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Environmental quality in a differentiated duopoly facing a minimum quality standard

Abstract

In a differentiated duopoly where firms compete in environmental quality, the authors examine the effects of a minimum quality standard (MQS) on firms' quality choices, profits, the average quality offered to consumers, and social welfare. Deviating from some of the previous results, the paper shows that in general the effects are ambiguous and depend critically on how strongly the products are substitutes, the difference in firms' unit costs of quality provision, and their market shares. In particular, the authors show that while at low levels the MQS has *no* effects on industry, at intermediate levels it can benefit the *high-cost* firm by forcing it to raise its quality while always causes the low-cost firm to *reduce* its quality and *lose* profits. It is found that only under quite restrictive conditions does an MQS policy unambiguously increase welfare.

Keywords: duopoly, minimum quality standard, overcompliance, profits, welfare.
JEL Classification: Q58, L13, L51, D43.

Introduction

The past three decades have witnessed two broad trends in concerns about environmental quality. On the one hand, consumers have become increasingly concerned about the environmental quality and impact of products they consume. They have often expressed these concerns both by showing willingness to pay a price premium for the so called "green" or environmentally-friendly products and by pressuring policy-makers to subject the polluting industries to environmental quality standards. On the other hand, responding to consumers' preferences and public pressure for environmental regulations, producers have more than ever become environmentally proactive[1]. At the same time, firms have been increasingly competing with one another on the basis of environmental quality either directly, by adopting more environmentally friendly technologies to improve the environmental quality of their production processes and products, or indirectly, by engaging in, or supporting, pro-environment activities in general to enhance their environmental image or reputation (see, for example, Videras and Alberini (2000) and Antona et al. (2004)). In these fashions, firms have been increasingly tending to environmentally differentiate their brands and public image from those of their rivals. Examples indicating these trends abound and include agricultural products differentiated by the degree of their genetic modification (GM), or by the degree of their organic content (organic versus conventionally produced products), or the extent of their bio-degradability (recyclability). Gasoline of different octane or lead content, electricity generated by different processes (fossil fuel-based, solar based, hydro or thermal based) or inputs (coal, oil, natural gas, biomass), and cars driving on different mixes of bio-fuel (ethanol) and gasoline, or electricity, are all only a few among numerous other examples. In this last respect, it is perhaps interesting to note that to further differentiate itself environmentally from its rival auto companies such as Toyota, Honda Motor Co. in 2006 announced that it was going to mass-produce compact cars that run solely on bioethanol, becoming the first Japanese automaker to do so, and that together with Japan's Research Institute of Innovative Technology for the Earth it had developed a new process to efficiently produce ethanol fuel from soft biomass, a renewable resource derived from plants (see *The Daily Yomiuri*, Tokyo, September 15, 2006, p. 8).

The trends noted above raise several important questions. For example, what factors determine the firms' choices of environmental quality of their brands if they are left to freely compete by differentiating their products, that is, in the absence of any minimum quality standard (MQS)? More importantly, faced with a MQS, do firms have an incentive to overcomply? How would the introduction of a MQS affect the firms' quality choices, their profits, and the average environmental quality provided to consumers? How would it affect social welfare?

We examine these questions in a simple model of quality choice in a duopoly in which the firms compete in environmental quality of their brands while facing a MQS set by a regulatory agency. In our model the firms are assumed to differ only with respect to their unit costs of environmental quality provision, perhaps due to having access to different pollution abating technologies or as a result of operating the same tech-

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[1] For an interesting historical account of corporate environmentalism, see Hoffman (1997). British Petroleum (BP), Dow Chemical, and Heinz are the examples of corporations whose pro-environment actions have benefited them both financially and in reputation, thereby giving them a competitive edge over their rivals. In contrast, Shell Oil Company, McDonalds, and Monsanto are among the corporations whose less environmental-friendly approaches have harmed their public image and profitability.

nology with different efficiencies. Accordingly, each firm produces a brand of a commodity at a different unit quality cost. Consumers are assumed to be identical. A typical consumer deems the two brands different only in their environmental quality attribute. She derives utility from the quality levels of the brands and is willing to pay a hedonic price for each brand, which is assumed to be proportional to each brand's quality. She chooses the quality levels to maximize her utility subject to budget constraint. The resulting revenue functions for the firms imply that the two products are strategic substitutes. In this setting, and in parallel with the standard result for quantity choices in a vertically differentiated duopoly (see, for example, Dixit, 1979; Singh and Vives, 1984; and Shy, 1995, we show that in the absence of a MQS each firm chooses an equilibrium quality level that varies inversely with its own quality cost and directly with the quality cost of its rival. Accordingly, the low-cost firm chooses a higher quality level than that the quality level chosen by the high-cost firm, and quality differentiation increases with quality cost difference and the degree to which the products are strategic substitutes. When a MQS is introduced, we show that for low levels of the standard up to the unregulated equilibrium level chosen by the high-cost firm, both firms choose to *overcomply*, thus rendering the MQS ineffective. Interestingly, for an intermediate range of standards, we show that the high-cost firm complies with the MQS, and hence raises its quality relative to its unregulated equilibrium level whereas the low-cost firm, although still overcomplying, *lowers* its quality. More interesting, within this intermediate range, the MQS may *increase* the high-cost firm's profit relative to its unregulated profit whereas it *always reduces* the profit of the low-cost firm. In fact, we show that for some sets of values of the parameters representing the quality costs difference and the degree to which the products are substitutes, for the *entire* intermediate range of MQS, the introduction of the MQS enables the high-cost firm to enjoy *higher* profit levels that it would under no regulation. Furthermore, we show that for a wide range of MQS levels, the effects of the MQS on average environmental quality provided to consumers and on social welfare are *ambiguous*. We identify the range of standard levels and market conditions under which a MQS unambiguously increases (reduces) the average quality and/or social welfare. These results suggest that when firms compete in environmental quality care needs to be exercised in using a MQS to regulate environmental quality. In particular, for the minimum quality policy to be successful close attention needs to be paid to, among other things, firms' quality cost difference, their market shares, and the degree to which the industry's products are substitutes, as these conditions may significantly restrict the range of standards that would result in intended (favorable) outcomes.

The paper proceeds as follows. To highlight the contribution of the present paper, in the next section we selectively review the literature closely related to the questions examined in this paper. In section 2 we set out the model and present the firms' equilibrium choices of environmental quality of their products in the absence of regulation. Section 3 examines the effects of introducing a MQS on firms' quality choices, where we show that depending on the level of the standard, either both, or only one, or none of the firms ma overcomply with the standard. Sections 4.1 and 4.2 respectively examine the effects of the MQS on the firms' profits and the average environmental quality enjoyed by consumers, while section 4.3 discusses the qualitative effects of the MQS on social welfare. Concluding remarks are presented in the final section.

1. Review of related literature

Several studies have explored the specific question of firms' environmental quality choice in a differentiated industry facing a mandatory standard. For example, Maloney and McCormick (1982) study the effect of a mandatory environmental quality regulation on profits in an atomistic competitive industry where the regulation increases a typical firm's costs but has no direct effect on industry demand. They show that with restricted entry to the industry, the regulation can result in increased profits for all firms in the industry by creating a scarcity rent from the right to use the environmental assets. Further, in contrast to the present paper, they show that when the firms differ in their production costs, the environmental regulation may increase the profits of the low-cost firms while lowering those of the high-cost firms, and that this intraindustry transfer can happen even if entry is not restricted. Farzin (2003) examines the effect of a mandatory emissions standard on a polluting oligopolistic industry with identical firms where a higher environmental quality standard raises both the firms' compliance costs and the demand for the industry's output. He identifies conditions under which a stricter standard leads to a larger profit in the industry, a larger number of firms, a greater industry output, and a lower total pollution in the long run. However, none of these studies considers strategic environmental quality differentiation and possibility of voluntary overcompliance with the standard. On the other hand, Arora and Gangopadhyay (1995) analyze a model in which firms overcomply in order to attract high-income consumers, and thereby raise consumers' welfare[1]. As such, in their model overcompliance derives from the demand side due the heterogeneity of consumers' willingness to pay for environmental quality, which

[1] A strand of literature on motives for corporate environmentalism has emphasized self-regulation as a strategic means of preempting otherwise higher future government regulations. For an excellent survey of this literature, see Lyon and Maxwell (2000).

arises from differences in income levels. In contrast, our model explains overcompliance from the supply side by considering heterogeneity of firms' pollution control technologies, which lead to differences in their unit costs of environmental quality improvement.

Salop and Scheffman (1983, 1987) consider a dominant firm-competitive fringe model of an industry where a lower-cost dominant firm acts as price leader. They show that a cost-raising action controlled by the dominant firm, which could be interpreted as controlling product standards or other government regulations, or expenditures on advertising or research and development, can increase the dominant firm's profit at the expense of the fringe's profit and possibly consumer welfare[1]. Interestingly, however, in our model of a quality differentiated duopoly, raising the MQS can increase the profit of the *high* cost firm while it always decreases that of the *low* cost firm.

