“Pricing life settlement portfolios with credibility adjustment”

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Pricing life settlement portfolios with credibility adjustment

Abstract

This work suggests an intuitive approach for pricing life settlement portfolios with credibility adjustment. The mortality assumption for pricing is dynamically adjusted so that the actual mortality experience is within a pre-defined likelihood. As the life settlement market matures and more experience data become available, this work will provide a general guidance of how to incorporate experience into pricing of the portfolio. The paper is intended to be for a general audience, possible participants in the life settlement market.

Keywords: life settlement portfolio, credibility theory, mortality experience.

Introduction

You thought that event A is one in 500 month event two days ago, but then it happened yesterday. Will you change your view? Maybe it is not a one in 500 month event, but a one in 50 month event? Flip flop, as used on TV debates.

This is what happens in the cat modeling. Before Kristina, a loss event of the size of Kristina is thought to have very remote probability. After Kristina, all modeling agency changed their model so that loss of Kristina is more probable.

Let us look at another example. If you have a new coin, it is assumed to be fair. You toss it 10 times. You got all heads. This is a one in 1024 event. Will you start to doubt the coin in unfair? Maybe not. A new coin is very unlikely to be unfair. What if you got 20 straight heads? This is a one in a million event. You might start to question. After you get 30 heads, that is a 1 in a billion event, you might conclude that there is some thing wrong with the coin. It might not be fair. Then you need to adjust your view on this coin. But how? And by how much?

What if it is an old coin? Will you adjust your belief sooner? When the actual experience defer from the prior beliefs, we need to adjust our beliefs. How much to adjust depends on how strongly we feel about our prior beliefs. Cat modeling agency does not believe their model as much as we believe a coin is fair.

One way to measure how strongly you believe could be defined as follows. It is the probability of the actual experience occurring under the prior believes. In the coin example, you could set it at one in billion for a new coin. Only after getting 30 straight heads you will change your view. You could set at one in thousand, if it is a bent old coin. We will call this measure the probability tolerance (PT).

In the life settlement business, how much do you believe a medical underwriter?

Let us say we have a bent old coin, I set my belief at one in a thousand. After 10 straight heads, I start to question its fairness. Then I got another head, I no longer believe it is fair, and I think the coin is biased. Before I thought head has 50% probability, now do I think it is 60%? 70%? Or it is so bent, the probability is 100%?

One simple and easy way to adjust your belief is to keep the probability at one in thousand. We will solve for the head probability so that the chance of 11 heads is still one in a thousand. A simple excel goal seek tells that it is 53.4%.

What if I get a head again? I will adjust my belief to 56.2% head after 12 straight heads.

What if I get a head again? I will adjust my belief to 58.8% head after 13 straight heads.

# of straight heads	Head probability
10	50.0%
11	53.4%
12	56.2%
13	58.8%

If the 14th toss is a head, we will continue to adjust upwards.

But what if the 14th toss is a tail? Now what we have is 13 heads and 1 tail in total of 14 tosses. What is its likelihood with a fair coin? It is .085%, which is still below our probability tolerance (PT) of .1%. We still think the coin is not fair, but it is not as biased as 58.8%. Keeping the probability at .1%, we have the head probability at 50.7%.

This method of credibility adjustment is intuitive. It is a sort of combination of maximum likelihood estimator concept with the credibility theory concept.

In this work, we will apply the credibility method to the life settlement portfolio. The life settlement market experienced growth in 2003-2008, and slow down in 2009-2011. The market is stable recently and interest in the asset class renewed. Research on the topic has a broad range, from market and finance view [1], [4] and [5], to the actuarial and risk management view [2] and [3]. This work addresses both topics to some extent. It is on pricing and valuation aspect of the business based on actuarial and mortality experience.

1. Application to life settlement portfolio

In a life settlement portfolio, each insured is assigned a mortality multiple (MM) against the baseline table (say, 2008 VBT). The MM measures the degree of impairment of the insured in the portfolio. One common way to adjust or stress test the underwriting accuracy is to haircut the excess mortality multiple. The adjusted MM with a hair cut of h is $\{1 + (1\text{-}h)^*(MM\text{-}1)\}$.

