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On market value of liability and life insurance securitization

Abstract

In the paper it is proposed a new formula to calculate the market value of liability (MVL) of a block of life insurance business. Luke Girard in his work reconciles the actuarial appraisal method and the option pricing method. This new MVL formula based on the actuarial appraisal method reconfirms Girard's result and decomposes the MVL into two components: the direct value of liability cash flows and the value of regulatory protection. The direct value of liability cash flows is the discounted liability cash flow at the weighted average cost of capital. The value of regulatory protection consists of the value of reserve requirement, the value of capital requirement, and a tax adjustment. In the second part of the paper, it is applied the MVL formula to a generic life insurance securitization structure. For blocks of term life insurance or universal life insurance with secondary guarantees, the value of reserve requirement is relatively high. Innovative capital market solutions of funding those reserves, such as securitization, could reduce the amount, thus create positive value for the company. A securitization transaction modifies the distributable earning of the block, thus changes the fair value of the block, and market value of the liability. The formulas and arguments work similarly for all external funding arrangements of redundant reserves.

Keywords: fair value accounting, market value of liabilities, life insurance securitization.

Introduction

In the study, we will show how a life insurance company could create value through securitization of XXX (AXXX) redundant reserves. Here we use the word "securitization" to mean any external funding instruments to meet the regulatory requirement. It could be true securitization by issuing notes to investors raising actual assets. It could be just a letter of credit from a bank, or any other means.

Securitization has been used by life insurance industry in past decade to transfer risks, to realize embedded profits, and to provide alternative financing, such as funding XXX (AXXX) redundant reserves. In this paper, we will work with a generic securitization model to illustrate the value added with such transaction. For a good survey of life insurance securitization in its early years (see Alex Cowley and J. David Cummins, 2005).

Fair value of a block of life insurance business has been a focus of lots of research and policy papers in recent years. See American Academy of Actuaries (2005), Girard and Luke N. (2000), Girard and Luke N. (2002), International Accounting Standards Board (2007), Perrott, Godfrey, and William Hines (2002), Don Solow (2006), Marsha Wallace (2002). In this work, we will use the actuarial appraisal method. Following Girard in Girard and Luke N. (2000), we define the fair value of a block as the distributable earning discounted at the weighted average cost of capital (WACC).

Our main result in section 1 is a new formula for the market value of liability (MVL). The new formula shows that MVL is a sum of two components. The first component is the direct value of liability cash flows (DVLCF), which is a sum of liability cash flows discounted at WACC. The second component is the value of regulatory protection (VRP). The value of regulatory protection consists of the value of reserve requirement, the value of capital requirement, and a tax adjustment. The value of reserve requirement is the value, lost by the company, while maintaining product assets (assets backing reserves) at the rate of return lower than WACC of the company. The value of capital requirement is the value, lost by the company, while maintaining capital assets at the rate of return lower than WACC of the company.

For blocks of term life insurance business or universal life insurance with secondary guarantees, the statutory reserve is relatively high, as some industry practitioners believe. It is much higher than a more realistic (with some degree of conservatism) measure of the company's obligation to the policyholders. This more realistic measure is often referred to as economic reserves. The difference of statutory and economic reserves is redundant reserves. Companies try to fund redundant reserves more efficiently with innovative capital market solutions, such as securitization. A securitization transaction could finance redundant reserves at a lower cost, thus create value for the company.

In section 2, we reconcile our main result in section 1 with option pricing method as defined by Girard in Girard and Luke N. (2000).

In section 3, we compute the value added of block from a securitization transaction. Calculations are based on a generic simplified deal structure. For some of the specific deal structures please see Alex Cowley and J. David Cummins (2005), Richard Leblancand and Dimitry Stambler (2007). In this simplified structure, the ceding company sets up a captive reinsurance company, a special purpose vehicle, and reinsures the block with large XXX (AXXX) redundant reserves to the captive. The captive is owned by the ceding com-

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pany. The ceding company provides funding for economic reserves and initial capital. The captive issues surplus notes to raise funds, or obtains letter of credit from banks, to fund the redundant reserves. Initial capital of the captive is determined by desired rating of the surplus note, and some other factors. The profit of the block is retained in the captive initially. It will be distributed to the ceding company only if total surplus of the captive satisfy certain restrictive criteria.

The fair value of the block for the ceding company is then the discounted distributable earning (dividend) out of the captive after the securitization transaction. The value added from the transaction is the difference of the fair value before and after the transaction.

We have a numerical example in section 4. The MVL of the block and value added from the transaction are calculated for different levels of capitalization of the captive.

In this work, we do not consider the impact of the securitization on the firm's weighted average cost of capital (WACC); we assume that WACC remains the same pre and post transaction. Please see Section 5.1 for some discussions on reasonableness of this assumption.

We use the deterministic approach of calculating the fair value based on the best estimate projections. This method works relatively well for term insurance. For the unlimited liability (UL), the stochastic approach will provide further insight of the business because of the interest rate sensitivity of the policy cash flows.

1. Market value of life insurance liabilities

We will use the actuarial appraisal method (AAM), following Girard and Luke N. (2000). We will assume it is a closed inforce block, and there is no adjusted net worth (ANW) with the block. The asset backing the block is exactly the product assets plus required surplus assets. The product assets are the assets backing the reserve. The statutory value of the product assets is the same as the statutory reserves. The statutory, market and tax value of required surplus assets are the same. It equals some multiples of the risk-based capital.

