“Solvency indicators for partially unfunded pension funds”

AUTHORS	Roberta Melis Alessandro Trudda
ARTICLE INFO	Roberta Melis and Alessandro Trudda (2012). Solvency indicators for partially unfunded pension funds. *Investment Management and Financial Innovations*, *9*(4)
RELEASED ON	Friday, 14 December 2012
JOURNAL	"Investment Management and Financial Innovations"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
0	0	0

© The author(s) 2025. This publication is an open access article.

BUSINESS PERSPECTIVES

businessperspectives.org

Roberta Melis (Italy), Alessandro Trudda (Italy)

Solvency indicators for partially unfunded pension funds

Abstract

This study addresses the issue of the management of a partially unfunded pension fund in a stochastic framework. In particular it focuses on the sustainability of a pay-as-you-go pension fund with a funded component. The model proposed is developed in a continuous time and the two stochastic variables are the return on the assets and the intensity of new entrants. The former are driven by a Vasiceck process and the latter is modelled by a mean reverting process. The goal is to monitor the solvency of the fund, namely its capability to pay future obligations. To check the financial stability of the fund, risk indicators are proposed.

The methodology is applied to the pension fund of Italian Professional Orders, pension funds that follow a not pure pay-as-you-go pension scheme, but have a funded component.

Finally, some examples illustrate how the management of the fund can activate the correctives to rebalance the fund solvency.

Keywords: pension funds, partial unfunded system, solvency indicators, force of new entrants.
JEL Classification: G23, H55, J11.

Introduction

The aim of this paper is to analyze the financial sustainability of a pension fund financed by pay-as-you-go system with a funded component.

Pension schemes can be classified, from the financing method, into pay-as-you-go (PAYG) or funded. In a PAYG scheme contributions paid by the workers are used to finance the current pensions. By contrast, in a funded scheme, the contributions are invested in the capital markets and the accrued amount is used to finance benefit upon retirement (see Davis, 1995).

In Europe, PAYG systems are generally used for public pension schemes, whereas private schemes (relative to the first and the second pillar) are managed in accordance to funded system.

In this paper we focus on *mixed* pensions schemes financed by PAYG with a funded component.

Among the public pension system, an example of PAYG system with a funded component is provided by the Swedish pension system. In the Swedish pension system there are two components, both with defined contribution, the first financed on PAYG and the second one funded. Recently also for the public PAYG pension system it has been attempted to apply actuarial solvency analysis methodology, used in the insurance field, to the public PAYG pension system management by introducing an Automatic Balance Mechanism (ABM). The ABM is a set of predetermined measures established by law to be applied immediately as required by the solvency indicator. Its aim is to restore the solvency or financial sustainability of PAYG systems, through successive applications, avoiding in this way the intervention of the Legislator (on this topic see Settergren (2001) and Vidal et al. (2009). The ABM can then be seen as an adjustment mechanism of the pension benefits adopted to maintain the soundness of the pension financing. This kind of mechanism is useful to re-establish the financial equilibrium of a PAYG pension system without the intervention of the Legislator.

An example among private pension funds is provided by the pension funds of Italian Professional Orders. In Italy each professional group recognized by a board (lawyers, doctors, accountants, etc.) manages its own retirement fund. The Italian Legislation for these funds, which were privatized in 1995, imposes to draw up actuarial balances and risk indicator to monitor the evolution of the fund in the long run (50 years). On this topic see Melis and Trudda (2010). These funds do not follow a pure PAYG pension scheme, but they have a funded component with the accumulation of wealth in a reserve fund. It is possible to observe for these pension funds a "life cycle" depending on the demographic trends. During the accumulation phase of the fund, the funded component prevails on the PAYG one, with the accumulation of a reserve fund on which returns are accrued which are used during the second phase, when the pension disbursements exceed the contribution revenues and the returns on assets.

