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AUTHORS	Roberta Melis Alessandro Trudda
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SECTION 1. Macroeconomic processes and regional economies management

Roberta Melis (Italy), Alessandro Trudda (Italy)

Demographic risk indicators in pay-as-you-go pension funds

Abstract

This paper deals with demographic risk in private pay-as-you-go pension systems. In particular, it analyzes the financial sustainability of the fund in a stochastic framework. The authors present a model to investigate the dynamics of these types of pension funds which operate according to the pay-as-you-go rule, focusing on the behavior of the demographic variable "new entrants" and on its influence on the future evolution of the fund. The global asset return and the new entrants variation rate are modelled by autoregressive processes. The goal is to propose risk indicators that can be employed to monitor the solvency of the fund. A numerical application is carried out using the data provided by the pension funds of Italian professional orders. The analysis highlights how the variable "new entrants" influences the final value of the fund and the application shows that the proposed controlling model appears effective at providing advance warning of the financial insolvency of the fund.

Keywords: pension funds, pay-as-you-go, demographic risk, stochastic new entrants.
JEL Classification: G23, H55.

Introduction

The purpose of a pension scheme is to collect workers' savings during their working life in order to give them back in the form of a life annuity at retirement. As far as the financing mechanism is concerned, pension schemes can be divided into pay-as-you-go (PAYG) and funded systems. In the former, contributions paid by the workers are used for financing current pensions (in a pure PAYG system, revenues exactly equal outlays each year, Trowbridge, (1952)), while in the latter there is not any intergenerational transfer or redistribution because contributions are used to purchase assets that finance benefits upon retirement.

In Europe, a PAYG system is generally used for public pension schemes, whereas private schemes (relative to first and second pillar) are managed in accordance to funded system. In this paper we study the private pension system financed by PAYG: in Italy, this is how the pension funds of Italian professional orders work. In Italy each professional group recognized by a board (such as lawyers, doctors, accountants, engineers) administrates its own retirement fund. These organizations can be assimilated to closed pension funds, because they have a restriction on membership, as it is necessary to be admitted to the professional order to join the fund. They are financed on a PAYG basis. While in general PAYG systems are applied to open and not closed groups, these "professional body" funds are anomalous in that they have a barrier to entry, as only those who pass a public examination can exercise the profession and consequently become member of the fund. The Italian legislation for these funds, privatized in 1995, imposes to draw up actuarial balances and risk indicators to monitor the evolution of the fund in the long run (30-50 years).

It is well known that pension funds are exposed to financial and demographic risks. The investment risk is linked to the randomness of the rates of return. It derives from changes in the financial markets where the plan invests and occurs in the deviations of the real rate of return from its expected values. This issue has been discussed in numerous papers. Among these Dufresne (1988), Haberman (1994), Haberman (1997), Cairn and Parker (1997) analyze investment risk for a defined benefit pension fund. Blake, Cairns and Dowd (2001; 2003) propose a model for a defined-contribution pension fund with stochastic wages and returns, in a discrete and continuous time respectively.

Regarding the demographic risk, longevity risk derives from improvements in mortality trends and determines systematic deviations in the number of deaths from its expected value. This kind of risk applies to all types of pension plans and life-insurance products. On this topic, an exhaustive literary review can be found in Pitacco (2004).

For PAYG pension funds, in which the financial sustainability is related to a balance between the active and retired members, there is a further demographic risk source to take into account: the risk relates to future monetary cash flows necessary to ensure payments of future pensions. This risk is related to the demographic variable "new entrants" and to their future contribution capacity.

In this paper we concentrate on demographic risk faced by a fund which operates in a PAYG system and which can be subscribed to only by certain employees. The impact of the demographic variable "new entrants" is studied and it is analyzed how its variation rate influences the future cash flows of the fund.

Over the past few years, a vast literature regarding demographic risks and the sustainability in public PAYG system has been developed (see Feldstein (1996; 2005) on the reform of the Social Security in the USA). Pemberton (2000) analyzes the transition from PAYG to funded pensions. Sinn (2000) and Blake (2000) illustrate the characteristics and the differences between PAYG and funded system. There are few contributions on private PAYG scheme. Ferrara (2002) analyzes the equilibrium of a PAYG pension fund with defined benefit, where new entrants are random variables depending on the employment rate. Menoncin (2005) studies the allocation problem for a pension fund, which behaves according to a PAYG rule: considering the total number of workers and pensioners as random variables. He shows how the demographic risk affects the optimal portfolio in a cyclical way. Melis and Trudda (2009) propose a model for the evolution of a PAYG pension fund, with stochastic new entrants and global asset return. Actuarial literature on pension population is based above all on deterministic models (Bower et al., 1976; Winklevoss, 1993), which consider the population as stationary. In the literature there are contributions which take into account the fluctuation in the number of "new entrants". Mandle and Mazurova (1996) use spectral decomposition of stationary random sequences to investigate a defined benefit pension scheme under randomly fluctuating rates of return and number of entrants. Chang and Cheng (2002) use a semi-Markov process in structuring the transition pattern of the plan's population. Iyer (2003) derives algebraic expressions for the variances of, and covariance among, the important aggregates that characterize the development of a pension scheme. He takes into account the stochastic variation of the "new entrants". Colombo and Haberman (2005) analyze the impact of the stochastic evolution of the active membership population on the mismatch between assets and liabilities of a defined pension scheme.

