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Implied volatility and future market return

Abstract

This study examines the predictability of the implied volatility (IV) of stock option contracts on the future market return. Using return and options data of the S&P 100 index between 1996 and 2008, we find that when the market return drops significantly, a high IV strongly predicts a future market reversal. On the other hand, when the market return drops only modestly, a high IV actually predicts a continuing market loss. We, then, develop and explore two trading strategies based on our findings, which yield much higher risk-adjusted returns than the S&P 500 index.

Keywords: implied volatility, predictability, contrarian trading strategy, momentum trading strategy.

Introduction

Does the implied volatility (IV) of stock options predict stock returns? The answer to this question particularly pertains to a strand of trading strategy called volatility timing, and could potentially help investors to make more accurate asset allocation in the portfolio.

Previous studies (e.g. Giot, 2005; Doran et al., 2010) found that IV was a weak predictor of the future market return[1], although many practitioners suspect that a stronger relation exists between current IV and future return. In addition, it is widely accepted that volatility timing can improve portfolio returns (see Eraker et al., 2003; Fleming, Kirby and Ostdiek, 1999; and Johnnes et al., 2001). These studies have shown that current state of the conditional volatility is very informative about future daily or weekly returns. Given that IV is a natural measure of the conditional volatility, there may be a stronger correlation between IV and future returns.

However, the current literature has mixed results regarding this issue. Backus and Gregory (1993) report a decreasing or zero relation between future market risk premium and conditional variance of market return. Whitelaw (1997) also calibrated reasonable parameters for a negative relation in a single factor model. But Scruggs (1998) shows that there could be a positive relation, if more factors are included in the model.

In this paper, we investigate the relation between future market return and market's conditional variance based on a different approach. Using a standard dynamic factor model of return proposed by Campbell and Yogo (2006), and Fama and French (1988), we show analytically that the sign of the relationship is nonlinear, i.e., the prediction of IV is not universal across all states of the market, instead it depends on current market return. We attribute this nonlinear relation to the fact that market will change its course when it reaches a reference point, otherwise, it will continue its trend[2]. We show analytically that such reference point depends on the level of IV. Therefore, identical IV value may forecast future return differently depending on how the market is currently performing.

Our method of empirical analysis is inspired by the regime-switching[3] method that is used extensively in modeling nonlinearity. Using return and options data of the S&P 100 from 1996-2008, we examined the relationship between future weekly market return and the IV on S&P 100 for both near- or at- the money call and put options. We ran simple OLS regressions of future market return onto the current IV conditional on immediate return of the S&P 100 index. Our regression results confirm the theoretical hypothesis of nonlinearity. To be specific, when current weekly return on S&P 100 is below -2%, the regression coefficient is positive, implying that a high IV predicts a possible future market reversal when current weekly return on S&P 100 is between -1% and -2%, the regression coefficient is negative, implying that a high IV predicts a continuing future market loss; when current weekly return on S&P 100 declines by less than 1% or rises, there is no clear relationship between level of the IV and the subsequent market return. In contrast to the previous papers, these results suggest there is not a simple, uniform relationship between the future market return and the conditional volatility across all market conditions.

Our result reaffirms that market timing decision based on options implied volatility is profitable, but it differs from previous studies in two important aspects. First, in contrast to Giot (2005) and Doran et al. (2010), our result strongly supports the notion that the conditional volatility derived from options predicts future returns. Timing based on the IV is potentially profitable. Second, levels of the IV point to different directions of movement in the future return under different state of the market. It implies that under specific conditions,

[1] Giot (2005) reported high level of VIX predicted of future market return reversal weakly. Doran et al. (2010) showed that a specific type of skewness in implied volatility predicted future returns. Other related research include: Copeland and Copeland (1999) reports strong negative correlation between contemporaneous market return and implied volatility. Banerjee et al. (2007) reported that VIX is negatively correlated with stock market returns, a risk factor affecting the stock market. But these results did not establish clear predicting direction of implied volatility as they are about the contemporaneous relationship.

[2] Models in behavioral finance, such as Daniel et al. (1998) and Hong and Stein (1999), found that stock market return may behave differently depending on the state of the market.

[3] See, for example, James Hamilton (2008).

reducing market exposure when the market volatility increases could be detrimental to investors. A smart investor should in fact add exposure to market when it just had a big loss even the volatility is high.

