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Measurement of efficiency in the airline industry using data envelopment analysis

Abstract

Data envelopment analysis is applied to US airlines industry for the period of 1985-1995 to determine technical efficiency of each airline for each year. Results of efficiency analysis are applied to determine if efficiency and stock returns are related. Two portfolios, one consisting of efficient airlines, and the other consisting inefficient airlines, are formed each year. The efficient portfolio outperforms the inefficient portfolio by an annual margin of 23% using raw returns.

Keywords: data envelopment analysis, airlines, stock returns, technical efficiency.
JEL Classification: C14, C44, C67, D24, L93.

Introduction

In this paper we estimate efficiency of the airline industry using data envelopment analysis (DEA) and then use the results of the DEA scores to test financial and accounting behavior of efficient and inefficient firms. The results of this study make an important contribution to the extant literature in accounting and finance. Most researchers in these areas have relied on financial accounting numbers to study stock return performance. The technical efficiency of a firm is ignored in these studies[1]. The implicit assumption in these studies is that a firm is operating at its maximum technical efficiency[2]. This assumption is controversial, and its resolution is an empirical issue.

Based on the estimation of efficiency derived from DEA, the paper attempts to answer the following questions: Do financial markets identify the efficiency of operations of a firm and if so, how is this information reflected in the share prices? In other words, is the technical efficiency of a firm priced by the market? These questions require an examination of the relationship between technical efficiency and profitability. This study examines whether a portfolio of technically efficient firms earns superior returns than a portfolio of inefficient firms. It uses publicly available data to identify efficient and inefficient airline companies and relate the resulting efficiency measures to financial performance. Due to a history of regulation in the airline industry, much airline data necessary to measure technical efficiency is available publicly. While such an analysis could be applied to any industry, the airline industry is chosen as a test case due to its homogeneous mix of inputs and outputs and the ready availability of physical input-output data.

The remainder of the paper is organized as follows. Section 1 provides an overview of the DEA and the airline industry. Section 2 selects appropriate input and output variables to be used for measuring efficiency of an airline. It then applies DEA to the airline data and obtains efficiency scores. Section 3 describes the methodology for comparing stock returns of efficient and inefficient firms and discusses the results of the stock return comparison. The final section concludes the paper. The basic DEA model, its associated linear programming problem, and the solution thereof, is provided in Appendix.

1. An overview

1.1. Data envelopment analysis (DEA). DEA was introduced in Charnes, Cooper, and Rhodes (1978) and further developed in Banker, Charnes and Cooper (1984). It is a generalization of the Farrell (1957) single-output/input measure of technical efficiency to mutli-output/multi-input measure case. DEA achieves this objective by constructing a single virtual output and a single virtual input by calculating optimal weights for each output and each input of a firm. Unlike other methodologies, the weights are not assigned apriori in an arbitrary manner. Rather, these weights are optimally determined for each firm through separate linear programming problems so as to maximize the resulting efficiency of each firm. Constraints for determining optimal weights of each firm are that (1) *using same weights that maximized the efficiency of this firm*, no firm has an efficiency greater than 1; and (2) that these weights are non-negative. Calculated this way, if the maximum achievable efficiency of a firm *j* is less than 1 given the constraints, then it must be the case that for a given level of inputs, this firm either produced less than some other firm *k*, or, for a given level of outputs, it used more inputs than some other firm *m*. Otherwise alternate weights would have been feasible that would increase firm *j*'s efficiency further.

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[1] The only exception to this is a paper by Alam and Sickle (1998).
[2] The concept of technical efficiency was first developed in a seminal paper by Farrell (1957).

Similarly, if weights are such that resulting efficiency of firm j is 1, and the same weights satisfy the constraints described earlier, then firm j is operating efficiently[1].

In contrast to DEA, parametric approaches to measure efficiency of a firm involve either apriori defining a production function of a firm and then comparing the actual performance of a firm with the estimated production function, or using a linear regression analysis for determining an average relationship between inputs and output. Parametric techniques have some limitations though. First, they are not very useful in a multi-output environment. Second, they require the imposition of a specific functional form like a regression equation, or a production function on the data. Third, the functional form also requires specific assumption about the error term. Fourth, these techniques assume that the optimized regression equation applies to each firm.

