“A dynamic price index theory for deflating green net national product: an illustrative application using data from the United States”

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A dynamic price index theory for deflating green net national product: an illustrative application using data from the United States

Abstract

This paper is concerned with the welfare significance of real comprehensive net national product based on an exact dynamic price index. The authors extend the Konus classical index number theory by taking into account current environmental externalities and investment for future consumption enhancement. It is shown that, when deflated with the proper dynamic price index, the real green net national product becomes an ideal measure for welfare comparisons over time. The authors demonstrate the application of the theory using time series data from the United States over the period from 1959 to 2008.

Keywords: dynamic price index theory, deflating green NNP, data from US.
JEL Classification: Q5, E19.

Introduction

It has been long known that traditional net national product (NNP) is not an exact welfare indicator for several reasons. The textbook arguments behind this view contain a couple of obvious reasons. One is related to the definition of net investments: the only information about net investments in the conventional NNP refers to physical, man-made capital. This means that changes in other important stocks, such as natural resource stocks, environmental stocks and the stock of human capital are not included. Another related flaw in NNP is that external effects are not handled in an appropriate manner. When present, the market data on which NNP is based are flawed because prices do not reflect the true underlying scarcities. A third example is that traditional NNP, because it is an aggregate number, does not reveal how consumption opportunities are distributed between individuals or generations. However, all three of the above reasons can be assumed away by moving to an ideal situation, where it is assumed that all types of capital stocks are correctly priced and included in NNP. We can also assume that all consumption services produced by capital goods are included in the consumption vector, and that the corresponding correct rental prices are available. Moreover, we can exclude externalities, and duck distributional issues by assuming that an intertemporal welfare function supports the efficient market solution. Now, in what sense will an augmented NNP concept, the green NNP, which does not include the above listed flaws be a welfare indicator? More specifically, will a higher comprehensive NNP indicate a welfare improvement?

Weitzman's (1976) result on dynamic welfare measurement, showing that for a special case NNP is a perfect welfare indicator, was obtained in a first best setting with a single aggregate consumption good, multiple capital goods, and a utility function equal to aggregate consumption. Normalizing the price of the consumption good to one the Hamiltonian will coincide with real NNP and, consequently, the money and utility metrics will be equivalent. The reason is that the utility function is linear, which means that the consumer surplus is equal to zero. In a more general setting this is no longer true. The Hamiltonian based on utility measurement will always be a perfect indicator of future welfare in a utility metrics. However, utility is not observable, and if we want an observable measure of the static welfare equivalent a practical problem arises. In the literature this has been dealt with by approximating the Hamiltonian by linearizing the utility function. However, the approximation will be poor if the utility function deviates strongly from linearity. In addition, a realistic case with more than one consumption goods will always give rise to a price index problem. Hence, it is relevant to ask how the index number problem changes in an intertemporal world where consumption takes place simultaneously with capital accumulation. The latter means that the prices of capital goods enter the picture as well as the prices of the consumption services that are rendered by the capital stocks[1]. The reason is that we need not only a measure of consumption today, but also a measure of what net investment today yields in terms of future utility.

The price of the consumption services can be handled by rental prices. As we will show below the financial asset property generates a Konus-like expenditure function that is modified to include the value of net investment (financial) saving. This expenditure function is based on the Hamiltonian of the optimal control problem, and it has one important flaw. The marginal utility of income is not con-

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[1] Owner occupied housing is an example of a financial asset that produces consumption services as well as gives utility in terms of future welfare.

stant over time, and, hence, preferences will vary over time. To handle this we apply an ideal consumer price index based on both real and virtual prices (Weitzman, 2001) that enables us to make preferences stationary over time. The main result of the paper is that comprehensive NNP, deflated by a composite index that consists of a dynamic Konus index based on the Hamiltonian, multiplied by the ideal index, is a perfect welfare indicator. We also show that the ideal index is measurable at any point in time, given the knowledge of the constant utility discount factor (the rate of time preference), and information about nominal interest rates.

We demonstrate the application of the theory with the time series data of NNP growth from the United States over the period from 1959 to 2008 (cf. Jorgenson et al., 2015). Based on individual expenditures on different commodity groups and savings as well as the aggregated consumer demand system by Jorgenson (1990), we are able to calculate our dynamic price index for exact (intertemporal) welfare measurement. The results indicate that the real sustainable individual income derived from price index is considerably higher than the corresponding figure based on the conventional consumer price index, and more importantly this real sustainable income serves as a better welfare indicator in comparison. For this particular dataset, we found that the growth both in the conventional real income measure deflated by the chained consumer price index and that in our sustainable income indicate welfare improvement. However, the conventional measure has a lower power about 3-4 percent in explaining the exact change in welfare level as compared to our sustainable income.

