“Exposure-based volatility: an application in corporate risk management”

AUTHORS	Athanasios P. Fassas Vasil Rumenov Lyaskov
ARTICLE INFO	Athanasios P. Fassas and Vasil Rumenov Lyaskov (2016). Exposure-based volatility: an application in corporate risk management. *Investment Management and Financial Innovations*, *13*(2-1), 235-245. doi:10.21511/imfi.13(2-1).2016.10
DOI	http://dx.doi.org/10.21511/imfi.13(2-1).2016.10
RELEASED ON	Monday, 04 July 2016
JOURNAL	"Investment Management and Financial Innovations"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
0	0	0

© The author(s) 2025. This publication is an open access article.

BUSINESS PERSPECTIVES

businessperspectives.org

Athanasios P. Fassas (Greece), Vasil Rumenov Lyaskov (Bulgaria)

Exposure-based volatility: an application in corporate risk management

Abstract

This study develops a non-traditional measure of risk, an exposure-based volatility, for the non-financial company and applies this measure to capture both the downside potential of cash-flows and the probability of requiring additional external financing under most foreseeable conditions. The empirical analysis is applied on a particular Bulgarian transport company and concludes that the proposed measure of exposure-based volatility manages to capture effectively the peaks and troughs in the variance of cash-flows, thus, significantly outperforming the historical standard deviation. This non-traditional downside risk estimate is by itself extremely useful as it contains significant information about a given company. Furthermore, it can be used as a valuable input in several risk management tools; in the current paper, a robust measure of CFaR and an original interpretation of Merton's credit risk model are presented.

Keywords: exposure-based volatility, Cash-flow-at-Risk (CFaR), Merton option pricing model, liquidity risk, corporate risk management.
JEL Classification: G30, G31, G32, G33.

Introduction

This paper presents an exposure-based measure of volatility (EBV) of cash-flows that captures both external (macroeconomic factors) and internal risks. We achieve this by calculating a forward-looking value for the standard deviation of EBITDA (Earnings before Interest, Depreciation and Amortization) through a methodology that combines strong economic reasoning and reliable econometric techniques. Subsequently, we use the EBV estimate in two risk management tools: the Cash-flow-at-Risk (CFaR) and the Merton option pricing model. We thoroughly show the application of exposure-based CFaR and the modified Merton methodology to a particular company, but we believe that our method provides a reliable tool for calculating the cash flow risk exposure of any company. This study demonstrates the application of exposure-based CFaR and the modified Merton model to a particular company, but the presented method provides a reliable tool for calculating the cash flow risk exposure of any enterprise.

The particular paper extends existing methodologies in corporate risk management by including several additional econometric steps in order to derive a proprietary measure of risk. In particular, we employ a VAR specification in order to determine exogeneity, we use the Dickey-Fuller test to examine the long-term relationship and finally we run an ARCH model in order to account for non-linearity; furthermore, we follow an ARIMA modelling to allow for linear dependencies of CF on their lagged values and/or a white noise element. Moreover, in order to estimate the probability distribution of future cash-flows, we run Monte Carlo simulations with two significant additional feature[1]. First, the volatility estimate is intentionally intentionally simulated out of a set of confidence intervals, giving more certainty to its extremity (after all we are seeking excessive downturn potentials). Second, we follow two major ways for calculating the mean value applied in the simulation: a forecasted return based on a model of both company specific and external factors (e.g. an ARMA time-series model) or, alternatively a slightly altered bootstrapping technique that allows superior accuracy in the estimation, as compared to the classical drift measures. Out of these possibilities we focus heavily on the former modelling technique (due to the availability of internal company data), but we argue that both of the above methods results in a robust estimation of CFaR. Another significant contribution of the current paper is the proposed interpretation of Merton's credit risk model for liquidity management. In particular, we modify the seminal structural model of default by Merton (1974) in order to derive, not the distance to default, but rather, the "distance to liquidity deficit/crisis".

The rest of the paper is structured as follows: the next section includes a succinct review of the relevant literature. The third section includes a thorough analysis of the specific steps for the calculation of the exposure-based volatility (EBV) estimate, while the fourth section presents two applications of the metric within the non-financial entity – the cash-flow-at-risk measure and Merton's (1974) model for credit risk. Finally, the last section

---

© Athanasios P. Fassas, Vasil Rumenov Lyaskov, 2016.
Athanasios P. Fassas, Lecturer, Business and Economics Department, International Faculty, University of Sheffield, City College, Greece.
Vasil Rumenov Lyaskov, International Faculty, University of Sheffield, City College and PIMK Holding Group JSC, Bulgaria.

[1] Andren et al. (2005) apply simulations in which the values for the macroeconomic and market variables identified as explanatory variables are picked randomly from the variance/covariance matrix.

includes the final remarks and summarizes the current analysis.

1. Literature review

Cash-flow-at-Risk (CFaR) attempts to estimate what is the most a company's cash-flows can suffer within a given time horizon at a given confidence level and it is considered to be the cash flow equivalent of Value-at-Risk (VaR), which is a widely used risk management tool for financial institutions. Its calculation is similar to VaR[2] with two differences: first, CFaR attempts to estimate the probability distribution of corporate cash-flows (instead of the total value of a firm) and second, its time horizon is considerably longer – three months to a year (whereas VaR is usually calculated in the very short-term – usually 10 days to a month). Although CFaR is becoming increasingly more popular with non-financial, industrial companies, there is exceptionally limited academic research regarding this risk management tool. The first ever developed measure of CFaR is usually credited to Risk Metrics (1999), but Oxelheim and Wihlborg (1987) actually developed methods for decomposing cash-flows and successfully suggested hedges to tackle various risks that arise from macroeconomic factors. Their methodology is based on strong economic reasoning combined with a regression approach to arrive at coefficients for each individual risk factor. A decade later, Risk Metrics (1999) coined the term CFaR and proposed a bottom up methodology for its calculation; they derive the cash flow distributions from random sampling (using a variance-covariance matrix) of certain defining factors (major building blocks), such as production volumes, revenue, costs, and exchange rates.

