Tetsuo Kurosaki (USA), Young Shin Kim (Germany)

Mean-CoAVaR optimization for global banking portfolios

Abstract

This paper proposes mean-CoAVaR portfolio optimization to mitigate the potential loss caused by systematic risk. CoAVaR is a natural extension of CoVaR, and is defined as the average value at risk on the condition that the market index is in distress. In the same way as CoVaR, CoAVaR accounts for the extent of how affected the institution is by systematic distress. The authors expect that the potential loss of the portfolio arising from systematic risk is mitigated by minimizing the CoAVaR of the portfolio against the market index. The paper investigates the effectiveness of the mean-CoAVaR optimization by using the stocks of global systemically important financial institutions (G-SIFIs). The reason for choosing G-SIFI stocks as trial samples is that they are both highly interconnected and potentially affected by systematic risk. The joint stock return distribution is predicted by an autoregressive moving average generalized autoregressive conditional heteroscedasticity model with multivariate normal tempered stable distributed innovations, which is shown to be a better model for G-SIFI stocks in the authors' separate paper. Throughout the empirical study, the authors observe that the mean-CoAVaR portfolio incurs relatively smaller cumulative loss in most cases compared with the mean-AVaR and mean-variance portfolios. This implies that the mean-CoAVaR strategy is effective during a financial crisis. The results open the applicability of CoVaR methodology to risk management.

Keywords: mean-CoAVaR optimization, ARMA-GARCH model, multivariate normal tempered stable distribution, portfolio selection, G-SIFIs, global banking stock markets.
JEL Classification: F37, G01, G11, G15, G17, G32.

Introduction

The recent financial turmoil has so severely deteriorated the global investment environment that traditional portfolio management theory currently has less effect. The failure of Lehman Brothers in September 2008 and the subsequent financial crisis, referred to as the "Lehman shock", had an adverse impact on global financial markets, causing massive spillover effects and bringing attention to systemic risk. In such a situation, portfolio managers are exposed to and need to address undiversifiable risk; loss is more or less inevitable no matter how they construct a portfolio. Undiversifiable risk is likely to be especially applicable to the banking sector because global banks are now closely interconnected. Undiversifiable risk is also called systematic risk.

The benchmark of modern portfolio theory is Markowitz's mean-variance optimization theory (Markowitz, 1952). The framework of mean-variance optimization is to construct a portfolio minimizing the variance under the expected return. However, it has been revealed that the variance is not always an appropriate risk measure to be minimized. Subsequently, several alternative approaches have been proposed to replace the variance in Markowitz's theory with other risk measures such as Value at Risk (VaR) and Average Value at Risk (AVaR), which are called mean-VaR and mean-AVaR optimization, respectively. Such approaches are collectively called mean-risk portfolio optimization. In general, AVaR is preferable to VaR in terms of the optimization problem. While VaR optimization is basically a nonconvex and nonsmooth problem with multiple local minima, AVaR optimization is a convex and smooth problem. See Rachev et al. (2008) for a general description and history of mean-risk portfolio optimization problems.

Recently, Adrian and Brunnermeier (2011) proposed CoVaR or ΔCoVaR measure for systemic risk. CoVaR, specifically $\mathrm{CoVaR}^{j|i}$, is defined between two institutions i and j. $\mathrm{CoVaR}^{j|i}$ is the VaR of j on a certain condition of i. $\Delta\mathrm{CoVaR}^{j|i}$ is the difference between the VaR of j on the condition of i being distressed and "normal". Note that either i or j can be the entire system. While the case of j being the system usually attracts more attention because $\Delta\mathrm{CoVaR}^{system|i}$ can quantify the marginal risk contribution of i to the overall system, Adrian and Brunnermeier (2011) also mention the case of i being the system. They refer to $\Delta\mathrm{CoVaR}^{j|system}$ as "exposure CoVaR" in the sense that it can be interpreted as j's exposure to systemic risk. CoVaR and ΔCoVaR are directly extended into the counterparts of AVaR, which we call CoAVaR and ΔCoAVaR[1]. We apply the CoVaR methodology to stock markets, where the financial system is approximated by the market index.

