“Power law in tails of bourse volatility – evidence from India”

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POWER LAW IN TAILS OF BOURSE VOLATILITY – EVIDENCE FROM INDIA

Abstract

Inverse cubic law has been an established Econophysics law. However, it has been only carried out on the distribution tails of the log returns of different asset classes (stocks, commodities, etc.). Financial Reynolds number, an Econophysics proxy for bourse volatility has been tested here with Hill estimator to find similar outcome. The Tail exponent or $\alpha \approx 3$, is found to be well outside the Levy regime ($0 < \alpha < 2$). This confirms that asymptotic decay pattern for the cumulative distribution in fat tails following inverse cubic law. Hence, volatility like stock returns also follow inverse cubic law, thus stay way outside the Levy regime. This piece of work finds the volatility proxy (econophysical) to be following asymptotic decay with tail exponent or $\alpha \approx 3$, or, in simple terms, 'inverse cubic law'. Risk (volatility proxy) and return (log returns) being two inseparable components of quantitative finance have been found to follow the similar law as well. Hence, inverse cubic law truly becomes universal in quantitative finance.

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INTRODUCTION

Bachelier's trailblazing random walk model (inspired from Brownian motion of particles) had an assumption such as the price changes emerge out as a result of many independent and external shocks. Therefore, he predicted the resulting distribution of returns to be Gaussian (Bachelier, 1900). Since an additive random walk model may lead to negative stock prices, an appropriate mathematical construct could well be a multiplicative random walk. Moreover, the price changes could be measured by logarithmic return. Return data from stock market often show larger deviation from its usual Gaussian distribution when taken for a relatively small length of time (Eugene & Fama, 1965). Commodity prices are no exception to this as well. Cotton price was found to follow a Levy-stable distribution (Mandelbrot, 1963). Contradiction surfaced as the long-run distribution of asset returns weren't found to follow Gaussian distribution. Levy-stable distribution emerged as an apt alternative to Gaussian distribution. Surprisingly, it was found that while the major portion of the return distribution for S&P 500 fits well apparently (Mantegna & Stanley, 1995) in a Levy distribution, however, it tends to experience a much faster exponential decay. Subsequently, it has been found that the tails in case of return distribution for any index interestingly found to follow the power law. Similar behavior has been reported from other prominent global indices as well while deploying Hill estimator (Hill, 1975) for calculating the pattern and exponent of the decay of the tail.

The current work focusses to find the asymptotic decay behavior in the tails of a completely different econophysical proxy for volatility. Firstly, the financial Reynolds number (Re) has been calculated for both regular and high frequency domains following an Indo-Bosnian quest for finding econophysical bourse volatility, this has been described in detail in the 'Methodology'; section subsequently, the same has been tested using Hills estimator (Hill, 1975) to find the tail index 'α'. Most of the relevant studies have been found to focus on the log returns and other return measures for finding both volatility and even testing the tail index in such a case. This study (due to its unusual premise) has been focused completely differently on the same problem. The study found similar tail index for 'Risk' (represented by volatility) that of 'Return'. This in turn completes the 'Risk-Return' characteristics comparison. Both 'Risk' and 'Return' ware found to experience the same decay coefficient in its tails.

1. LITERATURE REVIEW

The cardinal question remains unanswered though: "What causes power law link in stochastic systems?"

Power law has many underlying theories and various schools of thought underneath its strong premise. The first of it reads as:

1. Random growth

1.1. If initial distribution of a set of time series from a same domain (such as firms) is random, so there is growth and decay in a periodic manner satisfying Gibrat's law of proportional effect (Gibrat, 1931). Despite having the same standard deviation and the same growth rate, it (the distribution) tends to become lognormal with larger variance. This breeds the power law. However, the exponent remained a question of concern. Thus, any economic model with a random growth will somehow have an embedded power law; however, the exponent may vary (Gabaix, 2016; Plerou et al., 2004).

2. Economics of superstars

2.1. In any walks of life, there will be extremely high earners, earning substantially higher than their counterparts. A qualitative explanation was suggested by an eminent researcher (Rosen, 1981). Often these are attributed to the 'talent' of the top management of those firms, which are precisely non-quantifiable in nature. Using extreme value theory of probability, the tail possibilities of the talented individuals could be traced out even though the distribution remain uncertain.

2.1.1. Cross-sectional, cross country and time series predictions all are plausible, as well as possible under this broad concept.

2.1.2. In an innovative way, extreme value theory and power laws could well be the natural language to decipher quantitative answers from the economic superstars.