Let t be time period, where $t = 0$ to N. At the valuation date, $t = 0$. End of projection time, N, is chosen large enough so that the block is null beyond time N. In particular, there is no distributable earning beyond time N.

Let k be WACC of the company, and let $MVA(t)$, $SVA(t)$, $TVA(t)$ be market, statutory, tax value of product assets, respectively. Let $i(t)$ be return (rate) on the market value of product assets. Product assets are those assets backing reserves. Let $A(t)$ be cash flow at time t from product assets, such as coupon income, maturities, including reinvestments and sales of assets. We have:

$$A(t) = i(t) \times MVA(t-1) - \Delta MVA(t-1).$$

Let $F(t)$, $i(t)$ be any given schedules of quantities. Let us define:

$$NC(i,k,F) = \sum_{t=1}^{N} (k - i(t)) \times F(t-1)/(1+k)^t, \quad (1)$$

where $F = F(t)$, $i = i(t)$, $t = 0, 1, \ldots, N$, and k is a scalar. When $k > i(t)$, we have $NC(i, k, F) > 0$. The function $NC(i, k, F)$ will be frequently used through out the paper. It makes the formulation of key results of the paper more convenient.

It is easily verified that:

$NC(i, k, F + G) = NC(i, k, F) + NC(i, k, G)$,

$NC(i + j, k, F) = NC(i, k, k, F) + NC(j, k, G)$,

$NC(i, k, F) = NC(2k - i, k, F)$.

We could explain the "economic" intuition of $NC(i, k, F)$ as follows. If one borrows funds at an interest rate k (cost of funding, time value of money for the borrower), and keep a balance $F(t)$ in an account which earns interest rate $i(t)$, then $NC(i, k, F)$ is the value lost, (if $k > i(t)$), by maintaining such an account. It is quantitative definition of the "negative carry" concept.

Using this notation, we could simplify the following summation for the present value of asset cash flow $A(t)$ of product assets.

$$\sum_{t=1}^{N} A(t)/(1+k)^t = \sum_{t=1}^{N} \{i(t) \times MVA(t-1) - [MVA(t) - MVA(t-1)]\}/(1+k)^t =$$
$$= \sum_{t=1}^{N} \{(1+k) \times MVA(t-1) - MVA(t)\}/(1+k)^t + \sum_{t=1}^{N} (i(t) - k) \times MVA(t-1)/(1+k)^t = \quad (2)$$
$$= MVA(0) - NC(i,k,MVA).$$

We could interpret the equation (2) as follows. The product assets have current market value $MVA(0)$. It is legally required that assets of value $MVA(t)$ to be held by the company at time t with certain investment restrictions. There are rebalancing, sales and purchases of the assets at time t, so that the balance remains at $MVA(t)$. The rebalancing will change $A(t)$ and $MVA(t)$ simultaneously. However, it will not change $i(t)$ of the period.

The return on assets is $i(t)$, which is usually lower than the WACC k. Then the present value of cash flows generated by these product assets is no longer $MVA(0)$, but $MVA(0) - NC(i, k, MVA)$. Because the asset is not "free", it could not be liquidated immediately, thus it is not "distributable", the economic loss of $NC(i, k, MVA)$ is inevitable.

The equation (2) is a pure algebraic manipulation. First, we add and subtract $k \times MVA(t-1)$, so we obtain $NC(i, k, MVA)$. Second, in the summation

$$\sum_{t=1}^{N}\{(1+k)\times MVA(t-1) - MVA(t)\}/(1+k)^t,$$

all terms cancels out except the term MVA (0).

Let $II(t)$ be the investment income as defined in the statutory income statement. We have $II(t) = A(t) + \Delta SVA(t)$. Similarly define $TII(t)$ – taxable investment income, we have $TII(t) = A(t) + \Delta TVA(t)$.

Let $I(t)$ be the after tax income at time t of the block, $RS(t)$ be required surplus at time t, and $DE(t)$ be the distributable earning at time t. We have $DE(t) = I(t) - \Delta RS(t)$.

Let j be the return rate on required surplus assets.

Let T be the corporate tax rate. Tax losses are assumed to be utilized as they are incurred.

Let $MVL(t)$, $SVL(t)$, $TVL(t)$ be the market, statutory, tax value of liabilities. SVL is the statutory reserves with some adjustments. TVL is the tax reserve with some adjustments. We will assume $SVA(t) = SVL(t)$ for all t. The difference between SVA and SVL will flow through earnings and distributed.

Let $L(t)$ be the net policy cash flows, including premiums, benefits, reinsurance adjustments etc., and $E(t)$ be expense cash flows.

The after tax income is then:

$$I(t) = [II(t) + j\times RS(t-1) - L(t) - \Delta SVL(t-1) - E(t)] - T\times[TII(t) + j\times RS(t-1) - L(t) - \Delta TVL(t-1) - E(t)] = (1-T)\times[A(t) + j\times RS(t-1) - L(t) - E(t)] - \Delta TBA(t-1), \quad (3)$$

where $TBA(t)$ is defined as $T\times(TVA(t) - TVL(t))$.

Let $DDE(t)$ be the sum of present value at time t of all future ($T > t$) distributable earning $DE(T)$ discounted at WACC, and ANW be adjusted net worth, market value of distributable asset at time 0.

Let FV be the fair value of the block at time 0. FV – equals sum of ANW and DDE (0).