DEA overcomes many of these problems. In contrast to parametric techniques, DEA optimizes each firm individually, with an objective of calculating a discrete piecewise frontier determined by a set of Pareto-optimal firms. It then calculates the best efficiency measure of a firm with the only stipulation that a firm should lie on or below the production frontier enveloped by Pareto-efficient firms. Any firm that lies *on* the frontier is deemed efficient. A firm that lies below the frontier is deemed inefficient. Additionally, DEA results provide estimates for desired changes in inputs and outputs that would propel inefficient firms on to the efficient frontier. Thus, DEA provides not only the identity of an inefficient firm, but also suggests ways to make it efficient. Note that the DEA efficiency is measured with respect to a best-performing firm from actual data, rather than against an "average" firm implied by a regression line.

More than 1,500 journal articles on DEA have appeared in international journals since its introduction by Charnes et al. (1978). DEA has been used in myriad settings. For instance, it has been used by Charnes, Cooper and Rhodes (1981) to investigate efficiency of educational programs; by Bessent and Bessent (1980) to study comparative efficiency of schools; by Banker, Conrad and Strauss (1986) to study hospital programs; by Sherman and Gold (1985) to study efficiency of bank branches; and by Callen and Falk (1993) to study efficiency of charitable organizations. Banker and Johnston (1993) have used it to evaluate impact of operating strategies on efficiencies in the airline industry. Mazur (1994) used it to evaluate relative efficiency of baseball players. Cummins and Zi (1998), and Brokett, Cooper, Rousseau and Wang (1998) have used it to investigate if "mutual" or "stock owned" insurance companies were more efficient. Mester (1989) used it to compare mutual versus stock savings and loans. Porter and Sully (1987) have used used it to assess efficiency of cooperatives. For an extensive list of DEA related literature with respect to its application and further developments in the technique, refer to Cooper, Seiford and Tone (2000).

1.2. Airline industry in the United States. In 1938, Civil Aeronautics Board (CAB) came into existence and was responsible for regulating airlines in the United States. As a regulator, CAB decided the routes that an airline could fly and the fares it could charge from its passengers. CAB controlled both the entry and the exit by an airline from a route, using the "public convenience and necessity" criteria. The most significant step towards deregulation of the airline industry took place in October 1978, when the Congress passed the Airline Deregulation Act of 1978. By 1982, entry was granted to any carrier that was fit, willing, and able. On January 1, 1983, all regulations on fares were eliminated. Subsequent to the deregulation, the industry was characterized by increased competition, price wars, increased operating costs, diminished returns, mergers and bankruptcies. Many small 'No-frill" carriers appeared and disappeared. Reputed big carriers of old times faltered and filed for bankruptcy. The shake-out of the industry is not yet complete, as seen from current mergers and take-over moves initiated by major airlines[2].

Survival in this environment requires tight operating controls and high managerial efficiency. Airlines have attempted to reduce costs by using a more fuel-efficient fleet, buying airplanes of optimum passenger capacity, using hub-and-spoke routing etc. Industry analysts use accounting ratios PE ratios, EPS, ROA, and other techniques to measure financial performance of an airline. To measure operating performance of an airline, statistics like passenger load factor, operating cost per passenger mile, revenue passenger miles, and others are used for comparative analysis. Such analysis, however, is ad-hoc at best.

One-dimensional comparisons are not very effective in multi-input, multi-output settings. The ratio analysis assumes a linear relationship between the variables being used. Also comparisons based on dollar value of the input and outputs are based on the assumption that inputs and outputs were purchased or

[1] An exception to this conclusion is noted in Appendix.

[2] United and US Air; American and TWA.

sold in a perfectly competitive market. If the input market were not perfectly competitive, then the dollar amount spent on input factors would incorporate price inefficiency in the sense that two firms consuming the same dollar amount of an input may not be consuming the same physical amount of that input.

