

“Risk and performance attribution”

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Risk and performance attribution

Abstract

This paper develops a method based upon Sharpe's (1990 and 1992) Asset Class Factor Model for decomposing both the risk and return of an actively managed portfolio into independent categories associated with passive asset allocation, active asset allocation, and security selection. Because the risk measures for each category are additive, they can be used to separately evaluate asset allocation and security selection performance. Indeed, we are able to decompose the Sharpe ratio of a portfolio into a ratio attributed to passive asset allocation and incremental ratios associated with active asset allocation and security selection. In this way, it is possible to independently examine in a risk-return framework the efficacy of asset allocation and security selection.

Keywords: Sharpe ratio, performance measurement, linear factor model.

JEL Classification: G11.

Introduction

The performance of an actively managed portfolio is commonly measured with respect to a passive benchmark that tracks the active portfolio. Treynor and Black (1973) showed how a benchmark can be determined from a combination of risk-free asset and market index. Rudd and Clasing (1982) showed how a linear factor model can be used to construct a benchmark. Sharpe (1990 and 1992) developed the Asset Class Factor Model (ACFM) under which the underlying factors represented asset classes available to an investor. Under this model, an investor's universe of securities is divided up into a small number of asset classes. The return on each asset class is represented by an index, which is a portfolio of the component securities. Then, regression analysis is used to determine the effective exposure to each asset class, the performance alpha, and the residual risk.

Treynor and Black's approach is appropriate if two conditions are satisfied: (1) an investor's universe is well represented by the market index; and (2) an investor believes that the market portfolio is efficient – or approximately efficient, to allow for the chance of superior performance. The first condition is typically not satisfied because bonds, a major asset class, are typically excluded from a market index. Also, investors often adjust for risk and income preferences by the relative allocation to bonds and stock. The second condition is debatable. However, if we take taxes into account, non-marketable assets, and unique holdings, it is unlikely that a market index would be efficient for all investors, even if they had homogeneous expectations.

Sharpe's ACFM does not require the first condition to hold because the asset classes and the representative indexes are tailored to an investor's situation. The second condition is also not required.

If we do not believe the market portfolio is efficient with respect to our universe of securities, then presumably we can establish a fixed allocation among the asset classes that we believe is efficient. The fixed allocation can be thought of as a strategic allocation¹ based upon long-term capital market expectations. Active asset allocation then involves deviations from strategic weights, perhaps based upon short-term tactical considerations. The strategic allocation establishes a benchmark against which to judge active asset allocation. Active allocation does not require active security selection because the manager is simply choosing the amount to invest in each asset class index. Active security selection involves choosing a portfolio of securities within each asset class index that differs from the index. The benchmark for active security selection is a benchmark that tracks the investor's style as determined by the ACFM.

This paper develops a method based upon Sharpe's (1990 and 1992) ACFM for decomposing both the risk and return of an actively managed portfolio into independent categories associated with passive asset allocation, active asset allocation, and security selection. Because the risk measures for each category are additive, they can be used to separately evaluate asset allocation and security selection performance. Indeed, we are able to decompose the Sharpe (1966 and 1994) ratio of a portfolio into a ratio attributed to passive asset allocation and incremental ratios associated with active asset allocation and security selection. In this way, it is possible to independently examine in a risk-return framework the efficacy of asset allocation and security selection.

1. Asset class factor model

Sharpe (1990 and 1992) developed the Asset Class Factor Model (ACFM) to determine effective

¹ See Sharpe (1990) for a complete discussion of strategic and tactical asset allocation.

exposure of a manager's portfolio to each asset class, the performance alpha, and residual risk. First, an investor's universe of securities is divided up into a small number of asset classes. Ideally, stocks within each class should be highly correlated and the across-class correlations should be low. Second, the return on each asset class is represented by an index, which is a portfolio (ideally value weighted) of the component securities. Finally, regression analysis is used to determine the effective exposure to each asset class, the performance alpha, and the residual risk.