When $ANW = 0$, the fair value is:

$$\begin{aligned}FV = DDE(0) &= (1-T)\times\sum_{t=1}^{N}\{[A(t) + j\times RS(t-1) - L(t) - E(t)] - \Delta TBA(t-1) - \Delta RS(t-1)\}/(1+k)^t = \\ &= (1-T)\times\{\sum_{t=1}^{N} A(t)/(1+k)^t - \sum_{t=1}^{N}(L(t)+E(t))/(1+k)^t\} - \{\sum_{t=1}^{N}\Delta TBA(t-1)/(1+k)^t\} + \\ &+ \{\sum_{t=1}^{N}[(1-T)j\times RS(t-1) - \Delta RS(t-1)]/(1+k)^t\} = (1-T)\times\{MVA(0) - NC(i,k,MVA) - \\ &- \sum_{t=1}^{N}(L(t)+E(t))/(1+k)^t\} + \{TBA(0) - NC(0,k,TBA)\} + \{RS(0) - NC((1-T)j,k,RS)\} = \\ &= RS(0) + (1-T)(MVA(0) - MVL(0)) + T\times(TVA(0) - TVL(0)),\end{aligned} \quad (4)$$

where

$$MVL(0) = LE(0) + NC(i,k,MVA) + NC(0,k,TBA)/(1-T) + NC((1-T)j,k,RS)/(1-T)$$

and

$$LE(0) = \{\sum_{t=1}^{N}(L(t)+E(t))/(1+k)^t\}.$$

Note $TBA(t) = T\times(TVA(t) - TVL(t))$. It is the tax adjustment.

The above equation (4) is a reformulation of the basic identity in Girard and Luke N. (2000). The fair value of the block is a sum of the market value of surplus assets (RS(0)), after tax difference of market value of product assets and market value of liability ($(1-T)\times(MVA(0) - MVL(0))$), and a tax adjustment.

In the fair value accounting, the assets are accounted at the market value. The question is on the market value of liability.

The liability, this block of life insurance policies, provides future cash flows $L(t)$ and $E(t)$ for the company. Intuitively, the market value of liability should be $LE(0)$, sum of the discounted liability cash flow. But the situation is more complicated due to regulation. To operate this block of life insurance business, insurance regulation requires the company to maintain statutory reserves, and the capital. The asset backing reserve and capital has return $i(t)$ and j, generally lower than WACC of the company. The asset has the market value $MVA(0)$ and $RS(0)$. But it is not immediately distributable. This delay of distribution reduced the value of asset. The reduction in value of assets is a part of the cost of operating this block of life policies. It should be a part of market value of liability.

The reserve requirement and capital requirement increase the market value of liabilities by $NC\ (i, k, MVA)$ and $NC\ ((1-T)\ j, k, RS)\ /\ (1-T)$. Of course, these regulations provide the policyholder with extra protection against insurer insolvency and other adverse events. We could refer the increase in MVL as the value of regulatory protection (VRP). The third term $NC\ (0, k, TBA)\ /\ (1-T)$ exists only if tax values are different from statutory values. It could also make sense to group it with $T \times (TVA(0) - TVL(0))$ as a part of the tax adjustment. To be consistent with Girard and Luke N. (2000), we keep it in MVL.

In summary, MVL is a sum of direct value of liability, which is a sum of discounted liability cash flows ($DVLCF = LE(0)$), and the value of regulatory protection $VRP(0)$. The value of regulatory protection has three components.

$$VRP(0) = NC(i,k,MVA) + NC(0,k,TBA)/ \\ /(1-T) + NC((1-T)j,k,RS)/(1-T), \qquad (5) \\ MVL(0) = LE(0) + VRP(0).$$

In the next section, we will try to reconcile equation (5) with the option pricing approach defined by Girard in Girard and Luke N. (2000).

2. Reconciliation with option pricing method

We include D7 and D8 of Girard and Luke N. (2000) here for easy reference. It is shown in Girard and Luke N. (2000) that the market value of liabilities $MVL(t-1)$, using option pricing method (OPM), could be obtained by directly discounting the liability cash flow recursively. The discount rate is the asset return $i(t)$. The formula D7 of Girard and Luke N. (2000) states:

$$MVL(t-1) = \frac{MVL(t) + L(t) + E(t) + RP(t)}{1 + i(t)},$$

and $MVL\ (N) = 0$, (6)

where the $RP\ (t)$ is called the required profit. It is defined as in D8 of Girard and Luke N. (2000),

$$RP(t) = (\frac{k}{1-T} - j)RS(t-1) + (k - i(t))(MVA(t-1) - \\ - MVL(t-1)) + \frac{k}{1-T}T(TVA(t-1) - TVL(t-1)). \qquad (7)$$

We will show that the MVL equation (5), proposed in this work, is equivalent to equations (6) and (7).

We expand the definition of $NC(i, k, F)$ to any point of time n, as follows.

$$NC\ (i, k, F, n) = \sum_{t=1+n}^{N} (k - i(t)) \times F(t-1)/(1+k)^{(t-n)},$$

where $F = F\ (t)$, $i = i\ (t)$, $t = 0, 1, \ldots, N$. Note the definition override, we have $NC\ (i, k, F) = NC\ (i, k, F, 0)$.

Definition: $LE(n) = \sum_{t=1+n}^{N} (L(t) + E(t))/(1+k)^{(t-n)}$.