In view of this, a DEA efficiency measure based on physical units of inputs and outputs can provide meaningful insights. Prior research has used DEA to calculate the efficiency of airlines. Banker and Johnston (1993) used it to determine the impact of business strategies on the operating performance of airlines. They focused on the popular hub-and-spoke arrangement of flights used by national airlines. Chan and Sueyoshi (1991) used it to determine the impact of environmental changes on airline efficiency. Alam and Sickles (1998) used it to examine the relationship between change in quarterly efficiency measures and quarterly stock returns. Charnes, Gallegos and Li (1996) used multiplicative DEA to measure international and domestic operations of Latin American airlines. Gillen and Lall (1997) used DEA to measure airport productivity. Good and R.C. Sickles (1995) used DEA to measure the impact of EC regulation on European and American airlines. Li (1992) uses stochastic models and variable returns to scales in DEA with an application to airline industry. Ray and Hu (1997) use it to analyze resource allocation at airline industry level, rather than at firm level. Schefczyk (1993) uses it to evaluate strategic performance of international airlines. Most of these studies focus on operational efficiencies within a division of an airline. While this analysis is useful to managers of the airline, its usefulness to shareholders from an investment point of view is limited. To the best of our knowledge, no study has attempted to determine if a relationship exists between a firm's allocative efficiency and its value to shareholders. Although Alam and Sickles (1998) examine the relation between stock prices and technical efficiency, they use input-output data to form quarterly portfolios. It is unclear that input-output data is available at the time of formation of the portfolio. Yearly formation of portfolio, used in this paper overcomes this problem.

2. Input-output data for airline and DEA efficiency results

2.1. Model. This paper uses the original Charnes, Cooper and Rhodes (CCR) model of DEA. CCR model is suitable for industries that show a constant return to scale. The choice of CCR model is based on the evidence from prior literature that the airline industry shows a constant return to scale[1]. As mentioned in Appendix, DEA reduces to the following linear programming problem:

$$\underset{a_{ikt},\, b_{jkt}}{\text{Maximize}} \sum_{i=1}^{N} a_{ikt} Y_{ikt}, \tag{1}$$

Subject to:

$$-\sum_{i=1}^{N} a_{ikt}\, Y_{ikt} + \sum_{j=1}^{M} b_{jkt} X_{jkt} \geq 0 \text{ for } k = 1, K, \tag{2}$$

$$\sum_{j=1}^{M} b_{jkt}\, X_{jkt} = 1, \tag{3}$$

$$a_{ikt},\, b_{jkt} \geq \varepsilon, \tag{4}$$

where i are the ith output goods and services, from 1 to N; j are the jth input goods and services, from 1 to M; k is the kth firm in the industry; t is the year of data, from 1986 to 1995; X_{jk} is the amount of input j used by firm k; Y_{ik} is the amount of output i produced by firm k; a_{ikt} is the implicit price of output i, for firm k, for year t; b_{jkt} is the implicit price of input j, for firm k, for year t; ε is an arbitrary, very small positive number.

The number of firms need not be the same for each year, i.e. K is actually K_t. However, for the sake of brevity, the subscript t is dropped for K, and henceforth, it will be dropped for other variables as well. For each year t, the above linear program is solved K times, once of each of the K firms, to obtain the implicit prices of each input and each output, that maximizes the efficiency of a firm. This linear program can be easily solved using available software packages once the input and output variables are identified.

2.2. Input-output data. Based on a survey of literature mentioned at the beginning of this section, a study of annual reports of airlines, and discussions with airline analysts, I have selected the following input and output factors to measure efficiency of an airline.

Input: Number of planes, number of employees, and gallons of fuel consumed.

Output: Revenue passenger miles, number of departures, number of passengers, and available ton-miles.

Number of departures is selected as an output variable because it indicates the network level of an airline. A higher level of network indicates a higher level of service to customers.

Data for these variables was collected from the Department of Transportation, Form-41 schedule, reports of industry analysts, annual reports and industry trade journals. Form-41 requirement is very

[1] It must be noted that many models have been developed which are variation of the basic DEA. Evaluation of other models of DEA is beyond the scope of the current paper.

stringent for major airlines, known as Trunk Carriers, in DOT's classification. These airlines are also known as Group III airlines (Group I and Group II are smaller airlines for which the data reporting is less frequently monitored). All input and output variables measure physical quantities[1]. Complete input and output data were available for ten airlines, for the period from 1986 to 1995. Data for Eastern Airlines, which is a Group III airline was not available at this time. Table 1 provides details of the airline data used in this study for two years (1986, and 1987) of the sample period.