Financial markets often witness powerful traces of power law. Each periodic crash in the bourses globally can be linked with power law connection. Stock market returns, volumes under consideration and more interestingly stock price jump were found to have a clear trace of power law (Botta et al., 2015; Laloux et al., 1998; Bree & Joseph, 2013; Sornette, 2003). Stock volumes and returns (read as lognormal returns) were convincingly proved to have clear power law thread for two decades or so (Nirei, Stachurski, & Stachurski, 2018; Botta et al., 2015; Kelly, 2004). These various and thematic studies led researchers to believe that stock returns should ideally follow a α-stable distribution instead of the popularly assumed Gaussian distribution. Several studies dating back to early 1970s till 2016 showcase that α-stable distribution exists (Kyle, 2013; Kyle & Obizhaeva, 2016); however, the value of that α is definitely not '1'. These studies find that, in most cases, asset returns tend to converge to normality with time aggregation (Wu, 2006). Continuing the similar work a group of researchers found asymptotic normalization in tails of a stochastic distribution in a unique way; further they found similar trend in non-financial and heavy tailed time series as well (Gardes, 2008, 2010).

The daunting question that loomed over was the measurement of that 'α' exponent. Respite to the research world came through in form of 'Hill estimator'. Hill estimator is no newcomer to the world of tail risk identification; MIT professors have ap-

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plied Hill estimator (Jansen & Vries, 1991) to daily returns of US stocks and stock indices for a substantial length of time (from 1962 to 1986). They've found estimates for the tail index α within the range of 3.2-5.2. Another group of researchers (Loretan & Philips, 1994) performed the same estimator on daily returns of the S&P 500 (1962–1987) to generate α in a range of 3.1-3.8; they worked on a really long monthly stock index data comprising from 1834 till 1987 and found α in a narrow band of 2.5-3.2. Another trailblazing study appeared using the same tool in 1995 (Abhyankar, 1995), when a group of eminent American researchers have investigated a data set of daily stock return series from 1985 to1990 covering various different frequencies. They too found the tail exponent α in a band of 3-4. Similar study has been carried out by Lux (1996) using the Hill estimator on European index DAX for a reasonable length of time and found tail exponent α as 2.3-3.8. Indian researchers (Pan & Sinha, 2008) too found tail exponent α as 2.93-3.33.

Many research projects of the past have successfully found an inverse cubic law in the fat tail part of the distribution of stock market returns. This piece of humble attempt has been created bemoaning the dearth of necessary studies relating market volatility. Market volatility has been put into test and inverse cubic law was found to exist in the fat tails. Markets being a complex system ideally relate with asymptotic power law exponent. However, return and volatility are two completely diverse parts of the domain. One resembles return and the other depicts risk. If it has been found to follow the same law, then it's confirmed to be universal rather unusually. Whether developing or developed market returns follow power law connection, especially in the fat tails. However, it was never tested from the volatility perspective. This study not only finds the cubic law connection from volatility perspective, but also reaffirms the universality of this result across two of its different segments. Hill estimator has been deployed here to estimate the tail exponent α with an asymptotic power law connection.

2. METHODOLOGY

Financial Reynolds number (Re) has been established as an apt proxy for 'Risk' in a typical bourse or stock market. Daily data have been analyzed for regular financial Reynolds number (total observations were 3,918 from January 2001 to December 2016) and financial Reynolds number from HFT (total observations were tick by tick 2.8* 109 from February 2012 to December 2016) segment based on the works of an Indian Bosnian research project. The cardinal objective is to find out the hidden power law pattern embedded in the tail of an apt proxy for risk (financial Reynolds number). Furthermore, comparing that 'α' or tail exponent for risk with the 'α' or tail exponent of 'return' (covered in many studies across the globe).

Financial Reynolds number (Re) was formulated as a combination of 'Relative Volatility Index' representing 'momentum of the stock-market and 'Ease of Movement' (Arms, 1996) representing viscosity of the stock market by an Indo-Bosnian collaborative work in 2018. Donald Dorsey in 1993 designed and possibly bettered 'Relative Strength Index' as 'Relative Volatility Index' (Dorsey, 1993).

Conceptually speaking, RVI represents the magnitude (mass) and change in magnitude (rate of change of market mass or momentum) of relative volatility in a bourse condition; internationally '50' has been kept as a critical point, if the RVI clocks over the critical point it enters bullish zone, else it'll be in the bearish zone:

$$\Gamma' = 100 \cdot \frac{\ddot{\upsilon}}{(\ddot{\upsilon} + \delta)}, \quad (1)$$

where $RVI = \Gamma'$, $\ddot{\upsilon}$ – Wilder's Smoothing of USD and δ – Wilder's Smoothing of DSD.

Welles Wilder developed Welles Wilder's Smoothing Average (WWS) that is part of the Relative Strength Index (RSI) indicator usage.