1. A multi-sector growth model

To start with, we need a general multi-sector growth model. Following Weitzman (2001), let $C(t) = [C_1(t), C_2(t), ..., C_n(t)]$ denote the n-dimensional vector of consumption goods, and $K(t) = [K_1(t), K_2(t), ..., K_m(t)]$ the m-dimensional vector of capital stocks at time t. The former contains all consumption goods relevant for human welfare. This means, in particular, that the consumption services rendered by the capital stocks are included. The vector of capital goods is comprehensive in the sense that it contains all goods that are relevant for the productive capacity of the economy. It means, for example, that it contains human capital stocks, natural resource stocks as well as the services of the environment. Moreover, let $P(t) = [P_1(t), P_2(t), ..., P_n(t)]$ be the nominal efficiency prices of consumption goods, including the rental prices of the consumption services rendered by capital goods at time t, and let $Q(t) = [Q_1(t), Q_2(t), ..., Q_m(t)]$ be the corresponding efficiency prices of capital goods. For this economy, we define comprehensive NNP at time t by $Y(t) = P(t)C(t) + Q(t)\dot{K}(t)$, where $\dot{K}(t)$ denotes the vector of net investment at time t. A consumption-investment pair $[C(t), \dot{K}(t)]$ is attainable at time t from the capital stock $K(t)$ if and only if $\{C(t), \dot{K}(t), K(t)\} \in A$, where A denotes the attainable possibility set, assumed to be a strict convex. The general multi-sector growth problem can now be formulated in the following manner (Weitzman, 2001; Arrow et al., 2013).

$$\max \int_0^\infty U\left[C(t)\right] \exp(-\theta t) dt, \tag{1}$$

subject to the attainability constraints $\{C(t), I(t), K(t)\} \in A$ and the differential equations $K(t) = I(t)$ with initial conditions $K(0) = K_0$. The maximum principle is valid and it requires that the current value Hamiltonian:

$$H(t) = U(C(t)) + \psi(t) I(t), \tag{2}$$

be maximized with respect to $[K(t), I(t)]$ subject to the attainability restriction. Here $\psi(t)$ is an m-dimensional vector of utility shadow prices of capital goods (the co-state variables) which satisfies $\dot{\psi}(t) = \theta\psi(t) - H_k^*(t)$, where $H_k^*(t)$ is the gradient of the maximized current value Hamiltonian with respect to the capital stocks along the optimal path. In the Ramsey growth model, the nominal interest rate is determined by the marginal productivity of capital. In our model things are a bit more complicated, since the technology is very general and there are many capital stocks. A no-arbitrage argument is, however, available. Let $\lambda(t)$ denote the marginal utility of income, then we have $\psi(t) = \lambda(t)Q(t)$ and $\dot{\psi}(t) = \dot{\lambda}(t)Q(t) + \lambda(t)\dot{Q}(t)$. By substituting these expressions into the co-state dynamics equation, we obtain after a rearrangement:

$$\theta(t) - \frac{\dot{\lambda}(t)}{\lambda(t)} = \frac{H_i^* / \lambda(t) + \dot{Q}_i(t)}{Q_i(t)}, \tag{3}$$

where H_i^* is the i-th element of the gradient vector H_k^* for $i = 1, 2, ..., m$. Note that the right-hand-side expression in equation (3) represents the nominal rate of return to investment, i.e. the nominal interest rate, $r(t)$, Thus, we have:

$$\dot{\lambda}(t) = [\theta - r(t)]\lambda(t), \tag{4}$$

which is the differential equation for the marginal utility of income along the optimal path. The solu-

tion is $\lambda(t)\exp(-\theta t) = \lambda(0)\exp\left(-\int_0^t r(\tau)d\tau\right)$ which can be used to transfer the utility discount factor into the money discount factor. However, more importantly in this context, it gives a clue how to handle an index number problem by estimating a consumer index based on virtual prices by an indirect method.

2. The Hamiltonian as a quasi-linear utility function

Conditional on the market prices along the first best path of the economy, one can represent consumer choice at time t as the solution to the following optimization problem:

$$\max_{C(t),\kappa(t)} H(t) = U(C(t)) + \lambda(t)\kappa(t), \tag{5}$$

subject to:

$$P(t)C(t) + \kappa(t) = Y(t), \tag{6}$$

where $\kappa(t) = Q(t)I(t)$ is the total aggregate money value of net investments in the m capital stocks. The marginal utility of income is treated as a constant during the period as is money NNP, $Y(t)$. Since the objective function in (10) is quasi-linear, the solution for current consumption is $C(t) = d(P(t),\lambda(t))$, where $d(\cdot)$ is the m-dimensional vector of demand functions. The corresponding net investment value is $\kappa(t) = Y(t) - P(t)C(t)$. We now define an expenditure function:

$$E(P(t),\lambda(t),H(t)) = \min_{C(t),\kappa(t)} [P(t)C(t) + \kappa(t)]. \tag{7}$$