The linear program consisting of equations (1) to (4) was then formulated and solved separately for each airline, and for each year, resulting in 100 iterations. Table 2 (in Appendix) provides a year-wise summary of efficiency scores of each airline. Each year, anywhere from 3 to 7 airlines are on the efficient frontier.

Table 3 (in Appendix) provides an interpretation of DEA results for 1986 and is discussed in detail for illustrative purposes. As shown in Table 3, DEA analysis identified six airlines as efficient: American, American West, Continental, Northwest, Southwest, and United Airlines. The remaining four firms, Alaska, Delta, TWA, and US Air were not on the production frontier, and were identified as inefficient firms. It means that the actual input-output data shows that an inefficient firm either produced less output for a given level of input, or consumed more input, for a given level of output. Unlike other methods of productivity analysis, this conclusion is not based on some theoretical or hypothetical figures. Rather, it is based on the actual performance of efficient firms who demonstrated that such improvements were indeed possible. It is also important to note that firms rated as efficient are not necessarily operating at their "theoretical" optimal level. DEA does not identify the optimal level of performance; rather it establishes a relative measure.

Table 3 provides the value of weights that were calculated by the linear program, so as to give each firm a chance to maximize its efficiency score. Weights are shown in the top panel of Table 3. Note that the weights are different for each airline. DEA also provides meaningful input to managers of inefficient firms by identifying the best-practice firms that are relevant for the manager. DEA also suggests how much improvement in efficiency is possible for inefficient firms, and which input variables or output variables need management attention. The second panel of Table 3 shows the reference best-practice airline for each inefficient airline. For example, Delta's reference of best practice firms consists of American, Southwest, and United. It means that the input-output of these airlines (or a convex combination of them) was such that they produced more output with less input than Delta. It also shows how much improvement Delta will have to make to become efficient itself. The numbers in parentheses are a measure of how much Delta needs to improve its operations. The third panel of Table 3 identifies what input (or output) Delta needs to decrease (increase) to be considered an efficient firm amongst its peers. These are the slack variables from the solution of the "dual" of the main linear programming problem.

3. Financial performance and DEA analysis

We expect that the profitability of an airline is closely related to its operational efficiency. The concept of operational efficiency can be easily interpreted as a measure of gross profit margin under two assumptions: (a) that all firms are price takers for both input and output; and (b) that all firms are operating at their production possibility frontier. To the extent that a firm is not operating at its production frontier, we should expect a relatively less gross margin for this firm. This means the firm should expect lower cash flow. Hence the firm's price should be lower. In other words, we should expect inefficient firms to perform worse than the efficient firms.

To investigate this, we calculate the annual returns of each airline in our sample. Then we rank these airlines each year by their DEA efficiency scores, described in Table 2. Each year, we divide all airlines into three groups, ranked by their efficiency scores. Next, we calculate mean annual return for each portfolio. A difference in the mean of the highest efficiency group, and the lowest efficiency score group is calculated for each year providing an indication whether a group of efficient firms earns superior returns than a group of inefficient firms.

It was pointed out that DEA restricts each firm's efficiency to be one or less. Thus, in Table 2, all efficient firms have an efficiency score of 1. In other words, the model does not allow for super-efficient firms (efficiency greater than 1), but not all efficient firms are equal. For example, some efficient firms will remain on efficient frontier, even if their inputs (outputs) were increased (decreased) by a large amount. Other efficient firms may not be as robust to an adverse change in input or output. This allows us to measure a relative ranking of an efficient firm. I calculated the robustness of each efficient firm to break the tie between efficient firms when there were more than three efficient firms for a year.

[1] For some airlines variable 'IF', the amount of fuel consumed was not available in gallons. For these airlines, fuel cost in dollars was obtained from financial statements and divided by average fuel cost per gallon to obtain gallons of fuel consumed.