Stein et al. (2001) follow a different methodology in order to estimate the CFaR, specifically, they group comparable companies based on various characteristics (size, profitability, riskiness of cash-flows, and volatility of stock prices) and thus, they are able to construct a sample of 85 000 observations using only six years of data. Out of this dataset, they calculate a forecast for cash-flows based on a simple autoregressive model and obtain the forecast errors. This top-down approach is more suitable for a cluster of companies and not individual entities and additionally cannot be applied on privately-owned corporations (since it requires stock prices). Andren et al. (2005)[3] propose an exposure-based CFaR measure which extends the Oxelheim and Wihlborg (1987) effort. In particular, they regress the company's cash-flows against certain macroeconomic variables that they are exposed to. Risk factors are further categorized in terms of value-adding and non-value-adding ones with the goodness of fit statistic (R^2) being used to estimate the non-value adding macroeconomic variables and $1 - R^2$ as an estimate of the value adding ones. Afterwards, in order to arrive at a risk distribution, they apply either random sampling from the variance-covariance matrix or historical simulation, thus obtaining data on the worst 5% tail event.

According to Andren et al. (2005) both the top-down approach of Stein et al. (2001) and the bottom up methodology proposed by Risk Metrics (1999) have significant limitations; thus, we estimate an exposure-based CFaR. Our methodology – which is described in the following section – includes several econometric steps resulting in a more robust CFaR estimate.

2. A methodology for estimating exposure based volatility: step-by-step specifications and results

Our methodology includes the following steps: first, we establish a solid econometric model to capture movements in the mean of the explained variable based on both market (macroeconomic) factors and the variable itself; second, we prove exogeneity of each explanatory variable and long-term relationship between the former and the dependent and then establish a strong ARIMA model to allow for the inclusion of company-specific risks; the fourth step includes the creation of an ARCH model to compute a robust estimate of "forward-looking" volatility (the EBV measure). Afterwards, based on EBV, we run a Monte Carlo simulation to calculate CFaR and estimate the probability for potential lack/excess of liquidity using a minor modification of the Merton model. For the purpose of estimation, monthly data of the EBITDA of company PIMK Ltd (a Bulgarian-based international transport and logistics company) has been gathered. Figure 1 (see in Appendix) provides a visual representation of the company's monthly EBITDA and also shows the cumulative historical and one-period ahead standard deviations. It can be observed how volatility tends to peak strongly at times and then immediately recede towards lower levels. Such behavior might make risk analysis especially difficult. Even though the risk measure settles at around a mean of 50-60% (such a high level of risk is to be expected when analyzing monthly data), it would hardly present a reliable measure of volatility for probability forecasting. This is why, as reasoned above, more advanced procedures should be applied.

---

[2] A detailed analysis of the Value at Risk (VaR) concept and methodology can be found in Duffie and Pan (1997) and Linsmeier and Pearson (2000).

[3] Andren et al. (2012) essentially draw upon their initial work (Andren et al., 2005).

2.1. Choosing the appropriate variables. A combination of economic and econometric rationale is employed in the process of picking out the most appropriate explanatory variables. PIMK is a freight transportation and forwarding company that began operating in 2006. During 2009-2011 the company's sales increased by over 40% annually, while decreased by 20% in the subsequent years; this performance made PIMK one of the fastest growing European companies in the field and among the biggest 150 companies for 2013 both in terms of revenues and net profit (source: Plimsoll, 2013, industry report). Since it is a relatively young company, with insufficient amount of data to estimate the mid-to-long-term relationship between its operations and the macroeconomy, we are using monthly data for the period January 2011-June 2014. The major risks the company faces can be summarized as follows:

- Business risk – the risk, associated with the company's operating earnings. Firstly, this will be measured by the volume of goods, available for transportation, through the examination of the relationship with EU's export, import and net export (data has been compiled from the ECB website). Secondly, the significance of the ability of consumers to actually purchase such goods, estimated through the level of unemployment; thirdly, exposure to the overall economy is examined through: 1) inflation, calculated as both the consumer price index (CPI) and the production price index (PPI), 2) the money supply in the economy, represented by the M3 aggregate (including all of M2 plus money market funds with greater than twenty-four hours maturity and longer-term time deposits), and 3) the overall economic development as provided by quarterly GDP. Since the series tested is monthly, interpolation[4] techniques are implemented to the higher frequency GDP series.
- Price risk – the company is exposed to the risk of changes in oil prices since it uses millions of tons of oil annually for its operations. The major benchmark that is used by the greater part of European fuel providers for setting their prices is the ULSD 10 Platts Fob (Med), which is a combination of the prices of various other oil benchmarks (among others Brent and WTI). In order to estimate the degree of correlation of EBITDA returns and the volatility in oil prices, monthly quotes of both Brent and WTI areused (data was compiled from Energy Information Administration information). An additional cost is tires purchases; their pricing (usually) depends on a combination of the value of rubber, oil, inflation and exchange rates (since the raw materials are quoted in USD). For this reason, we also regress rubber prices against cash-flows, in order to capture the degree to which the latter is dependent on the former (data on rubber prices was obtained from the Singapore Stock Exchange).
- Operating risk – the risk associated with the company's fixed and variable costs. Variations in variable costs occur as result of changes in the major materials the company uses in its core business activities and thus, they are already factored into the price risk. All fixed costs, on the other hand, are insignificant since they are constant and thus do not cause volatility. Since an estimated EBITDA is applied, financial costs which can also be deemed as part of the company's invariable expenses – are not included in the calculation, but are taken into consideration in the final value of probable "extreme" loss that is estimated.
- Exchange rate risk – the company is not exposed to the risks of floating exchange rates due to the currency board (with Euro) in Bulgaria. About 95% of the major non-EU clients have set payments in euros, as well. Nevertheless, for the sake of diligence only, the EUR/USD exchange rate is also incorporated in the model.
- Exposure of the company to certain equity indices (in particular, the US S&P 500, the German DAX, the EU Stoxx 50 and the Stoxx transportation index, the Bulgarian Sofix, and the Turkish XU 100) is also tested, since the company is affected by major economic trends in both Bulgaria and abroad and has very strong business relations with Turkey (respective index data was obtained from Yahoo Finance).