In this paper a stochastic model for the evolution of a mixed PAYG pension fund in a continuous time framework is presented by extending the model of Melis and Trudda (2012). It is characterized by two stochastic components: the global return on assets and the force of new entrants into the fund. The goal of the work is to monitor the solvency of the fund, that is the capability of the fund to cover the future

© Roberta Melis, Alessandro Trudda, 2012.

pensions, using risk indicators capable to capture the funded and the PAYG component. Numerical applications are implemented using the data from a real Italian pension fund, one of the privatized pension funds of Italian Professional Orders.

The paper is organized as follows. Section 1 presents the model for the pension fund and its stochastic components. In Section 2 risk indicators to monitor the solvency of the fund are described and analyzed. Section 3 contains an empirical analysis and the last section concludes the paper.

1. The model

The pension scheme is modeled by a continuous time model. The age x and time t are treated as continuous variables. We consider the evolution over a long time horizon T for a pension fund with an age-structured population. It is a defined contribution pension fund, but the scheme is extensible also to defined benefit.

The evolution of the fund is described by the following differential equation:

$$dF(t) = [F(t)\delta(t) + C(t) - B(t)]dt, \qquad F(0) = F_0 \geq 0, \tag{1}$$

where $F(t)$ is the fund value function at time t, $\delta(t)$ is the instantaneous rate of return, $C(t)$ and $B(t)$ are the total contribution function and the benefit function (paid to pensioners) at time t, respectively.

In a pure PAYG pension system revenues equal outlays each year and $dF(t)=0\, \forall t, 0 \leq t \leq T$. In our model we consider a spurious PAYG system, with the accumulation of partial reserves.

The solution of equation (1) is:

$$F(t) = e^{\int_0^t \delta(s)ds} \left\{ F_0 + \int_0^t [C(s) - B(s)] e^{-\int_0^s \delta(z)dz} ds \right\}. \tag{2}$$

Let $N(x,t)$ be the active population aged x at time t, α is the entry age into the scheme, π is the age of retirement and ω is the extreme age.

The total contribution is calculated as follows:

$$C(t) = \gamma \int_\alpha^\pi w(x,t) N(x,t) dx, \tag{3}$$

where γ is the fixed contribution rate and $w(x,t)$ is the wage function for members aged x at time t. The wage function $w(x,t)$ evolves as follows:

$$w(x,t) = w(x,0)e^{gt},$$

where g is the annual growth of income. In the numerical examples the inflation is used as a proxy for g.

Indicating with $A(x,t)$ and $A^*(x,t)$ the number of active people aged x at time t has been already member of the scheme at time 0 and, respectively, entered into the scheme after time 0, equation (3) becomes:

$$C(t) = \begin{cases} \gamma \left[\int_{\alpha+t}^{\pi} w(x,t)A(x,t)dx + \int_{\alpha}^{\alpha+t} w(x,t)A^*(x,t)dx \right] \text{ if } \alpha + t < \pi, \\ \gamma \int_\alpha^\pi w(x,t)A^*(x,t)dx \text{ if } \alpha + t > \pi. \end{cases} \tag{4}$$

The original active population evolves in this way:

$$A(x,t) = A(x-t,0)e^{-\int_{x-t}^{x} \mu(u)du}, \tag{5}$$

where $\mu(x)$ is the mortality intensity at age x.

The new entrants, entering into the scheme at the age α evolve as follows:

$$A^*(\alpha, t+dt) = A^*(\alpha,t)e^{\int_t^{t+dt} \theta(s)ds}, \tag{6}$$

where $\theta(t)$ is the intensity of new entrants at time t. Then, once entered into the scheme they evolve as follows:

$$\begin{aligned} A^*(x,t) &= A^*(\alpha, t-(x-\alpha))e^{-\int_\alpha^x \mu(u)du} = \\ &= A^*(\alpha,0)e^{\int_0^{t-(x-\alpha)} \theta(s)ds} \cdot e^{-\int_\alpha^x \mu(u)du}. \end{aligned} \tag{7}$$