Based on our finding, we explore two trading strategies conditional on current market returns and levels of the IV of S&P 100 options. Given a high level of the IV, a Contrarian Trading Strategy (CTS) bets the market is about to revert itself after it was crashed in the previous week and thus initiates a long position in the market. On the other hand, given the same high current level of the IV, a Momentum Trading Strategy (MTS) indicates that the market will continue to decline after its moderate drop recently, thus a short position in the market is established. Our performance test shows that both strategies generate very impressive (risk-adjusted) profits. Our finding indicates that traders, who desire to explore temperate market inefficiency, could benefit from trading signal provided by high levels of the IV. We provide a theoretical justification for these trading strategies as well.

1. Data and methodology

1.1. Data. We constructed our sample by using the S&P 100 index[1] in the CRSP database and the corresponding option data from Option Metrics. We calculated the weighted average of implied volatility on each Wednesday from June 5, 1996 to Sep 10, 2008 (634 data points overall). We investigated the relationship between this measure of IV and the weekly holding period return of the S&P 100. We dropped index options that trade for $0.05 or less. All options in the analysis had to have 100 or more contracts in the open interests. To avoid noise from far-term options and short-term options, we used only options that expired in 10-60 days.

1.2. Implied volatility. For all options, we used the implied volatility of at-the-money and near-the-money calls and puts as well as the volume of the open interest to construct the weighted average IV. The weighting scheme is suggested by Latane and Rendleman (1976), and Stewart (1995).

$$IV = \frac{\sum_{i=1}^{N} w_i IV_i}{\sum_{i=1}^{N} w_i},$$

where the weights w_i are the volume of open interests and IV_i is the implied volatility of a given option.

We define the moneyness of an option by the strike-to-spot ratio, $m_i = E_i / s$. An ATM option has a strike-to-spot ratio between and including 98% and 102%, while a near-the-money option has a ratio following in either 90%, 98% or 102%, 110%. We deleted the observations if the strike-to-spot ratio is outside these ranges.

1.3. Regression model. In this section, we show analytically that a nonlinear relation exists between the future market return and conditional variance. The nonlinearity comes from the state-dependent nature of the regression coefficient. For this reason, we justify that the empirical analysis of the relation must also be state-dependent.

We start with the following basic predictive regression:

$$r_{t+1} = \alpha + \beta\sigma_t^2 + \varepsilon_{t+1}, \qquad (1)$$

where r_{t+1} is the future market return. The predicting variable $\sigma_t^2 \equiv Var\left(r_t \mid I_t\right)$ is the conditional variance of market return r_t, where I_t denotes available information up to time t. In our empirical analysis the measure of the conditional variance is the IV.

The coefficient β in equation (1) measures the marginal effect of the conditional variance on the future market return. A positive (negative) β indicates a positive (negative) relation. The estimation of β is not performed on the whole sample. Instead, we separate our sample into several sub-samples according to current market returns r_t. To justify this procedure, we cite a standard information updating process as in Fama and French (1988), Pertoba et al. (1987), Timmermann, A. (1996) and Campbell and Yogo (2006). Following the above authors, we postulate that the market return follows a dynamic factor model:

$$\begin{aligned} r_t &= x_t + v_t, && v_t \sim N(0,\sigma_v^2), \\ x_t &= ax_{t-1} + w_{t-1}, && w_{t-1} \sim N(0,\sigma_w^2), \end{aligned} \qquad (2)$$

where x_t is the unobservable latent factor, that drives the return process. The parameter a measures the persistency of the return process. In an equilibrium model, the conditional variance will affect the future market return r_{t+1} through market participants' updating mechanism. A typical updating process will include the past estimates of market return and the conditional volatility through a non-linear functional form. To see this, we compute the expected future market return conditional on information available at time t using a recursive Bayesian updating for-

[1] We find similar results from S&P 500 index and its options. We do not supply these results here, but they are available upon request. In this sense, we believe our results reflect the genuine behaviors of market return and its related options. We use IV of the index options instead of VIX index as our predicting variable. VIX is a symmetrical volatility measure in that it treats sharp movement in both directions equally. Our interest is mainly to explore the asymmetrical response to past IV in conjunction with previous market movement.

mula. We define the expected return conditional on time t information as $\hat{x}_t \equiv E_t(x_t)$. Then we have the following recursive formula:

$$E_t(r_{t+1}) = a\hat{x}_t,$$
$$\hat{x}_t = \left(1 - k_t\left(\sigma_t^2\right)\right)a\hat{x}_{t-1} + k_t\left(\sigma_t^2\right)r_t, \tag{3}$$

where $k_t \equiv k_t\left(\sigma_t^2\right) = \sigma_t^2 / \sigma_v^2$ is called the Kalman filter gain and the conditional variance itself follows a recursive formula $\sigma_t^2 = \frac{\left(a^2\sigma_{t-1}^2 + \sigma_w^2\right)\sigma_v^2}{a^2\sigma_{t-1}^2 + \sigma_w^2 + \sigma_v^2}$.

