diagram. That he had chosen the mean return for the reward and the standard-deviation for the risk axis was more for convenience, because at the time it was not possible to efficiently deal with higher moments of the return distribution. Here we suggest a different perspective on implementing the reward-risk-principle (see De Giorgi, Hens and Mayer (2006) for a detailed description). From the investor's point of view, the reward of an investment is not its expected return as in the mean-variance analysis but the expected return over his reference point, its *average gain*. It is defined as the sum of all portfolio returns over the investor's reference point, weighted with the corresponding probabilities as perceived by the investors. More precisely, the *average gain* is defined as:

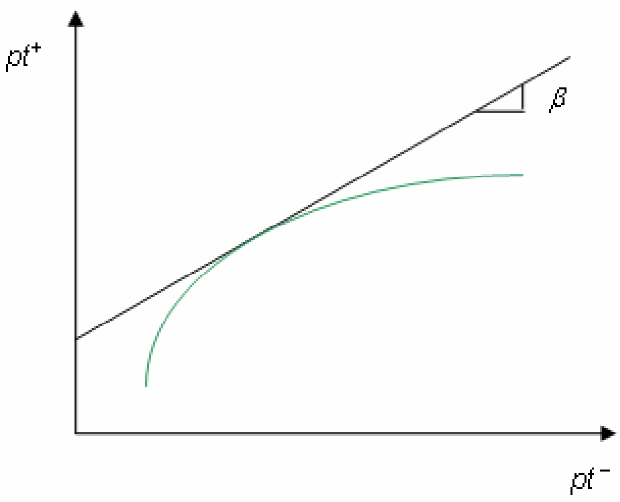
$$pt^{+} = \sum_{\substack{s=1 \\ \Delta s>0}}^{S} p_s v(R_s - RP),$$

where RP is the investor's reference point, R_s is the return of the portfolio in state s and $\Delta_s = R_s - RP$.

Respectively, the risk of the investment is not the deviation from the expected return as in the mean-variance analysis but the expected portfolio return below the investor's reference point. This is the portfolio's *average loss*, i.e.

$$pt^{-} = -\frac{1}{\beta} \sum_{\substack{s=1 \\ \Delta s<0}}^{S} p_s v(RP_s - R_s),$$

where, as above, β is the investor's loss aversion. Note that average gains and average losses are expressed in utility terms to account for investors' risk attitudes over gains and losses. Moreover, the average loss is multiplied by minus one to obtain a positive measure of risk. Finally, the average loss is normalized by the investor's loss aversion β, since β does not describe investors' attitude on losses, but the investor's tradeoff between gains and losses. Indeed, the utility over the average gains and losses is $PT = pt^{+} - \beta \cdot pt^{-}$ and β plays the same role of variance aversion in the mean-variance model. Graphically, the return-risk perspective can be represented as in Figure 2. Hence changing the degree of loss aversion, i.e., the slope of the straight line in Figure 2, different portfolios on the prospect theory efficient frontier can be selected.



Note: Risk is measured as the absolute value of the normalized value of loss, while reward is the value of gains. The trade-off is given by loss aversion β, which determines the optimal asset allocation on the prospect theory efficient frontier.

Fig. 2. Reward-risk diagram of prospect theory

2. Prospect theory and mean-variance analysis

Even when returns are normally distributed, it is not clear whether prospect theory and mean-variance analysis deliver the same set of efficient portfolios. Indeed, while it is clear that prospect theory decisions only depend on mean and variance when returns are normally distributed (since any risk utility integrated over a normal distribution only depends on mean and variance), in general, prospect theory does not imply variance aversion since prospect theory investors are risk seeking over losses. Levy and Levy (2004) show that the prospect theory efficient set is a strict subset of the mean-variance efficient set under the conditions that returns are normally distributed and portfolios are formed without restrictions, e.g., no short-sale constraints. Moreover, in this case, the subset of mean-variance efficient portfolios which are prospect theory inefficient is small. However, it is well known that for most assets the assumption of normally distributed returns has weak empirical support. Moreover, individual investors often face short-sale constraints. Therefore, we could expect relevant differences between the prospect theory efficient set and the mean-variance efficient set when more realistic assumptions are made concerning return distributions and portfolio restrictions. Whether this difference is large depends, for example, on how return distributions depart from the normal distribution and on how higher moments of the distribution impact the prospect theory value function. The latter point is obviously also related to the investor's degree of loss aversion and risk aversion or risk seeking behavior on gains and losses respectively.