Define M as the manager's portfolio return and F_1, F_2, \dots, F_K as the return on K indexes over a historical comparison period¹. The effective exposures are determined by the following regression:

$$M = \alpha + b_1 F_1 + b_2 F_2 + \dots + b_K F_K + u, \quad (1)$$

where u is the residual with zero mean. Sharpe calls the set of regression coefficients b_1, \dots, b_K the effective asset mix or investor's style. Each coefficient measures the effective exposure of the manager's portfolio return to a given index. Note the sum of the exposures does not necessarily equal one. For this reason, Sharpe (1992) used a constrained regression to ensure that the regression coefficients sum to one. In this case, one could (ex post) go back and construct a passive strategy by investing b_1 in index 1, b_2 in index 2, \dots , b_K in index K . The return on this passive benchmark is:

$$B = b_1 F_1 + b_2 F_2 + \dots + b_K F_K.$$

The α in equation (1) can then be interpreted as the difference between the average³ active portfolio and benchmark return:

$$\alpha = \bar{M} - \bar{B}.$$

Finally, the residual variance is the extra risk associated with choosing a portfolio different from the benchmark.

An alternative approach is to include a risk-free asset with return r in the ACFM. The exposure to the risky asset classes is determined by the ordinary least squares regression coefficients b_1, b_2, \dots, b_K .

The remainder $1 - b_1 - b_2 - \dots - b_K$ is the exposure to the risk-free asset. There are two advantages to this approach. First, it is no longer necessary to constrain the sum of the index exposures to one. Secondly, and more importantly, it allows us to account for a situation in which a manager's exposure to given asset class, represented by an index, is less (or greater) than the total exposure of the index to the asset class. For example, consider a single asset class represented by a market index with return R_m . Then the ACFM is:

$$M = \alpha + \beta R_m + u,$$

where β (beta) is the effective exposure to the market index. If the manager chooses low beta stocks, in a sense, she has less exposure to the asset class than the index. Consequently, it makes sense to include the risk-free asset in the passive benchmark strategy so that the benchmark also has reduced exposure to the market index. Now the benchmark consists of $(1 - \beta)$ invested in risk-free asset with return r and β invested in the market index. This framework leads to Jensen's (1969) alpha, which is determined by the following regression:

$$M - r = \alpha_J + \beta(R_m - r) + \varepsilon,$$

where ε is the residual error term and α_J is Jensen's alpha. Now the benchmark return is given by

$$B = r + \beta(R_m - r) = (1 - \beta)r + \beta R_m.$$

It is the return on a passive portfolio consisting of $(1 - \beta)$ invested in risk-free asset and β invested in the market index. Jensen's alpha can be interpreted as the difference between the average active and benchmark return:

$$\alpha_J = \bar{M} - \bar{B}$$

and the residual variance as the risk associated with the active strategy.

The same argument applies to a multiple-index model. To simplify the notation, define f_1, \dots, f_K as the excess (above the risk-free) rate of return of the K indexes. Then the Modified Asset Class Factor Model can be stated:

$$M - r = \alpha_S + b_1 f_1 + b_2 f_2 + \dots + b_K f_K + \varepsilon. \quad (2)$$

Because the remainder $(1 - b_1 - b_2 - \dots - b_K)$ is invested in the risk-free asset, the benchmark return is given by

¹ To simplify notation the subscript t indicating the particular period over which the historical return is computed is suppressed. All statistics are estimated over the period that performance is evaluated.

² Sharpe (1990 and 1992) did not include a constant term in the regression. The alpha, then, is the average of the residual.

³ The average is computed over the period that performance is being evaluated.

$$B = r + b_1 f_1 + \dots + b_K f_K.$$

The sum of the alpha and the residual is the excess return resulting from following an active strategy. The selection *alpha* is the excess average return of the active manager's return over the benchmark return:

$$\alpha_s = \bar{M} - \bar{B}.$$

The additional risk associated with the active strategy equals the residual variance¹, because the residual variance equals the difference (non-negative) between the variance of the manager's portfolio and benchmark return:

$$\text{var}(\varepsilon) = \text{var}(M - B).$$

The information ratio,

$$\frac{\alpha_s}{\sqrt{\text{var}(\varepsilon)}},$$

is sometimes used to adjust the alpha for the residual risk. In the following sections, we will develop a more comprehensive performance measure.