This paper focuses on private PAYG systems. We analyze the role of demographic risks on the financial sustainability of PAYG pension funds, by proposing risk indicators of the solvency of the fund, namely its capability to pay for future obligations. The active population evolves according to the process of "new entrants", which can be different for each fund. Therefore, the main problem is to analyze the flow of new entrants into the fund, to assess if the number of future taxpayers is sufficient to maintain the system in balance, and thus to ensure its solvency.

The objective of the model proposed is twofold. Firstly, to analyze the influence of the demographic variable "new entrants" on the future cash flows. Secondly, to monitor the solvency of the fund we propose risk indicators used to project the fund evolution and to anticipate its future dynamics. The aim of the work is to create risk indicators taking into account the dynamic patterns followed by new members into the fund and by the rate of return on assets.

The model is characterized by two stochastic components: the global asset return and the "new entrants" variation rate. The longevity risk is taken into account using projected life tables. In the model the scheme provides pensions only at retirement age, while invalidity and survivors' pensions are not considered. To verify the proposed methodology a numerical application is presented on the data provided by Italian Professional Orders. The dynamic analysis is performed by means of a Monte Carlo simulation in order to provide a year by year valuation.

The rest of the paper is organized as follows. In Section 1 the fund evolution mechanism is described. In Section 2 the model is analyzed. A risk indicator measure for the solvency of the fund is presented in Section 3. In Section 4 a numerical application to the data of *Cassa Nazionale di Previdenza e Assistenza dei Dottori Commercialisti* (CNPADC) is implemented. The final Section concludes the paper.

1. Fund evolution

In this Section we study the fund evolution, through the dynamical analysis of its single components. The pension scheme is modelled by a discrete time stochastic process. The fund value can be expressed by the following recursive equation[1]:

$$F_{t+1} = F_t\left(1 + r_{t+1}\right) + C_{t+1} - B_{t+1}, \qquad (1)$$

where F_t is the value of the fund at the end of the year t, r_{t+1} is the global asset return during the

[1] In a pure PAYG pension system revenues equal outlays each year (Trowbridge, 1952). Here we analyze a spurious PAYG scheme, in a growth phase (contributors greater than pensioners) where there is the accumulation of partial reserves.

year (t, t + 1), C_{t+1} is the annual contribution income at time t + 1 and B_{t+1} are the benefits paid to the pensioners (with all the cash flows, contribution income and benefit outgo, assumed to take place at the end of each year). Valuations are carried out at annual intervals. In the following subsections the characteristics of "new entrants", contributions and pensions will be analyzed.

1.1. "New entrants". As already stated, the purpose of the analysis is to study the impact of the evolution of new entrants in a pension fund financed with PAYG. There are different approaches to analyze the future flows of new entrants for these types of professions. An approach consists of studying variables related to the demographic evolution of the population, the development of education and the attractiveness of the profession, through the analysis of the transition probabilities from states of the population (university students, graduates, employment rates, active workers, members of the pension fund). This method is useful for short-term forecasting (5-10 years). As the aim is to study the fund dynamics in the long run, here we propose a model for the evolution of new entrants based on the analysis of the variable "new entrants" variation rate. This indicates the variation that occurs in the number of "new entrants" from one year to the next.

Let $ne_{(x,t+1)}$ be the function representing the number of new entrants with age x at time t + 1 and $\eta_{(x,t+1)}$ is the variation rate that occurs from time t to time t + 1 for each age x in the number of "new entrants". Thus, the number of "new entrants" for each age at time t + 1 depends on its value in t and on the variation rate as follows:

$$ne_{(x,t+1)} = ne_{(x,t)}(1+\eta_{(x,t+1)}). \quad (2)$$

In this approach the variable "new entrants" influences the evolution of the contributions and, in the long run, also the evolution of benefits. In the following paragraph the contributions and the pensions are expressed as a function of the "new entrants".