The current value Hamiltonian measures current utility that is obtained from current consumption plus future utility that is obtained from net investment today. In Weitzman (1976) it is shown that the current value Hamiltonian is directly proportional to the current value of future utility along the first best path of the economy, and the factor of proportionality being the utility discount rate, i.e.:

$$H^*(t) = \theta\int_t^\infty U(C^*(s)\exp[-\theta(s-t)]ds = \theta W^*(t), \tag{8}$$

where $W^*(t)$ is the optimal value function. In other words, keeping the present purchasing power (including that arising from capital formation) constant means that future consumption possibilities (ceteris paribus) are kept intact. Using equation (8), we can write the expenditure function in equation (7) in terms of the intertemporal value function (cf. Li and Löfgren, 2012).

$$E(P(t),\lambda(t),\theta W(t)) = \min_{C(t),i(t)} [P(t)C(t) + \kappa(t)]. \tag{9}$$

Although the expenditure function is defined in a similar way as the static theory, it is not clear that it can be used like the expenditure function in static index theory, comparing income compensation over time. The reason is that the current value Hamiltonian function, $H(t) = U(C(t)) + \lambda(t)\kappa(t)$, is not stationary due to the time dependent marginal utility of income $\lambda(t)$. From equation (4) it is evident that the marginal utility of income will change over time. This problem is also implicit in the standard Allen-Konus compensation index, but it disappears, since there are no intertemporal trade-off between present and future consumption; the utility function on which the expenditure function contains only consumption goods, $\kappa(t) \equiv 0$ in equation (5), which is a rather crude, but not so visible, simplification in standard index theory[1].

Since our main interest is to construct a dynamic index formula to facilitate welfare comparisons over time, it is necessary to first normalize the utility price of investment $\kappa(t)$, i.e. the marginal utility of income $\lambda(t)$. For this purpose, we define a new deflator (the Ideal Weitzman Index, IWI), invented by Martin Weitzman in (2001). This index is defined as:

$$\pi^0(t) = \frac{\lambda(t_0)}{\lambda(t)} = \frac{\tilde{P}(t)C(t_0)}{P(t_0)C(t_0)}, \tag{10}$$

where $\tilde{P}(t)$ denotes the virtual market clearing prices that would be observed at time t if the market basket of goods $C(t_0)$ were to be demanded along an optimal consumption path. The price vector $P(t_0)$ contains the market prices of the consumption bundle $C(t_0)$. It can be readily shown that the index is independent of the benchmark consumption vector. The term ideal measure is chosen by Weitzman (2001) to denote the ideal towards which the makers of a CPI type index strive when they try to select a representative market basket straddling two economies, or two points in time in the same economy. The practical imputation problems are difficult to solve. A direct approach would require an estimate of virtual prices for all consumption goods in the economy; a monumental task to put it mildly. However, according to equation (10) a sufficient statistic is an estimate of the quotient $\lambda(0)/\lambda(t)$. Rewriting equation (10) using equation (4) gives:

[1] The simplification, however, introduces problem for how private housing should be treated in CPI, since it is both an asset and provides rental services.

$$\pi^0(t) = \exp\left[\int_0^t r(s)ds - \theta t\right]. \tag{11}$$

It is seen that the ideal price index depends on the relative strength of the money rate of interest and the pure rate of time preference. Its value will increase when the interest rate is higher than the time preference rate, and decrease otherwise. To calculate the ideal price index, however, we will need an estimate of the scalar time preference or utility rate of discount θ. The money rate of interest is readily available even though the different risk premiums may add noise to its underlying value. The main problem is to assess the rate of time preference, which is not an easy task. One possibility, following Attanasio and Browning (1995) is to specify and estimate the marginal utility of income directly rather than starting from a utility function. Another possibility is to assume that the marginal utility of income is constant over time. This is implicitly done in traditional CPI practice that encompasses an approximation of a static Konus index.