Our empirical analysis addresses the following question: Assuming that prospect theory is the correct model to describe investors' preferences, what is the added value in monetary terms when an investor chooses an optimal portfolio from the prospect theory efficient set instead of choosing from the mean-variance efficient set? As discussed before, Levy and Levy (2004) show that in the case of normally distributed returns and no restrictions on portfolios, this added value is zero, i.e., the optimal asset allocation for a prospect theory investor belongs to the mean-variance efficient set. Therefore, Levy and Levy (2004) conclude that a prospect theory investor should not determine the prospect theory efficient set (as this is more complex to do), but simply optimize the prospect theory value function over the mean-variance efficient set. Does this result hold in general?

We use data from 792 private investors. For each investor in our dataset we calibrated an extended version of the prospect theory value function using the BhFS risk profiler[1,2]. We denote by V^i the calibrated prospect theory value function for investor $i = 1,...,792$. Therefore, we do not use the median parameters, as is usual in the behavioral finance literature, as the investors display a high degree of heterogeneity in our dataset. Using the calibrated value function we calculated for each investor two different asset allocations: 1) the optimal prospect theory asset allocation from the prospect theory efficient set (portfolio 1); 2) the asset allocation on the mean-variance efficient set with the highest value given the investor's prospect theory value function (portfolio 2). The optimization algorithm to find the optimal asset allocation in the prospect theory efficient set is described in De Giorgi, Hens and Mayer (2007). For investor $i = 1,...,792$, we denote by R_1^i the (random) return of portfolio 1 above and by R_2^i the (random) return of portfolio 2.

Since any positive linear transformation of a value function delivers the same optimal asset allocation as the original value function, the difference in utility levels can be made as small as possible for any two portfolios, and thus differences in utility are not very informative. Moreover, utility levels for two different investors cannot be compared in general. Therefore, for each investor we compare the certainty equivalents of the two optimal asset allocations instead of the prospect theory value. The certainty equivalent corresponds to the risk-free payoff that delivers the same prospect theory value as the risky portfolio, i.e., the risk-free return r such that $V^i(r) = V^i(R)$, where R is the (random) return of the risky portfolio. Note that certainty equivalents are not affected by any positive linear transformation of the value function.

For $k = 1,2$ and $i = 1,...,792$ let r_k^i be the certainty equivalent of portfolio *k*. Obviously, $r_1^i \geq r_2^i$ for all *i*, since portfolio 1 is prospect theory efficient and V^i is an increasing function. We call the difference $\Delta r^i = r_1^i - r_2^i$ the added value in monetary terms for using portfolio 1 instead of portfolio 2.

Figure 3 shows the distribution of annualized monetary added value in base points (bps) for our dataset with 792 investors.

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[1] BhFS stands for Behavioral Finance Solutions, a spin off firm of the University of Zurich that transfers research in Behavioral Finance into the banking industry, for details see: www.bhfs.ch.

[2] The extension of prospect theory used in this work relates to a different specification of the value function in order to solve the robustness problems of the piecewise-power function suggested by Tversky and Kahneman (1992).