Equations (3) are the Kalman filtering that are derived from the Bayesian updating. The formula indicates that the expected market return is a weighted average of the observed return and previous estimation of the factor[1]. Thus, the coefficient β in equation (1) is the derivative of market return with respect to the conditional variance σ_t^2:

$$\beta = \frac{\partial E_t(r_{t+1})}{\partial \sigma_t^2} = g\left[\frac{a\hat{x}_{t-1}}{\sigma_v^2} - \left(\frac{1}{\sigma_v^2} + k_t\right)r_t\right], \tag{4}$$

where $g = \frac{a}{a\frac{\partial\sigma_{t-1}^2}{\partial\sigma_t^2} - ak_t\frac{\partial\sigma_{t-1}^2}{\partial\sigma_t^2} - 1}$ is mostly a positive number.

The value of the Kalman filter gain k_t lies between 0 and 1. Based on equation (4), it is now straightforward to observe that the sign of β depends on the sign of the observed market return r_t. β tends to be positive when $\hat{x}_{t-1}$ (or r_t) is a large positive (or negative) number, and β tends to be negative or zero, otherwise. For instance, given a negative value of $\hat{x}_{t-1}$, the sign of β will become positive if the current return r_t is a large loss, otherwise it will be negative. Similarly, the sign of β can be analyzed analogously when given a positive value of $\hat{x}_{t-1}$. We recognize that $\frac{a\hat{x}_{t-1}}{\sigma_v^2}$ plays a reference point for the sign of β[2]. Hence, simply running a regression of r_{t+1} onto the conditional variance σ_t^2 would generate a spurious relation between the future market return and the conditional variance.

It follows that we should run simple regressions of equation (1) conditional on the value of current market return r_t. Similar to the regime-switching models, we therefore sorted weekly market returns into the following brackets[3] (-∞, -2%), [-2%, -1%), [-1%, 0), [0, 1%), [1%, 2%) and [2%, +∞). We term each bracket as one state of the return, which we denote as s_i. We perform analysis conditional on each state of return. Specifically, equation (1) is slightly modified into the following conditional regression:

$$r_{t+1} = \alpha + \beta_1 IV_t + \varepsilon_{t+1}, \quad \text{if } r_t \in s_i$$

for i = 1,2,…,6, where s_1 = (-∞, -2%), s_2 = [-2%,-1%), etc. Here, we use the implied volatility IV_t as the measurement of the conditional variance σ_t^2. In addition, the regression is performed across all moneyness (OTM, ATM and ITM), types of options (i.e. call and put), and six states of return. A total of 36 OLS regressions is performed.

2. Empirical results

Our empirical results include descriptive statistics, regression results, and performance of trading strategies based on state of the index return and the IV.

2.1. Descriptive results. Table 1 shows the summary statistics for the sample sorted by the state of weekly returns in the current week. The first column lists six groups (states) of current returns. Within each group we report the average of returns on Panel A and implied volatiles on Panel B. Average IV is reported according to the type of options (call put) in different moneyness as well as the number of observations.

Panel A shows that the average market return is about 0.89% (-0.06%) following a large negative (positive) return of less (more) than -2% (2%) in the previous week, which indicates a return reversal. In contrast, the market return generally shows momentum when the return is between [-2%, -1%) or [1%, 2%), because the return in the next week carries the same sign as that in the past week. We did not find clear direction in future return following a return between -1% and 1% in the current week.

The volatility statistics in Panel B show that a put option has higher IV than a call option with the same strike price all the time. For example, when the return is below -2%, the IV of an out-of-money call option is 21.55% in the same week, while the IV of an in-the-money put option is 23.62%. This finding is consistent with previous works on implied volatility skew (e.g., Doran and Kreger 2010).

[1] For detailed derivation, please referred to Hamilton (1994) or Green (2008).

[2] We term this phenomenon the reference effect.

[3] These brackets are selected through trials and errors.