1.2. Contributions. In the analyzed scheme, the contributions paid by each member depend on the contribution rate and on the average income. The function $R_{(x,t)}$ is used to indicate the average income for contributors aged x at time t. It is a not decreasing function with respect to the age x and time t. In the application from the analysis of the time series related to the income for the members of the fund we consider a constant rate increasing income for each age class. The same increasing function is assumed for each member's average income.

Indicating with α the minimum entry age, π with the retirement age (fixed for all the people), so (π − α) is the maximum length of the working life, the total contributions at time t + 1 depend on the number of active people, the contribution rate γ and the average income (at every age) as follows:

$$C_{t+1} = \sum_{x=\alpha+1}^{\pi} n_{x,t+1}\,\gamma\, R_{(x-1,t)}, \quad (3)$$

where $n_{(x,t+1)}$ is the membership function representing the number of people aged x at time t + 1 belonging to the scheme.

For the pensions it is necessary to take into account the number of years during which each member has contributed into the fund. For this purpose the membership function can be further decomposed using the function $n_{(x,c,t+1)}$ (see Colombo-Haberman, 2005) as in the following formula:

$$n_{(x,t+1)} = \sum_{c} n_{(x,c,t+1)}, \quad (4)$$

where c represents the number of years of contribution, but in order to simplify the notation equation (3) is used.

Thus, the membership function is expressed as follows:

$$n_{(x,t+1)} = n_{(x-1,t)} \cdot p_{x-1} + ne_{(x,t+1)}, \quad (5)$$

where p_{x-1} is the probability that a member aged x −1 is alive in the following year. The mortality is considered as deterministic and the longevity risk is taken into account by using projected life tables.

The membership function, which describes the evolution of the population of the fund can be expressed as a function of the values obtained for members and new entrants at time t.

$$n_{(x,t+1)} = n_{(x-1,t)} \cdot p_{x-1} + ne_{(x,t)}(1+\eta_{(x,t+1)}). \quad (6)$$

In this way equation (3) becomes:

$$C_{t+1} = \sum_{x=\alpha+1}^{\pi} [n_{(x-1,t)} p_{x-1} + ne_{(x,t)}(1+\eta_{(x,t+1)})]\gamma R_{(x-1,t)}. \quad (7)$$

1.3. Pensions. The approach developed so far can be used in several ways to calculate the benefits. It can deal with both defined benefits and defined contribution.

Indicating with $b_{x,t+1,c}$ the average pension for each member aged x at time t +1, with c years of contributions, the total pensions will be:

$$B_{t+1} = \sum_{x=\pi+1}^{\omega-1} \sum_{c=m}^{\pi-\alpha} n_{(x-1,c,t)} p_{x-1} b_{x,t+1,c} , \quad (8)$$

where m is the minimum number of years of service required for eligibility for retirement.

When t is higher than the contribution period, the function of new entrants influences also the total pensions, when active people will become pensioners.

In the applications we use a mixed method, where the pension received, for people already members of the scheme, is the sum of two components: the first calculated with defined benefit, the second with defined contribution.

In the defined benefit pension scheme, the pension benefit is calculated as follows:

$$br = iR_P N , \quad (9)$$

where i represents the average coefficient of revaluation, R_P the pensionable earnings[1] and N is the number of years of contributions accrued in the retributory[2] system.

In the defined contribution scheme, the pension is calculated by the conversion into annuity of the amount of the notional account of each workers with the following equation:

$$bc = M_c * cdt_\pi , \quad (10)$$

where M_c is the total amount accumulated associated with seniority, *cdt* is the transformation coefficient, i.e. the annuitization coefficient used for the conversion into annuity of the notional contribution amount accumulated by each worker. The *cdt* is function of the age, but in the present model it is constant because there is only one age to retire, π. For an exhaustive explanation of the argument we refer to Janssen and Manca (2006).

2. The model

2.1. The global asset return. The global asset return can be represented in different ways according to the asset composition of the fund's investment portfolio and the associated consequent risk. The funds analyzed in this paper deal with first pillar pension schemes and therefore they usually present prudential portfolios. The global asset returns associated usually show limited variation around their historical trends. For this reason (see Orlando and Trudda, 2004) the following model is used to represent the interest rate dynamics:

$$r_{t+1} = \hat{r}_{t+1} + X_{t+1} , \quad (11)$$

where the interest rate is the sum of a deterministic component $\hat{r}_{t+1}$ and a stochastic one X_{t+1} described by an autoregressive process of first order (AR1), represented by the following non-homogeneous equation:

$$X_{t+1} = \varphi X_t + \sigma_a a_{t+1} , \quad (12)$$

which expresses the autoregressive dependence of order one, where φ and σ_a are the parameters of the process and a_{t+1} are normal mutually independent random variables (with mean 0 and variance 1).