In this paper, we adopt CoAVaR as the objective function and propose mean-CoAVaR portfolio optimization. Even though the loss caused by systematic risk might be inevitable, we attempt to at least mitigate it through CoAVaR optimization. Because $\mathrm{CoAVaR}^{j|index}$ captures j's vulnerability to

[1] In Adrian and Brunnermeier (2011), CoAVaR is mentioned as CoES, where ES stands for expected shortfall.

the overall market risk, we expect to make the portfolio immune to systematic loss by minimizing the CoAVaR of the portfolio against the market index, i.e., $\text{CoAVaR}^{port|index}$. We perform an empirical study by using daily stock return data of 28 listed global systemically important financial institutions (G-SIFIs), as of November 2011. A G-SIFI stock is a good choice for testing the effect of a mean-CoAVaR strategy against systematic risk because G-SIFIs are specified by financial regulators as the institutions with a huge influence on the global financial system, and that potentially experience systemic risk in terms of their size, interconnectedness, and so on (Basel Committee on Banking Supervision, 2011; Financial Stability Board, 2011). By comparing the performance of the portfolio minimizing CoAVaR with that of the portfolio minimizing traditional risk measures such as variance and AVaR, we confirm the effectiveness of mean-CoAVaR optimization. This paper is a sequel to our separate paper (Kurosaki and Kim, 2013). We now focus on the management of systematic risk from the perspective of a portfolio manager, whereas we focused on the measurement of systematic risk in the separate paper. See our separate paper (Kurosaki and Kim, 2013) for more information on notations, description of datasets, and methodology because some of these are shared with this paper.

The rest of this paper is structured as follows. In section 1, we formulate mean-CoAVaR portfolio optimization. Section 2 provides an empirical study by using 28 G-SIFI stocks and an ARMA-GARCH[1] forecast. The final section concludes the paper.

1. Mean-CoAVaR optimization

In line with the concept of mean-risk optimization, we propose mean-CoAVaR portfolio optimization to minimize a portfolio's potential loss caused by systematic risk. Because exposure CoVaR is a measure of vulnerability to systematic distress, it is quite a natural idea to minimize it for the purpose of a defense against systematic risk. We select CoAVaR as the objective function rather than CoVaR because of the drawbacks of VaR optimization. Note that we also select CoAVaR rather than ΔCoAVaR for the following reason. While $\Delta\text{CoAVaR}^{port|index}$ focuses on the increase in the risk of a portfolio in the case of financial crisis, i.e., exposure to systemic distress, $\text{CoAVaR}^{port|index}$ accounts for the portfolio's idiosyncratic risk in addition to its exposure. The latter quantity should be minimized in terms of portfolio loss mitigation.

Let R_t^i be the return of stock i. The subscript t stands for a time period. We assume that any return distribution is continuous. Let $\mathrm{C}(R_t^i)$ be a certain condition of R_t^i. Then, the CoAVaR of stock j on the condition $\mathrm{C}(R_t^i)$ at the confidence level $1-q$ is defined as

$$CoAVaR_{q,t}^{j|\mathrm{C}(R_t^i)} = \frac{1}{q}\int_0^q CoVaR_{p,t}^{j|\mathrm{C}(R_t^i)}\mathrm{d}p = \\ = -E\left(R_t^j \middle| \left\{R_t^j < -CoVaR_{q,t}^{j|C(R_t^i)}\right\} \cap \mathrm{C}(R_t^i)\right), \quad (1)$$

where $CoVaR_{q,t}^{j|\mathrm{C}(R_t^i)}$ is the VaR of stock j on the condition $\mathrm{C}(R_t^i)$ at the confidence level $1-q$.

Let $\boldsymbol{w}_t = (w_t^1, w_t^2, ..., w_t^J)$ be a set of weights of stock 1, 2,…, J in the portfolio. We now formulate the mean-CoAVaR portfolio optimization as follows:

$$\min_{\boldsymbol{w}_t} CoAVaR_{q,t}^{port(\boldsymbol{w}_t)|\mathrm{C}^d(R_t^{index})}, \\ \text{s.t.} \sum_{j=1}^J w_t^j \mu_t^j = \overline{\mu}_t, \sum_{j=1}^J w_t^j = 1, w_t^j \geq 0, \forall j, \quad (2)$$

where μ_t^j is a conditional mean of R_t^j on the information up to $t-1$ and $\overline{\mu}_t$ is an expected return of the portfolio. Note that CoAVaR is defined for the portfolio return $R_t^{port(\boldsymbol{w}_t)} = \sum_{j=1}^J w_t^j R_t^j$ against the market index return R_t^{index}. The distress condition $\mathrm{C}^d(R_t^{index})$ is defined as the loss of the index being above its VaR:

$$\mathrm{C}^d(R_t^{index}) = \left\{R_t^{index} \leq -VaR_{q,t}^{index}\right\}. \quad (3)$$

Short selling is prohibited in line with common practice. For simplicity, we do not take transaction costs into account.