Table 1. Summary statistics, June 1996-September 2008

		Panel A	Panel B. Average IV in the current week					
If return in current week is		Return in the next week	OTM call	ITM put	ATM call	ATM put	ITM call	OTM put
<-2%	Average	0.89%	21.55%	23.62%	24.40%	25.11%	27.33%	28.69%
	n	91	91	89	89	91	71	91
[-2%,-1%)	Average	-0.23%	16.70%	18.88%	18.69%	19.67%	22.02%	22.99%
	n	75	75	62	73	75	61	75
[-1%,0)	Average	0.12%	15.12%	18.08%	17.24%	17.98%	20.42%	21.39%
	n	117	116	80	115	116	100	116
[0%,1%)	Average	-0.07%	14.25%	17.89%	16.23%	17.14%	19.33%	20.64%
	n	125	125	76	122	125	116	125
[1%,2%)	Average	0.15%	15.51%	18.62%	17.53%	18.14%	20.96%	21.62%
	n	117	117	63	116	116	113	117
>2%	Average	-0.06%	19.48%	22.41%	22.19%	22.57%	25.52%	26.19%
	n	102	103	72	102	102	98	103
Total	Average	0.13%	16.86%	20.06%	19.12%	19.83%	22.25%	23.32%
	n	627	627	442	617	625	559	627

Note: Panel A shows the average return in the following week for each sample. Panel B shows the average IV for each group. The size of each sample is indicated by *n*.

There is a common pattern of IV along the dimension of current return for all options. The IV is the highest when the current return is below -2%. It declines as the current return increases to between 0% and 1%, and then it starts to go up. For example, the volatility has its highest value of 28.69% for the out-of-money put option when the return is below -2%. The same IV drops to 20.63% when the current return is between 0 to 1%, and it creeps back to 26.18% when the market had a return of more than 2%. This U-shape movement of the IV along the dimension of return indicates that the IV is high when the market has more extreme returns, where investors may reflect on more uncertainty.

2.2. The relation between IV and market return. We are interested in finding how the IV predicts the future index return. We conditioned our OLS regression on each of the six different states of the market return. We report the results for both call and put options in separate tables. Since there are three types of moneyness for each option, we have 18 regressions in each table.

Table 2 reports estimation results for the 18 conditional regressions for call options. Each non-parenthesized number represents the estimate of slope for one type of moneyness conditional on one state of return. We suppress estimates of intercepts. Column A indicates that the subsequent return would start to increase if the return has dropped more than -2%. The size of increase is proportional to the IV because the regressions generate a positive coefficient with more than 99% of confidence level. The IV of out-of-money calls has the largest coefficient with a value of 0.188. Since the average IV is about 20%, it would add about 3.76% to the next return.

Surprisingly, column B shows that the subsequent return will continue to decrease if the current week's return has dropped by more than 1% but not greater than 2%. The IV of out-of-money calls predicts most decline with a negative coefficient of -0.131.

Table 2. Regression coefficients of call option IV on S&P 100 index return.

IV_t	A. $r_t \in (-\infty,-2\%)$	B. $r_t \in [-2\%,-1\%)$	C. $r_t \in [-1\%,0)$	D. $r_t \in [0,1\%)$	E. $r_t \in [1\%,2\%)$	F. $r_t \in [2\%,\infty)$
OTM call	0.188***	-0.131***	0.0566	-0.0428	-0.0381	0.0300
	(0.0508)	(0.0478)	(0.0385)	(0.0362)	(0.0416)	(0.0476)
ATM call	0.152***	-0.110**	0.0402	-0.0265	-0.0315	0.0281
	(0.0457)	(0.0449)	(0.0340)	(0.0317)	(0.0369)	(0.0441)
ITM call	0.144**	-0.100*	0.0497	0.00658	-0.0359	0.0294
	(0.0555)	(0.0546)	(0.0346)	(0.0305)	(0.0381)	(0.0402)

Note: The model is $r_{t+1} = c + \beta IV_t$, where IV_t is the implied volatility from one of the following call options: out-of-the-money, at-the-money and in-the-money calls. Each number without parentheses is an estimate of β for one of the 18 OLS regressions for call option. The numbers in parentheses under estimates are the standard deviations of the indicated variable. The sample of each regression is selected according to the return in previous week. The significance level of estimates is indicated by the number of asterisks: 1% (***), 5% (**) and 10% (*). All sample size of each regression varies from 61 to 124. Intercepts are not reported here. The suppressed intercepts are statistically negative related to column A and statistically positive related to column B with at least 10% significance level. All other intercepts are statistically insignificant.

Another contrasting result is that when the market return is greater than -1%, the IV has no predictive power across all types of moneyness because all estimates in column C-F are insignificant at even 90% confidence level.

The results indicate that information contained in options is useful for predicting the future return only when the weekly market return has dropped by more than 1%, but the predicted direction is completely opposite depending on whether the loss is more than 2% or not.