The model described by equation (11) is a discrete representation of the Vasicek's (Vasicek, 1977) model and it is suitable to represent the global asset return on a risky asset portfolio, since the return can reach a negative value, as there can be losses of capital. The choice of this type of process is due to the fact that the analyzed funds are characterized by prudential portfolios composed with low-risk assets (the heritage is in large part composed of real estate and liquidity and only in limited part of stock funds); subsequently portfolio's returns show low volatility around their historical trend.

2.2. "New entrants" variation rate. The rate of variation in the number of "new entrants" can be represented as an autoregressive and moving average (ARMA) process of order (p, q).

Through an accurate analysis of real data we noticed that for the analyzed funds ARMA(1,1) is well suited to describe the "new entrants" variation rate, splitting the population into males and females. Thus, we assume that the "new entrants" variation rate follows an autoregressive and moving average ARMA(1,1) described by the following equation:

$$\eta_{x,t+1} = \varphi_{0_x} + \varphi_{1_x} \eta_{x,t} + \varepsilon_{t+1} - \theta_{1_x} \varepsilon_t , \quad (13)$$

where φ_{0_x} , φ_{1_x} and θ_{1_x} are respectively the autoregressive and the moving average parameters, for every age x, and ε_{t+1} are normal mutually independent random variables with mean 0 and variance σ_ε^2 .

[1] Pensionable earnings is the mean of the earnings obtained over the final years of work, revalued at a given rate.
[2] In Italy the PAYG system with defined benefit is called "retributory", while the PAYG with defined contribution is called "contributory", whereas it is internationally known as Notional Defined Contribution (NDC). See Gronchi and Nisticò (2008).

The evolution of the function depends on the sign of the parameters of the process.

2.3. Fund value. The fund evolution (1) is rewritable as a function of the "new entrants" components. Substituting the expressions obtained in Section 1 for contributions and pensions and equations (11) and (13) into equation (1) we obtain the model for the fund:

$$F_{t+1} = F_t\left[1 + \hat{r}_{t+1} + \varphi X_t + \sigma_a a_{t+1}\right] + \sum_{x=\alpha+1}^{\pi}\left[n_{(x-1,t)}p_{x-1} + ne_{(x,t)}(1 + \varphi_{0_x} + \varphi_{1_x}\eta_{(x,t)} + \varepsilon_{t+1} - \theta_{1_x}\varepsilon_t)\right]\gamma R_{(x-1,t)} +$$

$$+ \sum_{x=\pi+1}^{\omega-1}\sum_{c=m}^{\pi-\alpha} n_{(x-1,c,t)}p_{x-1}b_{x,t+1,c}. \tag{14}$$

Both the total contributions and the pension benefits are influenced by the random variable "new entrants"; the first immediately, while the latter in the long run, when active people become pensioners.

The expected value conditioned at the information set at time t will be:

$$E\left[F_{t+1}\middle|I_t\right] = F_t\left[1 + \hat{r}_{t+1} + \varphi X_t\right] + \sum_{x=\alpha+1}^{\pi}\left[\ n_{(x-1,t)}p_{x-1} + ne_{(x,t)}\left[1 + \varphi_{0_x} + \varphi_{1_x}\eta_{(x,t)}\right]\ \right]\gamma R_{(x-1,t)} +$$

$$+ \sum_{x=\pi+1}^{\omega-1}\sum_{c=m}^{\pi-1} n_{(x-1,c,t)}p_{x-1}b_{x,t+1,c}. \tag{15}$$

We make the assumption that the random variable "new entrants" is independent of the random variable global asset return. From an economic point of view the hypothesis that the variation of the global asset return does not directly influence the variation of the "new entrants" rate and vice versa is realistic.

With this assumption, the conditional variance will be:

$$VAR\left[F_{t+1}\middle|I_t\right] = \left[F_t\right]^2\sigma_a^2 + \sum_{x=\alpha+1}^{\pi}\left[ne_{(x,t)}\right]^2\sigma_\varepsilon^2\left[\gamma R_{(x-1,t)}\right]^2 \tag{16}$$

3. A risk indicator for the solvency of the fund

The most used risk indicator in actuarial and financial literature to asses the solvency of the fund, for privately funded pension funds, is the funding ratio: the ratio of the value of the assets to the present value of net obligations or other correlated indices (see Van Gaalen, 2004). This type of indicators is not suited for PAYG pension schemes.