3. The ideal dynamic index theory

With help of the price index above, the static-like problem in (5-6) can be rewritten as:

$$\max_{\mathbf{C}(t),\bar{\kappa}(t)} H(t) = U(C(t)) + \lambda(t_0)\bar{\kappa}(t), \tag{12}$$

subject to:

$$\bar{P}(t)C(t) + \bar{\kappa}(t) = \bar{Y}(t), \tag{13}$$

where $\bar{\kappa}(t) = \kappa(t)/\pi^0(t)$ denotes the normalized value of investment, $\bar{P}(t) = P(t)/\pi^0(t)$ the normalized consumption prices, and $\bar{Y}(t)$ the normalized income (comprehensive NNP) at time *t*. With such a normalization, the price of investment $\bar{\kappa}(t)$ is made constant at the reference level $\lambda(t_0)$, and, thus, the current value Hamiltonian functional form (equation 17) becomes a stationary generalized utility function over time. It now becomes possible to define an intertemporal indifference map over the $(n+1)$-dimensional space $(C(t),\bar{\kappa}(t))$ by $H(C(t),\bar{\kappa}(t)) = H^0$. First, let us consider the base-year problem at time t_0, i.e. maximizing the current-value Hamiltonian $H(t_0) = U(C(t_0)) + \lambda(t_0)\bar{\kappa}(t_0)$ under the static-like budget constraint $\bar{P}(t_0)C(t_0) + \bar{\kappa}(t_0) = \bar{Y}_0$. Let $(C^0,\bar{\kappa}^0)$ denote the optimal solution, then the maximized current-value Hamiltonian can be expressed by $\hat{H}^0 = U(C^0) + \lambda^0\bar{\kappa}^0$ and the expenditure by $\bar{Y}_0 = \bar{P}(t_0)C^0 + \bar{\kappa}^0$. Now, our question is this: given a price vector $[P(t),\lambda_0]$ for consumption and normalized investment at any time t, what is the minimum expenditure $\bar{Y}_t$ which can support a current value Hamiltonian at the same level as $\hat{H}^0$? Following Konus (1924), we can express this expenditure by:

$$\bar{Y}_t = E(\bar{P}(t),\lambda(t_0),\hat{H}^0), \tag{14}$$

where $\hat{H}^0 = H(C^c,\bar{\kappa}^c)$ with C^c and $\bar{\kappa}^c$ as the compensating demands for consumption and investment such that with these devices we can now define Hamilton-Konus-dynamic price index by:

$$\pi(t) = \frac{\bar{Y}_t}{\bar{Y}_0} = \frac{\bar{P}(t)C^c + \bar{\kappa}^c}{\bar{P}(t_0)C^0 + \bar{\kappa}^0}, \tag{15}$$

which can be written as:

$$\pi(t) = \alpha\pi_c(t) + (1-\alpha)\pi_i(t), \tag{16}$$

where $\alpha = \frac{\bar{P}(t_0)C^0}{\bar{P}(t_0)C^0+\bar{\kappa}^0}$ and $1-\alpha = \frac{\bar{\kappa}^0}{\bar{P}(t_0)C^0+\bar{\kappa}^0}$ are the weights attached to the consumer price index, $\pi_c = \frac{\bar{P}(t)C^c}{\bar{P}(t_0)C^0}$ and the investment price index $\pi_i = \frac{\bar{\kappa}^c}{\bar{\kappa}^0} = \frac{\bar{Q}(t)I^c}{\bar{Q}(t_0)I^0}$, respectively.

Thus, the dynamic price index is a weighted average of two static-like indexes, one for the current consumption and the other for investment related to the value of future consumption[1]. The dynamic price index defined in (15) will prove valuable for welfare comparisons over time. Consider the following two situations, one with (normalized) prices $\bar{P}(t_0)$, $\bar{Q}(t_0)$ and national income (or comprehensive NNP) $\bar{Y}_0$ at time t_0, and the other with (normalized) prices $\bar{P}(t_0)$, $\bar{Q}(t)$ and national income (or comprehensive NNP) $\bar{Y}(t)$ at any other time t. To compare the intertemporal welfare between the two situations, we can now make use of the dynamic price index defined in (15) to arrive at a double-deflated real national income (real NNP) measure.

Let $\bar{Y}_r(t) = \bar{Y}(t)/\pi(t)$ be the deflated real income at time t, then the following claim is true: *When deflated by the composite dynamic price index* $\Pi(t) = \pi^0(t)\pi(t)$, *the real comprehensive* NNP, $\bar{Y}_r(t) = Y(t)/\Pi(t) = \bar{Y}(t)/\pi(t)$, *is a perfect welfare indicator. If* $\bar{Y}_r(t) > \bar{Y}_r(t_0)$, *intertemporal wel-*

[1] At a disaggregated level, the net financial position of the consumer, assets minus mortgages, would enter a dynamic true cost-of-living index, but aggregated over consumers this yields the value of net investments. See also Klevmarken (2004).

fare at time t is higher than at time t_0*; and if* $\bar{Y}_r(t) < \bar{Y}_r(t_0)$, *the intertemporal welfare at time t is lower than at time* t_0.