The following Lemma 1 provides some basic recursive relationship of LE and NC functions. It could be easily verified.

Lemma 1

$LE\ (n) = (1 + k)\ LE\ (n-1) - L\ (n) - E\ (n)$,

$NC\ (i, k, F, n) = (1 + k)\ NC\ (i, k, F, n-1) -$

$-\ (k - i\ (n))\ F\ (n-1)$.

Lemma 2

$MVL\ (n) = LE\ (n) + NC\ (i, k, MVA, n) +$

$+\ NC\ (0, k, TBA, n)\ /\ (1-T) +$

$+\ NC\ ((1-T)\ j, k, RS, n)\ /\ (1-T)$.

Note that MVL equation (5) is a special case of Lemma 2, when $n = 0$. We will try to prove Lemma 2 based on option pricing method of Girard and Luke N. (2000), which is described in equations (6) and (7).

Proof of Lemma 2

For $n = N$, it is trivial. Both sides are zero. We will use induction argument. Assuming it holds for n, we will show that it holds for $(n-1)$.

Let us write $MVL(n)$ in terms of $MVL\ (n-1)$, using Lemma 1.

$$MVL(n) = LE(n) + NC(i,k,MVA,n) + NC(0,k,TBA,n)/(1-T) + NC((1-T)j,k,RS,n)/(1-T) = \\ = (1+k)LE(n-1) - L(n) - E(n) + (1+k)NC(i,k,MVA,n-1) - (k-i(n))MVA(n-1) + \\ + ((1+k)NC(0,k,TBA,n-1) - k \times TBA(n-1))/(1-T) + ((1+k)NC((1-T)j,k,RS,n-1) - \qquad (8) \\ - (k-(1-T)j)RS(n-1))/(1-T) = (1+k)\{LE(n-1) + NC(i,k,MVA,n-1) + NC(0,k,TBA,n-1) + \\ + NC((1-T)j,k,RS,n-1)\} - L(n) - E(n) - RP(n) + (k-i(n))MVL(n-1).$$

Substituting equation (8) into equation (6), then write MVL $(n-1)$ in terms of other terms, (i.e., solving for MVL $(n-1)$).

The identity of equation (6) is prospective. The MVL at time $(t-1)$ is obtained by discounting the MVL at time (t), cash flows, and required profit (RP). But the identity of equation (7) for RP is retrospective. The RP at time (t) is obtained based on the quantities, RS, MVL, MVA, etc., at time $(t-1)$. In comparison, the new MVL equation (5) is only prospective and more straight-forward. The MVL is obtained by discounting the future cash flows and based on future quantities, such as statutory reserve and required capital levels in the future.

3. Value added with securitization

In the effort to reduce value of regulatory protection VRP (t), some insurance companies utilize securitization solutions to meet reserve and capital requirements at a lower cost.

To simplify, we assume that statutory, tax and market values of product assets are the same.

It is generally understood that there is some redundancy in XXX (AXXX) reserves. The degree of redundancy is up to debate. To measure the redundancy, we define economic reserve as a more realistic measure of company's obligation to policyholders. The difference between statuary reserves and economic reserves is redundant reserves.

If the product is priced appropriately, appropriately defined economic reserve could be supportable by policy cash flows. The different approaches of defining economic reserves are beyond the scope of this paper. Typically, the XXX redundant reserves reach the peak at 10-15 years for a closed block of term life business. For UL with secondary guarantee, AXXX redundant reserves reach the peak around year 20. See Keith Dall, Donna Megregian and Rob Stone for more details. The peak year depends on product design parameters, such as length of the level term period, charges in the shadow account, etc.

By definition, economic reserves should be able to support liability cash flows of the block, and redundant reserves are at a lower risk comparing to economic reserves. Thus, it should be possible to finance redundant reserves at a cost lower than cost of financing economic reserves.

In this work, we will focus on the actuarial and fair value aspect of the life insurance securitization transaction. For other details, such as legal, structural, regulatory concerns, please refer to other literatures.

A typical XXX (AXXX) securitization transaction involves a ceding company, which has a need to finance the redundant reserve. A simplified structure of such a transaction is as follows. The ceding company, with the help of an investment bank, sets up a captive reinsurance company to reinsure the block. The ceding company provides economic reserve of the block and initial capital to the captive. The captive issues surplus notes to investors to raise funds.

The proceeds of surplus notes, together with economic reserve assets from the ceding company, are kept in the reinsurance trust for the benefit of ceding company. Some structures have an account, called pre-funded account for proceeds, not yet needed in the reinsurance trust; to simplify, we group it to the reinsurance trust. The capital contribution of the ceding company is kept in a surplus account, which satisfies the capital requirement of the captive.

For the surplus note to be marketable, it is often rated by rating agencies. Often there is also a financial guarantee company (monoline) to guarantee the timely payment of the interest and ultimate payment of the principle, see Richard Leblanc and Dimitry Stambler (2007) for more on monoline participation.

To rate the note, the rating agencies use the traditional approach of rating a cash flow structured finance instrument. They apply various stress test, or stochastic test to the deal cash flow, and require certain credit enhancements, such as certain levels of over-collateralization and interest coverage, to get the desired rating. In the case of XXX (AXXX) securitization, the ceding company is asked to put extra capital to the captive. In some cases the extra capital is many times of the risk based capital (RBC), or required surplus (RS) previous held for this block. The regulator, rating agency, and the monoline also restrict the captive's ability to release the profit to the ceding company.