2. Empirical study

We evaluate a mean-CoAVaR strategy through an empirical study by using daily stock logarithmic return datasets of 28 out of 29 G-SIFIs, as of November 2011, where the only exclusion is Banque Populaire CdE because it is unlisted. The list of G-SIFIs is given in the Appendix. We refer to each stock by its ticker symbol or abbreviation. We use the S&P global 1200 financial sector index (SGFS) to represent the global banking stock market.

The procedure of evaluating a mean-CoAVaR strategy is as follows. First, we generate the one-period-ahead joint stock return distribution using the multivariate ARMA(1,1)-GARCH(1,1) model. We assume that the innovations of the ARMA-GARCH

[1] Autoregressive moving average generalized autoregressive conditional heteroscedasticity.

model follow the multivariate normal tempered stable (MNTS) distribution[1] because it is a better model for G-SIFI stocks compared with the Gaussian model in terms of both goodness of fit and accuracy of risk measure estimation (Kurosaki and Kim, 2013). Subsequently, under the predicted stock return joint distribution, we find the optimized portfolio $\boldsymbol{w}_t$ through three different strategies: mean-variance, mean-AVaR, and mean-CoAVaR optimization. In other words, we minimize the variance, AVaR, and CoAVaR of the portfolio under the same constraints for the three strategies, respectively, as given in equation (2). We regard an equally weighted portfolio as the benchmark, and thus set the expected return $\overline{\mu_t}$ as the simple average of conditional means μ_t^j. We rebalance the portfolio to the optimum each business day. Finally, we compare the performance among strategies in terms of long-run loss mitigation effects.

The operation period starts on January 1st, 2008 and ends at June 30th, 2012, during which systemic risk is of great concern. The operation days amount to 1174 in line with the United States business days. Each business day, the parameters of the ARMA-GARCH model are updated on the basis of the most recent 1250 days' historical stock return data. Historical returns are backfilled where missing (Kurosaki and Kim, 2013). The confidence level of risk measures is set as $1 - q = 0.95$. We use the Matlab *fmincon* command for optimization problems.

The portfolio is constructed from G-SIFI stocks. To see whether the effectiveness of strategies depends on portfolio size or regional specificity, we prepare three portfolios constructed from different number of stocks and another three portfolios constructed from different regional stocks. The three different-sized portfolios are referred to as large, middle, and small. The large group includes all 28 G-SIFI stocks; the middle group includes the following 12 stocks: BAC, BARC, BNP, C, CBK, CSGN, DBK, HSBA, MUFG, GLE, SMFG, and UBSN; and the small group includes the following 6 stocks: BAC, BARC, BNP, CBK, MUFG, and UBSN. For the middle and small groups, sample stocks are chosen from six countries, the United States, the United Kingdom, France, Germany, Switzerland, and Japan, which play critical roles in the global banking system in the sense that more than one institutions are selected as G-SIFIs from those countries. The three regional portfolios are constructed from G-SIFI stocks in each region: 8 stocks from the United States, 16 stocks from Europe, and 4 stocks from Asia.

The results are summarized in Tables 1 and 2 for different-sized portfolios and different regional portfolios, respectively. They report standard deviation, skewness, kurtosis of the realized daily returns of the optimized portfolios, the number of days on which the optimized portfolio outperforms the benchmark regarding the return, and cumulative return in percentage terms. The statistics of the market index and equally weighted portfolio are also presented as a reference. The main remark in Tables 1 and 2 is that the mean-CoAVaR and mean-AVaR portfolios generally incur smaller cumulative loss than the mean-variance portfolio. In Table 1, the mean-variance portfolio incurs an even larger cumulative loss than the simple equally weighted portfolio in the middle and small groups. We frequently observe that the mean-variance portfolio yields at most the same performance as the equally weighted portfolio. This supports the idea that the variance is not necessarily a proper risk measure during financial turmoil. Second, the mean-CoAVaR strategy still has loss mitigation effects compared with the mean-AVaR strategy in most cases in Tables 1 and 2. In Table 1, the loss mitigation effect is the least in the large group and the greatest in the small group. It can be explained by the size of the portfolio. When a portfolio is diversified by incorporating a larger number of stocks, the structure of the portfolio becomes closer to the market index. Therefore, the mean-AVaR optimization for a larger portfolio captures systematic risk well even without explicitly considering the comovement between the portfolio and entire market as CoAVaR does. In Table 2, the Asia group is the only exception out of all six portfolios where the mean-CoAVaR strategy is inferior to the mean-AVaR strategy, and moreover, the mean-AVaR strategy is inferior to the mean-variance strategy in terms of the cumulative loss. However, note that the mean-CoAVaR strategy incurs the smallest cumulative loss among the three strategies in the other five cases.