Table 3. Regression coefficients of put option IV on S&P 100 index return.

IV_t	A. $r_t \in (-\infty,-2\%)$	B. $r_t \in [-2\%,-1\%)$	C. $r_t \in [-1\%,0)$	D. $r_t \in [0,1\%)$	E. $r_t \in [1\%,2\%)$	F. $r_t \in [2\%,\infty)$
OTM put	0.162***	-0.120***	0.0324	-0.0211	-0.0250	0.0402
	(0.0475)	(0.0431)	(0.0331)	(0.0325)	(0.0367)	(0.0455)
ATM put	0.162***	-0.117***	0.0421	-0.0276	-0.0204	0.0407
	(0.0496)	(0.0437)	(0.0342)	(0.0330)	(0.0370)	(0.0469)
ITM put	0.162***	-0.158***	0.0713	-0.108*	0.0102	0.0647
	(0.0527)	(0.0510)	(0.0447)	(0.0645)	(0.0625)	(0.0709)

Note: See the notes to earlier tables for variable definition and model information. Sample size of each regression varies from 62 to 124.

Table 3 reports estimates of the 18 OLS regressions for put options. They are similar to those for the call options. Column A reports that if the S&P 100 index drops by more than 2% in current week, the IV predicts an increase in the index next week by a rate of 0.162 for each moneyness. Column B shows that when the index loses are between 1% and 2%, the IV predicts that the market will continue the losing streak for the next week because the regression coefficients are negative for all IV across different moneyness. All these estimates are significantly different from zero at a 99% confidence level. Similar to the results for call options, the IV does not have significant predicting power on the future return when the market return is greater than -1% in the current week. The estimates are reported in columns C to F.

2.3. Performance of contrarian and momentum trading strategies. According to our findings, the natural portfolio strategy will be timing buy or sell based on the market return and the level of the IV. One implication of the finding is that there is a significant positive return indicated by the IV after a more than 2% drop in the market return. A contrarian strategy could buy at this market downturn and profit on the subsequent reversal. On the other hand, the momentum strategy should short the market index when the market return drops more than 1%, but no greater than 2%. But both strategies should be executed only when the IV is high. Here, we provide an analytical argument why the strategies would profit under high IV.

2.4. Intuition for the strategies. Imagine that an investor wants to maximize her standard utility function $E_t\left(U\left(R_{t+1}\right)\right)$,

where $R_{t+1} = \alpha + w_t r_{t+1} + \sum_{i=1}^{n} \beta_i r_i + \varepsilon_{t+1}$ is the portfolio return, which includes the equity market return r_t. Under standard assumptions and the mean-variance analysis, the optimal weight to the equity market is equal to $w_t = \gamma \frac{E_t\left(r_{t+1}\right)}{\sigma_t^2}$, which depends on the conditional variance σ_t^2. Since obviously the weight to the equity market has a positive correlation with the market index, the conditional volatility σ_t^2 will affect the market index too.

We first examine how the conditional variance affects the investor's allocation in the equity market. Utilizing the definition of β in equation (4), the partial derivative of optimal weight w_t with regard to σ_t^2 is[1]:

$$\frac{\partial w_t}{\partial \sigma_t^2} = \frac{\gamma}{\sigma_t^2}\left[\beta - \frac{E_t\left(r_{t+1}\right)}{\sigma_t^2}\right]. \tag{5}$$

Equation (5) shows that the adjustment of the position w_t in equity depends on β and $\frac{E_t\left(r_{t+1}\right)}{\sigma_t^2}$. Our previous result indicates that β can be either positive or negative, so the investor's change of the weight on equity varies depending on the sign of β. When the current market was in a loss state of [-2%, -1%], we know that β is negative (from our empirical result). Applying this knowledge in equation (5), we find that investors will respond to reduce their position in equity because $\frac{\partial w_t}{\partial \sigma_t^2} < 0$ now. This implies that further price decline is more likely in that equity market and the best action is to follow the Momentum Trading Strategy, i.e., selling at the market mild drop to avoid further price decline.

On the other hand, when the market incurred a deep loss (a drop of 2% or more in a week), the positive sign of β from the regression may lead to an increasing size of the weight on equity in equation (5) if β is large

[1] Busse (1999) has a similar formula in studying the volatility timing of mutual funds.

enough to offset $\frac{E_t\left(r_{t+1}\right)}{\sigma_t^2}$, which creates a profit opportunity for following the Contrarian Trading Strategy, i.e., buying at the market dip to exploit possible price reversal.