Then,

$$N(x,t) = \begin{cases} A(x,t) \text{ if } t \leq x - \alpha, \\ A^*(x,t) \text{ if } t > x - \alpha. \end{cases}$$

The total pensions are calculated by the following equation:

$$B(t) = \begin{cases} \int_\pi^\omega \rho(x,t) \cdot P(x,t)dx \text{ if } 0 \leq t \leq \pi - \alpha, \\ \int_\pi^{t+\alpha} \rho(x,t) \cdot P^*(x,t)dx + \int_{t+\alpha}^{\omega} \rho(x,t) \cdot P(x,t)dx \text{ if } \pi - \alpha < t \leq \omega - \alpha, \end{cases} \tag{8}$$

where $\rho(x,t)$ is the average pension for people aged x at time t. Moreover, $P(x,t)$ denotes the number of pensioners aged x at time t who are already members of the fund at time 0, whereas $P^*(x,t)$ denotes the number of pensioners aged x at time t who entered into the fund after 0. Both groups of pensioners evolve according to the force of mortality. When the projection is shorter than the length of the working life if $\pi + t \le x \le \omega$ the member was already a pensioner at time 0, if $\pi \le x < \pi + t \le \omega$ he was an active member at time 0:

if $0 \le t \le \pi - \alpha$:

$$P(x,t) = \begin{cases} P(x-t,0)e^{-\int_{x-t}^{x}\mu(u)du} & \text{if } \pi + t \le x \le \omega, \\ P(\pi, t-(x-\pi))e^{-\int_{\pi}^{x}\mu(u)du} = A(x-t,0)e^{-\int_{x-t}^{x}\mu(u)du} & \text{if } \pi \le x < \pi + t \le \omega, \end{cases} \tag{9}$$

when the projection is longer than $\pi - \alpha$ then if $\pi \le \alpha + t \le x < \pi + t \le \omega$ the member has been already in the scheme as a contributor, and finally if $\pi \le x < \alpha + t \le \omega$ he has not been already a member of the scheme at time 0, but entered as $A^*(\alpha, t - x + \alpha)$ at time $t - x + \alpha$:

if $\pi - \alpha < t < \omega - \alpha$:

$$P(x,t) = P(\pi, t-(x-\pi))e^{-\int_{\pi}^{x}\mu(u)du} = A(x-t,0)e^{-\int_{x-t}^{x}\mu(u)du} \quad \text{if} \quad \pi \le \alpha + t \le x < \pi + t \le \omega \tag{10}$$

and

$$P^*(x,t) = P^*(\pi, t-(x-\pi))e^{-\int_{\pi}^{x}\mu(u)du} = A^*(\alpha, t-x+\alpha)e^{-\int_{\alpha}^{x}\mu(u)du} \quad \text{if } \pi \le x < \alpha + t \le \omega \tag{11}$$

In the applications we use a mixed method, called *pro rata* mechanism (see Baldacci and Tuzi, 2003), where the pension received by those who were already members of the scheme at time 0 is the sum of two components: the first is calculated according to a defined benefit rule, the second with a defined contribution rule. For the new members it is calculated entirely with the definedcontribution scheme.

For the $P^*(x,t)$ the average pension for a member aged x at time t is calculated as follows:

$$\rho(x,t) = \left[\int_{-x+\alpha}^{-x+\pi} w(x-t+s,s)e^{j(t-x+\pi-s)}ds\right] \cdot tc_{\pi} \cdot e^{(f+g)(\pi-x)}, \tag{12}$$

where f is the revaluation rate of pensions, j is the notional rate at which the contributions paid by the workers are capitalized and tc_{π} is the *transformation coefficient*, the annuitization coefficient used for the conversion into annuity of the notional amount accumulated by each worker. The transformation coefficient depends on the age of retirement, but in the present model it is constant because retirement age is fixed, π. For an exhaustive explanation of the argument we refer to Janssen and Manca (2006).