As an initial step to the construction of the metric, unit root tests are run with each variablein order to ensure stationarity. Following that, univariate OLS is conducted to rule out all non-significant factors. In particular, eighteen macroeconomic and market variables were regressed against the company's EBITDA. Tables 1 and 2 summarize information from the least squares tests and the respective auxiliary regressions. The empirical results suggest that only four variables (WTI, GDP, PPI and S&P 500) are statistically significant and were hence tested for exogeneity using the Granger Causality test (see next section).

---

[4] We are applying the linear method, although this is the simplest method available, it is the most appropriate one, given that there is no more than one data point available between missing observations and that both positive and negative returns are observed (making more advanced methods, such as log linear interpolation or cardinal spline unfitting).

Table 1. OLS estimates

Variable	Differenced/ Not differenced	p-value @level	Adj. R-sq @level	p-value @lag 1	Adj. R-sq @lag 1	p-value @lag 2	Adj. R-sq @lag 2	p-value @lag 3	Adj. R-sq @lag 3	p-value @lag 4	Adj. R-sq @lag 4
Export (rx)	Stationary at level	0.4308	-0.009	0.8431	-0.025	0.2785	0.006	0.2878	0.004	0.1974	0.020
Import (dri)	First differenced	0.4364	-0.010	0.0681	0.062	0.3922	-0.007	0.9758	-0.029	0.5163	-0.017
Net (E-I) (rn)	Stationary at level	0.8363	-0.025	0.7073	-0.022	0.5848	-0.019	0.1166	0.041	**0.0083**	**0.160**
Brent (rb)	Stationary at level	**0.0496**	**0.072**	0.7070	-0.022	0.1211	0.038	0.0774	0.059	0.2453	0.011
WTI (rw)	Stationary at level	**0.0227**	**0.104**	0.8489	-0.025	0.5537	-0.017	0.0095	0.150	0.1873	0.022
Rubber (rr)	Stationary at level	0.5193	-0.015	0.8182	-0.025	0.4483	-0.011	0.5057	-0.015	0.4524	-0.012
CPI (dcpi)	First differenced	0.9720	-0.026	0.9220	-0.026	0.4114	-0.008	0.6154	-0.020	0.8453	-0.027
PPI (rp)	Stationary at level	0.0643	0.062	0.5572	-0.017	0.7498	-0.024	0.0441	0.083	0.3030	0.003
Unempl (drun)	First differenced	0.5190	-0.015	0.8955	-0.027	0.9441	-0.028	0.3010	0.003	0.3309	-0.001
M3 (dm3)	First differenced	0.5991	-0.018	0.0989	0.0456	0.3065	0.002	0.4590	-0.012	0.4693	-0.013
EUR/USD (red)	Stationary at level	0.4644	-0.011	0.3634	-0.004	0.0972	0.048	0.2094	0.017	0.9722	-0.029
SPX500 (rspx)	Stationary at level	0.6735	-0.021	0.4965	-0.014	0.5444	-0.017	0.0530	0.075	0.5577	-0.018
STX50 (rstx)	Stationary at level	0.6878	-0.021	0.4899	-0.013	0.3703	-0.005	0.4154	-0.009	0.5868	-0.020
SFX (rsfx)	Stationary at level	0.8628	-0.025	0.8262	-0.025	0.4435	-0.011	0.3185	0.000	0.6545	-0.0238
DAX (rd)	Stationary at level	0.8490	-0.025	0.8097	-0.025	0.4693	-0.012	0.3585	-0.004	0.6892	-0.024
XU100 (drxu)	First differenced	0.6458	-0.021	0.6615	-0.021	0.1748	0.024	0.2780	0.006	0.9424	-0.023
STX Tr (rtr)	Stationary at level	0.9401	-0.026	0.8591	-0.025	0.3702	-0.004	0.3012	0.002	0.8929	-0.028
GDP (ddgdp_li)	Second differenced	**0.0054**	**0.173**	0.3279	-0.000	0.4578	-0.0123	0.3577	-0.004	0.0139	0.149

Table 2. Auxiliary regressions

Variable	$4\text{-}d_L$	$4\text{-}d_U$	DW statistic	Breusch-Godfrey LM test p-value	Jarque-Bera p-value	ARCH LM p-value
RN@ lag 4	2.79	2.68	2.61	0.3862	0.0558	0.8849
RB@ level	2.75	2.66	2.84	0.1725	0.0874	0.5674
RW@ level	2.75	2.66	2.92	0.1302	0.3080	0.4243
RP@ lag 3	2.78	2.68	2.81	0.4337	0.2007	0.9175
RSPX@ lag 3	2.78	2.68	2.94	0.1251	0.6034	0.8232
GDP@ level	2.75	2.66	2.64	0.1890	0.3403	0.5008