2. Return attribution

Under the framework of the ACFM, an active manager makes two decisions: asset allocation and security selection. Asset allocation refers to a manager determining the fraction of wealth to be invested in each asset class. Security selection refers to the selection of individual securities within each asset class. Asset allocation performance can be measured with respect to a strategic (or target) allocation. The strategic allocation weights define a passive asset allocation strategy. These weights can be determined in several ways. One approach is to choose the strategic weights so that the passive asset allocation portfolio is a mean-variance efficient portfolio of the indexes, determined by an optimizer with the covariance matrix and expected returns on the indexes as inputs (Markowitz, 1959). Another approach, in the spirit of the capital asset pricing model, is to choose the weights equal to relative capitalization of the assets in each index. Then, the market portfolio constructed from the indexes becomes the benchmark.

Once strategic weights are determined, a passive strategy allocates a fixed percentage w_1, \dots, w_K to each index with the remainder $1 - w_1 - \dots - w_K$

invested in the risk-free asset. The return on the passive asset allocation is:

$$P = r + w_1 f_1 + w_2 f_2 + \dots + w_K f_K.$$

The extra return associated with the risk of the passive asset allocation is determined by:

$$\alpha_p = \bar{P} - \bar{r}.$$

If a manager chooses to deviate from the strategic weights, he is assuming active asset allocation risk. The deviation from the strategic weights is determined by the effective asset exposure mix. Consequently, the active asset allocation *alpha* is:

$$\alpha_A = \bar{B} - \bar{P} = (b_1 - w_1) \bar{f}_1 + \dots + (b_K - w_K) \bar{f}_K. \quad (3)$$

It measures the extra return associated with effective exposures deviating from target weights. Finally, the security selection *alpha* is:

$$\alpha_s = \bar{M} - \bar{B}.$$

Return performance attribution involves decomposing the manager's average return into active security selection, active asset selection, passive asset selection, and risk-free return:

$$\bar{M} - \bar{r} = \alpha_p + \alpha_A + \alpha_s.$$

Extra risk is associated with the *alpha* for each component. This raises the question of how to adjust each *alpha* for the associated risk. Before we can answer this question, we first need to find an additive risk measure for each component.

3. Risk attribution

Performance attribution requires adjusting *alpha* for the risks associated with active asset allocation and security selection. To examine this problem, we need to develop a method of risk attribution. We start by decomposing the manager's return into passive asset allocation return, incremental active allocation return, and incremental active security selection return:

$$M - r = [w_1 f_1 + \dots + w_K f_K] + [(b_1 - w_1) f_1 + \dots + (b_K - w_K) f_K] + (\alpha_s + \varepsilon). \quad (4)$$

Security selection risk is the variance of the residual:

$$\gamma_s^2 = \text{var}(\varepsilon).$$

The passive asset selection risk is the variance of the first bracketed expression in equation (4):

$$\gamma_p^2 = \sum_{k=1}^K \sum_{j=1}^K w_j w_k \text{cov}(f_j, f_k).$$

¹ The residual risk in equation (2) is the tracking error of the active portfolio with respect to the benchmark portfolio.

The active asset selection risk is more difficult to measure. If we follow a passive strategy, the second bracketed term equals zero, and the total risk equals the sum of γ_p^2 and γ_s^2 . Consequently, any risk due to the correlation of the first and second bracketed term should be assigned to the active asset allocation. This leads us to the following definition of active allocation risk:

$$\gamma_A^2 = \sum_{k=1}^K \sum_{j=1}^K (b_j - w_j)(b_k - w_k) \text{cov}(f_j, f_k) + 2 \sum_{j=1}^K \sum_{k=1}^K (b_j - w_j) w_k \text{cov}(f_j, f_k).$$