The reason is straightforward; $\bar{Y}_r(t) > \bar{Y}_0$ implies that $\bar{Y}(t) > \pi(t)\bar{Y}_0 = \bar{Y}_t$, i.e. the normalized income at time t is greater than the minimum expenditure required to reach the reference welfare level $\hat{H}^0$, Since marginal utility of income is given by $\lambda(t_0) > 0$, the excess income $\bar{Y}(t) - \bar{Y}_t > 0$ also implies a higher welfare level at time t than at time t_0. Note that our dynamic price index in (15) was defined in terms of the normalized prices rather than the original nominal prices. This means that to arrive at real income we have used the Hamilton-Konus-Weitzman-chain index:

$$\Pi(t) = \pi^0(t)\pi(t), \tag{17}$$

such that:

$$\bar{P}_r(t) = \frac{P(t)}{\pi^0(t)\pi(t)} = \frac{\bar{P}(t)}{\pi(t)} \text{ and } \bar{Y}_r(t) = \frac{Y(t)}{\pi^0(t)\pi(t)} = \frac{\bar{Y}(t)}{\pi(t)}. \tag{18}$$

However, it is also possible to arrive at a welfare criterion by staying with only one index, the ideal Weitzman-index, i.e., using normalized prices. By the property of the static-like formulation in (12) and (13), it is possible to derive an exact expression of the compensating income and thereby the dynamic price index defined in (15). For this purpose, we consider the following two cases: given $\bar{P}(t_0)C(t_0) + \bar{\kappa}(t_0) = \bar{Y}_0$, the maximum attainable current-value Hamiltonian is $\hat{H}(t_0) = U(C^0(t_0)) +$, $+\lambda(t_0)\bar{\kappa}^0(t_0)$ and given another static-like budget constraint at time t, $\bar{P}(t)C(t) + \bar{\kappa}(t) = \bar{Y}_0$, with the same income level, the maximum attainable welfare is $\hat{H}(t)$. The difference in the maximized intertemporal welfare between the two points in time is:

$$\begin{aligned}\hat{H}(t) - \hat{H}(t_0) &= U(C^*(t)) - U(C^0(t)) + \lambda(t_0)\left[\bar{\kappa}^*(t) - \bar{\kappa}^0(t)\right] \\ &= \lambda(t_0)\int_{\bar{P}(t)}^{\bar{P}(t_0)} D(\bar{P})d\bar{P} \equiv \Delta(t),\end{aligned} \tag{19}$$

where the third equality follows from the definition of consumer surplus, the second equality is derived from the integration-by-parts formula (Weitzman, 2001; Li and Löfgren, 2002) as well as the assumption of equal income such that:

$$(\bar{P}(t)C^*(t) - \bar{P}(t_0)C^0(t)) + (\bar{\kappa}^*(t) - \bar{\kappa}^0(t)) = \bar{Y}_0 - \bar{Y}_0 = 0. \tag{20}$$

The relationship in equation (19) implies that maximizing $H(t) = U(C(t)) + \lambda(t_0)\bar{\kappa}(t)$ subject to a new budget constraint $\bar{P}(t)C(t) + \bar{\kappa}(t) = \bar{Y}_0 - \Delta(t)$ with the compensating income would yield the exact maximum utility level $\hat{H}(t_0)$. This means that the Hamilton-Konus-Weitzman dynamic price index defined in equation (15) can be rewritten as:

$$\pi(t) = \frac{\bar{Y}_0 - \Delta(t)}{\bar{Y}_0}. \tag{21}$$

Now, consider the following two situations, one with (normalized) prices $\bar{P}(t_0)$, $\bar{Q}(t_0)$ and national income (or comprehensive NNP) $\bar{Y}_0$ at time t_0, and the other with (normalized) prices $\bar{P}(t)$, $\bar{Q}(t)$ and national income (or comprehensive NNP) $\bar{Y}(t)$ at any other time t. If the real income at time t is greater than the base-year income $\bar{Y}_0$, i.e. $\bar{Y}_r(t) = \bar{Y}(t)/\pi(t) > \bar{Y}_0$, or $\bar{Y}(t) > \pi(t)\bar{Y}_0 = \bar{Y}_0 - \Delta$, then we know that welfare at t is higher than at t_0. This means that *"If the real income in normalized prices* $\bar{Y}_r(t) = \bar{Y}(t)/\pi(t)$ *at time t is greater than the base-year normalized income* $\bar{Y}_0$, *the welfare at time t is higher; or equivalently, if the sum of income change and the consumer surplus term* Δ *is positive in that* $\bar{Y}(t) - \bar{Y}_0 + \Delta(t) > 0$, *i.e.* $\Delta\bar{Y}(t) + \Delta(t) > 0$, *then welfare is higher at time t than that at* t_0*".*

This result is closely related to a result in Li and Löfgren (2002), where they show that the money metrics version of the Hamiltonian along an optimal path equals NNP plus the consumer surplus evaluated at normalized prices[1]. This entity, called Generalized Comprehensive NNP, is proportional to future welfare like in the utility metrics version of Weitzman's theorem in equation (13). The right hand side of equation (13) would in a money metrics equal the utility discount factor multiplied by money wealth in normalized prices[2].