Compounded annual returns for each airline were taken from CRSP database. Airline industry has gone through many organizational changes. In fact, of the ten airlines in the sample, only two had the same name and CRSP Permno variable for the entire sample period. Two airlines went private during this period, and re-emerged at a later date as a public company. Northwest was acquired by a group of private investors called Wing Holdings in August 1989 and stopped trading publicly. It re-emerged as a public company in March 1994. TWA was privatized by Carl Ichan in November 1988, and remained private for most of the study period. During the study period, Continental Airlines did not trade from September 1992 to September 1993. If a firm traded for more than 100 days during a year, it was included in the group. Thus Continental was included for 1992, but not for 1993.

Other airlines also went through some re-incarnations, mostly by being separated from their holding company, or joining with their parent company. Except as noted above, these airlines traded continuously either as a separate entity or a part of publicly traded holding company.

Table 4 provides results of efficient and inefficient groups of airlines. As shown, the group consisting of efficient firms outperforms inefficient firms nine out of ten years. The difference between efficient and inefficient firms ranges from a low of -36% to a high of 78.6%. Assuming each year as an independent observation, the ten-year average is 22.9%. This is significantly different than zero at better than 5% level.

Given a small sample size, results must be interpreted with caution. These results may be driven by one or two dominant observations. For example, in 1995, the return of America West stock was 650%, based on its stock price increase from 25 cent/share to $1.87/share. To mitigate the risk of influential observations, Wilcox Rank test was used. The results were qualitatively similar. Another method is to calculate efficiency for a pooled data. For example, we could consider the data of Table 2 as a group in which each firm-year is a separate entity. The interpretation of pooled data for DEA is that the best performance is now measured as the "all-time" best performance. This allows the possibility that in a given year, no firm may have reached the efficient frontier. While the year-wise methodology controls for time-period effects (say cancellation of flights and grounding of aircrafts due to severe weather conditions), the pooled data allows for the best performance (which DEA uses to benchmark other performance) to emerge over a period of time. From a methodology point of view, it allows for more observations for DEA and hence a smoother envelopment. This in turn leads to better bench-marking. This analysis is left for future research.

Conclusion and future research

A simple DEA model was used to evaluate operating performance of major US airlines. Publicly available data was used. While DEA can be applied to any industry, and in more complicated situations, airline industry was chosen for its homogeneity of input and output. Data for input-output is not easily available for non-regulated industries. Limitations of this study due to small sample size have already been noted. It is possible to expand the time horizon of the data. Future work consists of using some control variables which are non-discretionary (like size of the plane). In the long run, all variables are discretionary, but in the short run, an airline may be saddled with wrong types of planes. Another direction where this research may be taken is a comparison of accounting ratios and DEA efficiency score to determine which one is a better measure of a firm's performance.

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Appendix. Basics of data envelopment analysis

Data envelopment analysis (DEA) is a technique to measure the relative efficiency of k firms operating in the same industry. Each firm j produces m different types of outputs, using n different types of inputs. Let X_j be an n x 1 vector consisting of inputs of firm j. Similarly, let Y_j be an m x 1 vector of firm j's output factors. Each input and output can be assigned implicit prices (opportunity cost). Let U_j denote an n x 1 vector of implicit prices for inputs of firm j and V_j be an m x 1 vector of implicit prices for outputs. To make any economic sense, these prices should be positive. Thus:

$$U_j, V_j \geq 0. \tag{1}$$

With these prices, we can determine the total value of inputs and outputs for firm j. Then we can measure efficiency η of the firm j as follows:

$$\eta = (V_j'Y_j) / (U_j'X_j). \tag{2}$$

Note that vector U_j and V_j may be different for each firm $j = 1, k$, since opportunity cost for each firm will be different.

We need to make sure that the efficiency measure does not exceed 1. Thus the following condition is imposed for firm j:

$$(V_j'Y_j) \leq (U_j'X_j). \tag{3}$$

The above condition will apply to all the firms in the industry. To measure the relative efficiency of firm j, we would like to know how did the other firms perform if firm j's implicit prices were used. We still have to make sure that regardless of whichever implicit prices are used, the efficiency measure for any firm does not exceed 1. DEA determines the implicit prices for each firm that maximizes its efficiency measure. In other words, for firm j, we want the following:

$$\underset{X_j, Y_j}{\text{Maximize}} (V_j'Y_j)/(U_j'X_j), \tag{4}$$

$$\text{Subject to: } (V_j'Y_k) / (U_j'X_k) \leq 1 \text{ for } k = 1, k. \tag{5}$$

In addition, the non-zero constraints (1) are also applied.