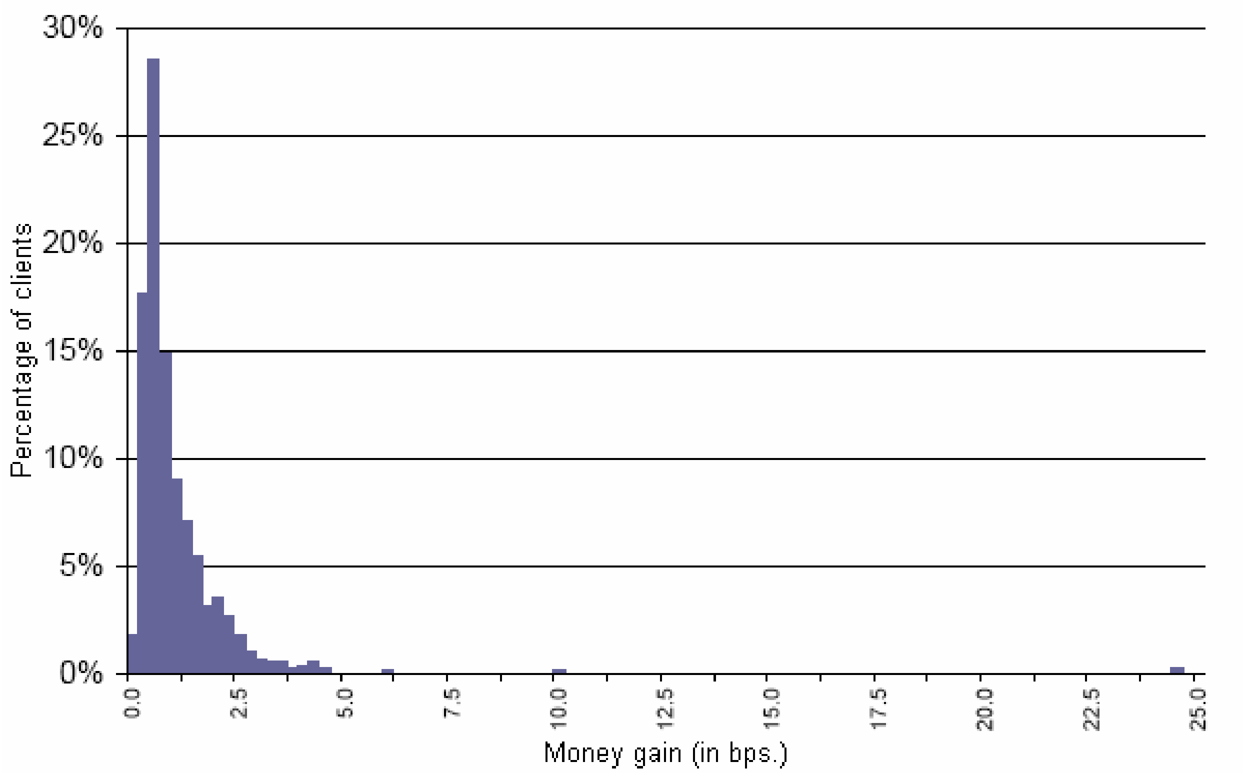

Fig. 3. Distribution of monetary added value Δr^i based on a comparison between the prospect theory efficient portfolios and mean-variance efficient portfolio with the highest prospect theory value

The average added value from investing in the prospect theory optimal portfolio (portfolio 1) instead of the optimal asset allocation on the mean-variance efficient frontier (portfolio 2) is 0.88 bps. About 10% of the investors experience an added value which is larger than 2 bps, while about 1% of the investors have an added value between 5 and 24 bps per annum.

We run a simple linear regression model to analyze the relationship between investors' characteristics (reference point, loss aversion, risk aversion) and the monetary added value for holding the prospect theory efficient portfolio instead of the mean-variance portfolio. We found that loss aversion, the reference point and the aspiration level have a statistically significant impact on Δr^i[1]. The difference between the aspiration level and the reference point is positively related to investors' risk aversion. When this difference is higher, investors' display, on average, a higher risk tolerance since they are willing to take more risk on order to achieve a higher average return above the reference point. In our dataset, the difference between the aspiration level and the reference point, as well as the aspiration level itself, are positively related to the added value, while loss aversion is negatively related to it. To summarize, investors who have a higher aspiration level display a higher risk and loss tolerance, also obtain a higher added value from the prospect theory efficient portfolio compared to the mean-variance portfolio. For comparison, investors in both the lowest 20% quantile for loss aversion and the highest 10% quantile for the aspiration level, have an average added value of 1.62 bps (almost twice the average added value over the whole sample). In contrast, investors both in the highest 20% quantile for loss aversion and the lowest 10% quantile for the aspiration level, have an average added value of 0.70 bps (which is less than the average added value over the whole sample).