Our model is cast in the specific form of competition in environmental quality. However, its results pertain more generally to the question of quality competition in a duopoly facing a MQS, which has been the subject of several studies, including those by Leland (1979), Bonanno (1986), Ronnen (1991), Motta and Thisse (1993), Crampes and Hollander (1995), Scarpa (1998), Garella (2006), and Kuhn (2007). Typically these studies focus on heterogeneity of consumers' tastes for quality and competition of firms both in price (or output quantity) and quality. In contrast, in order to focus on the role of quality competition for the outcomes of a MQS, our model assumes identical consumers' tastes and fixed market shares of products. These assumptions enormously simplify the analysis and allow us to isolate the role of quality competition from the confounded effects of both price and quality competition. They also allow us to explicitly determine the firms' choices of quality levels and their profits as functions of the MQS level, the firms' quality costs difference, and the degree to which the products are substitutes. These in turn enable us to obtain new insights and a clearer understanding of the results furnished by previous studies. At the same time, our model is somewhat more general in respects of specifications of firms' cost of quality provision and the typical consumer's preferences for qualities. Because of these differences in assumptions and modeling approach, our model leads to results that either deviate from, or modify, reinforce, or extend, those in the literature. Thus, for example, in a key study Ronnen (1991) develops a model of quality differentiated duopoly in which there are a continuum of consumers with differing tastes for quality[2] and two firms with access to a common technology for providing quality and hence identical quality development (fixed) cost functions that are increasing in quality. In Ronnen's model the firms compete first in quality choice and then in price. This give rise to what is crucial to his results, namely the interaction between price competition and quality competition, in that price competition is intensified – causing prices to fall – as the quality differentiation is reduced. Ronnen shows that, compared to the unregulated market, and as a result of introducing the MQS, if the standard level is set above, *but sufficiently close to*, the unregulated low quality (corresponding to α_2^* in our paper)[3], then (1)) the profit of the low-quality (corresponding to our high-cost) firm *increases* while that of the high-quality (corresponding to our low-cost) firm decreases with the standard; (2) the quality levels of *both* firms *increase* with the standard, which together with falling quality-adjusted prices makes all consumers better off; and (3) the consumers' gain is large enough to offset the industry's loss of profits so that social welfare *increases*. Crampes and Hollander (1995) analyze a similar model as Ronnen's except that they consider the cost of quality as a *variable* cost that is increasing and convex in quality. This assumption allows quality to determine prices directly through cost, and not just indirectly by shifting the demand. Again, for the MQS levels slightly above the unregulated low quality, they reach the same results as (1) and (3) above found by Ronnen. However, they found that the result (2) holds only if the response of the high-quality firm to the quality choice of it rival is weak (in our paper this condition corresponds to the requirement that either the products are weakly substitutes and/or the firms' quality costs difference is small)[4]. Motta and Thisse (1993) use a model virtually identical to that of Ronnen except that by specifying a quadratic convex (fixed) cost of quality provision, they solve numerically for the firms' equilibrium quality choices, the profits, consumers surplus and welfare. They obtain the same qualitative results ((1)-(3) above) as in Ronnen. In particular, they show that as long as the MQS level is set moderately enough to accommodate both firms in the market, the low-quality firm's

[1] For a good review of the literature on the use of regulation as a cost-raising strategy, see McCormick (1984). In a related but different model, Lutz et al. (2000) consider situations where a high quality firm in the industry takes the role of quality leader by credibly committing to a quality level that is higher than the anticipated standard to be set by the regulator. They show that by such a strategic action, the high-quality firm can influence the regulator to set lower standards, thereby leading to a *lower* social welfare than would be the case if the regulator were to lead in setting the industry standard.

[2] The formulation of differing consumers' tastes for quality in this literature usually follows that of Mussa and Rosen (1978).

[3] This condition ensures that in equilibrium with the MQS both firms continue to be in the market.

[4] For a study showing how in a vertically differentiated duopoly different assumptions about variable versus fixed quality cost and about price versus quantity competition affect the equilibrium quality choices and producer and consumer surplus, see Motta (1993).

profit first increases for the standard levels close to unregulated low quality and then decreases with it, while the high-quality firm's profits always decreases with the standard.

In studying the effects of a MQS regulation in the present paper, we explicitly solve for the firms' equilibrium choices of quality and establish three explicit and broad intervals of MQS levels. The first interval, specified by standards up to the unregulated choice of the low-quality firm [0, α_2^*] is similar to that identified in some of the previous studies, and marks the interval over which the MQS is *ineffective* as both firms choose to *overcomply* it. Although, such weak standards have no effect on the industry profits and consumer surplus, to the extent that their administration involves costs, they reduce social welfare[1]. Our second interval, containing intermediate levels of MQS, is the one that has been the focus of the previous studies. However, in several respects our results for this interval are distinguished from those discussed under (1)-(3) and their variants above. In our paper, over this intermediate interval, specified by $[\alpha_2^*, \hat{\alpha}^U)$, whereas in accord with the previous studies the low-quality (high-cost) firm complies with MQS and hence raises its quality as the MQS is raised, in contrast to previous literature, the high-quality firm *lowers* its equilibrium quality in response, although it still overcomplies. The reason for this difference is that in most of the previous studies the high-quality firm's best response to a quality improvement by the low-quality firm is to increase its own quality in order to ease off quality competition in the first stage of the game while stiffening the price competition in the second stage. In our model, since the firms compete only in quality, and given the unit cost advantage of the high-quality (low-cost) firm over its rival's, when the low-quality firm raises its quality in compliance with MQS, the best response of the high-quality firm is to *lower* its quality[2] (although still differentiating by overcomplying) in order to lower its total cost relative to its rival's and thereby reduce the inroads into its unregulated profit as a consequence of the low-quality firm being forced by the MQS to raise its quality. As a result, setting the MQS in this interval reduces the market quality differentiation by raising the quality of the low-quality firm while inducing the high-quality firm to *reduce* its quality. Accordingly, in contrast to previous studies, the introduction of a MQS in this interval does *not* necessarily enhance the quality enjoyed by consumers. We show that it may in fact *lower* the average quality offered to consumers if the high-quality firm dominates the market and the products are close substitutes.

As regards the effects on firms' profits, similar to previous studies, the high-quality firm's profit decreases with the MQS over this interval. However, our result regarding the effect on the low-quality firm's profit is much more general than that obtained in the literature and depends both on the firms' quality cost difference and the degree to which the products are strategic substitutes. We establish four sets of the combinations of the values of these parameters, resulting in four different cases in which the low-quality firm's profit (A) increases over the *entire* interval at increasing rates (*i.e.*, is convex and increasing in MQS), (B) increases over the *entire* interval but at decreasing rates (*i.e.*, is concave and increasing in MQS), (C) be quadratic and concave in the MQS; first increases and then decreases with the MQS, but remains positive over the entire interval, and (D) for the MQS levels very close to the unregulated low-quality level, increases as the MSQ is raised but then falls to zero at some MQS level in the interval. It is this *local* result – case (D) – that has been the focus of the literature. Given the general ambiguity of the effects of MSQ in the second interval on the average quality enjoyed by consumers and on the producers' profits, in our paper, the welfare effect of the MQS is generally ambiguous. In fact, contrary to most of the literature, in our model, even for the levels of MSQ in the second interval for which both firms remain in the market, the standard can *reduce* welfare if the products are weakly substitutes (implying a sufficiently strong dominance of the high-quality firm) and there is a sufficiently large quality costs differential between the firms. That the MQS can actually reduce social welfare is also noted by Crampes and Hollander (1995), Scarpa (1998) and Kuhn (2007) but for reasons different from that in our model. As noted earlier, in Crampes and Hollander it is a result of quality cost being a variable cost that is convex and increasing in quality, coupled with a strong response of the high-quality firm to quality improvement by its rival due to imposition of the MQS. In Scarpa's model it is because *three* or more firms compete in quality and price instead of the duopoly model typically studied in the literature. In Kuhn's model, the adverse welfare effect of the MSQ is a result of the assumption that consumers derive utility not just from quality (which is the standard assumption in other models) but that they also derive a baseline benefit, unrelated to quality, from consuming a product. He shows that when the baseline benefit dominates the willingness to pay for

[1] In a model of a differentiated duopoly with quality affecting fixed investment in R&D, Garella (2006) shows that a MSQ in this interval may not be so "innocuous" as it can reduce the quality-leading firm's incentive to invest in R&D.

[2] In our model, the marginal revenue from a quality improvement by one firm decreases in the rival's quality, so the two products may be considered strategic substitutes. Also, see Eales and Binkley (2003) and Garella (2006) for examples of such cases.

quality, then it is the *low-quality* firm that dominates in the market, in which case the MQS at intermediate levels has a negative effect both on consumer surplus and industry profits.

Finally, while previous studies have limited their attention to the intermediate MQS levels, we show that for standards beyond the intermediate interval (that is, $\hat{\alpha} \geq \hat{\alpha}^U$), as long as both firms still remain in the market, they just comply with the MQS, so that quality will no longer be differentiated, and profits decline as MQS level is raised. Despite requiring such high quality standards, it is still possible that for a sub-interval the MQS results in a *lower* quality than the quality consumers would enjoy without regulation, thus *reducing* the overall welfare. Furthermore, even when the MQS is so high that consumers unambiguously gain from its introduction, its welfare effect can still be ambiguous because of decreased producers' surplus.

2. The model. Unregulated quality choices

Consider an industry consisting of two firms, labeled i = 1 and 2, each producing a brand of a product. From consumers' perspective, the brands are different only with respect to their environmental quality attributes, otherwise are identical in all other respects. Let $\alpha_i \geq 0$ denote the environmental quality and $q_i \geq 0$ the quantity of firm i's output.