In general, there is a tax sharing agreement between the captive and the ceding company. The initial losses from the reserve increases are utilized as a tax deduction by the ceding company, or its affiliates. The tax saving of the ceding company is an important consideration in designing of the deal structure. In this paper, to simplify, we assume that the tax saving is passed back to the captive.

There are also a few capital market deals involved in funding of the capital requirements. For certain blocks, certain types of insurance business, the future cash flow are more predictable than the others. So, some people believe that the capital requirements are "redundant", and the redundancy could be financed more efficiently with capital market solutions.

In this section, we will try to compute the fair value of the block after the transaction. The fair value of the block for the ceding company is the distributable earning from the captive discounted at the WACC. Our model is based on the simplified deal structure.

Let SVL (t) be the statutory value of liability at time t, the statutory reserve. Let EVL (t) be the economic value of liability at time t, the economic reserves. Let $RVL(t)$ be the redundant reserves at time t. $RVL = SVL - EVL$.

Let $i(t)$ be the return on the assets in the reinsurance trust account. The reinsurance trust account is broadly defined. It holds the proceeds from the surplus notes and the economic reserves. It follows a more conservative investment strategy. It should have more assets than statutory reserves at all times.

Let MS (t) be the minimum surplus account balance threshold before profit transfer (/ dividend distribution) to ceding company is allowed. The least restrictive choice of $MS(t)$ could be $RS(t)$, or $RBC(t)$. The most restrictive choice of $MS(t)$ is infinite, meaning that before the end of the transaction, there will be no transfer of profit regardless of performance of the block. In section 4, we propose a possible MS formula. It is the sum of: (1) present value of all future extra death benefit to the desired mortality stress level; and (2) present value of negative liability cash flow of next 5 years. It is floored at $RS(t)$. In reality, the $MS(t)$ formula varies deal by deal. It is agreed upon by all parties after negotiation.

Let j be the return on surplus account. It follows a more aggressive investment strategy than the reinsurance trust account.

Let NB (t) be the surplus note balance at time t. The surplus note at time 0. NB (0) usually is the maximum RVL based on the baseline projection. There will be no redemption before the reserve reaches the peak. After the peak, the surplus note is partially redeemed, so that $NB(t) = RVL(t)$.

r (t) – surplus note interest rate. The interest paid on the note at time t is $BN (t-1) \times r (t)$.

The total investment income of the captive is $i(t)\times[NB(t-1)+EVL(t-1)]+j\times MS(t-1)$. The interest on the note is $r(t)\times NB(t-1)$.

Let Des (t) be the distributable earning (/ dividend) at time t from the captive to the ceding company.

Let EE (t) be the securitization transaction expense. Let $PvEE(0)=\sum_{t=1}^{N}\frac{EE(t)}{(1+k)^t}$.

The distributable earning is the total investment income, less liability cash flows, less interest payment on the note, less change of economic reserve ΔEVL and change of minimum surplus ΔMS, less tax and securitization expense. Note that the tax is calculated based in taxable income and tax reserves of the block. We have:

$$\begin{aligned}DEs(t) &= i(t)\times[NB(t-1)+EVL(t-1)]+\\&+ j\times MS(t-1)-L(t)-E(t)-r(t)\times\\&\times NB(t-1)-\Delta EVL(t-1)-\\&-\Delta MS(t-1)-TAX-EE(t),\end{aligned} \tag{9}$$

where

$$\begin{aligned}TAX &= T\times\{i(t)\times[NB(t-1)+EVL(t-1)]+\\&+ j\times MS(t-1)-L(t)-E(t)-r(t)\times NB(t-1)-\\&-\Delta TVL(t-1)-EE(t)\}.\end{aligned}$$

Let $DDEs$ (t) be the sum of present value at time t of all future ($T > t$) distributable earning Des (T) discounted at the WACC.

The $DDEs$ (0) is then,

$$\begin{aligned}DDEs(0) &= \sum_{t=1}^{N} DEs(t)/(1+k)^t = \sum_{t=1}^{N}[(1-T)\times j\times MS(t-1)-\Delta MS(t-1)]/(1+k)^t-\\&-(1-T)\times\sum_{t=1}^{N}[L(t)+E(t)]/(1+k)^t-(1-T)\times\sum_{t=1}^{N}EE(t)/(1+k)^t+(1-T)\times\\&\times\sum_{t=1}^{N}[(i(t)-r(t))\times NB(t-1)]/(1+k)^t+(1-T)\times\sum_{t=1}^{N}[i(t)\times EVL(t-1)-\Delta EVL(t-1)]/(1+k)^t+\\&+T\times\sum_{t=1}^{N}[\Delta TVL(t-1)-\Delta EVL(t-1)]/(1+k) = MS(0)-NC((1-T)\times j,k,MS)-\\&-(1-T)\times LE(0)-(1-T)\times PvEE(0)-(1-T)\times NC(i-r+k,k,NB)+(1-T)\times\\&\times[EVL(0)-NC(i,k,EVL)]+\{TBA(0)-NC(0,k,TBA)\}+RTA,\end{aligned} \tag{10}$$

where RTA is the life time tax adjustment of the redundant reserve, defined as:

$$RTA = T\times\sum_{t=1}^{N}\Delta RVL(t-1)/(1+k)^t.$$

The equation (7) requires some algebra manipulation. The change of SVL (ΔSVL) is spitted into the sum of ΔEVL and ΔRVL. The difference of SVL and TVL is grouped into TBA. And the algebra trick in equation (2) is applied to variables MS, NB, and EVL.