Even the literature on public PAYG pension system has recently attempted to apply actuarial solvency analysis methodology, used in the insurance field, to the public PAYG pension system management introducing automatic balance mechanism (ABM). The ABM is a selection of predetermined measures set by law to be applied immediately as required by the solvency indicator (financial assets+contributions/pension liabilities) in order to reestablish the solvency or financial sustainability of PAYG systems, through successive applications (Settergren, 2001; Vidal et al., 2009). It is an adjustment method of the pension benefits adopted to maintain the soundness of the pension financing. This kind of mechanism is useful to reestablish the financial equilibrium of a PAYG pension system without the intervention of the legislator. Angrisani (2008) proposes a logical mathematical model to manage a pension system with a structural funded component. In a PAYG pension scheme, where current pensions are financed through current contributions, it is necessary for the financial sustainability of the system that there be a balance between active and retired people. For this reason it is essential to monitor the ratio of the active members to the passive members (demographic ratio) and consequently the ratio of the contributions received to the benefits paid. The aim of the present paper is to develop a control mechanism to monitor the solvency of the fund with respect to the variable "new entrants".

The general equation of the fund (1) can be expressed as follows:

$$\frac{F_{t+1} - F_t\left(1 + r_{t+1}\right)}{B_{t+1}} = \frac{C_{t+1}}{B_{t+1}} - 1. \tag{17}$$

From equation (17) we can observe that if the ratio contributions/pensions is above 1, the fund is increasing its value (in addition to income from capital), while if the ratio is below 1, the fund is decreasing, since the pension disbursement is greater than the contribution income. Here we propose to employ the ratio of the contribution income to the pensions to monitor the solvency of the fund:

$$CPr(t+1) = \frac{\sum_{x=\alpha+1}^{\pi} [n_{(x-1,t)} p_{x-1,t} + ne_{(x,t)}(1+\eta_{(x,t+1)})]\gamma R_{(x-1,t)}}{\sum_{x=\pi+1}^{\omega-1} \sum_{c=m}^{\pi-\alpha} n_{(x-1,c,t)} p_{x-1}\, b_{x,t+1,c}},$$

where $CPr\ (t+1)$ is the ratio between contribution income and outgoing benefits at time $t+1$, as a function of the two stochastic components.

This index could be seen as an indicator of the fund's liquidity. If the ratio is below 1, the fund is in a situation of financial instability and it must be monitored properly. The index can be used to forecast because it checks the future projections of the fund value.

Another empiric indicator to measure the financial sustainability of a pension funds is the ratio of the heritage (fund value) to the current expenditure for pensions.

The Italian legislation proposes as indicator of the financial sustainability of the retirement funds of the professional orders analyzed in the next Section, the ratio of the heritage (fund value) to the current expenditure for pensions, which must be equal to five. In the applications, for comparative purposes we also calculate the stochastic projections of this index, calculating the ratio of the fund value to 5 times the current expenditure for pensions (that we call *FPr* index), which must be over one.

4. Numerical applications and results

In this Section we apply the proposed methodology to the data provided by *Cassa Nazionale di Previdenza e Assistenza dei Dottori Commercialisti* (CNPADC), the pension fund of Italian chartered accountants (data are available from 1976 to 2006). It is one of the Italian professional order retirement funds privatized by Legislative Decree (D.Lgs) 509/1994. Until 1995 these organizations, managing the social security of given categories of self-employed professionals, were administrated by the State that would step in, in case of insolvency. Since 1995 they have managed the security of a growing number of self-employed without being sponsored by the State[1]. The pension funds of professional orders are now self-managed and they continue to operate according to a PAYG financing mechanism. As already highlighted in the introduction, this is an anomaly because private closed schemes are usually funded. In this particular system the financial self-sufficiency is certainly guaranteed only in the initial phase, because there are many contributors and no actual pensioners. In the long run it is necessary for the financial sustainability of the pension plan that the number of pensioners remains proportional to the number of workers. If the ratio active/retired decreases, the increase in the financial burden can entail a situation of financial disequilibrium. This is most relevant for the retirement funds of each specific professional order for which, unlike in a public system, there is indeed no intergroup compensation.

The social security of professional orders is not marginal inside the Italian pension system. About 1.3 million workers, equal to 5.6% of total employment, are registered as contributors to the private pension system of Italian professional orders. The share of subscribers has increased notably for some funds (engineers and architects, lawyers and chartered accountants), others remain stationary (notaries), while other ones have experienced a reduction (accountants).