The time series of the cumulative return of the portfolios optimized by three different mean-risk strategies for the small and Europe groups is plotted in Figures 1 and 2, respectively. In addition, the difference of the cumulative return between the mean-CoAVaR and mean-AVaR portfolios is also plotted in Figures 3 and 4. Note that the small and Europe groups constitute the portfolio where the mean-CoAVaR strategy has the most pronounced effect of mitigating the loss among different-sized portfolios and different regional portfolios, respectively. It is observed in the figures that the mean-CoAVaR portfolio has a noticeable difference in the cumulative return from the mean-AVaR portfolio after the collapse of Lehman Brothers, which triggered financial turmoil and concern about systemic risk. From the observations above, we conclude that the mean-CoAVaR optimization is as effective or even better compared with the mean-AVaR optimization, especially when systematic distress is of great concern.

[1] See Kim et al. (2012) for the MNTS distribution.

Table 1. Portfolio performance of three mean-risk optimizations (by size)

Portfolio	Standard deviation	Skewness	Kurtosis	# of outperforming days	Cumulative return
SGFS	0.022	-0.041	8.035	N.A.	-71.580
Large group (28 G-SIFIs)					
Benchmark (equally weighted)	0.026	0.054	9.186	N.A.	-127.879
Mean-Variance	0.016	-0.060	6.612	359	-124.692
Mean-AVaR	0.014	-0.028	9.123	599	-78.316
Mean-CoAVaR	0.014	0.068	9.384	606	-77.237
Middle group (12 G-SIFIs)					
Benchmark (equally weighted)	0.027	0.206	8.323	N.A.	-126.879
Mean-Variance	0.022	-0.021	6.902	219	-127.338
Mean-AVaR	0.021	0.059	8.373	611	-99.424
Mean-CoAVaR	0.021	0.048	8.997	610	-96.641
Small group (6 G-SIFIs)					
Benchmark (equally weighted)	0.029	0.124	8.312	N.A.	-136.908
Mean-Variance	0.025	-0.052	6.885	179	-167.646
Mean-AVaR	0.023	-0.014	7.390	581	-144.940
Mean-CoAVaR	0.023	0.027	7.241	592	-127.064

Note: The cumulative return is the cumulative amount of the weighted average of logarithmic returns of stocks in the portfolio and is expressed as a percentage. Thus, it can be lower than -100. N.A. – not available.

Table 2. Portfolio performance of three mean-risk optimizations (by region)

Portfolio	Standard deviation	Skewness	Kurtosis	# of outperforming days	Cumulative return
SGFS	0.022	-0.041	8.035	N.A.	-71.580
United States group (8 G-SIFIs)					
Benchmark (equally weighted)	0.038	-0.041	14.612	N.A.	-87.213
Mean-Variance	0.034	-0.580	18.598	165	-130.506
Mean-AVaR	0.032	0.139	14.051	579	-89.881
Mean-CoAVaR	0.033	0.150	14.232	583	-89.520
Europe group (16 G-SIFIs)					
Benchmark (equally weighted)	0.030	0.162	7.370	N.A.	-155.552
Mean-Variance	0.025	0.099	5.989	313	-104.376
Mean-AVaR	0.024	0.126	6.989	621	-38.265
Mean-CoAVaR	0.024	0.173	6.866	620	-34.009
Asia group (4 G-SIFIs)					
Benchmark (equally weighted)	0.022	0.113	7.674	N.A.	-98.517
Mean-Variance	0.020	0.250	7.027	82	-73.763
Mean-AVaR	0.019	0.013	7.862	571	-75.031
Mean-CoAVaR	0.019	-0.012	7.939	570	-76.736

Note: The cumulative return is the cumulative amount of the weighted average of logarithmic returns of stocks in the portfolio and is expressed as a percentage. Thus, it can be lower than -100. N.A. – not available.