Let $ANWs$ be the adjusted net worth after the transaction at time 0. It is the same as surplus relief provided by the transaction. Let FVs be the fair value of the block after the transaction.

The initial reserve contribution of the ceding company will be $EVL(0)$, and initial capital contribution is $MS(0)$. The reserve to be released is $SVL(0) - EVL(0)$ (= $RVL(0)$). The distributable asset at the transaction, is $ANWs = (1 - T) \times RVL(0) + RS(0) - MS(0)$. To some ceding companies, this surplus relief effect is also a motivation for the transaction. As always, $FVs = ANWs + DDEs(0)$.

There will be different investment guidelines for the ceding company and the captive. For simplicity, let us assume that surplus accounts have the same return "j" as define above. The product asset has return $i_1(t)$ for the ceding company, and $i_2(t)$ for the captive.

Let i_1 (t) be the return on product assets in the ceding company. Usually, it is the return of the general account of the ceding company, and i_2 (t) be the return on product assets in the captive. It is the same as i (t) defined in section 1.

The value added (VAs) by the transaction is:

$$\begin{aligned} VAs = FVs - FV &= -(1-T)\times PvEE(0) - RTA - NC((1-T)\times j, k, MS - RS) - \\ &- (1-T)\times \{NC(i_2 - r + k, k, NB) + NC(i_2, k, EVL) - NC(i_1, k, SVL)\} = \\ &= Term_1 + Term_2 + Term_3 + +Term_4 + Term_5, \end{aligned} \quad (11)$$

where $Term_1 = (1-T)\times NC(r,k,RVL)$,

$Term_2 = -NC((1-T)\times j, k, MS - RS)$,

$Term_3 = -(1-T)\times NC(i_2 - r + k, k, NB - RVL)$,

$Term_4 = (1-T)\times NC(i_1 - i_2 + k, k, SVL)$,

$Term_5 = -(1-T)\times PvEE(0) - RTA$.

The $Term_1 = (1-T)\times NC(r,k,RVL)$ is the major term in VAs. In general, $k > r$, and this term is positive and serves as purpose of the transaction. It is the value created for the company by financing RVL at rate r instead of the WACC k. Please see discussions in section 5 for more discussion on value creation in a structured finance securitization transaction.

In general, $(1 - T) \times j < k$, $MS \geq RS$, the term $Term_2 = -NC((1-T)\times j, k, MS - RS)$ is negative. The after tax return of surplus asset is usually lower than the WACC. This is the extra negative carry of the minimum capital of captive (MS) over require surplus (RS). It is difficult to compare i_2 (t) and r (t), it depends on the rating of the surplus note and the investment strategy.

In general, $i_2 < r$, $NB \geq RVL$, the term $Term_3 = -(1-T)\times NC(i_2 - r + k, k, NB - RVL)$ is negative. The return of the assets in the reinsurance trust is smaller then interest rate of the surplus note. This is the amount of negative carry of the extra note balance over the redundant reserves.

In general, $i_2 < i_1$, the term $Term_4 = (1-T)\times NC(i_1 - i_2 + k, k, SVL)$ is negative. Return of the ceding company general account i_1 is greater than return of assets in the reinsurance trust i_2. It is the lost investment income of product assets by the captive with more conservative investment strategy.

We see that the value created by the transaction depends on the note balance NB, the amount of capital MS hold by the captive. The ceding company desires MS to be as small as possible (at the level of RS), and NB to be as small as possible as well (at the level of RVL). On the opposite side, regulators, and financial guarantors Richard Leblanc and Dimitry Stambler (2007) require MS and NB to be at a certain level. In Section 4 below, we list few possible capital requirements for the captive.

4. A numerical example

Data used in this section is based on a block of term life insurance in the public market. The policies in the block are in duration 4 or 5 on average. It will reach the reserve peak in another 6 years. The projection is slightly modified to protect privacy of data source. It is proportional rescaled, then projected with some extra lapse. The modified projection still has similar characteristics as the original. We will use the modified projection as our baseline projection. We will compute fair value of the block without securitization, fair value of the block after securitization under structures with different capital requirement.

Basic assumptions are in Table 1. We assume that returns of surplus account of both ceding company and captive are 6.5%. The return of product asset in the ceding company is also 6.5%. The return of product asset of captive is 5%. We assume that interest rate of surplus note is 5.5%. Thus, negative carry of the surplus note is 50BP (if this is a letter of credit deal, this 50BP is the letter of credit fee). The WACC is 11%. The securitization transaction cost (EE (t)) is zero.

Table 1. Assumptions

Notation	Value	Remarks
i_1	6.5%	Return on product assets at ceding company
i_2	5.0%	Return on reserve type accounts at captive
r	5.5%	Interest ratesurplus note
k	11.0%	Cost of capital
T	35.0%	Tax rate
j	6.5%	Investment return on surplus type accounts

The initial balance sheet items of the block are in Table 2. There is no ANW. The assets backing the block are exactly product assets of $385.5M, and surplus assets of $19.3M.

Table 2. Initial balance sheet items (in thousands)

Items	Value
Statutory reserve	385 480
Economic reserve	91 356
Redundant reserve	294 124
Tax reserve	365 380
Required capital	19 274

Fair value of the block is $148.3M based on the equation (4). In particular, MVL is $197.8M. Major part of the MVL is the value of reserve requirement at $162.7M. The direct value of liability cash flow (DVLCF) LE (0) is relatively small at $6.0M.