1.1. Extensions to heterogeneous population. It is possible to distinguish different populations indexed by $i \in I$ (see Gabay and Grasselli, 2012). In the applications we describe the population according to gender, using N_1 for males and N_2 for females. This way we can use different wage, different mortality, survival rate and intensity of new entrants as in Melis and Trudda (2012). The result is that the total contribution and the total pension processes are the aggregation over the different categories:

$$C(t) = \sum_{i=1}^{2} C_i(t), \quad B(t) = \sum_{i=1}^{2} B_i(t) \tag{13}$$

1.2. Stochastic evolution. *1.2.1. A model for the force of new entrants.* Let $\eta(t)$ (with $0 \le t < T$) be a deterministic function. We propose the following stochastic model for the force of new entrants:

$$\theta(t) = \eta(t) + X(t), \tag{14}$$

where $\eta(t)$ is the baseline for the process θ and $X(t)$ is described by an Ornstein Uhlenbeck process, characterized by the following stochastic differential equation:

$$dX(t) = -\beta X(t)dt + \sigma dW_d(t) \tag{15}$$

with $X(0) = 0$, β and σ strictly positive real numbers, and $W_d(t)$ denoting a Wiener process. Substituting (14) into (6) we obtain then:

$$A^*(\alpha, t+dt) = A^*(\alpha, t) \cdot e^{\int_t^{t+dt}[\eta(s)+X(s)]ds}. \tag{16}$$

1.2.2. Global asset return. The following model is used to represent the interest rate dynamics:

$$\delta(t) = \hat{\delta}(t) + X_r(t), \tag{17}$$

where $\delta(t)$ is the stochastic force of interest, $\hat{\delta}(t)$ is the deterministic component of the force of interest and $X_r(t)$ the stochastic component described as follows:

$$dX_r(t) = -\beta_r X_r(t)dt + \sigma_r dW_r(t), \qquad (18)$$
$$X_r(0) = 0.$$

The two Brownian motions $W_d(t)$ and $W_r(t)$ are assumed to be independent.

We use the Vasicek model (Vasicek, 1977) to describe the return on assets of the fund. The Vasicek model is suitable to represent the global return on a risky asset portfolio, that can reach also negative values as there can be losses of capital. The choice of the process (17) is due to the fact that the analyzed funds are characterized by prudential portfolios composed with low-risk assets (the heritage is in large part composed of real estate and liquidity and only in limited part of stock funds); subsequently portfolio's returns show low volatility around their historical trend.

2. Solvency indicators

In actuarial and financial literature different risk measures are used to assess the solvency of a pension fund. One of these is the *funding ratio*, the proportion between the fund value and the present net value of future obligations of the fund:

$$Fr(t) = \frac{F(t)}{AL(t)}, \qquad (19)$$

where $AL(t)$ is the actuarial liability, the present net value of the future obligation of the fund. This ratio indicates the fraction of the actuarial liabilities covered by the funded component. The complement to 1 measures the amount that must be covered by PAYG, in case of partially unfunded pension schemes.

As already highlighted in the introduction, in a PAYG pension scheme, where the current pensions are financed through the current contributions, it is essential for the financial sustainability of the system that there be a balance between active and retired people. An indicator of the solvency of the fund is the expected value of the ratio between the contributions and the pensions:

$$C\Pr(t) = E\left[\frac{C(t)}{B(t)}\right]. \qquad (20)$$

Melis and Trudda (2010) explain that this index could be seen as an indicator of the fund's liquidity. If the ratio is below 1 the fund is in a situation of financial instability and it must be monitored properly.

For the management of the pension fund it is useful to have time indicators that measure the patterns of the cash-flows with respect to the life cycle of the pension fund. A first time indicator is t^* for which the fund reaches its maximum value. From this moment the funded component is used to cover the current pensions.