The first term is the tracking error between the active and passive portfolio. The second double summation is the result of the correlation between the two bracketed terms in equation (4). The above equation reduces to

$$\gamma_A^2 = \sum_{k=1}^K \sum_{j=1}^K (b_j + w_j)(b_k - w_k) \text{cov}(f_j, f_k), \quad (5)$$

which can be thought of as the variance of the tracking error corrected for the correlation between the bracketed expressions in equation (4). Consequently, total risk attribution formula can be written as:

$$\sigma^2 = \gamma_p^2 + \gamma_A^2 + \gamma_s^2.$$

4. Performance attribution

The total excess active return per unit of risk is given by the Sharpe (1966) ratio:

$$S = \frac{\bar{M} - \bar{r}}{\sigma}$$

Our objective is to break the Sharpe ratio into components associated with passive asset allocation, active asset allocation, and active security selection. Passive asset allocation Sharpe ratio is:

$$S_p = \frac{\alpha_p}{\gamma_p} = \frac{\bar{P} - \bar{r}}{\gamma_p}$$

The Sharpe measure for the active allocation is given by

$$S_A = \frac{\alpha_A + \alpha_p}{\sqrt{\gamma_A^2 + \gamma_p^2}}$$

So we define the incremental Sharpe ratio for active allocation as

$$\Delta S_A = \frac{\alpha_A + \alpha_p}{\sqrt{\gamma_A^2 + \gamma_p^2}} - S_p.$$

A positive number means that active asset allocation has increased the excess return per unit of risk. Finally, for active stock selection, we define the incremental ratio as:

$$\Delta S_s = \frac{\alpha_A + \alpha_p + \alpha_s}{\sqrt{\gamma_A^2 + \gamma_p^2 + \gamma_s^2}} - \frac{\alpha_A + \alpha_p}{\sqrt{\gamma_A^2 + \gamma_p^2}}$$

Therefore, the Sharpe performance measure can be decomposed into three components:

$$S = S_p + \Delta S_A + \Delta S_s.$$

The components are associated with passive allocation, active asset allocation, and active security selection.

Example. An investor's universe is divided up into two asset classes, represented by indexes F_1, F_2 with excess returns f_1, f_2 . Suppose $\text{var}(f_1) = 25$, $\text{var}(f_2) = 36$, $\text{corr}(f_1, f_2) = 0.5$, $\text{cov}(f_1, f_2) = 15$, $\bar{f}_1 = 2\%$, $\bar{f}_2 = 6\%$. Strategic asset allocation results in an equally weighted portfolio of the two asset classes ($w_1, w_2 = 0.5$). Based upon the regression of the active return on the two indexes, the effective asset mix is given by $b_1 = 0.4$, $b_2 = 0.6$, the average residual equals 2%, and the residual variance equals 2.45% ($\sqrt{6}$).

Based upon the strategic allocation, the passive asset allocation alpha is

$$\alpha_p = 0.5 \times 2 + 0.5 \times 6 = 4\%.$$

The active allocation *alpha* is the difference between the average return on the portfolio with the effective asset mix and the passive portfolio:

$$\alpha_A = (0.4 - 0.5)2 + (0.6 - 0.5)6 = 0.4\%.$$

Finally, the active selection *alpha* equals the average residual:

$$\alpha_s = 2\%.$$

The average excess active return can be decomposed into three components:

$$\alpha_p + \alpha_A + \alpha_s = 4 + 0.4 + 2 = 6.4\%.$$

The next step is to determine the risk associated with each *alpha* component. The passive asset allocation risk is the variance of the strategic allocation:

$$\gamma_p^2 = 0.5^2 \times 25 + 0.5^2 \times 36 + 2 \times 0.5 \times 0.5 \times 15 = 22.75.$$

The active asset allocation risk is the variance of the tracking error adjusted for the correlation between the tracking error and the passive portfolio. Based upon equation (5), the active allocation risk is given by

$$\gamma_A^2 = (0.9)(-0.1)(25) + (0.9)(0.1)(15) + (1.1)(-0.1)(15) + (1.1)(0.1)(36) = 1.41.$$