2.2. Testing for exogeneity. Once a significant set of factors has been established, a Vector Autoregressive (VAR) model is estimated to determine whether the macroeconomic variables are exogenous to cash-flows. This is especially important as it proves that the explanatory variables are in fact non-stochastic, that is their values are determined outside of the observed equation. In general, a simple VAR can be expressed as:

$$Y_1 = \alpha + \beta_{11}\gamma_{1t\text{-}1} + \beta_{12}\gamma_{2t\text{-}1} + u_T. \qquad (1)$$

If all above variables are stationary (or properly differenced so as to be made stationary) then the equation can easily be estimated using OLS to obtain the unrestricted residual sum of squares (RSS). After that, restrictions can be introduced and the model is reestimated in order to generate the restricted RSS and test for Granger causality. Additionally, by converting the model into a vector moving average, VAR allows for tracing of the responsiveness of the dependent variable to shocks on the independent. With its qualities, this test is to bring high credibility to the implementation of any of the significant variables in the ARCH model since if the existence of unidirectional causality towards cash-flows is proven then it would be reasonable to assume that the given variable "is responsible" for changes in the expected deviations from the series' mean.

Table 3. VAR estimates

Dependent	t-stat @lag1	t-stat @lag2	t-stat @lag3	Adj R-sq.	H_0 of autocor.	H_0 of heterosk.	H_0 of norm.
Model 1							
RCF	-2.999**			0.163	Not rejected	Not rejected	Rejected
RN	0.5104						
Model 2							
RCF	-3.967**	-1.627	-0.011	0.392	Not rejected	Not rejected	Not rejected
WTI	-0.743	-0.688	3.105**				
Model 3							
RCF	-2.984**			0.158	Not rejected	Not rejected	Not rejected
RP	-0.247						

Table 3 (cont.). VAR estimates

Dependent	t-stat @lag1	t-stat @lag2	t-stat @lag3	Adj R-sq.	H_0 of autocor.	H_0 of heterosk.	H_0 of norm.
Model 4							
RCF	-3.001**			0.163	Not rejected	Not rejected	Rejected
RSPX	0.513						
Model 5							
RCF	-2.697**			0.152	Not rejected	Not rejected	Rejected
DDGDP	0,231						

Note: ** indicates statistical significance at the 95% confidence interval.

Based on the result of the OLS regressions, the return series on export, the producer price index measuring inflation, the return on West Texas Intermediate (WTI) and S&P 500, and the twice differenced series of GDP, are all tested individually with cash-flows in a VAR specification. Table 3 presents data from the VAR outputs, while Table 4 includes the results from the causality tests. Amongst all variables that have previously proven significant under the OLS assumptions, only WTI proves exogenous based on the Granger statistic. In addition, only shocks in oil cause any significant changes in cash-flows as is proven from the impulse responses and variance decompositions (results not presented here, but are available upon request). None of the other variables prove significant under the outlined assumptions and are thus, discarded from the model.

Table 4. Granger causality tests

Dependent	Independent	Probability	Causality
RCF	RN	0.6097	X
RN	RCF	0.7595	
RCF	WTI	0.0044	Unidirectional WTI=>CF
WTI	RCF	0.595	
RCF	RP	0.8049	X
RP	RCF	0.9155	
RCF	RSPX	0.6077	X
RSPX	RCF	0.2651	
RCF	DDGDP	0.818	X
DDGDP	RCF	0.1731	

2.3. Testing for long-run relationship. In order to obtain a reasonable long-term volatility estimate, the macroeconomic variables to which cash-flows are exposed are further tested for cointegration. The Dickey-Fuller test conducted on all variables shows that both series (CF and WTI) fail to reject the null of stationarity when in returns. However, using log forms, both variables reveal the presence of unit root when neither trend, nor constant is applied. Even though such results pose the question of whether cointegration tests should be implemented, following Lütkepohl and Krätzig (2004), we run the Johansen-Jusilius test on the natural log series. Both the Trace statistic and the Max eigenvalues suggest the presence of a cointegrating equation. The Engle-Granger technique also confirms the results. The latter test is more appropriate since both variables are I (1) in the case of no trend and, therefore it could be expected that if they were to comove then a linear relationship between them should eliminate this order of integration (as indeed happens). Even though for volatility estimation purposes the series are applied in returns, the fact that cointegration is present when logged can be considered transferrable. This further substantiates the choice of including oil in the ARCH model.

2.4. ARMA and ARCH model construction. The Autoregressive Moving Average model, although not theoretically grounded, has gained high popularity in practice. In general terms, the two processes (AR and MA) can be written as follows:

An autoregressive process:

$$X_t = c + \Sigma\varphi x_t - i + \varepsilon_t.$$

A moving average process:

$$X_t = \mu + \Sigma\theta_t - i + \varepsilon_t. \tag{3}$$

A combined (ARMA) process:

$$X_t = c + \Sigma\theta_t\varepsilon_t\,\Sigma\varphi x_t - i + \varepsilon_t. \tag{4}$$

The estimation applied in this work follows the methodology outlined in the seminal paper of Box and Jenkins (1976). The information criteria suggest that an ARMA (1,1) is the most appropriate, followed by a simple MA(1) and AR(1), respectively. As additional steps, stationarity in the models is tested via inspection of the inverses of the AR and MA roots and the existence of autocorrelation – through the Breusch-Godfrey statistics. A further indicator considered is the root mean squared error (RMSE) calculated from an in-sample forecast of the series from all three models. As can be seen from the results, outlined in Table 5, the value of RMSE is lowest in the case of the ARMA (1,1), however, not significantly so as compared to the rest of the models.