We find the time t^* for which the fund reaches its maximum value:

$$F(t^*)\delta(t^*) + C(t^*) = B(t^*). \qquad (21)$$

This is an expected time indicator because when $\delta(t)$ and $\theta(t)$ are stochastic, condition (21) is not more valid.

From equation (21) it is evident that when the fund reaches its maximum value, then the pension disbursement is equal to contributions plus the returns on fund assets. By numerical methods in the applications we find t^*.

Another more restrictive risk indicator is the time t^{**}, which precedes the time t^* for which:

$$C(t^{**}) = B(t^{**}) \qquad (22)$$

for which the *CPr* index is equal to one. This indicator considers the demographic risk of the PAYG component, neglecting the financial component.

With respect to the life cycle of the fund, evidently $t^{**} \le t^*$.

3. Empirical results and analysis

In this section the model presented above is applied to areal pension fund of Italian Professional Orders. Data are available from 1976 to 2010. By means of Monte Carlo simulations (10000) we estimated the probabilistic structure of the fund. It is a "young retirement fund", with a high component of young members: the main age class is represented by the 35-45 category. From this demographical situation a dynamic fund evolution has been developed.

The following assumptions are adopted:

- the starting population is the actual population of the pension fund on January 1, 2010; the initial value of the fund is known and it is that resulting from the 2010 balance sheet; its value is 2,060,000,000 euro;
- evolution of the population based on *IPS55* male and female mortality tables;
- for new entrants fixed entry age $\alpha = 30$, retirement age $\pi = 65$;
- a subjective contribution rate is equal to 12% of annual professional income;
- the actual transformation coefficients have been employed;

- professional incomes are appreciated at rate of inflation; the inflation rate is fixed at 2%;
- benefits are calculated with the mixed method;
- administrative costs are considered resulting from the 2010 balance sheet, appreciated at 3% annual rate.

To estimate the parameters of the return on financial asset, we used a portfolio of bonds using the data from 1998 until 2010. As the discrete representation of the A(1) model used in section 1.2 to represent the force on new entrants and the global rate of return is the AR(1) model, we used the well known principle of equivalence of covariance, obtaining the value of 0.491023 for β_r and 0.022862 for σ_r, while the deterministic component $\hat{\delta}(t)$ was estimated by a linear regression. On this topic an exhaustive explanation can be found in Orlando and Trudda (2004).

The same methodology was used to estimate the parameters of the force of new entrants, but in this case we used different values for man and female category.

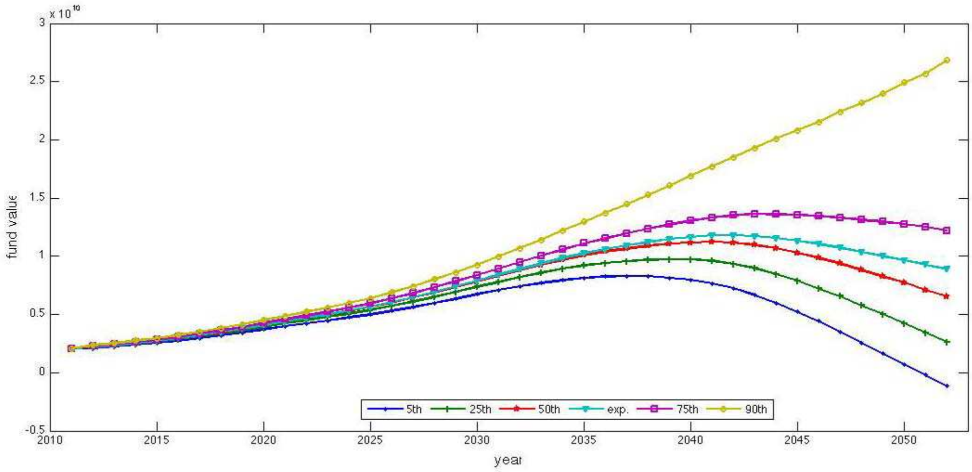

Fig. 1. Evolution of the fund: percentiles of the distribution and expected value

Figure 1 shows the percentiles of the frequency distribution and the expected value.