In general, each firm's revenue is a function of the quantity demanded of the firm's own product and that of its rival firm's product. It also is a function of both firms' choices of environmental quality. Formally, the revenue function of firm i can generally be represented by $R_i = R_i\ (q_i, q_j, \alpha_i, \alpha_j) = p_i\ (q_i, q_j, \alpha_i, \alpha_j)\ q_i$. To concentrate on firms strategic behavior with regard to the choice of environmental quality and their responses to the MQS set by an environmental regulatory agency, we abstract from firms' strategic behavior with regard to the choice of output quantity. This simplification can be justified, for example, by considering situations where consumers' aggregate income spent on the products is large enough and the firms make short-run decisions, so that consumers' demand for each product is determined by the firm's available output capacity, which is assumed to be fixed at $\overline{q}_i$ in the short run. In other words, we are assuming that in the short-run the firms inherit their existing capacities and hence their historical market shares. Thus, the revenue function of each firm simplifies to

$$R_i = R_i\left(\overline{q}_i, \overline{q}_j, \alpha_i, \alpha_j\right) = p_i\left(\alpha_i, \alpha_j\right)\overline{q}_i = R_i\left(\alpha_i, \alpha_j\right).$$

The environmental quality of the firm in our model can be interpreted broadly to represent not only the environmental quality associated with any stage of production of the final products (that is, from input acquirement to production processing, packaging, and distribution). It can also represent a firm's environmental activities which may not necessarily be related to its product *per se*, but could be pro-environment activities which, for example, improve the firm's public environmental image and reputation. The firm's incentive to engage in such activities is to attract consumers who support its pro-environment stance by their willingness to pay a premium price for the firm's product. In other words, α_i in our model can be interpreted broadly enough to encompass the notion of firm's environmental responsibility. We are thus treating α_i in our model as firm's environmental reputation which can from consumers' perspective be distinct from how much of the firm's product they may consume. Accordingly, our notion of the environmental standard set by the regulator may also be interpreted broadly. It may not only represent the minimum environmental standard that firms have to observe in production of their products. It can more generally be viewed as a composite index of a firm's environmental friendliness.

To simplify the model, we make two further assumptions. First, we assume that the choice of environmental quality by a firm does not affect its output level. That is, the firm's environmental quality activity is like end-of-pipe pollution abatement and as such is separate from the firm's production process, so that there is no spillover effect from environmental quality activity into the production activity and vice versa. An implication of this assumption is that the production cost is not affected by choice of environmental quality. This is consistent with the assumption of constant unit production costs of the products that we shall also be making shortly. Second, we assume that inputs employed in production and environmental activities are specific to each activity. An implication of this assumption is that a firm can not by reallocating some of the inputs from production into environmental quality activity reduce the level of its output to improve the environmental quality of its product, thereby obtaining a higher price for its product.

For analytical convenience, we adopt the following quadratic revenue functions, which in Appendix A we explicitly derive from consumer's utility maximization problem.

$$R^i(\alpha_i, \alpha_j) = -\frac{1}{2}a\alpha_i^2 - b\alpha_i\alpha_j + r\alpha_i,\ i = 1,2,$$
$$j \neq i,\ a,b,r > 0, \tag{1}$$

where $\partial^2 R^i(\alpha_i,\alpha_j)/\partial\alpha_i^2 = -a < 0$ and $\partial^2 R^i(\alpha_i,\alpha_j)/\partial\alpha_i\partial\alpha_j = -b < a$. The first inequality indicates that for each firm there are diminishing marginal returns to quality improvement. The second inequality indicates a quality improvement by one firm reduces its rival's marginal revenue, implying that the qualities are strategic substitutes.

It is plausible to assume that a firm's marginal revenue is more sensitive to a change in its own quality than to a change in its rival's; that is

$$a > b. \tag{2}$$

In fact, for a given value of a, the magnitude of b indicates the degree to which the consumers perceive the two products as substitutes, or inversely, how closely they are strategic substitutes from the firms' perspective. In the extreme case of $b = a > 0$ (*i.e.*, $a - b = 0$, or $b / a = 1$) the two products become homogeneous (perfect substitutes or zero degree of differentiation) and the firms' profits would drop to the lowest level. In the other extreme case, when $b = 0$ (*i.e.*, $a - b = 0$, or $b / a = 0$), quality differentiation is the highest, and the two products become independent of each other. In this case, each firm behaves like a monopolist in choosing its quality level. As such, one could consider $(a - b)$ as an index of product differentiation, or b / a as the degree to which the products are strategic substitutes.

To focus on the role of quality competition, we assume that the unit production costs of products are the same, and normalize them to be zero. Let A_i be the constant unit cost of achieving environmental quality α_i[1]. We assume that the two firms differ only with respect to this cost, for example, due to differences in their pollution abatement technologies. More specifically, we assume that firm 1 has an advantage over firm 2 in cost of quality provision, *i.e.*,

$$A_2 > A_1 > 0. \tag{3}$$

Then, the profit functions are expressed as[2]

$$\pi^i(\alpha_i,\alpha_j) = \left(-\frac{1}{2}a\alpha_i^2 - b\alpha_i\alpha_j + r\alpha_i\right) - -A_i\alpha_i \; (i, j = 1,2,\; i \neq j). \tag{4}$$

To ensure that both firms can coexist in the market, we need to assume that

$$r > A_2 \tag{5}$$

Otherwise, the profit of firm 2 will always be negative and thus not entering the market.

The two firms play a Nash-Cournot game in qualities of their products. The problem of firm $i = 1, 2$ is

$$\max_{\alpha_i} \pi^i(\alpha_i,\alpha_j) = -\frac{1}{2}a\alpha_i^2 - b\alpha_i\alpha_j + r\alpha_i - A_i\alpha_i, \tag{6}$$

α_j $(j \neq i)$ given.

Suppose that in the absence of any environmental regulation, there exists an equilibrium $\left(\alpha_1^*,\alpha_2^*\right)$[3]. At equilibrium, the following equation holds:

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{pmatrix} \alpha_1^* \\ \alpha_2^* \end{pmatrix} = \begin{pmatrix} r - A_1 \\ r - A_2 \end{pmatrix}. \tag{7}$$

With condition (2) one has $a^2 - b^2 > 0$, which ensures that a Nash-equilibrium is unique and stable (see Dixit, 1986). The firms' equilibrium quality choices are

$$\alpha_1^* = \frac{a(r-A_1) - b(r-A_2)}{a^2-b^2} > 0 \text{ (by (2) and (3))}, \tag{8a}$$

$$\alpha_2^* = \frac{a(r-A_2) - b(r-A_1)}{a^2-b^2}. \tag{8b}$$

Notice that whereas α_1^* is always positive, to ensure that α_2^* is positive we need the condition

$$\frac{b}{a} < \frac{r-A_2}{r-A_1} < 1, \tag{9}$$

that is, the adverse effect of an increase in the rival's quality on the firm's marginal revenue should not be too strong, or, equivalently, the two products should be sufficiently differentiated.

As to be expected, from (8a) and (8b) it is seen that the equilibrium choice of the quality by each firm varies inversely with its own cost of quality and directly with that of its opponent.

The associated profits at the equilibrium are calculated as

$$\pi^i(\alpha_i^*,\alpha_j^*) = \frac{a}{2}\left(\frac{a(r-A_i) - b(r-A_j)}{\left(a^2-b^2\right)}\right)^2 > 0, \tag{10}$$

$i = 1,2, \quad j \neq i,$

which ensures that both firms coexist in the market.

An interesting finding here is (from (8a) and (8b))

$$\alpha_1^* - \alpha_2^* = \frac{A_2 - A_1}{a - b} > 0, \tag{11}$$

[1] A_i can also be interpreted, for example, as a constant unit cost of (end-of pipe) pollution abatement.

[2] We could more generally write the profit function to include a constant term c_i, as $\pi^i(\alpha_i, a_j) = R^i - A_i\alpha_i - c_i$, where c_i can be interpreted either as a unit cost of production or as a tax (subsidy) per unit of output respectively when c_i is positive (negative). This generalization would not affect the results as long as both firms remain in the market.

[3] It is shown below (see equation (10)) that the firms' profits at equilibrium are positive. Therefore, both firms can coexist under no regulation.

which we state as the following proposition.

Proposition 1: In a differentiated duopoly with no regulation, (1) the firm with the lower quality cost ($A_2 < A_1$) adopts a higher quality level than that chosen by its high-cost rival ($\alpha_1^* > \alpha_2^*$), and (2) the extent of quality differentiation in the market increases with the quality costs difference ($A_2 - A_1$) and the degree to which the products are strategic substitutes (b/a).

These results are similar to those of output quantity choices in a differentiated duopoly (see, for example, Dixit, 1979; Singh and Vives, 1984; and Shy, 1995) and may serve as a theoretical basis for empirical tests of market efficiency with duopoly. For reference, in the following sections we refer to firm 1 as either low-cost or high-quality firm and to firm 2 as high-cost or low-quality firm.

3. Quality choices with a minimum quality standard

In this section we analyze the firms' equilibrium quality choices in the face of an MQS[1]. Let $\hat{\alpha} > 0$ denote the minimum quality standard set exogenously by the regulatory agency. Taking this standard and the rival firm's choice of quality as given, the profit maximization problem for firm i is

$$\max_{\alpha_i}\left(\frac{-1}{2}a\alpha_i - b\alpha_j + r - A_i\right)\alpha_i, \tag{12}$$

subject to $\alpha_i \geq \hat{\alpha},\ \ \alpha_j (j \neq i)$ given.