These conditions ensure that none of the firm is more than 100% efficient. Also if the objective function is less than 1, then in comparison to firm j, one or more of the firms, denoted by constraints in equation (5) is producing more output using the same level of inputs, or producing the same level of output using less input, or both. Such a result would show that firm j is relatively inefficient with respect to these firms.

This maximization problem of (3) subject to (1) and (5) is a nonlinear programming problem. Fortunately, Charnes et al. (1978) show that this problem can be transformed into the following linear program:

$$\underset{U_j, V_j}{\text{Max}}\,(V_j'Y_j) \tag{6}$$

Subject to:

$$- V_j'Y_j + U_j'X_j \geq 0 \text{ for } j = 1,k \tag{7}$$

$$U_j'X_j = 1 \tag{8}$$

$$U_j, V_j \geq \varepsilon, \tag{9}$$

where ε is small positive number.

Note that U_j, V_j are new implicit price vectors for inputs and outputs respectively. They are different from U_j and V_j of equations (4) and (5).

Formulation of linear programming problem consisting of equations (6) to (9) constitute the basic DEA technique. This maximization problem is solved for each firm in the group. The firms which have objective functions value equal to 1 are deemed relatively efficient, while those less than 1 are deemed relatively inefficient. The maximization problem can easily be solved by a simple linear program.

Table 1. Input output data and firms (1986 and 1987)

		Output factors				Input factors		
		ASM	ATM	DEP	Passengers	FUEL	EMP	EQP
American	1986	79,265.4	10,430.1	618.3	51,095	1,519.2	52,652	334.2
Alaska	1986	6,359.2	914.1	85.4	4,479	144.5	3,904	40.0
AmWest	1986	5,296.1	623.2	106.8	7,140	100.8	3,298	37.0
Continental	1986	59,629.5	7,522.8	545.9	42,776	1,147.8	28,327	285.2
Delta	1986	73,237.6	9,474.1	707.9	53,272	1,532.5	50,568	336.4
Northwest	1986	61,155.3	9,736.1	560.6	35,153	1,423.9	33,376	292.1
Southwest	1986	12,574.40	1,642.50	262.2	15,277	251.10	5,605	72.9
TWA	1986	51,402.10	6,874.40	331.4	24,191	1,043.20	32,975	213.7
US AIR	1986	43,360.60	5,233.10	947.7	55,242	1,018.40	36,732	347.9
UAL	1986	91,272.40	12,148.90	627	50,479	1,955.00	57,012	345.2
American	1987	89,828.5	11,984.9	680.6	56,888	1,740.1	59,971	383.6
Alaska	1987	6,892.3	989.4	94.3	4,698	157.6	4,302	43.6
AmWest	1987	10,318.1	1,235.8	177.2	11,232	181.1	6,213	56.2
Continental	1987	64,174.9	8,827.9	545.1	40,148	1,289.0	32,434	327.1
Delta	1987	82,844.4	10,810.1	778.0	57,006	1,695.8	51,507	361.5
Northwest	1987	61,420.5	9,882.2	517.3	37,247	1,421.3	34,194	306.8
Southwest	1987	13,331.10	1,749.20	276.40	15,643	258.20	6,045	74
TWA	1987	51,810.70	6,979.50	309.70	24,623	1,066.10	30,800	208.2
US AIR	1987	46,949.50	5,835.60	1,022.20	61,312	1,102.70	41,877	380.5
UAL	1987	101,312.90	13,499.20	669.80	55,183	2,148.90	60,871	369.9

Notes: Output: ASM – available seat miles, in thousands; ATM – available ton miles, in millions; DEP – number of departures, in thousands; Passengers – in thousands. Input: FUEL – in million gallons; EMP – number of total employees; EQP – number of aircrafts.