The chart indicates a probability for the lower percentiles that the fund could reach a peak in 2035, then taking a downward trend until it reaches the default. This is a consequence of the initial structure of the population considered. When the "hump" of the members aged 35-45 years in Figure 1 reaches retirement age, then the fund begins to decrease because of the increase in pension outlays. Developing the projections for a long-term horizon (40 years) we can observe that, if there is not a sufficient number of new entrants and, therefore, a sufficient flow of contributions, the fund value will tend to decrease rapidly, as the contributions and the investment performance is not sufficient to cover pensions and administrative costs and the fund will move towards zero and hence default. It is possible to observe that the fund distribution presents a high variability, the gap between the percentiles is, in fact, very large, which denotes the risk to which the fund is exposed. Further, the percentiles have a different pattern: the lowest percentiles show that after 40 years the fund tends to be zero, while in the upper percentiles it continues to grow.

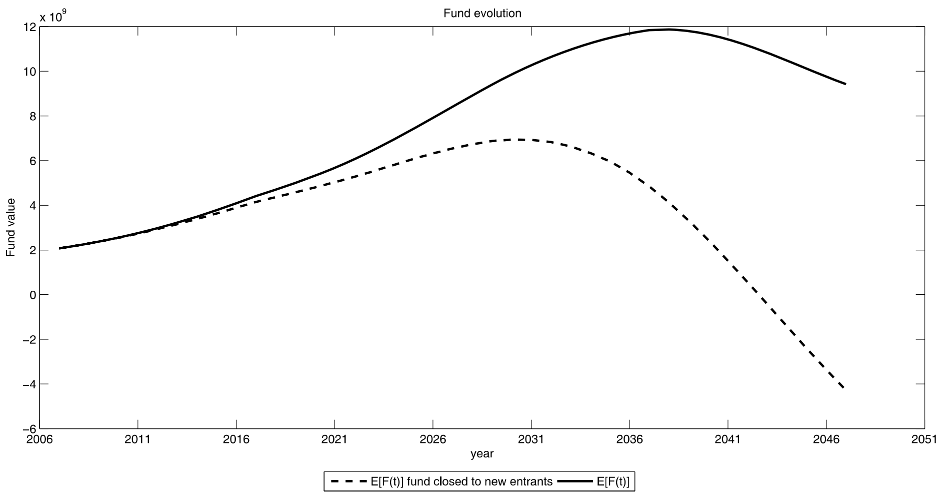

Fig. 2. Evolution of the fund: expected values

Figure 2 shows the evolution of the fund. The continuous line represents the expected value of the fund, while the dotted line represents the expected value of the fund closed to new entrants. In the case of absence of new entrants the fund goes to zero very rapidly. In case of absence of new member the distribution present less variability, as in this case there is only one stochastic variable.

Subsequently, we calculated the *CPr* index: i.e. the ratio between contributions and pensions. As already stated in the previous section, this is an index of liquidity. Figure 3 shows the percentiles of the frequency distribution and the expected value. By observing the *CPr* index we can see its particular shape, due to the initial demographic structure of the population.