At equilibrium $\left(\alpha_1(\hat{\alpha}), \alpha_2(\hat{\alpha})\right)$, it holds that

$$-\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{pmatrix} \alpha_1(\hat{\alpha}) \\ \alpha_2(\hat{\alpha}) \end{pmatrix} + \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = -\begin{pmatrix} r - A_1 \\ r - A_2 \end{pmatrix},$$

$$\left(\alpha_1(\hat{\alpha}) - \hat{\alpha}\right)\mu_1 = 0,\ \left(\alpha_2(\hat{\alpha}) - \hat{\alpha}\right)\mu_2 = 0, \tag{13}$$

where $\mu_1, \mu_2 \geq 0$ are the Lagrange multipliers. Notice that for now we have left aside the possibility that the standard may render production by one or both firms unprofitable. Later, we will take this possibility into account and show how a sufficiently high standard may force one of the firms or both of them out of the market.

Using the Kuhn-Tucker conditions (13), the equilibria are classified into three types:

1. *Both firms overcomply*. In this case, $\mu_1 = \mu_2 = 0$, implying that $\alpha_1(\hat{\alpha}) = \alpha_1^* > \hat{\alpha}$ and $\alpha_2(\hat{\alpha}) = \alpha_2^* > \hat{\alpha}$, where, as before, α_1^* and α_2^* are given by (8a) and (8b). The equilibrium exists if $\min(\alpha_1^*, \alpha_2^*) = \alpha_2^* \geq \hat{\alpha}$. We term the interval $[0, \alpha_2^*)$ as *Interval 1.*

Proposition 2: With an MQS in Interval 1, (1) both firms overcomply, (2) their equilibrium choices are the same as those in the unregulated case, implying that within this interval the MQS is ineffective, and (3) the low-cost firm overcomplies by a larger extent than the high-cost firm does (see Figure 1).

An important implication of equation (2) is that since the MQS does not affect the unregulated quality choices, to the extent that its administration involves costs, imposing no standard should be preferred to weak standards, those for which

$\hat{\alpha} < \alpha_2^* = \left[a(r - A_2) - b(r - A_1)\right]/(a^2 - b^2)$[2].

2. *Only one of the firms overcomplies*. In this case $\mu_1 = 0$ and, so $\mu_2 > 0$ that

3. $\alpha_1(\hat{\alpha}) = \left(-b\hat{\alpha} + r - A_1\right)/a > \hat{\alpha}$ and $\alpha_2(\hat{\alpha}) = \hat{\alpha}$[3]. This equilibrium exists if $\alpha_2^* \leq \hat{\alpha} \leq \hat{\alpha}^U$, where $\hat{\alpha}^U$ is defined by

$$\hat{\alpha}^U = \frac{r - A_1}{a + b} > \alpha_2^{*}.\text{[4]} \tag{14}$$

So, calling $[\alpha_2^*, \hat{\alpha}^U)$ as *Interval 2*, we have the following proposition.

Proposition 3: For MQS levels in Interval 2, the high-cost firm just complies with the standard so that its quality rises with the MQS. The low-cost firm, however, still overcomplies but reduces its quality as the standard is raised. As a result, the quality difference would be less than that in the unregulated case and narrows further as the MSQ is raised (see Figure 1).

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[1] A minimum quality standard (MQS) is the highest level of a pollutant that is legally allowed, and it can take various forms. *Performance standards* specify a certain environmental quality outcome per unit of product, and may be achieved by process changes, output reduction, or changes in polluting inputs. An examples of these standards is the US EPA (Environmental Protection Agency) mandated fuel economy standards (miles per gallon of gasoline) for passenger cars and light trucks. *Emission and ambient standards* are usually measured in terms of the quantity or volume of a pollutant discharged or its concentration in the ambient environment. For example, for water quality standards, it is measured in the unit of µg/l: micrograms of the pollutant (e.g., Mercury or Arsenic) per liter of water, or in the unit of cfu/100ml: colony forming units of the pollutant (e.g., Fecal Coliform) per 100 milliliters of sample. For the air quality standards, examples of the ambient standards are the California's 24-hour average standard of 50 µg/m^3 for PM10 and 8-hour average standard of .07 ppm (parts per million) for Ozone.

[2] Also, see Farzin (2004) who analyzes the social welfare effects of a stricter environmental standard and identifies situations in which the regulator may prefer no standard to weak standards.

[3] The other overcompliance case where $\mu_1 > 0$, $\mu_2 = 0$ implying that $\alpha_1(\hat{\alpha}) = \hat{\alpha},\ \alpha_2(\hat{\alpha}) > \hat{\alpha}$ can never happen because otherwise one would have $a\hat{\alpha} + b\alpha_2(\hat{\alpha}) = r - A_1,\ b\hat{\alpha} + a\alpha_2(\hat{\alpha}) > r - A_2$. By (2), this implies that $a\hat{\alpha} + b\alpha_2(\hat{\alpha}) > b\hat{\alpha} + a\alpha_2(\hat{\alpha})$. Since $\alpha - b > 0$, we should have $\alpha_2(\hat{\alpha}) < \hat{\alpha}$, which is a contradiction.

[4] $\hat{\alpha}^U - \alpha_2^* = \frac{r - A_1}{a + b} - \frac{a(r - A_2) - b(r - A_1)}{a^2 - b^2} = \frac{a(A_2 - A_1)}{a^2 - b^2} > 0$ (since $A_1 < A_2$ and $\alpha > b$).

The reason for the high-cost firm no longer overcomplying is simple: being the high-cost firm, at standards higher than α_2^* the cost of overcompliance becomes too large to be affordable. The explanation for the response of the low-cost firm to the MQS, which contrasts that found in the literature, is more subtle and involves two parts. First, given its quality cost advantage over its high-cost rival, it still finds it profitable to overcomply. However, rather than widening its quality gap with its rival, it *lowers* its quality (and hence its quality difference) as the MQS is raised over Interval 2, although its response, at the rate of $b/a < 1$, is less than one-for-one and is be less strong the less strong strategic substitutes the two products are. The reason for this behavior of firm 1 is as follows. Since its rival now has to adopt and stick to a higher quality (*i.e.*, $\alpha_2(\hat{\alpha}) = \hat{\alpha} > \alpha_2^*$) than it would under the unregulated case (or equivalently over Interval 1) and since its best response is negatively related to its rival's quality choice (recall that $\alpha_1(\alpha_2) = (-b\alpha_2 + r - A_1)/a$ so that $\frac{d}{d\alpha_2}\alpha_1(\alpha_2) = -b/a$), it follows that the best strategy of firm 1 is to reduce its quality below its unregulated level. In fact, the introduction of a relatively high MQS in Interval 2 alters the nature of the game of quality competition from a Nash-Cournot one to a game akin to the Stackleberg's leader-follower game in that, by complying with the regulator's standard, the high-cost firm behaves as the "first mover" by setting its quality at the MQS level and letting the low-cost firm react to this strategy. As such, the low-cost firm's cost advantage no longer gives it the incentive to choose as high a quality (and hence as high a degree of differentiation) as it would have chosen in the absence of the MQS. In other words, by requiring a high enough standard ($\hat{\alpha} > \alpha_2^*$) through the MQS, the regulator sends a "credible signal" to the low-cost firm that the high-cost firm is committed to, at least, comply with this higher standard. Therefore, contrary to the equilibrium choices over Interval 1 (or in unregulated case), the low-cost firm can no longer by choosing a much higher quality (and hence a greater differentiation) disadvantage the high-cost firm to lower its quality below $\hat{\alpha}$ ($> \alpha_2^*$) and thereby increase its own profit at the expense of the high-cost firm's. It should be noted that although the "first-mover" and "credible-signaling" effect of the MQS explained here are also noted by Ronnen (1991) and Crampes and Hollander (1995), our results that the MQS causes the low-cost firm to *reduce* its quality and *narrow* the quality difference contrast theirs. The reason for this difference is in modeling approach. In their models in the presence of the MQS the firms find it in their best strategic interests to ease off the quality competition in the first stage of the game by increasing the quality disparity while stiffening their price competition (by lowering their price difference) in the second stage. In our model, the firms compete in quality only, and the force of MQS strengthens the hand of the high-cost firm in that competition, thus inducing the low-cost firm to lower its quality and reduce the quality difference.

At the limit when $\hat{\alpha} = \hat{\alpha}^U$ quality differentiation no longer pays off and both firms just comply with the minimum standard.

4. None of the firms overcomplies. In this case, $\mu_1 > 0, \mu_2 > 0$, implying that $\alpha_1(\hat{\alpha}) = \hat{\alpha}$ and $\alpha_2(\hat{\alpha}) = \hat{\alpha}$. This equilibrium occurs on *Interval 3*, defined as $[\hat{\alpha}^U, \infty)$. We therefore have the following proposition.

Proposition 4: At sufficiently high standards $\left(\hat{\alpha} > \hat{\alpha}^U = (r - A_1)/(a+b)\right)$, *none of the firms has an incentive to differentiate its quality by overcomplying. Therefore both firms choose to comply with the minimum standard* (see Figure 1).

The explanation of this result is simple: when the standard is too high it becomes too costly even for the low-cost firm to differentiate its quality and use overcompliance as a strategic means of competition.