Table 2. DEA efficiency scores for US airlines (1986-1995)

	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Alaska	93.83%	91.15%	82.60%	82.31%	81.24%	80.23%	81.47%	90.39%	96.50%	99.65%
AMR	100.00%	100.00%	100.00%	100.00%	98.55%	95.44%	95.31%	94.73%	100.00%	100.00%
AmWest	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Continental	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	99.14%	87.05%	85.39%	82.60%
Delta	95.60%	97.53%	100.00%	100.00%	100.00%	99.53%	98.36%	97.15%	96.66%	100.00%
Northwest	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
Southwest	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
TWA	99.38%	100.00%	100.00%	100.00%	100.00%	88.31%	93.82%	80.67%	83.84%	83.39%
UAL	100.00%	100.00%	100.00%	99.43%	100.00%	100.00%	100.00%	100.00%	100.00%	88.43%
US AIR	88.25%	90.66%	81.86%	80.79%	83.91%	87.25%	86.64%	89.12%	88.85%	100.00%

Notes: DEA scores are based on Charnes Coopers, and Rhodes (CCR) model using constant returns to scale and input orientation.

Table 3. Details of DEA scores for 1986

Panel A. Virtual input and output prices									
DMU No	Name		Output factors				Input factors		
		SCORE	DEP	PASS	ASM	ATM	FUEL	EMP	EQP
1	AMR	100.00%	0	0.1	0.38	0.51	0.68	0	0.32
2	Alaska	93.83%	0	0.0	0	1.0	1.0	0.0	0.0
3	AmWest	100.00%	0	1.0	0	0.0	1.0	0.0	0.0
4	Continental	100.00%	0	0.0	1	0.0	0.0	0.5	0.5
5	Delta	95.60%	0	0.4	1	0.0	0.3	0.0	0.7
6	Northwest	100.00%	0	0.0	0	1.0	0.0	0.5	0.5
7	Southwest	100.00%	1	0.0	0	0.5	0.0	1.0	0.0
8	TWA	99.38%	0	0.00	1	0.47	0.47	0.22	0.32
9	US AIR	88.25%	1	0.00	0	0	0.94	0.00	0.06
10	UAL	100.00%	0	0.08	0	0.69	0.00	0.00	1
Panel B. Reference best-practice firms for inefficient firms, and their radial distance									
			Reference firms						
1	AMR	100.00%							
2	Alaska	93.83%	1 (0.03) 6 (0.03) 7 (0.19)						
3	Amwest	100.00%							
4	Continental	100.00%							
5	Delta	95.60%	1 (0.47) 7 (1.13) 10 (0.24)						
6	Northwest	100.00%							
7	Southwest	100.00%							
8	TWA	99.38%	1 (0.38) 4 (0.03) 6 (0.06) 10 (0.18)						
9	US AIR	88.25%	3 (5.96) 7 (1.19)						
10	UAL	100.00%							
Panel C. Potential slack for inefficient DMUs									
DMU No	Name		Output factors				Input factors		
		SCORE	DEP	PASS	ASM	ATM	FUEL	EMP	EQP
1	AMR	100.00%							
2	Alaska	93.83%	0	1042	251	0	0	0.02	4.9
3	AmWest	100.00%							
4	Continental	100.00%							
5	Delta	95.60%	27.94	0	0	182.93	0	3673.91	0
6	Northwest	100.00%							
7	Southwest	100.00%							
8	TWA	99.38%	61.03	7307	0	0	0	0	0
9	US AIR	88.25%	0	5442	3127	430.47	0	6109.66	0
10	UAL	100.00%							

Table 4. Year-wise return of inefficient and efficient groups

Year	Inefficient group		Efficient group		Difference return
	Return	Std. dev	Return	Std. dev	
1986	0.190	0.115	0.296	0.485	0.106
1987	-0.199	0.127	-0.119	0.436	0.080
1988	0.297	0.256	0.560	0.180	0.263
1989	0.018	0.056	0.693	0.582	0.675
1990	-0.279	0.215	-0.172	0.385	0.106
1991	0.012	0.343	0.070	1.254	0.058
1992	-0.091	0.201	-0.043	1.097	0.048
1993	0.010	0.256	0.531	0.528	0.521
1994	-0.593	0.068	0.194	0.927	0.787
1995	1.541	0.649	1.182	0.798	-0.359
				Mean	0.229
				Std. dev	0.343
				t-value	2.110