Some funds remained with defined benefits, the other ones have shifted from defined benefits to defined contribution. This is the case of CNPADC, the pension fund of chartered accountants, which since 2004 has worked with defined contributions. A reform of the benefits system starting in 2004 has been passed, aimed at guaranteeing the financial sustainability of the fund in the long run. The benefits are calculated with a mixed method (pro-rata) and the pension is the sum of two components: the first calculated with defined benefits for the contributions paid until 2003, while the second for contributions paid since 2004 is on a contributory basis.

The CNPADC is a "young retirement fund", meaning that there is a high component of young members: the main age class is represented by the 35-45 years category. Figure 1 shows the distribution of the actual population classified by age and gender. From this demographical situation a dynamic fund evolution has been developed.

[1] The privatization of retirement funds of self-employed was implemented by the Legislative Decree (D.Lgs) 509/1994, which has enabled the transformation of 16 institutions in private associations or foundations from the first January, 1995. The two key legislative actions immediately following are: Law 335/1995 and D.Lgs 103/1996. Thus it is possible for these institutions to acquire legal personality, making its members independent of the public welfare. The new funds, built by D.Lgs 103/1996, follow a fully funded financial scheme. In this paper we analyze only the privatised funds (PAYG).

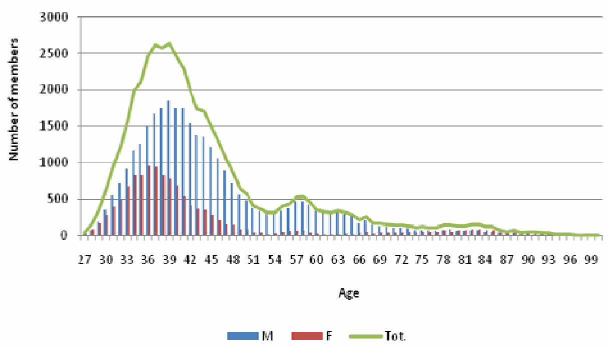

Fig. 1. Population classified by age and gender

The following assumptions are adopted:

- the starting population is the actual population of CNPADC pension fund on January 1, 2006;
- evolution of the population based on IPS55 male and female mortality tables[1];
- for "new entrants" fixed entry age α = 30, retirement age π = 65;
- for the initial population real age of ingress and contributory seniority is considered;
- the initial value of the fund is known and it is that resulting from the 2005 balance sheet;
- the inflation rate is fixed at 2%;
- a subjective contribution rate γ is equal to 10.7% of annual professional income[2];
- the transformation coefficients ex lege 335/1995 have been employed;
- professional incomes are appreciated at rate of inflation;
- benefits are calculated with the mixed method;
- administrative costs A_t are considered resulting from the 2005 balance sheet, appreciated at 3% annual rate. Thus, the general equation of the fund becomes:

$$F_{t+1} = F_t\left[1 + r_{t+1}\right] + C_{t+1} - B_{t+1} - A_{t+1}.$$

All the variables ("new entrants", contributions, pensions) have been calculated separately for males and females, because the two gender categories are characterized by different variation rates and, as we can see from Figure 1, the distribution of population is different, they present different mortality, different income and consequently different pensions. As a result in equation (14) the two summations can be split into two parts, one for females and one for males. Ten thousand simulations have been carried out, with both stochastic interest rates and "new entrants". "New entrants" in the fund have been estimated with an ARMA(1,1), where negative values below 0 have been set equal to 0. The results show that for males the number of "new entrants" begins to decrease, while for females it increases, assuming similar values for the long run. Finally, the total number of "new entrants" decreases along time.

Figure 2 shows the percentiles of the frequency distribution and the expected value of the fund value (all values are expressed in million euros).

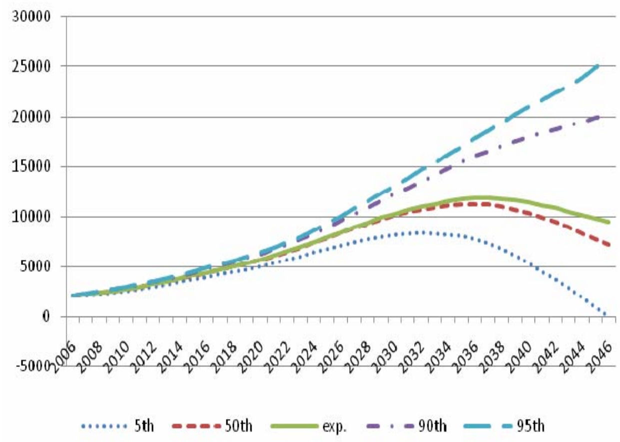

Fig. 2. Evolution of the fund (percentiles of the frequency distribution and expected value)

Figure 2 indicates that there is a probability that the fund could reach a peak in 2035, then taking a downward trend until reaching default. This is a consequence of the initial structure of the population considered. When the "hump" of the members aged 35-45 years in Figure 1 reaches retirement age from 2026 to 2036, then the fund begins to decrease because the pension outlays increase. If there is not a sufficient flow of "new entrants", and therefore a sufficient flow of contributions, the fund will tend to decrease as the contributions and performance of the fund are not sufficient to cover pensions and administrative costs and the fund will tend to grow towards achieving zero and thus default.