While these numbers are small, consider a bank which uses the prospect theory approach for the 792 clients in our dataset instead of choosing clients' optimal portfolios from the mean-variance efficient frontier. Suppose that a typical private banking client holds $1 million, remains at the bank for at least five years, and asset allocations are updated annually. Then assuming an interest rate of 2%, the present value of the total added value from using the prospect theory efficient portfolio instead of the mean-variance portfolio is $324'950. Setting this in relation to the current assets under management of $792 million, we find that the bank could ask an additional fee of 4 bps if it uses prospect theory efficient portfolios

[1] The aspiration level differs from the reference point and is used to calibrate investors risk aversion. The aspiration level is higher than the reference point and determines the average gain above the reference point the investor wants to achieve.

instead of mean-variance portfolios. Put into a different perspective, the added value delivered to a large number of clients is worth the additional cost of implementing the prospect theory approach.

In typical implementations of mean-variance analysis in wealth management, clients' profiles are mapped into a few master portfolios on the mean-variance efficient frontier. Risk profile questionnaires based on mean-variance are designed accordingly. One reason for this might be that it is difficult to calibrate client's variance aversion, partially because variance differs from investors' perception of risk. A (personalized) optimal asset allocation is superior only for those investors who are able to express their volatility aversion, and who's advisor is able to accurately calibrate their volatility aversion in the first place.

Recently, Das et al. (2008) have proposed a way to determine investors' aversion to variance starting from a notion of risk which is more familiar to investors, i.e., the possibility of missing a given target return or reference point. Das et al. (2008) state that investors are better calibrated about their tolerated probability of missing the target return than about their variance aversion. Similarly, the BhFS risk profiler used to calibrate the prospect theory value function addresses investors using their own notions of risk, e.g., losses below a target return or reference point. Therefore, we expect that a move from the current scenario to a prospect theory approach with more accurate risk profiling offers even more value than what is described above.

Assuming that prospect theory is the correct model to describe clients' preferences and that prospect theory is well calibrated given that it incorporates investors' notions of risk, we now address the question of the added value of using prospect theory efficient portfolios instead of master portfolios on the mean-variance efficient frontier.

We define five master portfolios on the mean-variance efficient frontier and using the calibrated value function we obtain for each investor in our dataset the master portfolio with the highest prospect theory value (portfolio 3). For investor $i = 1,...,792$, we denote by R_3^i the (random) return of portfolio 3 and by r_3^i the corresponding certainty equivalent. Recall that r_1^i is the certainty equivalent of the optimal asset allocation from the prospect theory efficient set. Again, $r_1^i \geq r_3^i$ and for each investor $i = 1,...,792$ we define the added value in monetary terms for using the optimal asset allocation from the prospect theory efficient set instead of mean-variance master portfolios as $\widetilde{\Delta} r^i = r_1^i - r_3^i$.

Figure 4 shows the distribution in our dataset of the annualized values for $\widetilde{\Delta} r^i$ in bps.