Table 3. Fair value of the block (in thousands)

Terms in (4)	Value	Remarks
LE (0)	6 039	Discounted liability cash flow
NC (i_1, k, MVA)	162 742	Cost of reserve requirement
NC (0, k, TBA)/(1 – T)	8 350	Tax adjustment
NC ((1 – T) j, k, RS)	20 680	Cost of capital requirement
MVL (0)	197 810	Market value of liability
MVA (0)	385 480	Market value of product asset
RS (0)	19 274	Market value of surplus asset
T × (TVA (0) – TVL (0))	7 035	Tax adjustment
DDE (0)	148 294	Discounted distributable earnings
ANW	–	Adjusted net worth
FV	148 294	Fair value

Table 4 has fair value of the block after securitization transaction with different levels of capital requirement. The transaction defines the economic reserve as the gross premium reserve under best estimate assumption with limited conservatism. The note issued has legal maturity of 30 years, weighted average life of 15 years. The cash flow of the block after N = 30 will be immaterial due to high lapse rate after the level term period of 20 years. The initial capital contribution is determined in large part by the surplus note rating desired, and the mortality stress and other stress tests associated with such ratings.

There are no distributions of earnings on Case 1 and Case 3. The surpluses are accumulated in the surplus account until the end of the transaction. The minimum surplus (MS) requirement in the Case 2 and Case 4 is the sum of: (1) present value of all future extra death benefit of corresponding mortality stress; and (2) present value of next 5 year liability cash flow deficiencies. The MS is floored at RS. Any amount in the surplus account over MS is distributed to the ceding company.

We only consider the mortality stress. Generally speaking, mortality stress is the most critical stress for term life insurance. The initial capital contribution in Case 1 and Case 2 is $94M. The amount is based on 120% of mortality stress. It is enough to cover the liabilities of the captive if the experience mortality is 120% of the base assumption. The initial capital contribution of Case 3 and Case 4 is $77M. The amount is based on 115% of mortality stress.

The surplus note issued is the maximum of redundant reserve over the years. There is no redemption before the reserve peak. It is redeemed so that the balance equals the redundant reserve after the peak.

Table 4. Fair value with securitization

Items in fair value	Base	Case 1	Case 2	Case 3	Case 4	Remarks
MS (0)		94 000	94 000	77 000	77 000	Initial capital contribution to captive
(1 – T) × RVL (0)		191 180	191 180	191 180	191 180	Reserve released
RS (0) – MS (0)		(74 726)	(74 726)	(57 726)	(57 726)	Capital released (or required)
AVW		116 454	116 454	133 454	133 454	Surplus relief at transaction
DDE	148 294	11 375	39 527	8 962	35 553	Future distributable earning
FV	148 294	127 829	155 982	142 416	169 008	Fair value with securitization
VA		(20 465)	7 687	(5 878)	20 713	Valued added with securitization

Among all the structures, Case 1 is the most restrictive. It actually destroys value. But companies might consider undertaking such transactions just for the purpose of surplus relief and of obtaining external funding. Case 4 is under the most liberal conditions. It creates $20.7M value. The Cases 1 and 2 provide $116.5M of surplus relief at the transaction. The Cases 3 and 4 provide $133.5M surplus relief transaction.

Table 5 decomposes the value added to four components based on equation (11). We use the best estimate assumptions. If the experience differs from the best estimate assumptions, the values in Table 5 certainly will be different. We see that the capital requirement of the captive is the most important factor of a securitization structure. Saving on funding redundancy is \$92.9M. The cost of funding for the extra capital varies. It is \$74.2M in Case 1, and \$33.0M in Case 4.

Table 5. Value added with securitization

Items in fair value	Case 1	Case 2	Case 3	Case 4	Remarks
RTA	(2 938)	(2 938)	(2 938)	(2 938)	Tax adjustment
$NC(i_1 - i_2 + k, k, SVL)$	(35 261)	(35 261)	(35 261)	(35 261)	Lost investment income
$-NC((1-T)\times j, k, MS - RS)$	(74 240)	(46 087)	(59 653)	(33 061)	Cost of funding extra capital
$-NC(i_2 - r + k, k, NB - RVL)\times(1-T)$	(888)	(888)	(888)	(888)	Cost of funding extra syrplus note
$NC(r, k, RVL)\times(1-T)$	92 862	92 862	92 862	92 862	Saving on funding redundancy
VA	(20 465)	7 687	(5 878)	20 713	Valued added with securitization

5. Discussions

5.1. Is constant WACC assumption reasonable? WACC is weighted average cost of capital of both debt and equity. It is a company (entity) specific concept. If capital management of the company is not unusually efficient or inefficient, WACC should reflect risk and uncertainty of distributable earnings.

Without the securitization transaction, it is necessary for the company to raise the funds (or LOC) to fund the reserves. This obligation of funding redundant reserves in coming (5, 10, 15) years creates uncertainty for the block and for the company. One could argue that the pre-transaction WACC should be higher than the post-transaction WACC because of this risk and uncertainty. On the other hand, one might argue that there is a redundancy in the pre transaction reserves, which is removed with the transaction, so the post-transaction WACC should be higher. The direction of the movement of WACC is not obvious. It warrants a detailed investigation.