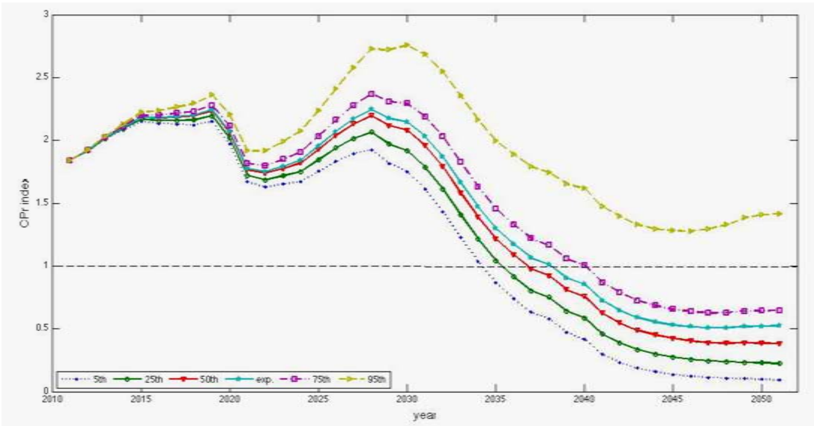

Fig. 3. *CPr* index, percentiles of the distribution and expected value

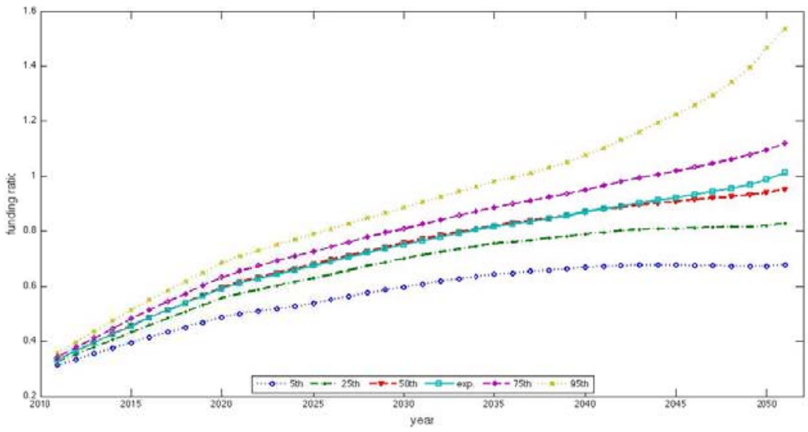

Fig. 4. Funding ratio, percentiles of the distribution and expected value

We calculated the funding ratio, the proportion of the actuarial liabilities covered by the funding component. Figure 4 shows the percentiles of the frequency distribution and the expected value.

As it is possible to observe the funding ratio is low, but it is increasing during the period of the simulation. This is because the fund is increasing its funding component, to the aim of balance the disequilibrium between active and retired people. For the pension funds of Italian Professional Order, an objective of the management is to increase the funding ratio.

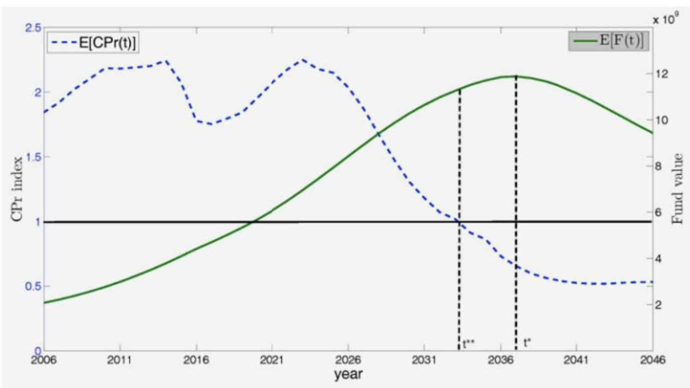

Fig. 5. Fund value and *CPr* index

Figure 5 shows the expected value of the fund and the expected value of the *CPr* index. When the members aged 35-45 at time 0 reaches retirement age from 2030 to 2040, then the fund begins to decrease because the pension outlays increase.

When the index reaches the value 1, in t^{**}, the fund begins to decrease 4-5 years after, in t^{*}. The reason is because in the period $t^{**} - t^{*}$ returns on assets cover the difference $C(t) - B(t)$.