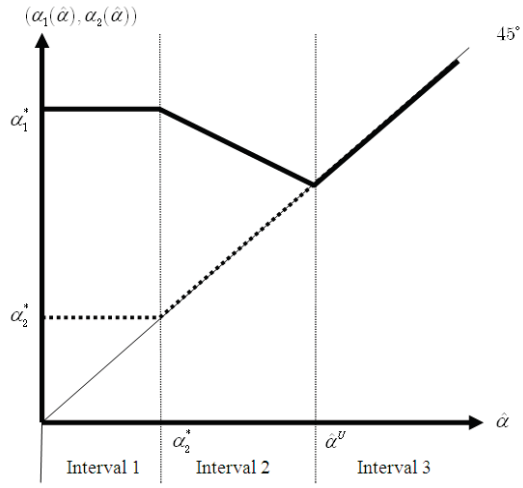

Fig. 1. Firms' equilibrium quality responses to environmental standard $\hat{\alpha}$

4. Profits, average quality and welfare effects of a MQS

4.1. Profits under a MQS. It would be interesting to examine how the duopoly profits would be affected by the MQS levels in the three intervals. Over Interval 1, the MQS is ineffective, so the firms' profits remain constant at their unregulated levels, given by (10), regardless of the standard level. It is easy to verify from (10) that $\pi^{*1} > \pi^{*2}$ (see Figure 2). Over Interval 2, the firms' profits, denoted by $\pi_{II}^{*1}(\hat{\alpha})$ and $\pi_{II}^{*2}(\hat{\alpha})$ and illustrated by solid and dashed lines in Figure 2, are calculated as

$$\pi_{II}^{*1}(\hat{\alpha}) = \frac{-1}{2} a\left[\alpha_1(\hat{\alpha})\right]^2 + \left(-b\hat{\alpha} + r - A_1\right)\alpha_1(\hat{\alpha}) \quad (15a)$$

$$\pi_{II}^{*2}(\hat{\alpha}) = \frac{2b^2 - a^2}{2a}\hat{\alpha}^2 + \frac{a(r - A_2) - b(r - A_1)}{a}\hat{\alpha} \quad (15b)$$

Using (15a) and the envelop theorem, we have for firm 1's profit

$$\frac{d\pi_{II}^{*1}(\hat{\alpha})}{d\hat{\alpha}} = -b\alpha_1(\hat{\alpha}) < 0. \quad (16)$$

That is, the low-cost firm's profit monotonically *declines* as the standard is tightened over Interval 2, although the profit always remains positive over this interval (see Appendix B).

The response of the high-cost (low-quality) firm's profit to MSQ levels in this interval is more complicated and, as formally derived in Appendix C, depends critically on (1) size of b / a, the degree to which the products are strategic substitutes, (2) the magnitude of $(r - A_2) / (r - A_1)$, the unit quality costs difference, and (3) the sign of the expression $2b^2 - a^2$, which determines the curvature of the low-quality firm's profit function. Figure 2 illustrates possible responses of firm 2's profit to MQS levels in Interval 2. As can be seen, there are four possible cases, labeled A, B, C, and D. Figure 3 shows the sets of combinations of b / a and $(r - A_2) / (r - A_1)$ values that correspond to each of the four cases (for formal derivations of these, see Appendix C)[1].

These lead us to the rather interesting result summarized in the following proposition.

Proposition 5: For any standard level in Interval 2, the MQS causes the low-cost (high-quality) firm's profit to fall below its unregulated level and to decline as the standard is raised. On the other hand, regardless of the degree to which the products are strategic substitutes (i.e., the size of b/a) and/or the firms' relative quality costs difference (i.e., the size of $(r - A_2) / (r - A_1)$), there is a sub-range of Interval 2 for which the MQS raises the high-cost (low-quality) firm's profit relative to its unregulated level and profit increases as the standard is tightened.

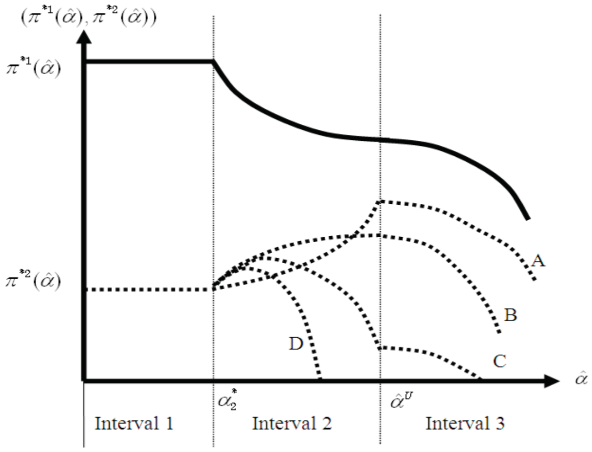

Fig. 2. Firms' profit responses to MQS level $\hat{\alpha}$

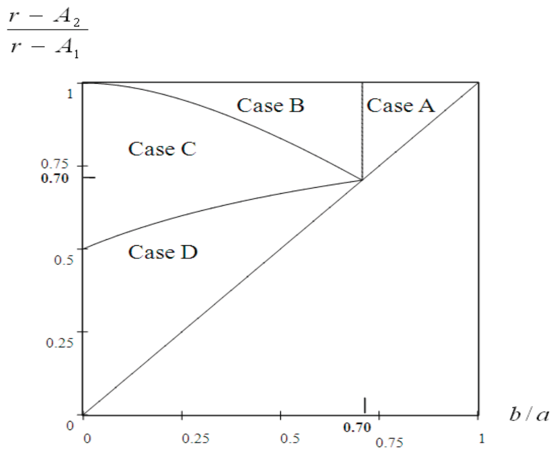

Fig. 3. Classification of the response of firm 2's profit to MQS in Interval 2

That by raising the standard, the regulator causes the high-cost firm's profit to increase and the law-cost firm's to decrease is interesting and counter intuitive for two reasons. First, it shows that the industrialists' claim that a higher environmental standard reduces a firm's profit is not always true[2]. Second, following Salop and Shceffman's (1983) argument of "raising the rival's cost", one may have expected that by raising the compliance cost for the high-cost firm, a higher standard should benefit the low-cost, not the high-cost, firm. The explanation for our counter-intuitive and contrasting result is as follows. Mandating a relatively high MQS in Interval 2 $\left(\alpha_2^* \le \hat{\alpha} \le \hat{\alpha}^U\right)$ has both a negative and a positive effect on firm 2's profit, compared with its unregulated profit (or profit under MQS in Interval 1). First, given firm 1's equilibrium choice, $\alpha_1 = \alpha_1^*$, a higher standard, $\hat{\alpha} > \alpha_2^*$, lowers firm 2's profit compared to its unregulated profit (*i.e.*, $\pi^{*2}(\hat{\alpha}, \alpha_1^*) < \pi^{*2}(\alpha_2^*, \alpha_1^*)$. Second, as we noted earlier (see Proposition 3), forcing firm 2 to adopt a higher

[1] Note from (9) that the relevant values of b / a and $(r - A_2) / (r - A_1)$ are those to the left of the 45- degree line to ensure positive α_2^*, and that the firm 2's profit function is convex (concave) for the standard levels in this interval if $2b^2 - a^2 > (<)\ 0$, or if $1 > b / a > \sqrt{2} / 2 \approx 0.70$ $(0 < b / a < \sqrt{2} / 2)$.

[2] See Farzin (2003) for a similar result but derived in the context of an oligopoly with identical firms and where a tighter environmental quality standard increases both the marginal pollution abatement cost and the marginal willingness to pay for the product.

MQS, $\hat{\alpha} > \alpha_2^*$, induces firm 1 to lower its quality choice to a level below α_1^*. The products being strategic substitutes, this has a positive effect on firm 2's profit (it shifts its profit curve $\pi^{*2}(\alpha_2, \alpha_1^*)$ upward so that $\pi^{*2}(\hat{\alpha}, \alpha_1(\hat{\alpha})) > \pi^{*2}(\hat{\alpha}, \alpha_1^*)$ for $\alpha_1(\hat{\alpha}) < \alpha_1^*$). As stated in Proposition 5, the *net* effect is always positive for at least some initial range of standards in Interval 2. However, as shown in Figure 3, for other standard levels in this interval the net effect depends on magnitudes of b / a and $\frac{r - A_2}{r - A_1}$. On the other hand, as should be expected, the effect on firm 1's profit of any MQS in Interval 2 is to reduce it from its unregulated level, $\pi^{*1}(\alpha_2^*, \alpha_1^*)$.

The contrast between Salop and Shceffman's (1983) argument and our result derives from the fact that whereas they model the game of quality competition as the Stackleberg's leader-follower variety in which a *low-cost* dominant firm raises the costs for a high-cost competitive fringe, our model characterizes the market as a Nash-Cournot differentiated duopoly in Interval 1, and a kind of leader-follower game in interval 2 *but* with the difference that over Interval 2 it is the high-cost, and not the low-cost, firm which moves first and leads by setting its quality choice at the MQS level and letting the low-cost firm react to it.

A rather striking and novel result of our analysis is that

Proposition 6: The situations where the MQS raises the profit of the high-cost (low-quality) firm over the entire Interval 2 (Case A and Case B) occur when the products are sufficiently strong strategic substitutes ($b / a \geq \sqrt{2} / 2 \approx 0.70$) or/and the quality costs differential between the firms is sufficiently small ($(r - A_2) / (r - A_1) > 0.70$ and sufficiently close to 1) (see Figure 3).

The intuition behind this proposition is straightforward: it is precisely under those conditions that the positive effect on firm 2's profits of tightening the MQS is the strongest. In the opposite situation of Case D, where the products are highly differentiated and the firms' quality costs difference is large, the favorable effect of a stricter standard on firm 2's profit is rather weak, thus leading to the possibility of firm 2's profit to decline to zero as the standard is raised to some level within Interval 2. Between these extremes is Case C, representing moderate degrees of product differentiation and costs differentials.