4. An application to the US time series data

Our dataset contains the average individual consumption expenditures and prices of some broadly aggregated commodity groups, as well as the saving rates in the United States for the period from 1959 to 2008 (BEA's website: www.bea.gov). In the same way as in Jorgenson and Slesnick (1990), we define five commodity groups as described in the following:

1. Energy: expenditures on electricity, gas, gasoline, fuel oil, and other energy goods.

[1] See also Aronsson et al. (2004) chapter 3.

[2] Note that in this paper we consider NNP as a welfare indicator and in future research we will extend our theoretical framework to accommodate for wealth accounting models as in Arrow et al. (2013).

2. Food: expenditures on all food products including beverage and tobacco.
3. Consumer goods: expenditures on clothing, shoes and all other non-during goods.
4. Capital services: the service flow from housing and other consumer durables such as motor vehicles, furniture and household equipment, and so on.
5. Consumer services: expenditures on consumer services such as household operation, transportation, recreation and medical care, and so on.

Table 1 below shows a part of the dataset with prices for the five commodity groups, the overall consumer price indices, the individual consumption expenditure and saving for the years from 2000 to 2008.

Table 1. Price data, individual expenditure and saving

Year	PEN	PF	PCG	PK	PCS	CPI	M	I
2000	1.00	1.00	1.00	1.00	1.00	1.00	23864	596
2001	1.03	1.02	1.01	1.01	1.03	1.02	24729	463
2002	0.97	1.04	1.01	1.02	1.06	1.04	25518	641
2003	1.09	1.06	1.01	1.02	1.09	1.06	26498	601
2004	1.21	1.10	1.01	1.03	1.13	1.09	27939	619
2005	1.43	1.12	1.02	1.05	1.17	1.13	29368	109
2006	1.59	1.15	1.04	1.07	1.21	1.16	30812	236
2007	1.70	1.19	1.04	1.09	1.25	1.20	32181	190
2008	1.95	1.26	1.06	1.12	1.28	1.25	33024	633

The reason for defining commodity groups in such a way is to take advantage of Jorgenson and Slesnick's estimated demand system for the US economy, which is based on exact aggregation from individual preferences (cf. Gorman, 1953; Muellbauer, 1975). The system of budget shares for each of the commodity groups is specified as:

$$w = \frac{1}{d(p)}\left(\alpha + \beta \ln(p) - \gamma \ln(M) + \delta A\right), \tag{22}$$

where $w = [w_1, w_2, ..., w_5]'$ denotes a 5x1 vector of budget shares satisfying $\sum_{i=1}^{5} w_i = 1.0$. The column vector α represents the five intercepts, γ the parameters associated with the total consumption expenditure M, and β a 5x5 symmetric matrix of parameters belonging to the 5x1 logarithm of the price vector $\ln(p) = [\ln(PEN)$, $\ln(PF)$, $\ln(PCG)$, $\ln(PK)$, $\ln(CS)]'$ for energy, food, consumption goods, capital service and consumer services, respectively. The 5x1 vector A describes some individual characteristics and δ its associated 1x5 vector of parameters. The function $d(p)$ is constrained to be $d(p) = -1 + e\beta \ln p < 0$ and a scalar with $e = [1,1,1,1,1]$. The demand system here over all individuals implies the following representative individual annual welfare function:

$$V = \alpha' \ln(p) + 0.5 \ln(p)' \beta \ln(p) - d(p) \ln(M). \tag{23}$$

Using earlier time series data on budget shares, commodity prices, consumption expenditure and other personal characteristics from 1947 to 1985, Jorgenson and Slesnick (1990) have estimated the parameters as:

$$\alpha = \begin{bmatrix} -0.1954 \\ -0.6789 \\ -0.6789 \\ -0.5347 \\ 0.3796 \end{bmatrix},$$

$$\beta = \begin{bmatrix} 0.0406 & -0.0731 & 0.0228 & 0.0369 & -0.0407 \\ -0.0731 & 0.1726 & -0.0899 & -0.0738 & 0.0048 \\ 0.0228 & -0.0899 & 0.14575 & -0.0676 & 0.0090 \\ 0.0369 & -0.0738 & -0.0676 & 0.1846 & -0.0950 \\ -0.0407 & 0.0048 & 0.0090 & -0.0950 & 0.1899 \end{bmatrix}.$$