Table 5. ARMA model specifications estimates

AR/MA	p-value AR(1)	p-value AR(2)	p-value AR(3)	p-value MA(1)	p-value MA(2)	p-value MA(3)	Joint probability	RMSE	Adj. R^2	AIC	SIC
ARMA (1,1,0)	**0.0038**							**0.4690**	**0.179**	**1.4236**	**1.5080**
ARMA (2,1,0)	0.0020	0.0890					0.0077	0.4532	0.194	1.4092	1.5371
ARMA (3,1,0)	0.0010	0.0833	0.3358				0.0114	0.4445	0.210	1.4272	1.5996
ARMA (0,1,1)				**0.0000**				**0.4315**	**0.338**	**1.2549**	**1.3385**
ARMA (0,1,2)				0.0000	0.2308		0.0000	0.4200	0.357	1.2496	1.3750
ARMA (0,1,3)				0.0000	0.2531	0.5049	0.0000	0.3864	0.441	1.1314	1.2986
ARMA (1,1,1)	**0.1902**			**0.0000**			**0.0000**	**0.4015**	**0.382**	**1.1631**	**1.2898**
ARMA (1,1,2)	0.8633			0.2002	0.9157		0.0002	0.4015	0.365	1.2129	1.3818
ARMA (1,1,3)	0.7454			0.4188	0.6004	0.5492	0.0006	0.3990	0.353	1.2525	1.4636
ARMA (2,1,1)	0.3141	0.6955		0.0000			0.0007	0.4076	0.330	1.2481	1.4187
ARMA (2,1,2)	0.8100	0.5958		0.2980	0.6313		0.0018	0.4061	0.315	1.2924	1.5057
ARMA (2,1,3)	0.7500	0.7921		0.0005	0.9371	0.4996	0.0000	0.3235	0.552	0.88874	1.1447
ARMA (3,1,1)	0.3197	0.6147	0.9936	0.0000			0.0029	0.4115	0.303	1.3255	1.5410
ARMA (3,1,2)	0.0678	0.2023	0.5890	0.6130	0.0011		0.0046	0.4051	0.303	1.3466	1.6052
ARMA (3,1,3)	0.8086	0.0943	0.5160	0.1132	0.8709	0.1228	0.0000	0.3280	0.528	0.9774	1.2790

The next step is to estimate the volatility of the cash flow series using an ARCH specification. All three ARMA variationsare applied together with WTI in GARCH (1,1), TGARCH, and EGARCH models in order to derive the best-fitting model. Following Hsieh (1993) the effectiveness of each model is further tested through a BDS test, applied on the standardized residuals. The test, designed by Brocket al. (1996), detects any remaining non-linearity in the time series by measuring the frequency of repetition of temporal patterns. Firstly, the ARMA (1,0) is applied in combination with WTI. The results show that the GARCH model under the assumption of normal distribution is most appropriate. All coefficients are highly significant (except the constant) with the residual regressions being satisfactory as well. In addition, the BDS test conducted on the standardized residuals confirms that they are i.i.d. However, the conditional standard deviation graph plots consistently high values which are in discord with the actual volatility. In the second specification – the ARMA (1,1) with WTI – all coefficients (with the exception of the AR term and the constant in the variance equation) are highly significant. The auxiliary regressions are all significant with the observed R-squared from the ARCH LM test being close to 97%. Nevertheless, as with the previous model, the conditional standard deviation does not account for the peaks and troughs in volatility. In addition to this, the BDS test results (although significant under the bootstrap probability) are insignificant under the normal probability. Finally, in the third case – the ARMA (0,1) with WTI – the EGARCH (Nelson, 1991) model is chosen with all coefficients being highly significant. The auxiliary tests detect no autocorrelation, no non-normality, and no heteroskedasticity in the residuals. A further prove of the model's appropriateness are the results from the BDS test which show that non on-linearity is detected in the standardized residuals. In addition, all criteria, namely the AIC and SIC point to the normally, rather than t-distributed exponential model which is why the former is applied. In concluding, the EGARCH model with ARMA (0,1) and WTI, under the assumption of normal distribution, provides the best results. Although the other models are also significant, they fail to capture the occasional falls and rises in volatility in the series. Furthermore, despite the criteria being lowest with the ARMA+WTI GARCH (1,1) model, firstit would be mathematically incorrect to assume that between models comparisons are reasonable since the number of observations are different under the different specifications and second, the BDS test shows remaining non-linearity in the residuals in the case of the ARMA GARCH under the normal distribution whereas non is observed with the residuals from the MA EGARCH. Table 6 summarizes the diagnostic tests results on the ARCH models.

Table 6. ARCH models diagnostic tests

Specification	Autocorrelation	Heteroskedasticity	Normality	BDS	Conditional SD	AIC/SIC
GARCH (1,1) WTI+ARMA (1,0)	Not rejected	Not rejected	Not rejected	Not rejected	Does not follow ups and downs	1,09/1,34
GARCH (1,1) WTI+ARMA (1,1)	Not rejected	Not rejected	Not rejected	Not rejected	Does not follow ups and downs	0,94/1,24
EGARCH WTI+ARMA (1,1)	Not rejected	Not rejected	Not rejected	Not rejected	Follows ups and downs	0,95/1,24

2.5. Calculating the exposure-based volatility (EBV). Based on the estimated ARCH model, the exposure-based volatility (EBV) is 73%. EBV is preferable over more traditional estimates of standard deviation – e.g. historical standard deviation and the exponentially-weighted moving average (EWMA) – because it incorporates more effectively two significant and well documented volatility properties, namely volatility clustering and mean reversion. A forward-looking estimate, such as EBV, allows for actual expectations to be included in the analysis, thus, providing important insight into potential events, whereas historical variance does not take into account trends and/or potential trend changes and as a result it very often under/over-estimates reality. The EWMA can theoretically account for this property, but it only does so through its lambda term, which is difficult to construct and susceptible to change, thus misleading in the case of more abrupt changes in returns. The ARCH and GARCH models are taking into consideration these volatility properties and that's the reason they are very often preferred in financial forecasting. The EBV estimate is calculated out of a ARCH specification and so, has superior forecasting power as compared to its peers, allowing for more informed decisions on corporate development issues to be made within the environment of various risk measures. In the following section we will use EBV in two risk management tools: the Cash-flow-at-Risk and the Merton credit risk models.