The active selection risk is the variance of regression residual:

$$\gamma_A^2 = 6.$$

The total active risk is the sum of the three components:

$$22.75 + 1.41 + 6 = 30.16.$$

Direct calculation of total risk yields the same result:

$$6 + 0.4^2 \times 25 + 0.6^2 \times 36 + 2 \times 0.4 \times 0.6 \times 15 = 30.16.$$

The final step is assigning a performance measure to each return component based upon risk attribution. The Sharpe ratio for the passive allocation is given by

$$S_p = \frac{4}{\sqrt{22.75}} = 0.8386.$$

The incremental Sharpe ratio for the active allocation is

$$\Delta S_A = \frac{4 + 0.4}{\sqrt{22.75 + 1.41}} - 0.8386 = 0.0567.$$

Finally, the incremental Sharpe ratio for the active selection is

$$\Delta S_S = \frac{4 + 0.4 + 2}{\sqrt{6 + 22.75 + 1.41}} - \frac{4 + 0.4}{\sqrt{22.75 + 1.41}} = 0.2702.$$

We can easily verify that the total Sharpe ratio is the sum of the three components:

$$S = S_p + \Delta S_A + \Delta S_S = 0.8386 + 0.0567 + 0.2702 = 1.1655.$$

The analysis shows that the active asset allocation added 0.0567 to the total Sharpe ratio and active stock selection added 0.2702. In this example, both asset allocation and stock selection added value, but stock selection added more value than active asset allocation.

It is important to realize that even if the *alpha* is positive the incremental Sharpe ratio could be negative. For example, suppose the average residual value is 0.5%. Although the manager is earning 0.5% more than the active asset allocation, the

portfolio risk is also greater. The incremental stock selection *alpha* is given by

$$\frac{0.5 + 0.4 + 4}{\sqrt{6 + 22.75 + 1.41}} - \frac{0.4 + 4}{\sqrt{22.75 + 1.41}} = -0.0293.$$

This means that active stock selection reduced the total Sharpe ratio. The manager would have obtained superior results by selecting the active asset allocation. The manager was not adequately compensated for the 2.45% residual risk associated with 0.5% *alpha*.

5. Performance of a balanced mutual fund

In this section, we examine the performance of the actively managed Fidelity Balanced (FBALX) mutual fund. The objective of this fund is to “... seek income and capital growth consistent with reasonable risk. The fund invests approximately 60% of assets in stocks and other equity securities and the remainder in bonds and other debt securities, including lower-quality debt securities. It invests at least 25% of total assets in fixed-income senior securities” (Fidelity Mutual Funds, 2010). Based upon the fund’s objective, FBALX’s strategic allocation is 60 percent stock and 40% bonds. The actual exposures as of March 31, 2010 are 59.88% stocks, 33.52% bonds, 4.53% cash¹, and 1.99% other.

The asset allocation for a balanced mutual fund is split between stocks and bonds. The stock asset class is represented by the SPDR S&P 500 index exchange traded fund (SPY), and the bond asset class by the iShares Barclays Aggregate Bond exchange traded fund (AGG). The two exchange traded funds have lower expense ratios compared to the actively managed fund.

The fund’s effective exposure over the last four years is estimated by regressing the fund’s excess monthly return on the SPY and AGG monthly returns. The monthly return on the FBALX, SPY, and AGG were computed from adjusted closing prices, which include dividends, provided by Yahoo Finance (2010). The monthly risk-free rate is the effective rate on a four week Treasury Bill, computed from the discount yield provided by Federal Reserve Statistical Release (2010).