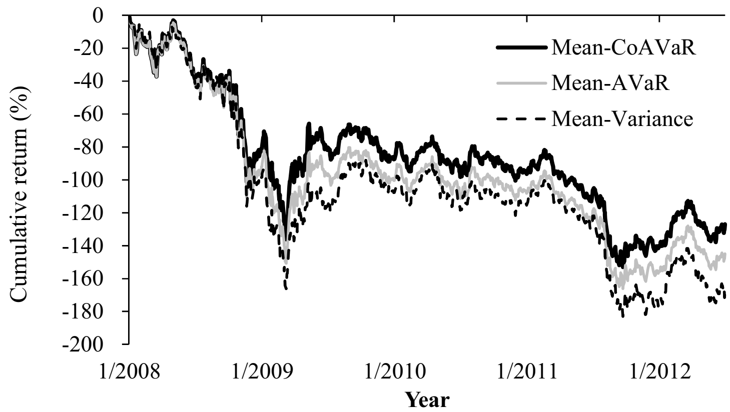

Fig. 1. Cumulative return of the portfolios optimized by different strategies (small group)

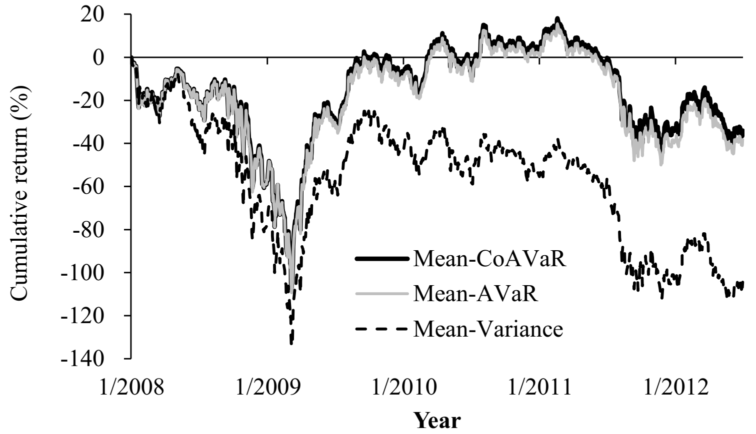

Fig. 2. Cumulative return of the portfolios optimized by different strategies (Europe group)

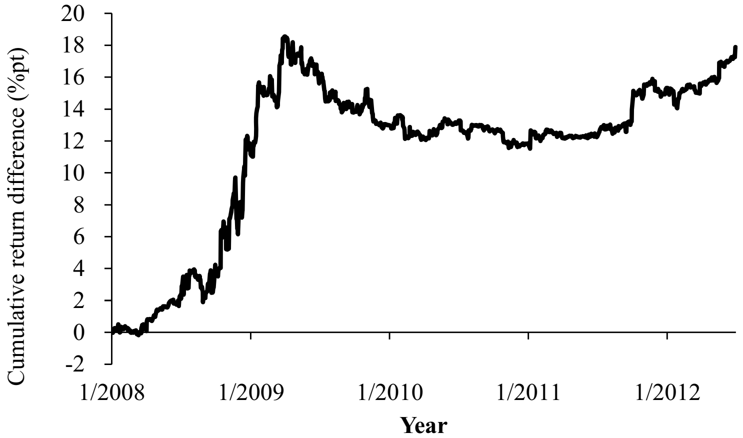

Fig. 3. Difference of the cumulative return between the mean-CoAVaR and mean-AVaR portfolios (small group)

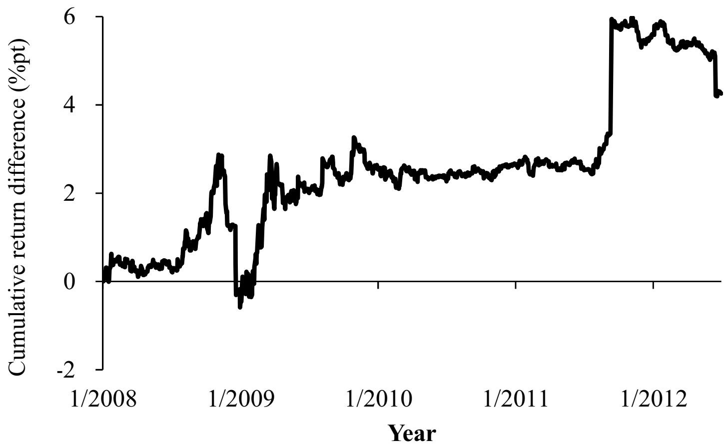

Fig. 4. Difference of the cumulative return between the mean-CoAVaR and mean-AVaR portfolios (Europe group)