Conclusions

This paper analyzes pension schemes financed by PAYG with a funded component. With the aim of monitoring the solvency of the fund, we developed risk indicators used to project the fund evolution and to anticipate its future dynamics are developed. Those indicators take into account the dynamic patterns followed by the new members into the fund and by the rate of return on assets. Numerical applications are carried out by means of a simulation methodology, on the Italian Professional Orders pension funds.

This kind of pension funds can operate through a mixed PAYG financing mechanism. They usually are not "pure PAYG" pension schemes as they do not distribute all the revenues between the beneficiaries every year.

With respect to the demographic evolution, a "life cycle of the fund" emerges, in which there is an initial phase of accumulation because there are few pensioners and many contributors. The variable "new entrants" has a relevant impact on the financial growth of the fund because it influences PAYG component of the fund. During the growth phase of the system, the accumulation phase, with accumulation of reserves, the funded component prevails over the PAYG one, as the pension disbursement is limited in the short run. The flow of new entrants over time has a strong relevance to guarantee the financial sustainability of the fund.

The analysis highlights the importance to monitor the financial stability of the fund, by checking that the fund maintains a high probability to cover the future obligations to pay pensions and the operating costs.

The risk indicators presented can be used not only for forecasting and also for monitoring purposes. With respect to the projections of the fund value, these indicators can anticipate trend reversal in cash-flows. The management of the fund can promptly activate the correctives to rebalance, by using the technical levers at its disposal. In the case analyzed in the empirical application, the funding ratio increase over the time. This is due to the fact for the new member it is used the defined contribution method to calculate the benefit, and the pension is linked to the contribution paid by the workers during the active life. Using the transformation coefficient more actuarial fair like that obtained using projected mortality table, the funding ratio increase more than using the actual coefficient of Italian Law.

References

1. Baldacci, E. and Tuzi, D. (2003). Demographic trends and pension system in Italy: an assessment of 1990s reforms, *Labour*, 17, pp. 209-240.
2. Davis, E.P. (1995). *Pension Funds*, Oxford University Press.
3. Gabay, D. and Grasselli M. (2012). Fair demographic risk sharing in defined contribution pension systems, *Journal of Economic Dynamics and Control*, 36 (4), pp. 657-669.
4. Holzmann, R., Palmer, E. and Robalino, D. (2012). *Nonfinancial Defined Contribution in a changing pension world*, Vol. 1, progress, lessons and implementation, The World Bank, DC, Washington.
5. Janssen, J. and Manca, R. (2006). Notional defined contribution pension method and the construction of annuitization coefficients. In Proceedings ICA 2006.
6. Melis, R. and Trudda, A. (2010). Demographic risk indicators in pay-as-you-go pension funds, *Problems and Perspectives in Management*, 8 (4), pp. 117-126.
7. Melis, R. and Trudda, A. (2012). Financial and demographic risks impact on a pay-as-you-go pension fund. In Sibillo, M. and Perna, C. ed., *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, Springer.
8. OECD (2009). "Pensions at a glance 2009: retirement-income systems in OECD Countries", OECD Publishing.
9. Orlando, A. and Trudda, A. (2004). Some remarks on first and second order stochastic processes choice, *Investment Management and Financial Innovations*, 3, pp. 118-131.
10. Settergren, O. (2001). The automatic balance mechanism of the Swedish pension system, *Wirtschaftspolitishe Blätter*, 4, pp. 339-349.
11. Trudda, A. (2012). Sostenibilità ed Adeguatezza: il Trade-off del Sistema Previdenziale Italiano, *Giappichelli*.
12. Vasicek, O.A. (1977). An equilibrium characterization of the term structure, *Journal of Financial Economics*, 5 (2), pp. 177-188.
13. Van Gaalen, R. (2004). Pension Funds, funding index, mismatch risk premium and volatility, 14th Afir Colloquium.
14. Vidal, C., Boado, M. and Settergren, O. (2009). Automatic balance mechanism in pay-as-you-go pension system, *The Geneva Papers*, 34 (2), pp. 287-317.