3. Applications of the exposure-based volatility estimate

3.1. Cash-flow-at-Risk (CFaR). The calculation of Cash-Flow-at-Risk requires an estimate of the probability distribution of future cash-flows. For a concise analysis of the existing approaches in estimating the Cash-Flow-at-Risk measure see Adrenet (2005). In this paper, the Monte Carlo simulation method is applied due to its flexibility and the lack of constraints regarding the underlying distribution assumption. In order to determine next period's cash-flows we apply the following method proposed by Jorion (2007):

$$S = \mu + \zeta + \sigma. \quad (5)$$

In which:

- ζ is a random, normally distributed (as per the EGARCH specifications), variable which is the Brownian element in the equation, driving the random shocks. In addition, the process is also geometric as all parameters are scaled by the current value of the variable;
- μ denotes the drift. Three major theories assume different values for the drift – a volatility eroded risk-free rate of return (namely, the no riskless arbitrage theory), zero drift (derived from the random walk hypothesis, set forth by Bachlier (1900)), and the volatility eroded historical average return. Even though all these drift approaches present reasonable estimates for expected return, this paper will apply a forecasted return of EBITDA (forecast estimation is irrelevant for this explanation). Applying this expected return (based on realistic expectations of volumes, assets, cost factors, etc. rather than random draws and/or statistics) results in a more reliable estimate of CFaR. Due to its wide applicability, however, this model may well be employed to any given company for which relevant data (for such forecasts) is not available to the analyst. In such cases the results, obtained from the above ARMA models, can be utilized to come up with an expected value for the next period return. Alternatively, we suggest sampling from the pool of historical returns (e.g. bootstrapping) with additional randomness added to each individual draw, which would allow for a combination between the inherent distribution and the ARCH multivariate normal.
- σ is the standard deviation. In this application we intend to apply the EBV measure within the Monte Carlo simulation environment. Using the information, provided by the EGARCH output, we calculate 1% confidence intervals for each coefficient. Our reasoning for this is the following: there exists a statistically greater chance of the actual one-period ahead volatility falling within the confidence bands than being simply the output value from the model; therefore, applying random draws from the bands for each individual parameter to arrive at a set of deviations and then converging each expected deviation to the most appropriate probability distribution (as suggested by the model) would give a more reliable downturn risk measure. An additional advantage of this simulation is that it possibly "corrects" the very probable pattern in simulated random variables. This is to say that even though modern software provide reliable random series they are all guided by certain algorithms that are bound to repeat at some point. By using random deviations from the mean in the Monte Carlo equation (rather than a fixed value for every single simulation), the possibility of such cycles to occur is significantly decreased (if not eliminated). We argue that this combination makes our method more realistic in its presentation of a lower bound tail event.

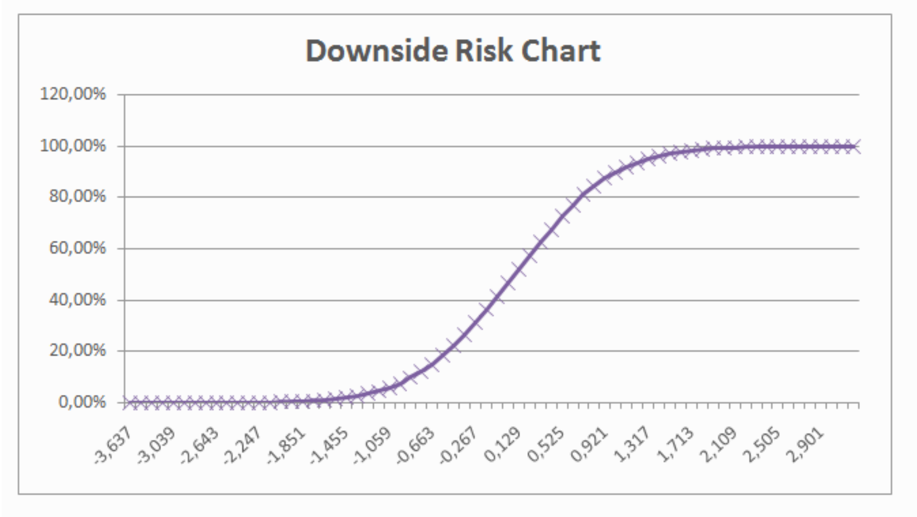

Fig. 2. Cash-flow-at-Risk (CFaR)

Table 7 includes the results of our empirical analysis and Figure 2 visually depicts the respective results. In particular, we estimate a 7% expected return of EBITDA over the next month (the amount of 11 768 838 is the expectation of cumulative EBITDA over the next three months period) and a 5% worst loss of around BGN 417400 and BGN 723 000 over the next one month and three months, respectively. In other words, the CFaR analysis indicates that there exists a 5% risk that cash-flows will fall by BGN 3 306 558 less of the expectations over the next month. Whether the company can overcome such extreme events can be determined from information on its balance sheet. In this case, PIMK has access to a credit line of BGN 8 mil and cash-in-hand to the amount of BGN 1.2 mil. Additionally, the company is lightly leveraged compared to its peers and has an equity of over BGN 50 mil. All these result in a confident outlook regarding the company's financial health.