Concluding remarks

In this paper, we propose mean-CoAVaR portfolio optimization to mitigate the potential loss arising from systematic risk. Since the CoAVaR of the portfolio accounts for the intrinsic risk and extent of its vulnerability to systematic downturn, on the condition that the market index is in distress, CoAVaR is expected to be a good candidate for the objective function to be minimized against undiversifiable risk. Note that CoAVaR is more appropriate than CoVaR for the optimization problem because of convexity.

We examine the effectiveness of the proposed mean-CoAVaR optimization by using 28 listed G-SIFI stocks. G-SIFIs are good trial samples to test the mean-CoAVaR strategy because they are both highly interconnected and potentially affected by systematic risk in global financial markets. We utilize the ARMA(1,1)-GARCH(1,1) model with the MNTS distributed innovations to forecast the one-period-ahead joint distribution of stock returns, which is revealed to be a better model for G-SIFI stocks in our separate paper. Throughout the empirical study, we observe that the mean-CoAVaR portfolio incurs smaller cumulative loss than the mean-AVaR and mean-variance portfolios in most cases. Therefore, we conclude that the mean-CoAVaR optimization is effective during the time of global bear markets. Until now, CoVaR has been considered primarily a macro-prudential tool for measuring the systemic importance of an institution. Our results open its applicability to risk management usage.

References

1. Adrian, T. and M.K. Brunnermeier (2011). "CoVaR", *Federal Reserve Bank of New York Staff Reports*, No. 348.
2. Basel Committee on Banking Supervision (2011). "Global systemically important banks: Assessment methodology and the additional loss absorbency requirement", *Bank for International Settlements*, Basel.
3. Financial Stability Board (2011). "Policy Measures to Address Systemically Important Financial Institutions", Basel.
4. Financial Stability Board (2012). "Update of group of global systemically important banks (G-SIBs)", Basel.
5. Kim, Y.S., R. Giacometti, S.T. Rachev, F.J. Fabozzi and D. Mignacca (2012). "Measuring financial risk and portfolio optimization with a non-Gaussian multivariate model", *Annals of Operations Research*, Vol. 201, Issue 1, pp. 325-343.
6. Kurosaki, T. and Y.S. Kim (2013). "Systematic risk measurement in the global banking stock market with time series analysis and CoVaR", *Journal of Investment Management and Financial Innovations*, Vol. 10, Issue 1, pp. 184-196.
7. Markowitz, H. (1952). "Portfolio Selection", *The Journal of Finance*, Vol. 7, No. 1, pp. 77-91.
8. Rachev, S.T., S.V. Stoyanov and F.J. Fabozzi (2008). *Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization: The Ideal Risk, Uncertainty, and Performance Measures*, John Wiley and Sons.

Appendix

Table A1. List of 29 G-SIFIs as of November 2011[1]

United States	Europe	Asia
Bank of America (BAC) Bank of New York Mellon (BK) Citigroup (C) Goldman Sachs (GS) JP Morgan Chase (JPM) Morgan Stanley (MS) State Street (STT) Wells Fargo (WFC)	Banque Populaire CdE Barclays (BARC) BNP Paribas (BNP) Commerzbank (CBK) Credit Suisse (CSGN) Deutsche Bank (DBK) Dexia (DEXB) Group Crédit Agricole (ACA) HSBC (HSBA) ING Bank (INGA) Lloyds Banking Group (LLOY) Nordea (NDA) Royal Bank of Scotland (RBS) Santander (SAN) Société Générale (GLE) UBS (UBSN) Unicredit Group (UCG)	Bank of China (3988) Mitsubishi UFJ FG (8306) Mizuho FG (8411) Sumitomo Mitsui FG (8316)

Note: Characters in parentheses stand for the ticker symbols in each domestic market. We refer to G-SIFIs by their ticker symbol except the Asian G-SIFIs. We refer to the Asian G-SIFIs by their abbreviations: BOC (Bank of China), MUFJ (Mitsubishi UFJ FG), MHFG (Mizuho FG), and SMFG (Sumitomo Mitsui FG).

[1] The most recent list contains revisions owing to the update on November 2012. See Financial Stability Board (2012).