Over Interval 3, the profits of both firms monotonically decrease as the standard is raised, although firm 1's profit always exceeds firm 2's and eventually they fall to zero within the interval (see Appendixes B and D and Figure 2). This interval could represent situations where low quality (for example, severe environmental pollution or product safety) is deemed socially too serious a problem, thus calling for sufficiently high MQS levels to be imposed. In such situations, the regulator trades off the firms' profits for consumers' quality gain and possibly for increased social welfare. In extremely serious cases, the regulator may in fact set the standard so high as to render the high-cost (low-quality) firm unprofitable or even so high as to force both of the firms out of the market[1].

4.2. Average quality under a MQS. Of particular interest is the effect of a minimum quality standard on the average quality enjoyed by consumers. To examine this effect, we define the average quality as the weighted average of the firms' quality choices $(\alpha_1(\hat{\alpha}), \alpha_2(\hat{\alpha}))$ under the MQS regime (denoted by an over bar, dotted line) where the weights are the firms' output shares of the products consumed $(\bar{q}_1, \bar{q}_2)$, *i.e.*,

$$\bar{\alpha}(\hat{\alpha}) = \frac{\bar{q}_1 \alpha_1(\hat{\alpha}) + \bar{q}_2 \alpha_2(\hat{\alpha})}{\bar{q}_1 + \bar{q}_2}.$$

We then compare this average with the average quality which would have provided in the absence of any regulation (*i.e.*, the laissez-faire average quality), which is calculated as:

$$\bar{\alpha}_{LF} = \frac{(\bar{q}_1 - \bar{q}_2)(a-b)r - (a\bar{q}_1 - b\bar{q}_2)A_1 + (b\bar{q}_1 - a\bar{q}_2)A_2}{(\bar{q}_1 + \bar{q}_2)(a^2 - b^2)}.$$

Figure 4 presents this comparison for different intervals of the MQS level.

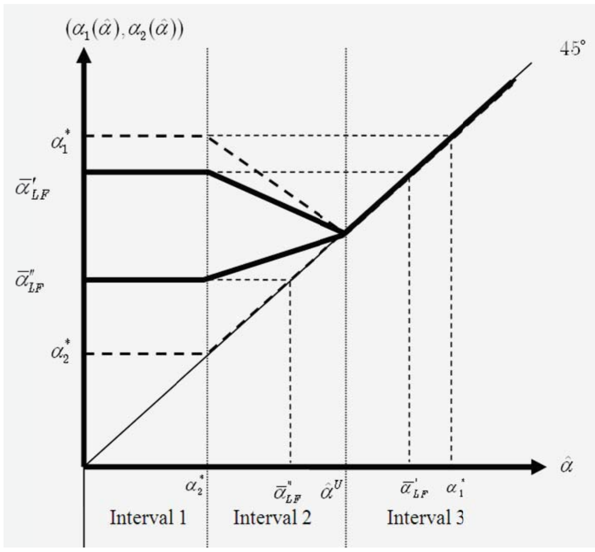

Fig. 4. Average quality with and without MQS

[1] For an analysis of the socially optimal level of emissions standard in a polluting oligopoly with identical firms, see Farzin (2004).

As can be seen from Figure 4, over Interval 1 where the standard is relatively weak, the average quality would be lower than the unregulated quality if the standard is *mandatory*. Otherwise, both firms overcomply so that an MSQ would have no effect on firms' qualities, and hence the average quality consumed. In the mandatory case, the extent to which the average quality would differ from the unregulated quality level depends on the firms' market shares, *i.e.*, whether $\bar{q}_1 \gtrless \bar{q}_2$, and on the b/a ratio. In Figure 4, $\bar{\alpha}_{LF}'$ represents the case where the high-quality firm dominates the market whereas $\bar{\alpha}_{LF}''$ denotes the opposite case. The solid lines present the average quality over different intervals of the standard. Regardless of which firm dominates, it is clear that when firms compete in quality and the likely negative social effects (externalities) of their quality choices is a serious problem to necessitate stringent standards, then, as far as the average quality is concerned, having no mandatory standard at all may serve the society better than imposing weak standards. The same policy implication also holds when the quality regulation is in the form of a *minimum* quality standard, given the administration costs of the required standard.

For MQS levels in Interval 2, the effect on the average quality is ambiguous. Indeed, when the high-quality firm dominates in the market, it would lead to a *lower* average quality if $\bar{\alpha}_{LF}' > \hat{\alpha}^U$, or equivalently if $1 > b/a > \bar{q}_2/\bar{q}_1$. Inversely, it would lead to a higher average quality if $\bar{\alpha}_{LF}'' < \hat{\alpha}^U$. The effect of the MQS on the average quality is ambiguous over Interval 3 too. When the relevant benchmark average is $\bar{\alpha}_{LF}'$, then as long as $\bar{\alpha}_{LF}' > \hat{\alpha}^U$ and $\hat{\alpha} < \bar{\alpha}_{LF}'$, imposing a MQS would *reduce* the average quality. Together with the same qualitative effect over Interval 2, this result cautions against setting the MQS in the range of

$$[\alpha_2^*, \bar{\alpha}_{LF}'] = \left[\frac{a(r-A_2)-b(r-A_1)}{a^2-b^2}, \frac{(\bar{q}_1-\bar{q}_2)(a-b)r-(a\bar{q}_1-b\bar{q}_2)A_1+(b\bar{q}_1-a\bar{q}_2)A_2}{(\bar{q}_1+\bar{q}_2)(a^2-b^2)}\right]$$

However, in Interval 3, any standard level such that $\hat{\alpha} > \bar{\alpha}_{LF}' > \hat{\alpha}^U$ or $\hat{\alpha} > \hat{\alpha}^U > \bar{\alpha}_{LF}''$ would lead to a higher average quality than that under no regulation.

The effects of the MQS policy on average quality over Intervals 1 and 2 can be summarized as proposition 7.

Proposition 7: In a differentiated duopoly, (1) too weak minimum standards will be overridden by the firms' voluntary quality choices, and (2) for intermediate levels, the MQS can even lower the average quality below the unregulated level.

Finally, regardless of how strongly the products are strategic substitutes (*i.e.*, of the size of b/a) and which firm dominates the market (*i.e.*, whether $\bar{q}_1 \gtrless \bar{q}_2$), as long as at least the low-cost firm remains in the market, an MQS higher than $\alpha_1^* = [a(r-A_1)-b(r-A_2)]/(a^2-b^2)$ improves the average quality relative to unregulated quality level.

4.3. Welfare effect of a MQS. Drawing on the preceding analyses, in this section we briefly discuss the qualitative effects of introducing the MQS on welfare by comparing the consumers' quality gain and producers' profits under the regulated and unregulated regimes. Clearly, to the extent that some of our results regarding the effects of MQS on firms' profits and the average quality enjoyed by consumers differ from those in the literature, it is natural to expect that some of our welfare effects to be different too.

To begin with, for weak standards, that is, the MQS levels in interval 1,

$$0 < \hat{\alpha} \le \alpha_2^* = \left[a(r-A_2)-b(r-A_1)\right]/(a^2-b^2),$$

since there is no effect on the firms' unregulated quality choices and profits, to the degree that legislating, monitoring, and enforcing an MSQ entails administrative costs, setting such low standards *reduces* social welfare. For the intermediate levels of the MSQ, that is the levels in Interval 2, $\alpha_2^* < \hat{\alpha} \le \hat{\alpha}^U = \dfrac{r-A_1}{a+b}$, the welfare effect depends critically on the benchmark unregulated average quality chosen for comparison, that is, $\bar{\alpha}_{LF}''$ or $\bar{\alpha}_{LF}'$, on how strongly the products are strategic substitutes, and on the firms' unit quality costs difference, where the last two factors (indexed by b/a and $(r-A_2)/(r-A_1)$) determine the behavior of firms' profits over this interval. Taking $\bar{\alpha}_{LF}''$ as the benchmark unregulated average quality, introducing an MQS in this interval increases the consumers' quality gain but since it always reduces the high-quality firm's profit its effect on the total firms' profits is ambiguous, even though for MQS levels sufficiently close to α_2^* (the low-quality firm's unregulated quality), the low-quality firm's profit increases with the MQS. Accordingly, for this case, the welfare effect of the MQS is ambiguous. This is in contrast with the welfare enhancing effect of an intermediate MQS found, for example, by Ronnen (1991), Motta and Thisse (1993), and Crampes and Hollander (1995)[1]. When $\bar{\alpha}_{LF}'$ is the benchmark unregulated average quality, the introduction of an MQS in Interval 2 and at levels that sufficiently exceeds α_2^* *lowers* the average quality offered to consumers and for Case D and Case C it also *reduces* the low-quality firm's profits, so that coupled with decreasing profits of the high-quality firm, it unambiguously *reduces* welfare[2]. Under this benchmark case and for values of *b/a* or $(r - A_2)/(r - A_1)$ close to 1 (implying Case A or Case B) so that the low-quality firm's profits increases, the welfare effect of the MQS remains ambiguous. Clearly, with $\bar{\alpha}_{LF}'$ as the benchmark unregulated average quality, even for standard levels close to α_2^*, the welfare effect of the MSQ will be ambiguous, this time because of the reductions both in quality enjoyed by consumers and in high-quality firm's profit. Thus, in contrast with previous studies, one may conclude that irrespective of which of the two firms dominate in the market (*i.e.*, whether $\bar{\alpha}_{LF}''$ or $\bar{\alpha}_{LF}'$ is taken as the benchmark for welfare comparison), the welfare effect of an MSQ sufficiently close to α_2^* is generally ambiguous. In Interval 3, if the benchmark unregulated average quality is $\bar{\alpha}_{LF}'$, then for MQS levels less than that (*i.e.*, for $\hat{\alpha}^U < \hat{\alpha} < \bar{\alpha}_{LF}'$), the MQS policy would unambiguously reduce welfare because, compared with the unregulated situation, it would reduce both the quality enjoyed by consumers and the producers' profits. For MQS levels higher than $\bar{\alpha}_{LF}'$, the welfare effect remains ambiguous as consumers' gain of higher quality must be weighed against the producers' loss of profits. When the benchmark unregulated quality is $\bar{\alpha}_{LF}''$, any MSQ level in Interval 3 (*i.e.*, for all $\hat{\alpha} > \hat{\alpha}^U = (r - A_1)/(a + b)$) benefits consumers while worsens off the produces, thus resulting in an ambiguous net welfare effect.