We calculated the conditional value at risk (cVaR) 95% confidence level of the pension fund value. The cVaR, known also as expected shortfall (ES), is a quantile based measure of risk. It can be calculated by: $CVaR = \frac{1}{1-\alpha}\int_{\alpha}^{1} q_p dp$ being q_p the p-quantile of the fund distribution. The cVaR is the average of the worst $1-\alpha$ cases lower than the α-quantile: $CVaR = E[F_t | F_t \le Q_\alpha(t)]$. The cVaR takes into account the extreme lost.

For the sake of comparison we calculated the cVaR also in the case of fund closed to "new en-

[1] IPS55 are projected life tables for Italian males and females, cohort 1955.

[2] Subjective contribution rate varies electively between 10% and 17% of annual professional income. In 2005 the average rate was 10.71% (Source: CNPADC).

trants". Figure 3 shows the cVaR in both cases (stochastic evolution of "new entrants" and fund closed to "new entrants").

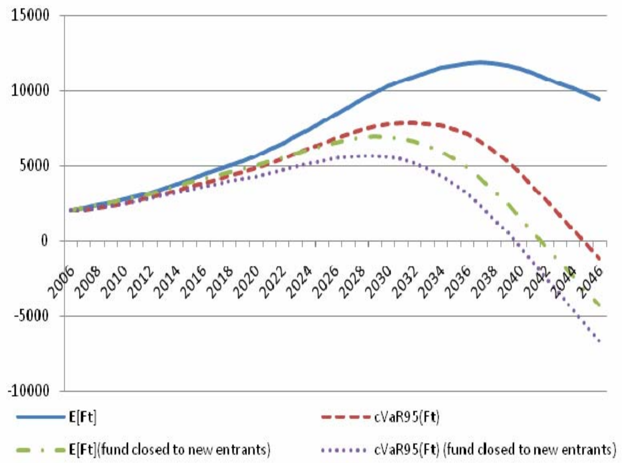

Fig. 3. Evolution of the fund

From Figure 3 it emerges that the expected value of the fund is always positive along the period analyzed, while analyzing the cVaR the risk to which the fund is exposed to becomes evident.

The high influence of the demographic variable new entrants is highlighted in the case of absence of new contributors, where the fund moves to zero rapidly. This highlights the importance of the demographic variable "new entrants". If we look at the figure, the expected value of the fund closed to "new entrants" is lower than the cVaR in the case of stochastic evolution of "new entrants".

Subsequently, we calculated the CPr index (i.e. the ratio between contributions and pensions). As already stated in the previous Section, this is an index of liquidity. In Figure 4 the cVaR at 95% confidence level and the expected value of the index are represented.

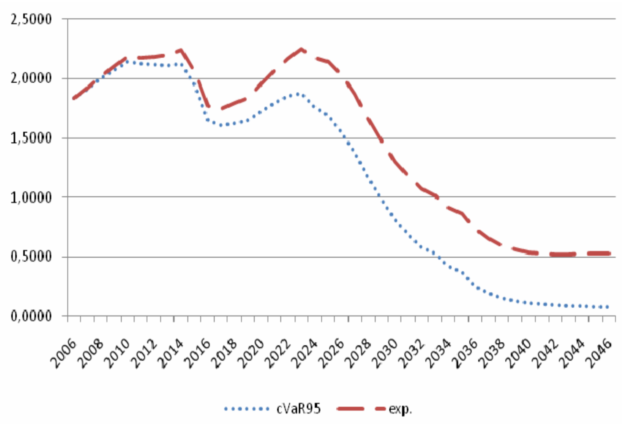

Fig. 4. CPr index, cVaR at 95% confidence level and expected value

Observing the CPr index in Figure 4 we see its particular shape, due to the demographic structure of the population. The chart highlights that the expected value of the CPr index is higher than 1 only until 2036, after decreasing and becoming stable around the value of 0.7. But observing the cVaR we can see that this index is below 0 already around 2030, and taking values close to 0 (about 0.1). This confirms that the fund is exposed to default risk. In fact, if the CPr index is stable below 1, it means that the fund is progressively reducing the accumulated resources, since the fund begins to decrease.