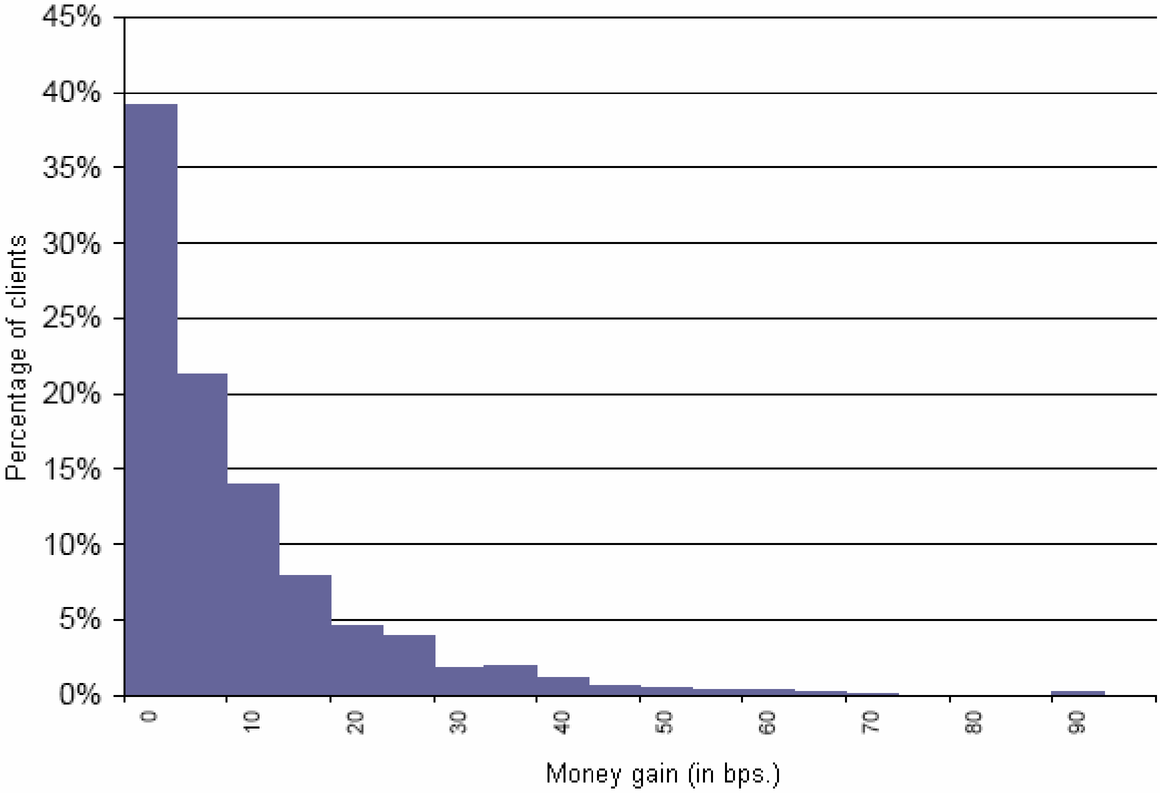

Fig. 4. Distribution of monetary added value (in basis points) based on a comparison between prospect theory efficient portfolios and master portfolios on the mean-variance efficient frontier

Again, consider a bank with our 792 clients and suppose that the average wealth of a private banking client is $1 million. Then, the average added value in dollar terms when using the prospect theory optimal allocation instead of mean-variance master portfolios corresponds to $1'087. The total added value for all 792 clients is $868'735. If clients update their asset allocation annually, this figure refers to annual gains. If clients remain at the bank at least five years, we find that the bank could ask an additional fee of 52 bps if it uses prospect theory efficient portfolios instead of mean-variance master portfolios.

Conclusion

We see prospect theory as a major breakthrough in decision theory that – if modeled carefully – can improve on the mean-variance analysis in particular with respect to wealth management applications such as client risk profiling and creating client's asset allocations.

A careful modeling of prospect theory can ensure that the decisions based on it are fully rational – even in the case of non-normally distributed returns like that of structured products where mean-variance analysis is found to fail. Moreover, nothing is lost in terms of simplicity as prospect theory can easily be formulated in a reward-risk way similar to mean-variance analysis, and thus a simple tool to communicate with clients can be used. Finally, to support our qualitative arguments, we measured the clients' added value from holding a prospect theory optimal portfolio as compared to a mean-variance asset allocation. Using real data, we show that a bank following the prospect theory approach could increase its management fees with 4 bps or 52 bps depending on the degree of personalization the bank offers to its clients when determining their optimal asset allocations. Note that offering personalized asset allocations depends on having the investors' preferences well calibrated. This is possible with prospect theory, being a model that uses investors' notions of risk, and more difficult with mean-variance. Depending on a banks' assets under management, it can expect to gain considerably from using prospect theory instead of mean-variance analysis.

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