As the foregoing welfare effects make clear, it is only under some strict conditions that a minimum quality standard would *unambiguously* enhance welfare; namely when (1) the standard is neither too weak nor too strict; (2) the low-quality product dominates the market; (3) the products are close substitutes, and (4) the difference in firms' unit costs of quality is sufficiently small. Outside of these conditions, the welfare effect of a MQS policy is most likely ambiguous or could even be negative. Of course, for other reasons such as internalizing negative externalities or attaching more importance to consumers' quality gain than to producers' profits, the regulator may still find it desirable to regulate quality through a MQS. However, when merits of such additional considerations are not obvious or strong enough, the preceding analysis suggests exercise of caution in using MQS as a regulatory instrument.

Conclusions

In a simple model of a differentiated duopoly in which firms compete in quality only, we have examined the effects of introducing a minimum quality standard (MQS) on the firms' quality choices, profits, and the average quality offered to consumers. We have shown that the effects depend on the degree to which the two products are strategic substitutes, on the firms' quality costs difference, and critically on the standard level, and that they can differ from those obtained in previous studies.

Specifically, we have shown that at too low standards both firms overcomply, thus rendering the standard ineffective and implying that no standard may be preferred to too weak standards. Interestingly, and contrary to common intuition, we have shown that at intermediate levels, the MQS can benefit the *high-cost* firm and harm the *low-cost* one and that it can also lower the average offered to consumers. We have identified the precise conditions regarding the degrees of product differentiation, the quality costs differential, and, most importantly, the level of standard under which an MQS policy can lead to unintended outcomes. Aside from suggesting exercise of caution in using the MQS as a regulatory instrument and in setting of the standard level, our results can explain (1) why in situations where environmental quality and reputation of firms matter to consumers, the "greener" firms may prefer voluntary pollution control to regulating emissions by imposing an MQS, and (2) why environmental advocacy groups cry for strict environmental standards, fearing that weak standards can alter the behavior of an otherwise more environmental friendly firm (motivated, of course, to outcompete its rivals) to become less environmental friendly by becoming content with merely complying with the standard.

Obviously, our results derive from a simplified model of industry and consumers' behavior, thus suggesting further research in several respects. A natural exten-

[1] Other things equal, an MQS is more likely to be welfare enhancing in this case if the products are strong substitutes (*b / a* close to 1) or the firms' difference in unit quality costs is negligible ($(r - A_2) / (r - A_1)$ close to 1), implying that the low-quality firm strongly dominates the market and its profit increases with the MQS level (conditions that ensure Case A and Case B in Figure 3 and Figure 2).

[2] Note that values of $b / a < \sqrt{2} / 2 \approx 0.70$ and $r - A_2) / (r - A_1) < 70$ ensure Case D or Case C to occur. As mentioned in section 2, in models that significantly differ from ours, Scarpa (1998) and Kuhn (2007) also identify conditions under which the MSQ would reduce welfare.

sion of our model could along with Scarpa's (1998) work examine whether the main results obtained here for a duopoly would generalize for a differentiated oligopoly with three or more firms. Another extension would endogenize the level of MQS to examine whether it would be socially optimal to set the standard so high as to drive the low-quality (high-cost) firm out of market and have the low-cost (high-quality) firm to act monopolistically in providing the quality. Could the optimal MQS level exceed the low-cost firm's quality level in the absence of regulation in that case? The present model can also be extended by allowing (a) each firm to produce both a low-quality and a high-quality product; and (b) consumers to differ in their tastes, along Mussa and Rosen's (1978) formulation, or in their incomes and hence willingness to pay for quality, along the work of Arora and Gangopadhyay (1995). An interesting question then would be: Can the introduction of MQS lead firms to specialize, *i.e.*, the low-cost firm only producing the high-quality product and the high-cost firm only the low-quality one?

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Appendix A. Derivation of the system of revenue functions

Consider a continuum of identical consumers where a typical consumer faces the following utility maximization problem: For given pair $(p_1, p_2) > 0$,

$$\max_{(q_1,\alpha_1),(q_2,\alpha_2),y} u\left[(q_1,\alpha_1),(q_2,\alpha_2)\right] + y$$

subject to $I - (p_1\alpha_1 q_1 + p_2\alpha_2 q_2 + y) = 0,$

$q_1, q_2 \in \{0,1\},$

where a_i is the environmental quality of good $i = 1,2$, and I is the consumer's income. We assume:

1. The amount of consumption q_i is chosen in 0 or 1 (discrete choice).

2. The price of each good is a hedonic price and is proportional to its environmental quality: $p_i a_i$.

3. The sub-utility function u is given by:

$$u(q_1,\alpha_1,q_2,\alpha_2) = q_1\alpha_1\left(-\frac{1}{4}a\alpha_1 - \frac{1}{2}b\alpha_2 q_2 + r\right) + \alpha_2 q_2\left(-\frac{1}{4}a\alpha_2 - \frac{1}{2}b\alpha_1 q_1 + r\right),$$

where a, b and r are positive constants. Assume that income I is sufficiently large to avoid a corner solution with $y = 0$. Then, there are four partial problems whose maximal utilities are compared with one another. These problems are:

1. *Both goods are purchased* ($q_1 = q_2 = 1$):

$$\tilde{u}^* = \max_{\alpha_1,\alpha_2,y} \tilde{u}(\alpha_1,\alpha_2,y) = \alpha_1\left(-\frac{1}{4}a\alpha_1 - \frac{1}{2}b\alpha_2 + r\right) + \alpha_2\left(-\frac{1}{4}a\alpha_2 - \frac{1}{2}b\alpha_1 + r\right) + y,$$

subject to $I - (p_1\alpha_1 + p_2\alpha_2) - y = 0,\ \ \alpha_1,\alpha_2 \ge 0.$

2. *Only good 1 is purchased*:

$$\tilde{u}^1{*} = \max_{\alpha_1,y} \tilde{u}^1(\alpha_1,\alpha_2,y) = \alpha_1\left(-\frac{1}{4}a\alpha_1 + r\right) + y$$

subject to $I - p_1\alpha_1 - y = 0,\ \ \alpha_1 \ge 0.$

3. *Only good 2 is purchased*:

$$\tilde{u}^2{*} = \max_{\alpha_2,y} \tilde{u}^2(\alpha_1,\alpha_2,y) = \alpha_2\left(-\frac{1}{4}a\alpha_2 + r\right) + y$$

subject to $I - p_2\alpha_2 - y = 0,\ \ \alpha_2 \ge 0.$

4. *No good is purchased*:

$\tilde{u}^0{*} = I.$

Notice that the second problem coincides with a special case of the first problem with an *additional constraint* $a_2 = 0$. A similar argument is applicable to the third and the fourth problems. Therefore, $\tilde{u}^* \ge \max\{\tilde{u}^1{*}, \tilde{u}^2{*}, \tilde{u}^0{*}\}$. This implies that the consumer's problem is virtually represented by the first problem.

The first order conditions for the first problem are:

$$-\frac{1}{2}a\alpha_1 - b\alpha_2 + r - p_1 + \lambda_1 = 0,$$

$$-\frac{1}{2}a\alpha_2 - b\alpha_1 + r - p_2 + \lambda_2 = 0,$$

$$I - (p_1\alpha_1 + p_2\alpha_2) - y = 0,$$

and $\lambda_1\alpha_1 = \lambda_2\alpha_2 = 0,\ \ \lambda_1,\lambda_2 \ge 0,$

where λ_i, $i = 1,2$ are the Lagrange multipliers. Notice that if the solution contains $a_i = 0$ for a given (p_1, p_2), then $\tilde{u}^* = \tilde{u}^{i*}$. Therefore, we can interpret this as the case that the consumer chooses to purchase only one good $j \ne i$.