Table 7. Cash-flow-at-Risk (CFaR) estimate

Percentile	1 month	3 months
5% worst cash flow	- BGN 417 403	- BGN 722 964
Most probable	BGN 3 693 145	BGN 11 768 838

3.2. Modified Merton model. Another noteworthy application of the exposure risk metric relates to Merton's (1974) model for credit risk. In his seminal work, Merton applies the option pricing theory of Black and Scholes (1973) by treating an equity investment as a call option. He proposes a calculation of the distance to default, by setting a reasonable lower limit below which the assets should not fall, an expected value of the assets and their volatility. By converting this value into a standard normal distribution, he finds the probability of default.

Table 8. Modified Merton model representation

Synthetic assets	Synthetic liabilities
Operating Profit (EBITDA) plus Cash-in-hand	WC
	Current financial liabilities
	Current portion of long-term financial liabilities
	Capex

In terms of liquidity management, a slightly altered Merton model can be applied by using the EBV in a framework of an indirect cash flow statement. In particular, we take the expected value of EBITDA on the one side (for example, the three months ahead forecast) and any potential changes in working capital, expected payments on financing operations for the given period, and all expected investments (Capex), on the other (see Table 8). This would then represent a "synthetic" balance sheet where the left side (EBITDA) would be considered as synthetic assets and all on the right side as synthetic liabilities which would also sum up to the threshold value. This allows for an application of the volatility estimate of EBITDA in a Merton model environment, in which the result will show not exactly the distance to default but rather, in simple terms, the "distance to liquidity deficit/ crisis". In other words, given the value of BGN 8 mil. of short-term financial liabilities and current portion of long-term financial liabilities, assuming no change in the structure of non-cash working capitaland putting into the equation a value of BGN 2 mil. for expected investment for the period, a forecasted EBITDA of BGN 12 mil. with a current cash balance of BGN 1.5 mil. and at volatility of 73%, this yields a modified distance to default of 0.141 and a respective probability of default (PD) of 44.41%. This result is particularly important since it will allow management to evaluate more properly its capital strategy for the year in order to receive the lowest possible result of PD or simply put off certain investments, if it is considered that they might lead to high risk of lack of liquidity.

Table 9. Probability of default under three different scenarios

	Base case	Scenario 1	Scenario 2	Scenario 3
EBITDA	12	12	12	12
Cash in the bank	1.5	1.5	1.5	1.5
Total synthetic assets	13.5	13.5	13.5	13.5
Expected increase in EBITDA	7%	7%	7%	7%
WC	0	-5	0	-5
Current financial liabilities	1	1	1	1
Current portion of long-term financial liabilities	7	7	7	7
Capex foreseen	2	2	2	2
Volatility	73%	73%	63%	63%
Nominator	0.10	0.80	0.63	0.63
Denominator	0.73	0.73	0.63	0.63
Distance to “liquidity crisis”	0.14	1.09	0.28	1.39
Probability of “liquidity crisis”	44.41%	13.82%	39.02%	8.27%

It is especially interesting to observe how PD changes under different circumstances (Table 9). In particular, a change in working capital (the financial crisis has led to a chain of indebtedness, resulting in a big amount of uncollected receivables, averaging at BGN 10 mil.) can have significant effect on the PD (lowering it to 13.82% from 44.41% before, a downward correction of 68.8%). In this way, sensitivity analysis can be done to create a list of priorities with the goal to improve the company’s capital structure (one of the most important, yet difficult to tackle, issues in corporate finance). Just as an example, considering that the company manages to ideally hedge its oil exposition for the period, then EBV would fall to 63% (a decrease of 14%) and would result in (given normal working capital conditions) a probability of liquidity crisis of 39.02% (a decrease of 12.15%). It should be noted of course that a one-month-ahead volatility forecast is applied over a three month period, however, it is reasoned that monthly returns tend to be significantly more volatile than quarterly due to the limited amount of time to react to events and the fact that the company runs its business by setting its goals in the short-term thus, managing stable quarterly EBITDA. All these indicate that a volatility based on lower frequency data will tend to be lower compared tovolatility based onhigher frequency data, substantiating the choice of extending the monthly EBV over the three-month horizon.

Conclusion

This paper presents a non-traditional method for calculating an exposure-based volatility (EBV) estimate of corporate cash-flows, which is subsequently applied in the calculation of, what we consider to be, two highly reliable metrics of liquidity risk; namely, Cash-flow-at-Risk (CFaR) and a modification of the Merton model for liquidity management.

In particular, this paper proposes a methodology for calculating an alternative to VaR for the non-financial corporation, the Cash-flow-at-Risk (CFaR). Value at Risk (VaR) is an essential and intuitive risk management tool. Its concept and use is relatively new as major financial institutions began using VaR in the late 1980s to measure the market risks of their trading portfolios. It was the J.P. Morgan’s attempt in 1994 to set a market standard (by proposing its proprietary Risk Metrics system) that led to the explosion of its acceptance (Linsmeier and Pearson, 2000). VaR’s popularity has grown to the extent that it has been included in the 1996 Amendment of the 1988 BIS Accord of the Basel Committee as a method for calculating capital requirements.VaR estimates the maximum amount of total value a firm is expected to lose under a given confidence level, while CFaR attempts to estimate the maximum cash flow shortfall that an industrial company is willing to accept, again given a specified level of statistical confidence. CFaR is an intuitive and practical corporate risk management tool, as it sums up all the company’s risk exposures in a single number.

In the present empirical research, after obtaining an extreme value of EBV through random sampling from the confidence intervals of the variance equation coefficients of the ARCH model, the EBV is included in a Monte Carlo simulation from which the downside risk distribution of cash-flows is obtained. From this environment, the 5% CFaR is calculated. Furthermore, an unorthodox modification of the Merton (1974) model is built up to calculate liquidity risk. By creating a "synthetic" balance sheet, which takes after the logic behind an indirect cash flow statement, we apply EBV within the Merton equation to arrive at a probability of falling into a "liquidity crisis".