Let us say we have a sample portfolio of identical 75 insured with $MM = 4$. The insureds, based on age and sex, have the life expectancy (LE) of 15 months based on the baseline table. The LE is 7.3 with $MM = 4$.

For simplicity, let us assume that the cost of insurance charge is the same as the baseline mortality. Premium payments are the COI charge and the baseline mortality. Assuming a 15% IRR, the life policy is worth 0.32 of $1 face amount.

Based on the baseline table, in 36 month, the expected number of death is 4.39 and STDEV = 2.03. The likelihood of no death at all is 3.08%. For simplicity, we use the normal distribution as an approximation for the sum of the 75 binomial distributions.

But if we believe the underwriter, and it's $MM = 4$ for each insured. We expect 16.1 deaths with STDEV of 3.56. The likelihood of getting zero death is only 0.006%, a one in 170,000 event. We could believe a new coin to this PT, but few people will trust life settlement underwriters to this degree. It will be reasonable to adjust the prior beliefs of $MM = 4$, and reduce the MM with a haircut.

We will use the method describe above to do the adjustment. As the MM is reduced with a hair cut, the price of the policy will reduce as well.

2. Impact on a sample portfolio

Table 1 is the table of the hair cuts of assuming there is no death in 36 month. The probability tolerance is 1%, one in a 100 event. We will start to do the haircut at month 15. The probability of zero death for the first 15 month is 0.765%. To adjust this probability to be within the probability of tolerance, we need to haircut excess mortality by 9%, and this reduces the price of the policy from 32% of face to 30%.

To keep PT within the target, the haircut will continue to grow if there is no death. At month 36, the haircut is at 87%. The MM after haircut is 140%. The price of the policy will reduce from 32 cents of $1 face to 7 cents of $1 face, a 25 cents reduction.

Table 1. Price reduction assuming zero death and $PT = 0.01$

Month	Probability of zero death, assuming $MM = 4$	Haircut	Adjusted MM	Adjusted price	Price reduction
15	0,756%	9%	373%	30%	-2%
16	0.561%	17%	348%	28%	-4%
17	0.414%	25%	326%	27%	-5%
18	0.305%	31%	307%	25%	-7%
19	0.224%	37%	289%	24%	-8%
20	0.164%	42%	273%	22%	-10%
21	0.120%	47%	259%	21%	-11%
22	0.087%	51%	246%	20%	-12%
23	0.063%	55%	234%	19%	-13%
24	0.045%	59%	223%	17%	-14%
25	0.032%	62%	214%	16%	-16%
26	0.023%	65%	204%	15%	-17%
27	0.016%	68%	196%	14%	-18%
28	0.012%	71%	188%	13%	-19%
29	0.008%	73%	181%	12%	-19%
30	0.006%	75%	174%	12%	-20%
31	0.004%	78%	167%	11%	-21%
32	0.003%	80%	161%	10%	-22%
33	0.002%	81%	156%	9%	-23%
34	0.001%	83%	150%	8%	-24%
35	0.001%	85%	145%	8%	-24%
36	0.001%	87%	140%	7%	-25%

Table 2 has the similar numbers at $PT = .001$, one in a thousand event. The haircut starts at month 22. At month 36, if there is no death, then the haircut is 59% to MM of 223%. And the price will be 17 cents, a reduction of 14 cents.

Table 2. Price reduction assuming zero death and $PT = 0.001$

Month	Probability of zero death, assuming $MM = 4$	Haircut	Adjusted MM	Adjusted price	Price reduction
22	0.087%	3%	391%	31%	-1%
23	0.063%	9%	372%	30%	-2%
24	0.045%	15%	355%	29%	-3%
25	0.032%	20%	339%	28%	-4%
26	0.023%	25%	325%	27%	-5%
27	0.016%	30%	311%	26%	-6%
28	0.012%	34%	298%	25%	-7%
29	0.008%	38%	287%	24%	-8%
30	0.006%	41%	276%	23%	-9%
31	0.004%	45%	266%	22%	-10%
32	0.003%	48%	256%	21%	-11%
33	0.002%	51%	247%	20%	-12%
34	0.001%	54%	239%	19%	-13%
35	0.001%	56%	231%	18%	-14%
36	0.001%	59%	223%	17%	-14%

Table 3 assumes the $PT = .001$, and then there is one death occurring at month 26. As before, there is no death by month 22, the haircut should be applied is 3%. The haircut percentage will grow to 20% at month 25, at which point the price reduced by 4%. When the death appears at month 26, it reduces the haircut to 5%. The price reduction is 1%.