To further examining equation (5), we found that the value of $\frac{E_t\left(r_{t+1}\right)}{\sigma_t^2}$ becomes relatively small at high conditional volatility, which minimizes its impact on the investor's adjustment of the equity weight. Therefore, the change in the equity position w_t relies more on the sign of β at high market volatility (IV), which provides a clearer signal for investors to follow either MTS or CTS.

2.5. Performance statistics. We have just shown that the MTS or CTS should be carried out under the condition of a high IV. To find a sensible gauge for a high level of the IV, we use the historical distribution of the IV for each option[1]. We consider the IV as high if it is above the cutoff of its 75% percentile[1] of its historical distribution. Table 4 displays the cutoffs of 75% percentile of the IV for each option. To avoid being arbitrary on the cutoff points, we perform similar analysis for cutoffs of other "highs" for robust check in the next subsection.

Table 4. The 75% percentile of implied volatility

OTM call	ITM put	ATM call	ATM put	ITM call	OTM put
0.20	0.23	0.23	0.24	0.26	0.27

Table 5 reports weekly returns from contrarian trading conditional on return being less than -2%. The strategy generates a weekly return of at least 1.4% if the IV falls in its top quarter of its historical distribution. Similar result holds under other measures of the IV. The standard deviations of these weekly returns under different IV are around 3%. Given a risk-free rate of 6% annually, the Sharpe ratio is between 2.87 and 3.56, which is very impressive compared to any other asset in the market. Given the same risk-free rate of 6%, a typical Sharpe ratio for S&P 500 is about 0.2 if it averages 10% return with a 25% standard deviation annually.

Although the coefficient is positive, we find that the contrarian strategy does not generate significant profit when the IV is not high (below its 75% percentile cutoff). It may indicate the positive effect of IV on return is not enough to overcome the reference point, which would continue its downside pull on the return.

Table 5. Time t+1 returns from contrarian trading when the time t market return was in (-∞, -2%)

	IV_t high				IV_t not high			
Type of option	Mean	Standard dev	Sharpe ratio	Sample size	Mean	Standard dev	Sharpe ratio	Sample size
OTM call	1.71%	3.22%	3.56	57	0.48%	2.00%	1.31	34
ATM call	1.51%	2.98%	3.37	53	0.31%	2.03%	0.70	31
ITM call	1.42%	3.27%	2.87	44	0.14%	2.00%	0.07	19
OTM put	1.61%	3.26%	3.30	57	0.31%	2.08%	0.67	34
ATM put	1.53%	3.33%	3.05	57	0.17%	2.00%	0.21	34
ITM put	1.57%	3.67%	2.87	43	-0.28%	2.12%	-1.34	48

Note: The Sharpe ratio is calculated assuming 6% risk-free rate. IV_t is high if it is above its corresponding cutoff in table 4, otherwise it is not high.

Table 6 reports returns from momentum trading that is conditional on that the market return is between -2% and -1%. Like in Table 5, the highest return is achieved when the IV is in the highest quartile (above its 75% percentile). The return ranges from 1.1% to 1.45%. The standard deviation ranges from 3.16% to 3.74%. The Sharpe ratio would range from 2 to 2.5. Again returns from other momentum strategies are much less when the IV is not high.

Table 6. Time t+1 returns from momentum trading when the time t market return was in [-2%, -1%]

	IV_t high				IV_t not high			
Type of option	Mean	Standard dev	Sharpe ratio	Sample size	Mean	Standard dev	Sharpe ratio	Sample size
OTM call	1.10%	3.66%	2.40	14	-0.03%	1.78%	-0.34	61
ATM call	1.06%	3.61%	2.34	14	0.03%	1.78%	-0.60	59
ITM call	1.00%	3.16%	2.55	16	-0.01%	1.84%	-0.40	45
OTM put	1.39%	3.49%	3.11	16	0.08%	1.70%	-0.83	59
ATM put	1.16%	3.65%	2.51	15	0.00%	1.72%	-0.48	60
ITM put	1.45%	3.74%	3.02	12	0.01%	1.83%	-0.48	52

See the notes to earlier table for definition of IV_{t-1} high.

[1] The empirical distribution of historical IV is quite stable over time.

We also experimented on trading under other circumstances, where the return falls into other brackets. The returns are all discouraging. We reported future returns when the current return is above -1% in Table 7. Returns from either strategy show insignificant or ambiguous results.