In concluding, we believe that our measure is a highly reliable and practically applicable addition to the corporate risk management toolkit. The model can be the building block (or at least an important consideration) when strategic decisions are made concerning capital structure, budgeting, and financial planning as a whole. It can further be applied for company valuation purposes where pro-forma financial statements, which are usually constructed, can include a worst-case scenario based on the metric or simply apply the exposure-based volatility estimate in the forecasted statements to come up with firm value estimates. Financial analysts and portfolio managers can also benefit from its applications as this can alleviate the decision-making process of whether the company is a sound asset or not (using it as a sort of stress test).

References

1. Andrén, N., Januarykensgård, H. and Oxelheim, L. (2012). Exposure-Based Cash-Flow-at-Risk for Value-Creating Risk Management under Macroeconomic Uncertainty, in Fabich, M., Firnkorn, L., Hommel, U. and Schellenberg, E. (Ed.), The Strategic CFO – Creating Value in a Dynamic Market Environment, Berlin Heidelberg: Springer, pp. 1-5.
2. Andren, N., Januarykensgard, H. and Oxelheim, L. (2005). Exposure-based-cash-flow-at-risk: an alternative for VaR for industrial companies, *Journal of Applied Corporate Finance*, 17(3), pp. 76-86.
3. Bachelier, L. (1900). The theory of speculation, *Annales Scientifiques de l'ecolenormale superieure*, 3(17), pp. 21-86.
4. Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities, *Journal of Political Economy*, 81(3), pp. 637-54.
5. Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, 31(3), pp. 307-327.
6. Box, G.E.P. and Jenkins, G.M. (1976). Time series analysis, forecasting and control, San Francisco, CA: Holden-Day.
7. Brock, W.A., Deckert, D., Scheinkman, H. and Le Baron, B. (1996). A test for independence based on the correlation dimension, *Econometric review*, 15, pp. 197-235.
8. Duffie, D. and Pan, J. (1997). An overview of Value at Risk, *The Journal of Derivatives*, 4(3), pp. 7-49.
9. Engle, R.F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation, *Econometrica*, 50, pp. 987-1008.
10. Granger, C.W.J. (1969). Investingating causal relations by econometric models and cross-specral methods, *Econometrica*, 37, pp. 424-438.
11. Hsieh, D.A. (1993). Implication of nonlinear dynamics for financial risk management, *Journal of Financial and Quantitative Analysis*, 28(1), pp. 41-64.
12. Jorion, P. (2007). Value at Risk: The new benchmark for managing financial risk, 3rd Edition, New York, NY: McGraw-Hill.
13. Linsmeier, T.J. and Pearson, N.D. (2000). Value at risk, *Financial Analysts Journal*, 56(2), pp. 47-67.
14. Lütkepohl, H. and Krätzig, M. (2004). Applied Time Series Econometrics, Cambridge, UK: Cambridge University Press.
15. Nelson, D.B. (1991). Conditional heteroskedasticity in asset returns: a new approach, *Econometrica*, 55, pp. 703-708.
16. Oxelheim, L. and Wihlborg, C. (1987). Macroeconomic uncertainty: intenational risk and opportunities for the corporation, Chichester, UK: John Wiley & Sons.
17. Risk Metrics Technical Group (1999). Corporate Metrics Technical Documents. New York: Risk Metrics Group.
18. Stein, J., Usher, S., La Gattuta, D. and Youngen, J. (2001). A comparables approach for measuring cash-flow-at-risk for non-financial firms, *Journal of Applied Corporate Finance*, 13(4), pp. 100-109.

Appendix

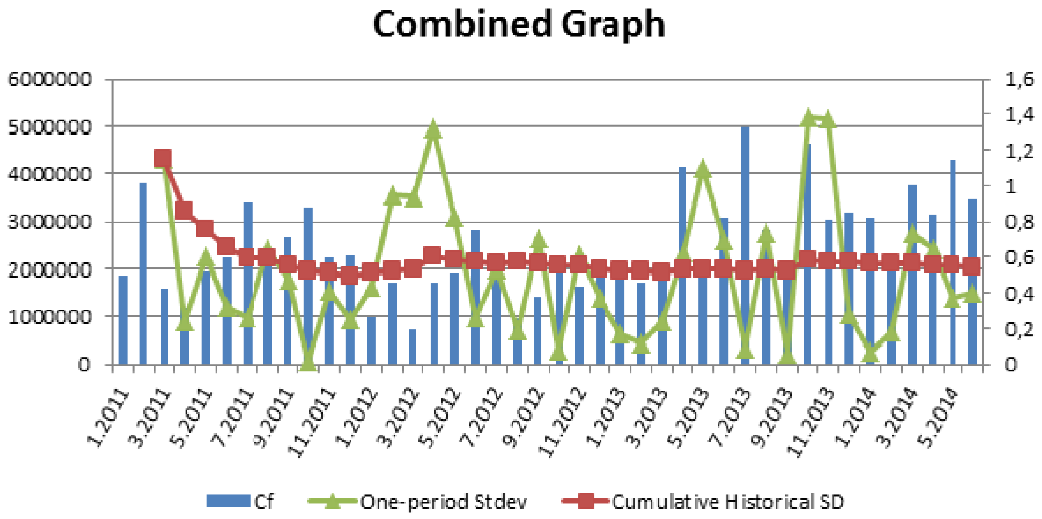

Fig. 1. Monthly EBITDA and one-period-ahead Standard Deviation