With the expression (23) as indirect utility function with respect to consumption, we can write the Hamiltonian value as:

$$H = V(p, M) + \lambda I, \tag{24}$$

where $\lambda = -d(p)/M > 0$ denotes marginal utility of money. Using the data as described above, we have calculated the Weitzman ideal price index π^0 defined in equation (10), the composite price index Π defined in equation (17) as shown in Table 2. The ideal index numbers indicate that, a nominal value of \$1.38 in year 2008, for example, is equal to \$1.00 in year 2000 in utility terms. This index enables us to normalize the prices in order to calculate the NNP deflator in equation (15), and finally our composite dynamic price index Π as shown in column 3. It is seen that, except in the base year of 2000, our dynamic price index numbers are smaller than the conventional consumer price indices, and the longer the comparison period is, the larger the difference. This is expected since our dynamic price index is a kind of intertemporal cost-of-living index allowing for substitutions among the commodity groups whereas the conventional CPI is a Laspyres type of index known with upwards bias. This difference leads to the higher real NNP based on our dynamic price index than that deflated by the conventional CPI for all years except the base year. Loosely speaking, the real income, or the constancy-equivalent of the future disposable income is higher than the real income derived from using the conventional CPI.

Now, we are interested in whether and how well the growth these real NNP series would indicate welfare improvement. The last three columns in Table 2 show that the Hamiltonian value and the two real NNP measures all increase over time, and we can

therefore conclude that growth in real NNP based on any of the two index series indicates welfare improvement. Although the CPI based one is less exact, its growth happens to indicate welfare changes in the right direction. The question now is: does the dynamic price index based measure perform better for welfare measurement? To answer this question, we run regressions between the annual change in the Hamilton value and the change in log-transformed real NNP and obtain the following results:

$$\begin{aligned} \Delta H &= 0.0034 + 0.9161 \cdot \Delta \ln\left(NNP_{true}\right) \text{ with } R^2 = 0.9543 \\ \Delta H &= 0.0009 + 0.9243 \cdot \Delta \ln\left(NNP_{cpi}\right) \text{ with } R^2 = 0.9141. \end{aligned} \quad (25)$$

Since the slope parameters are all positive, we can once again confirm that the growth in both of real NNP measures indicates welfare improvement. However, the growth based on our "true" dynamic cost-of-living index has a higher explanatory power with about 4% (the difference in the $R^2 - value$) as compared to the CPI based one. It is worth mentioning that the small enhancement in explanatory power should not be over interpreted due to the following two underlying reasons. One is that we have used broadly aggregated commodity groups in this study, and thus the substitution possibility among them becomes small. As a consequence, the bias in the Laspyres-type consumer price index becomes smaller as compared to the true cost-of-living index for consumption. The other reason is that the average individual saving rates in the United States have been rather small, with less than 2% of its income, and thus the weight of investment in the index calculation in equation (16) becomes tiny small. These two factors may have contributed to the relatively small difference between CPI and our dynamic index number. If more disaggregated commodity groups are used and when the saving rates are relatively high, the results may be rather different.

Table 2. True and proxy measures of welfare

Year	π^0	Π	CPI	Hamil-tonian value	Dynamic index deflated NNP	CPI deflated real NNP
2000	1.00	1.00	1.00	10.105	24460	24460
2001	1.04	1.02	1.02	10.115	24704	24662
2002	1.07	1.03	1.04	10.141	25339	25225
2003	1.11	1.05	1.06	10.159	25773	25591
2004	1.17	1.01	1.09	10.187	26501	26215
2005	1.22	1.11	1.13	10.191	26631	26165
2006	1.28	1.14	1.16	10.215	27282	26698
2007	1.34	1.17	1.20	10.233	27750	27008
2008	1.38	1.21	1.25	10.240	27881	26879