Table 7. Time t+1 returns from buy when the time t market return was greater -1%

	IV_t high				IV_t not high			
Type of option	Mean	Standard dev	Sharpe ratio	Sample size	Mean	Standard dev	Sharpe ratio	Sample size
OTM call	-0.06%	3.35%	-0.05	83	0.06%	1.78%	-0.03	377
ATM call	0.11%	3.26%	0.00	83	0.02%	1.82%	-0.05	376
ITM call	0.03%	3.09%	-0.03	81	0.02%	1.82%	-0.05	352
OTM put	-0.07%	3.28%	-0.06	83	0.06%	1.81%	-0.03	377
ATM put	-0.04%	3.28%	-0.05	82	0.07%	1.81%	-0.02	376
ITM put	0.02%	3.79%	-0.02	53	0.17%	2.04%	0.03	229

Note: All returns come from buy position. See the notes to earlier table for definition of IV_{t-1} high.

2.6. Performance under different high IVs. We provide results analogous to Table 5 and Table 6 under different levels of high IV. To save space, we only report the results when the IV is high. We also suppressed standard deviations. We define IV_t as high if it is above the corresponding cutoff points. They are shown in Table 8 and Table 9.

Table 8 reports contrarian returns, when the IV_t is high. The results resemble to those in Table 5 in that the weekly return is above 1% with very impressive Sharpe ratios. We also note that the average return and Sharpe ratio generally increase as we push up the cutoff for high IV. But the number of weeks for contrarian trading also decreases at the same time. Therefore, it is not necessary a good idea to increase the cutoff. A good balance between a higher average return and a good size of sample would need more investigation. A similar situation is in Table 9 compared to Table 6, which reports returns from the momentum trading.

Table 8. Time t+1 returns from contrarian trading when the market return was in (-∞, -2%) at time t

Type of option IV	Mean	Share ratio	Sample size	Mean	Share ratio	Sample size	Mean	Share ratio	Sample size
	High (>50%)			High (>67%)			High (>80%)		
OTM call	1.04%	2.14	80	1.33%	2.69	65	1.69%	3.33	50
ATM call	1.06%	2.21	76	1.25%	2.54	64	1.77%	3.55	48
ITM call	1.07%	2.25	59	1.19%	2.45	52	1.36%	2.60	38
OTM put	1.01%	2.04	79	1.30%	2.67	66	1.84%	3.57	48
ATM put	1.00%	2.07	82	1.32%	2.76	98	1.65%	3.25	52
ITM put	1.21%	2.50	72	1.40%	2.69	57	1.50%	2.55	35
	High (>87.5%)			High (>90%)			High (>95%)		
OTM call	1.62%	2.77	34	1.85%	3.01	28	2.51%	3.89	19
ATM call	1.89%	3.37	34	1.74%	2.84	28	2.64%	4.12	18
ITM call	1.42%	2.34	26	1.69%	2.69	22	2.69%	4.19	15
OTM put	2.06%	3.59	34	2.03%	3.40	28	2.82%	4.36	19
ATM put	1.90%	3.24	32	2.03%	3.33	28	2.65%	3.89	17
ITM put	2.39%	4.05	25	2.70%	4.25	20	2.47%	3.44	13

Note: The number in each pair of parentheses is the percentage cutoff for a high volatility from the historical distribution of the IV.

Table 9. Time t+1 returns from momentum trading when the market returns was in (-2%, -1%) at time t

Type of option IV	Mean	Share ratio	Sample size	Mean	Share ratio	Sample size	Mean	Share ratio	Sample size
	High (>50%)			High (>67%)			High (>80%)		
OTM call	0.40%	0.74	38	0.66%	1.27	25	1.31%	2.31	13
ATM call	0.29%	0.46	36	1.15%	2.55	24	1.13%	1.82	11
ITM call	0.55%	1.09	33	1.25%	2.64	21	1.81%	3.44	12
OTM put	0.71%	1.52	34	1.28%	2.58	19	1.42%	2.65	13
ATM put	0.74%	1.58	32	1.07%	2.12	21	1.31%	2.22	12
ITM put	0.98%	2.08	26	1.66%	3.04	13	1.58%	2.58	9
	High (>87.5%)			High (>90%)			High (>95%)		
OTM call	3.14%	7.73	7	3.51%	7.32	5	5.22%	15.21	3
ATM call	3.38%	7.00	5	3.56%	6.43	4	5.22%	15.21	3

Table 9 (cont.). Time $t+1$ returns from momentum trading when the market returns was in (-2%, -1%) at time t

Type of option IV	Mean	Share ratio	Sample size	Mean	Share ratio	Sample size	Mean	Share ratio	Sample size
	High (>87.5%)			High (>90%)			High (>95%)		
ITM call	1.79%	2.46	6	5.22%	15.21	3	5.22%	15.21	3
OTM put	2.46%	3.95	7	2.24%	2.94	5	6.51%	35.42	2
ATM put	1.96%	3.15	8	4.14%	12.44	5	5.22%	15.21	3
ITM put	3.19%	7.09	6	4.14%	12.44	5	6.51%	35.42	2

See notes in previous tables.