We could also argue that, without adequate information on the block, the market's perception of the company's business should remain similar, pre-transaction including this block, post-transaction excluding this block. Market has a sense of existence of reserve redundancy, but it is not clear on the degree of redundancy until the transaction. Post-transaction, this block are ring fenced with the captive, majority of the assets in captive are no longer part of the capital structure of the ceding company. (different accounting schemes have different treatments of the captive's asset and liabilities).

The rating agencies usually treat surplus note of the captives (in none recourse structures) as operating leverage of the ceding company. Income of captive is designated to service the note. Interest expense of the note might (should, could) be excluded when computing the financial measures (such as interest coverage ratio of its senior debt) for the ceding company. It is reasonable to ignore this surplus note of the captive when computing the WACC of the ceding company. The capital contribution of the ceding company to the captive still should be included as a part of the ceding company's capital structure. The return of this capital contribution is the distributable earning (dividend) out of the captive.

The majority of XXX transactions are done on early duration of the block. Some transactions are even done for new issues only. In this case, the block's existence or none existence should have small impact on the WACC of the company, besides the impact of intangible factors, as it is a relatively small part of the overall balance sheet of the ceding company. In all likelihood, the risk profile of the ceding company should remain similar.

In conclusion, the constant WACC assumption is reasonable, but impact of securitization on WACC is definitely worthy of further investigation.

5.2. Where is the value from? In my view, life insurance securitization transaction creates value from two aspects, increased transparency and risk restructuring.

When all blocks of business of a company are commingled together and each block of business is not well understood by the market, it is difficult for the company to obtain optimal level of WACC reflective true risk of each block. Life insurance securitization, with extensive involvement by third party actuarial consulting firms, financial guarantee companies, rating agencies, and regulators, increases the transparency of the inherent risk of the block. This transparency reduces the excess spread the investors demand.

5.3. What if there is no regulatory requirement? What happens in a hypothetical world of insurance without regulatory protection? Of course, if insurance business is only business of the company and there is no asset backing those insurance liabilities, no one will purchase any insurance policy from this company.

For arguments sake, let us assume that this block of insurance business is infinitely small portion of company's overall business, and the block has no impact on the overall companies operation, its credit standing, etc. Under these hypothetical assumptions, the optimal action for the company is not to hold any reserves. The company should use all assets and positive cash flows in earlier durations of the block to repay investors, or as capital for other vesture's. It could raise the capital when the claims are due in later years. In this case, fair value of the liability is the discounted liability cash flow at the WACC, and the value of regulatory protection disappears.

This paper focuses on the regulatory requirement. The arguments of the paper could be used to compare two insurance jurisdictions, one with high reserve requirement, and one with lower reserve requirement. Even if the liability cash flow is exactly the same, the fair value of liability is lower in the second jurisdiction. This could explain the heated debate on reinsurance collateral requirements between the US and EU carriers. It could also explain why reinsurance tends to move blocks of business to less regulated jurisdictions.

One of the purposes of developing fair value accounting model is to make insurer's financial statement more comparable with entities that are not insurers. For none insurers, in most of the cases, there could be immediate distribution of cash flow from the operation to (or infusion from) the owner. To make the comparison possible between insurers and non-insures, it is necessary to consider the delay in distribution caused by the insurance regulatory requirements.

Conclusions

In this work, we introduced the concept of value of regulatory protection. Value of regulatory protection has three components: value of reserve requirement, value of capital requirement, and a tax adjustment. It is the value lost while holding assets with earned rate lower than weighted average cost of capital. Those assets have to be held to satisfy the reserve and capital requirements. The direct value of the liability cash flows (DVLCF) is the liability cash flow discounted at the WACC. The market value of the liability is the sum of the direct value of liability cash flows (DVLCF) and the value of regulatory protection. This new formulation of the market value of the liability provides new possibility of reconciling the indirect method (embedded value or appraisal value approach) and direct method (which discounts the liability cash flow directly).

For blocks of term life insurance and universal life with secondary guarantees, the value of reserve requirement is relatively high. We could reduce the amount by innovative capital market solutions, such as securitization. The fair value of a block is the distributable earning discounted at the WACC. These capital market solutions change the pattern of the distributable earning of the block. As the result, companies could increase the fair value of the block by utilizing these solutions.

There are two approaches to reduce the value of the regulatory requirement. One approach is to keep the protection level as it is (no change to the reserve and capital requirements), and finance the redundancy more efficiently. Another approach is to reduce the level of protection through new reserve regulations, such as principle based reserving. The first approach introduces the insurance industry to the capital market participation, increases the industry efficiency and transparency. It provides value to policyholders with both cost savings and stronger protection against insurer insolvency.

In practice, there are many companies using the annual renewal letter of credit (LOC) to fund redundant reserves. For such companies, the issue will be volatility, flexibility and low cost of short-term solution vs the certainty of a long-term commitment. Future cost of annual letter of credit will depend on overall level of the LOC market, and future dynamics of credit worthiness of the company. To provide a theoretical framework for this decision-making process would be an interesting project. It should involve a stochastic model of the annual LOC cost, based on ceding company's credit migration, and stochastic market movements. The overall financial constraints of the firm, such as risk return profile, should be the other important consideration.

In this work, we assumed that the WACC is fixed, does not change upon securitization transaction. It would be interesting to study the impact of securitization on firms' WACC, and on firm's capital structure, both in theory and in reality.

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