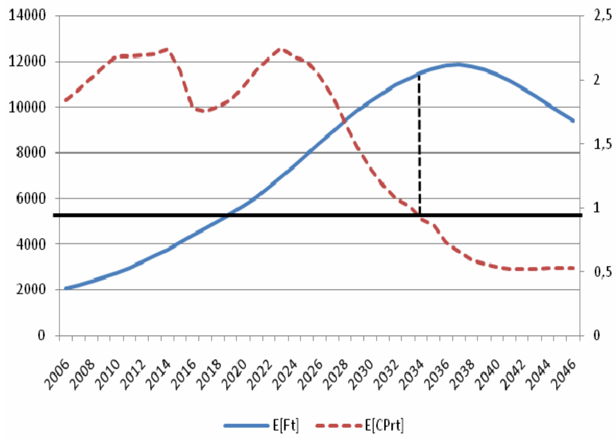

Fig. 5. Fund value and CPr index (expected value)

Figure 5 shows the expected value of the fund and CPr index. As it is possible to observe, the fund begins to decrease 4-5 years after the CPr index reaches 1, since CPr < 1. This is because during that period returns on assets cover the difference $C_t - B_t$.

The index can be used for monitoring purposes, but also for forecasting.

We have also calculated the stochastic projections of the indicator proposed by Italian rules: the ratio of the fund value to the current expenditure for pensions, which by law must be equal to five. For comparative purposes, we calculate the ratio of the fund value to 5 times the current expenditure for pensions (FPr index), which must be over one. Figure 6 shows the expected value and the cVaR of FPr index. This empiric index is not a good indicator for the solvency of the fund.

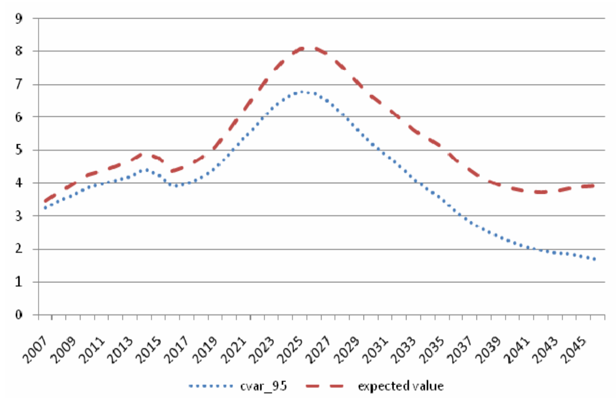

Fig. 6. FPr index, cVaR 95% and expected value

Conclusions

In this paper we analyze the demographic risks related to PAYG pension funds. In particular we focus on the risk that the flow of "new entrants" could be insufficient to guarantee the financial equilibrium in the long run. A specific model has been developed to analyze the fund evolution as a function of the futures active members. Numerical applications are approached by means of a simulation methodology, on the Italian Professional Orders' private pension funds.

We showed that the process of the "new entrants" variation rate is well represented by an autoregressive moving average process. Particularly, after dividing the active members into the male and female categories, the future evolution of the "new entrants" variation rate is well described by an ARMA(1,1) process. A Vasicek's process was used to describe the global asset return dynamics. In fact the Italian funds investigated usually assume prudential policies because of their social security function. Hence, the evolution of the global asset return presents lower volatility and limited variations around its historical trend.

We propose to monitor the solvency of the fund over the time by the CPr index, the ratio between contributions' and benefits' expected cash flows. It can be used to check the liquidity and consequently the stability of the fund for monitoring but also for forecasting purposes. We compare it with another index based on the ratio of the fund value to the current disbursement for pensions, which is proposed by Italian rules.

The model is applied to the data provided from CNPADC, the pension fund of Italian chartered accountants. The demographic structure of the fund is characterized, like most other analyzed funds, by a relevant presence of "young cohorts" and a small presence of pensioners. This means that in the short term the fund analyzed is in a strong growth phase without any financial equilibrium problems, because the positive cash flows of contributions are much higher than payments for pensions. By means of a Monte Carlo simulation we analyze the future cash flows. The analysis of the CVaR and the CPr index highlights that, after a period of increasing fund value, the fund is exposed to the risk of default, due to changes in the ratio contributions-pensions. As a result, the demographic variable "new entrants" has a strong influence on the future dynamics of the fund.

With respect to the projections of the fund value, the proposed indicator shows its capability to anticipate trend reversals in the cash flows. The risk indicators constructed in this way, respond in advance to the demographic crisis.

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