Conclusion

In this paper, we find an asymmetric pattern for the IV of the S&P 100 options as a predictor of the future market return. Practitioners have long suspected that a high IV signals an oversold market. Our finding supports the validity of such claim only when the weekly market return drops by more than 2% and the IV is at a high level. We believe this reversal phenomenon is robust because it has occurred at least 43 weeks during 1996-2008. In contrast to previous studies, we also discovered that when the loss in the market is moderate (i.e., weekly loss between 1% and 2%), the IV in fact predicts a continual loss.

Researchers have focused on the link between the implied volatility and the future realized volatility. Few studies deal with the possible relationship between the implied volatility and future returns. We hope our finding can make up some of the missing part in the empirical research on this aspect. In addition, the standard finance theory cannot explain readily why a high IV should predict significant market returns. The predictability of return and abnormal returns from our test strategies is against the hypothesis of market efficiency. We hope that future study may reconcile this anomaly with a judicious theory of finance.

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Appendix. Derivation of the regression coefficient β

The coefficient is defined as $\beta \equiv \dfrac{\partial E_t\left(r_{t+1}\right)}{\partial \sigma_t^2}$. From equation (1) we find that $E_t\left(r_{t+1}\right) = E_t\left(ax_t + w_t + v_{t+1}\right) = aE_t\left(x_t\right) = a\hat{x}_t$. Therefore, combining with the formula of Kalman filtering, the first derivative is $\dfrac{\partial E_t\left(r_{t+1}\right)}{\partial \sigma_t^2} = a\dfrac{\partial \hat{x}_t}{\partial \sigma_t^2} = \dfrac{a}{\sigma_v^2}\left(r_t - a\hat{x}_{t-1}\right) + a^2\dfrac{\partial \hat{x}_{t-1}}{\partial \sigma_{t-1}^2}\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2} + ak_t\left(r_t - a\dfrac{\partial \hat{x}_{t-1}}{\partial \sigma_{t-1}^2}\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2}\right)$.

Given that $\sigma_t^2 = \dfrac{\left(a^2\sigma_{t-1}^2 + \sigma_w^2\right)\sigma_v^2}{a^2\sigma_{t-1}^2 + \sigma_w^2 + \sigma_v^2}$, we have the following derivative, $\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2} = \dfrac{a\dfrac{\sigma_t^2}{\sigma_v^2} + \dfrac{\sigma_w^2}{\sigma_v^2} + 1}{a^2\left(1 - \dfrac{\sigma_t^2}{\sigma_v^2}\right)}$, where $\dfrac{\sigma_w^2}{\sigma_v^2}$ is typically interpreted as the singal-to-noise ratio in the return data. The typical value of $\dfrac{\sigma_w^2}{\sigma_v^2}$ is in the range of [100,200] (see Koop, 2003; Li, 2008). It can be shown that $\dfrac{\sigma_t^2}{\sigma_v^2} \in \left(0,1\right)$ since σ_t^2 is a weighted average of $\left(\sigma_{t-1}^2 + \sigma_w^2\right)$ and σ_v^2. We find that $\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2}$ is a very large positive number. A simulation for a=0.9 shows that the range of $\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2}$ is between 500 and 2000.

Assume the first-order effect of σ_t^2 on $\hat{x}_t$ converges to constant as time approaches the infinity, i.e. $\dfrac{\partial \hat{x}_t}{\partial \sigma_t^2} = \dfrac{\partial \hat{x}_{t-1}}{\partial \sigma_{t-1}^2}$. Collecting terms in the above equation, we arrive a simplified formula for the derivative: $\dfrac{\partial E_t\left(r_{t+1}\right)}{\partial \sigma_t^2} = g\left[\dfrac{a\hat{x}_{t-1}}{\sigma_v^2} - \left(\dfrac{1}{\sigma_v^2} + k_t\right)r_t\right]$, where $g = \dfrac{a}{a\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2} - ak_t\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2} - 1}$

since $\dfrac{\partial \sigma_{t-1}^2}{\partial \sigma_t^2}$ is very large, g is mostly positive given that a and k_t are both between 0 and 1.