From these equations, we have

$$0 = -\lambda_i \alpha_i = \left(-\frac{1}{2}a\alpha_i - b\alpha_j + r - p_i\right)\alpha_i,\ (i, j = 1,2,\ \ j \neq i).$$

Since $\alpha_i p_i$ is the consumer's willingness to pay for good i given (α_1, α_2), we have the system of revenue functions of firm i = 1,2:

$$R^1(\alpha_1, \alpha_2) = \left(-\frac{1}{2}a\alpha_1 - b\alpha_2 + r\right)\alpha_1,$$

$$R^2(\alpha_1, \alpha_2) = \left(-\frac{1}{2}a\alpha_2 - b\alpha_1 + r\right)\alpha_2.$$

Appendix B. Firm 1's profit in Interval 2

Firm 1's profit in Interval 2, $\pi_{II}^{*1}(\hat{\alpha})$, is

$$\pi_{II}^{*1}(\hat{\alpha}) = \max_{\alpha_1 \geq \hat{\alpha}} \pi^1(\alpha_1; \hat{\alpha}) = \pi^1(\alpha_1(\hat{\alpha}), \hat{\alpha}) = \left(\frac{-1}{2}a\alpha_1(\hat{\alpha}) - b\hat{\alpha} + r - A_1\right)\alpha_1(\hat{\alpha}) \cdot \tag{A.1}$$

Applying the envelop theorem, we have

$$\frac{d\pi_{II}^{*1}(\hat{\alpha})}{d\hat{\alpha}} = \frac{\partial \pi^1(\alpha_1(\hat{\alpha}), \hat{\alpha})}{\partial \hat{\alpha}} = -b\alpha_1(\hat{\alpha}) < 0 \cdot \tag{A.2}$$

That is, the low-cost firm 1's profit monotonically decreases as the standard is raised.

The minimum profit is attained at $\hat{\alpha} = \hat{\alpha}^U$. Since

$$\pi_{II}^{*1}(\hat{\alpha}^U) = \frac{a(r - A_1)}{a + b}\frac{\alpha_1(\hat{\alpha})}{2} > 0, \tag{A.3}$$

we conclude that Firm 1's profit remains positive over Interval 2.

Appendix C. Firm 2's profit in Interval 2

Interval 2 is defined as $[\alpha_2^*, \hat{\alpha}^U)$, where

$$\alpha_2^* = \frac{a(r - A_2) - b(r - A_1)}{a^2 - b^2},$$

and $\hat{\alpha}^U = \dfrac{r - A_1}{a + b} > \alpha_2^*$

α_2^* is positive if and only if

$$\frac{b}{a} < \frac{r - A_2}{r - A_1} < 1. \tag{A.4}$$

The firm 2's profit in Interval 2, $\pi_{II}^{*2}(\hat{\alpha})$, is given by

$$\pi_{II}^{*2}(\hat{\alpha}) = \max_{\alpha_2 \geq \hat{\alpha}} \pi(\alpha_2; \alpha_1(\hat{\alpha})) = \pi^2(\hat{\alpha}, \alpha_1(\hat{\alpha})) = \left(\frac{-1}{2}a\hat{\alpha} - b\alpha_1(\hat{\alpha}) + r - A_2\right)\hat{\alpha} \tag{A.5}$$

By substituting $\alpha_1(\hat{\alpha}) = \dfrac{-b\hat{\alpha} + r - A_1}{a}$, we have

$$\pi_{II}^{*2}(\hat{\alpha}) = \frac{2b^2 - a^2}{2a}\hat{\alpha}^2 + \frac{a(r - A_2) - b(r - A_1)}{a}\hat{\alpha}. \tag{A.6}$$

The second term on the right hand side is always positive by (A4). Therefore, we have the following four cases:

If $2b^2 - a^2 \geq 0$,

Case A: $\pi_{II}^{*2}(\hat{\alpha}) > 0$ over Interval 2. Furthermore, the profit is monotonically increasing in the standard $\left(d\pi_{II}^{*2}(\hat{\alpha}) / d\hat{\alpha} > 0\right)$.

If $2b^2 - a^2 < 0$, we classify the following three cases:

Case B: $\pi_{II}^{*2}(\hat{\alpha}) > 0$ and $d\pi_{II}^{*2}(\hat{\alpha}) / d\hat{\alpha} > 0$ over Interval 2.

Case C: $\pi_{II}^{*2}(\hat{\alpha}) > 0$ over Interval 2. There is $\tilde{\alpha}$ in the interval such that

$d\pi_{II}^{*2}(\hat{\alpha}) / d\hat{\alpha} > (<) 0$ if $\hat{\alpha} < (>) \tilde{\alpha}$.

Case D: There is $\hat{\hat{\alpha}}_{II}$ in the interval such that $\pi_{II}^{*2}(\hat{\alpha}) > 0$ if and only if $\alpha_2^* \leq \hat{\alpha} < \hat{\hat{\alpha}}_{II}$. In this interval, there is $\tilde{\alpha}$ such that $d\pi_{II}^{*2}(\hat{\alpha}) / d\hat{\alpha} > (<) 0$ if $\hat{\alpha} < (>) \tilde{\alpha}$.

We next derive the conditions under which each of the four cases occurs. The condition for Case B is

$$0 \leq \frac{d\pi_{II}^{*2}(\hat{\alpha}^U)}{d\hat{\alpha}} = \frac{\left(a^2 + ab\right)\left(r - A_2\right) - \left(a^2 + ab - b^2\right)\left(r - A_1\right)}{a}. \tag{A.7}$$

That is, Case B happens if

$$\frac{r - A_2}{r - A_1} \geq \frac{a^2 + ab - b^2}{a^2 + ab} > \frac{b}{a}. \tag{A.8}$$

Notice that the last inequality in (A.8) is equivalent to $2b^2 - a^2 < 0$, which is the case we are considering.

The conditions for Case C are

$$\frac{a^2 + ab - b^2}{a^2 + ab} > \frac{r - A_2}{r - A_1} > \frac{b}{a}, \tag{A.9}$$

and

$$\frac{r - A_2}{r - A_1} \geq \frac{a + 2b}{2\left(a + b\right)}. \tag{A.10}$$

The latter condition follows from

$$0 \leq \pi_{II}^{*2}(\hat{\alpha}^U) = \left(\frac{2a\left(a + b\right)\left(r - A_2\right) - \left(a^2 + 2ab\right)\left(r - A_1\right)}{2\left(a + b\right)}\right)\frac{\hat{\alpha}^U}{a}. \tag{A.11}$$

Putting (A.9) and (A.10) together, Case C occurs if

$$\frac{a^2 + ab - b^2}{a^2 + ab} > \frac{r - A_2}{r - A_1} \geq \frac{a + 2b}{2\left(a + b\right)} > \frac{b}{a}. \tag{A.12}$$

Notice that the last inequality in (A.12) is equivalent to $2b^2 - a^2 < 0$, which holds for the case under consideration.

Finally, Case D happens if

$$\frac{a + 2b}{2\left(a + b\right)} > \frac{r - A_2}{r - A_1} > \frac{b}{a}. \tag{A.13}$$

$$\hat{\hat{\alpha}}_{II} = \frac{a\left(r - A_2\right) - b\left(r - A_1\right)}{\left(1/2\right)a^2 - b^2} \quad \left(< \hat{\alpha}^U = \frac{r - A_1}{a + b} \text{ if } \frac{r - A_2}{r - A_1} < \frac{a + 2b}{2(a + b)} \right) \tag{A.14}$$

Appendix D. Firms' profits in Interval 3

In Interval 3, firm i 's profit is

$$\pi_{III}^{*i}(\hat{\alpha}) = \max_{\alpha_i \geq \hat{\alpha}} \pi^i(\alpha_i;\hat{\alpha}) = -\frac{1}{2}a(\hat{\alpha})^2 - b(\hat{\alpha})^2 + (r - A_i)\hat{\alpha}, \quad \hat{\alpha} \geq \hat{\alpha}^U . \tag{A.15}$$

Since $\pi_{III}^{*i}(\hat{\alpha})$ is a concave function,

$$\frac{d\pi_{III}^{*1}(\hat{\alpha}^U)}{d\hat{\alpha}} = -\frac{a+2b}{a+b}(r - A_1) + (r - A_1) = \frac{-b(r - A_1)}{a+b} < 0,$$

and

$$\frac{d\pi_{III}^{*2}(\hat{\alpha}^U)}{d\hat{\alpha}} = -\frac{a+2b}{a+b}(r - A_1) + (r - A_2) = \frac{(a+b)(r - A_2) - (a+2b)(r - A_1)}{a+b}\hat{\alpha}^U < 0,$$

imply that both firms' profits are monotonically decreasing over Interval 2. Notice that before firm 1's profit goes to zero, the rival's profit has already dropped to zero, since $A_1 < A_2$.

For Firm 2 to be operative over Interval 3 its profit at the standard level $\hat{\alpha}^U$ has to be nonnegative:

$$\pi_{III}^{*2}(\hat{\alpha}^U) = \left[-\frac{a+2b}{2}\frac{r - A_1}{a+b} + (r - A_2)\right]\hat{\alpha}^U = \frac{2(a+b)(r - A_2) - (a+2b)(r - A_1)}{2(a+b)}\hat{\alpha}^U \geq 0 \cdot$$

Combining this condition with the positivity condition of α_2^* ((A.4)), we have

$$\frac{r - A_2}{r - A_1} \geq \max\left[\frac{a+2b}{2(a+b)}, \frac{b}{a}\right]. \tag{A.16}$$

Under condition (A.16), there is a unique ceiling of the standard on Interval 3, at which Firm 2's profit is zero. This ceiling, denoted by $\hat{\hat{\alpha}}_{III}$, is given by

$$\hat{\hat{\alpha}}_{III} = \frac{r - A_2}{(1/2)a + b} \quad \left(\geq \hat{\alpha}^U = \frac{r - A_1}{a+b} \text{ if } \frac{r - A_2}{r - A_1} \geq \frac{a+2b}{2(a+b)}\right). \tag{A.17}$$