Say, in month 31, there are two more death to bring the total death to 3. The probability of this under assumption of $MM = 4$ is 0.13%, within the PT. No haircut is necessary.

Table 3. Price reduction assuming some death and $PT = 0.01$

Month	Accumulated # of death	Probability, assuming $MM = 4$	Haircut	Adjusted *MM*	Adjusted price	Price reduction
22	-	0.087%	3%	391%	31%	-1%
23	-	0.063%	9%	372%	30%	-2%
24	-	0.045%	15%	355%	29%	-3%
25	-	0.032%	20%	339%	28%	-4%
26	1	0.077%	5%	386%	31%	-1%
27	1	0.055%	10%	371%	30%	-2%
28	1	0.039%	15%	356%	29%	-3%
29	1	0.028%	19%	342%	28%	-4%
30	1	0.020%	24%	329%	27%	-5%
31	3	0.130%	0%	400%	32%	0%
32	3	0.093%	1%	397%	32%	0%
33	3	0.066%	6%	383%	31%	-1%
34	3	0.046%	10%	370%	30%	-2%
35	3	0.033%	14%	357%	29%	-3%
36	3	0.023%	18%	346%	28%	-4%

3. An analytical formula

In this section, we will describe the concept using mathematical formulas.

Let N be the number of insured in the portfolio. Let $MM(i)$ be the mortality multiplier of insured I, based on the original underwriting of the LE providers and $MM(I,h) = 1+(1-h)*(MM(i)-1)$ be the mortality multipier of insured I with haircut percentage h.

Let $p(j, i)$ be probability of survivorship to the end of month j for insured i. $p(j,i) = p(j,I,MM,h) = p(j,I,MM(I,h))$ is a function of the haircut percentage and the original $MM(i)$. it is given that it is function of the baseline mortality table.

Let $E(j,h)$ be the expected accumulated number of death by month j. We have

$$E(j,h) = \sum_{i=1}^{N} p(j,i,MM,h).$$

Let $Var(j,h)$ be the variance of the expected accumulated number of death by end of month j.

$$Var(j,h) = \sum_{i=1}^{N} p(j,i,MM,h) * (1 - p(j,i,MM,h).$$

Let $X(j,h)$ be the random variable equals to the accumulated number of death in the portfolio by the end of month j. We have $E(X(j,h)) = E(j,h)$ and $Var(X(j,h)) = Var(j,h)$.

Assuming the N is large enough and mortality rates of insureds are homogenous to some extent, we could reasonably approximate the sum of the N of binomial distribution with a normal distribution.

Let $\text{Ø}(\cdot)$ be the cumulative distribution function of the standard normal distribution be $N(0,1)$. Let $\alpha(j)$ the actual accumulated number of death by end of month j. As defined earlier, let PT be the probability tolerance.

For each j, we find $h = h(j)$, such that

$$\phi((\alpha(j) - E(j,h)) \big/ \sqrt{Var(j,h)}) = PT.$$

Note that, while the solution $h = h(j)$ of above equation could be negative; $\text{Ø}(\cdot)$ and $\alpha(j)$ are used in this formula. We should floor the solution of h at 0, so that we could have positive haircut and adjust MM down.

After $h(j) = \max(0,h(j))$ is determined, we could value the portfolio based on the credibility adjusted mortality assumption.

Conclusions

It appears that after the adjustment starts, the haircut will increase rapidly. We could reduce the pace by setting some other conditions, such as weighting, upper or lower bounds, a schedule for PT as the haircut % increases.

This method does appear to be reasonable. It provides an intuitive way to adjust the price of portfolio of life settlements based on the actual mortality experience. This study is more relevant today as the more actual mortality data is available.

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