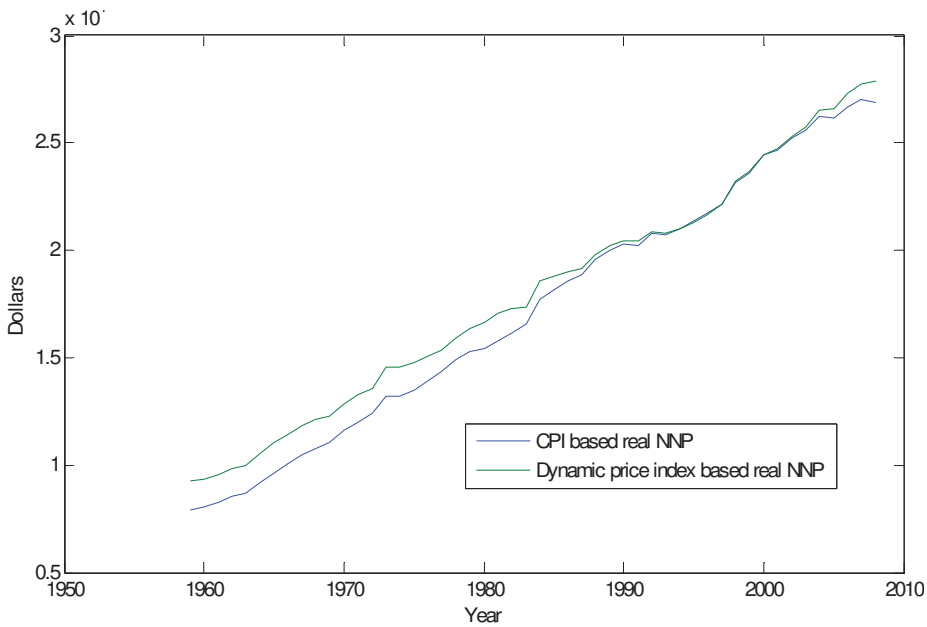

Fig. 1. CPI based versus the dynamic price index based real NNP

In equation (21), we have told the same story in terms of (normalized) income difference and consumer surplus. In Table 3 below, we show the corresponding numerical values. Column 2 indicates that the change in individual's monetary income from the base year 2000, measured in normalized prices, is considerably small. Much of the increase in nominal income was "eaten" by the more rapidly diminishing marginal utility of income, and thereby the normalized growth in income in utility numeraraire becomes small. Recall that a 2008 dollar is valued only 1/1.38 = \$0.72 as in year 2000. In terms of such normalized prices, there has been a deflation from 2000, or in other words, the utility prices have decreased since 2000. For this change, we have derived the consumer surplus as shown in column 3. Note that the sum of income difference and the

consumer surplus as shown in column 4 is positive and increasing. Once, again this trend implies that welfare has been steadily increasing from 2000 to 2008.

Table 3. Welfare change and its components

Year	Normalized income difference $\Delta\overline{Y}$	Normalized consumer surplus CS	Sum $\Delta\overline{Y} + CS$
2000	0	0	0
2001	6	374	380
2002	-30	855	825
2003	-16	1282	1266
2004	3	1896	1899
2005	35	2368	2403
2006	52	2782	2834
2007	78	3153	3231
2008	151	2915	3066

Conclusion

The main conclusion of this paper is that properly indexed comprehensive NNP can serve as a perfect welfare indicator. This is slightly surprising since it has been known since Adam Smith and Jules Dupuit (1844) that there are two value concepts; one is value in exchange, which loosely speaking is NNP; a second is value in use which corresponds to NNP plus the consumer surplus. More recently, Asheim and Weitzman (2001) has shown that growth in real NNP, deflated by a Divisa price index can be an accurate indicator of a local welfare improvement, provided that the real interest rate is positive. Li and Löfgren (2006) show that the Divisa index can be substituted for any consumption price index provided that a rate of return measure that can be interpreted as the net-investment weighted own rate of interest is positive. Moreover, as shown by Li and Löfgren (2002) a transformation of Weitzman's (1976) dynamic welfare theorem from a utility metrics into a money metrics typically requires a measure of the consumer surplus.

The qualifications that are necessary for a NNP measure to be an accurate measure of dynamic welfare are that we have to introduce a dynamic price index that contains price indices of the comprehensive vectors of consumption and net investment goods, and is conditional on that the future path of the economy is optimal. In addition, we need an estimate, at each instant of time, what it would cost to buy last periods consumption vector at a virtual price vector consistent with an optimal path. This is used to construct an ideal index that can be recovered, given the knowledge of today's and previous nominal interest rates, and the rate of time preference; a latent parameter that has to be estimated. This index is used to construct an indifference map based on the Hamiltonian that is stationary over time in the space of consumption goods and the value of net investment (saving). Given the resulting utility function we can proceed in the same manner as Konus (1924). The resulting expenditure function encompasses the consumer surplus.

We have also demonstrated the application of the dynamic price index theory to the case of the United States. The numerical results indicate that real NNP growth based on our dynamic price index has some more explanatory power, about 4%, than that based on the conventional consumer price index, to indicate welfare changes. The relatively good performance of the conventional CPI in the illustration probably depend on our used of highly aggregated commodity groups which may have undermined the substitution possibilities within each commodity group. Another reason is that the average saving rate in the US is rather small so that our composite intertemporal cost-of-living index (involving saving/investment) does not go any further from the static cost-of-living index. Further studies with more comprehensive datasets in other countries, especially developing countries, may lead to rather different conclusions about the adjustment effect.

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