The *r-squared* of the regression is 0.9450, suggesting that the SPY and AGG are good proxies for the stock and bond assets classes in which FBALX invests. The effective exposures in stocks and bonds are $b_1 = 0.7652$ and $b_2 = 0.1328$. By

¹ Cash is shorthand for short-term, low risk money market holdings.

construction, the remainder of 0.1020 ($1 - 0.7652 - 0.1328$) is the fund's effective exposure to cash. The alpha of the regression is $\alpha_s = 0.1505$ associated with a residual variance of $\gamma_s^2 = 1.0417$. The sample covariance matrix of the excess monthly returns for the two asset classes is:

$$V = \begin{bmatrix} 29.7425 & 2.2376 \\ 2.2376 & 2.3397 \end{bmatrix}.$$

The vector of mean monthly excess returns for the two asset classes is:

$$\bar{f} = [-0.235 \quad 0.345].$$

The mean excess monthly return and variance of the actively managed FBALX portfolio are 0.1850 and 18.950, resulting in a Sharpe ratio of 0.0425 ($0.1850 / \sqrt{18.950}$). Based upon the strategic allocation of 60% stocks ($w_1 = 0.6$) and 40% bonds ($w_2 = 0.4$), the variance of the passive allocation is:

$$\gamma_p^2 = w'Vw = 12.1557,$$

and the mean return is:

$$\alpha_p = w'\bar{f} = 0.1440.$$

The active asset allocation variance, determined by equation (5) in matrix form, equals:

$$\gamma_A^2 = [b + w]'V[b - w] = 5.7555,$$

and the active asset alpha, from equation (3), equals:

$$\alpha_A = [b - w]'\bar{f} = -0.1095.$$

We can see that the total excess return on FBALX equals the sum of the passive allocation alpha, the active allocation alpha, and the security selection alpha:

$$0.1850 = 0.1440 - 0.1095 + 0.1505.$$

Further, the total active variance on FBALX is equal to the sum of the passive allocation variance, the active allocation variance, and the security selection variance:

$$18.9529 = 12.1557 + 5.7555 + 1.0417.$$

The final step is assigning a performance measure to each return component based upon risk attribution. The Sharpe ratio for the passive allocation is given by

$$S_p = \frac{0.1440}{\sqrt{12.1557}} = 0.0413.$$

The incremental Sharpe ratio for the active allocation is:

$$\Delta S_A = \frac{0.1440 - 0.1095}{\sqrt{12.1557 + 5.7555}} - 0.0413 = -0.0331.$$

Finally, the incremental Sharpe ratio for the active security selection is:

$$\Delta S_s = \frac{0.1440 - 0.1095 + 0.1505}{\sqrt{12.1557 + 5.7555 + 1.0417}} - \frac{0.1440 - 0.1095}{\sqrt{12.1557 + 5.7555}} = 0.0343.$$

We can easily verify that the total Sharpe ratio is the sum of the three components:

$$S = 0.0413 - 0.0331 + 0.0343 = 0.0425.$$

The analysis shows that the active asset allocation reduced the total Sharpe ratio by 0.0331 and active stock selection added 0.0343. It is not surprising that active asset allocation reduced performance because the active asset allocation overweighted stocks during a period when the stock market had a negative return. On the other hand, active security selection added 0.0343 to the Sharpe ratio. The net gain in the Sharpe ratio due to active management is 0.0012. In other words, the Sharpe ratio of the actively managed portfolio exceeded the ratio of the passive portfolio with a 60/40 allocation by 0.0012. The gain due to stock selection is nearly offset by the higher allocation to stocks. Also, some of the gain due to active management is reduced by the higher expense ratio of the actively managed fund compared to the two index exchange traded funds.

Conclusion

This paper shows how the *alpha* of an actively managed portfolio can be decomposed into components associated with passive asset allocation, active asset allocation, and security selection. Further, we develop an additive risk measure for each component, so that the total risk is simply the sum of the risk of individual components. This is a departure from the normal breakdown between active and passive risk based upon tracking error, for which additivity does not hold. Because the risk measures for each component are additive, they can be used to separately evaluate asset allocation and security selection performance. Indeed, we are able to decompose the Sharpe ratio of the portfolio into a ratio attributed to passive asset allocation and incremental ratios associated with active asset allocation and security selection. In this way, it is possible to independently examine the efficacy in a risk-return framework of asset allocation and security selection.

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