USD = if close > close (1) then SD, S else 0; 10 day SD is in use

DSD = if close < close (1) then SD, S else 0; 10 day SD is in use

S = specified period for the Standard Deviation of the close (as per Dorsey's suggestion, it should be 10 days).

N = specified selected smoothing period (as per Dorsey's suggestion, it should be 14 days).

The second key operator in this piece of calculation we found a path breaking analogy of 'viscosity' being present in stock exchanges as well. Richard Arms formulated 'Ease of Movement' (EMV) calculation where the concept is derived from the fluid-mechanics concept of viscosity (Arms, 1996). The calculation is basically a ratio, with numerator being "Distance" and denominator being "Box Ratio".

To understand the concept, we find that high positive values of EMV indicating price increase due to low volume. Less of supply is helping the demand to go high. In a completely contrasting scenario, we find highly negative values of EMV, since the price is dropping owing to low volume. That means thin trades are dampening the spirit of the traders following unlikely sales. This shows ability to buy or sale freely is quite important. In absence of the freeness of movement (i.e. buy or sale), the operator EMV can be truly chaotic. That free movement can well be linked with the concept of 'viscosity'. It's perceived to be difficult to move in a highly viscous material.

$$Distance = g = \frac{1}{2}(High + Low) - \frac{1}{2}(Prior\ High + Prior\ Low). \quad (2)$$

Volume and current high-low range form the *Box Ratio*, which is quite similar to Equivolume charts.

$$Box\ Ratio = BR = \frac{Volume/100{,}000{,}000}{High - Low} \quad (3)$$

$$EMV = \frac{Distance}{Box\ Ratio} = \frac{\frac{1}{2}(High + Low) - \frac{1}{2}(Prior\ High + Prior\ Low)}{(Volume/100{,}000{,}000)\ /\ (High - Low)} = \frac{g}{BR}. \quad (4)$$

Fluid transmission from a laminar into a turbulent flow all on a sudden was a daunting question in fluid mechanics till 1883, however, it was majorly resolved (Reynolds, 1901). Navier raised the query of 'liner laminar flow of liquids suddenly becoming chaotic and turbulent' 60 years before Reynolds finally tackled the same. Osborne Reynolds invented a unit less number, which in turn can represent the delta or change of fluid flow. The number was named 'Reynolds number' in the honor of such a great scientist. Experimentally observed facts prove that while a particle passes through a fluid, it experiences forces against the flow, referred as drag (T and R). According to the research conducted by Reynolds, the pressure drag (R) and the viscosity drag (T) are represented as (Reynolds, 1883):

$$R = C\frac{\rho\vartheta^2}{2}S,$$

$$T = B\eta\vartheta l,$$

where ρ – density of the fluid, S – cross-sectional area of the object, perpendicular to the direction of the fluid flow, C and B – dimensionless constants, ϑ – certain mean velocity of the fluid, η – fluid viscosity, l – linear dimension of the object.

$$\frac{R}{T} = \frac{C\rho\vartheta^2 S}{2B\eta\vartheta l}.$$

After assuming $C = 2B$ and $S = l^2$, finally,

$$Re = \frac{\rho\vartheta l}{\eta}.$$

Arriving at the final version of financial Reynolds number, we find it's having viscosity indicators as denominator and momentum indicators as numerator. "RVI" behaves quite alike momentum of particles having wave-particle dualism inside a defined quantum well, placed inside a finite Hilbert space. On the other hand, "EMV" is similar to viscosity of particles having wave-particle dualism inside a defined quantum well, placed inside a finite Hilbert space. This striking resemblance allows RVI as numerator and EMV as denominator. This in turn redefines Osborne Reynolds number for financial markets and bourses.

In practical terms, using 'RVI' and 'EMV', it becomes:

$$R_e = \frac{100 \cdot \ddot{\upsilon}}{\ddot{\upsilon} + BR} \div \frac{g}{BR}.$$

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This is how the explosive term in a stochastic time series emerges as a new face of volatility. This gets its name from the great Osborne Reynolds. If this explosive number breaches a certain critical value, then the embedded volatility is beyond control, else the volatility is present, but within control. In reality, Gaussian distributions are rare to find. Most distributions are found with fat tails in financial markets. Hence, it would certainly be imperative that financial Reynolds number distribution would also have fat tail embedded. Each tail (both positive and negative) comes up with its own tail probabilities. Tail probabilities are generally defined in three categories, namely Medium Tailed, Fat Tailed and Thin Tailed. This is primarily done by the function, defined as the survival or tail probability function i.e. $\dot{F}$ (Pape, 2007). It has been an existing knowledge that kurtosis 'k' of an entirely stochastic variable 'x' stands as a measurement of dispersion around the two extreme values $\mu \pm \sigma$, where μ is the mean expected value and σ remains as the standard deviation of 'x'. In a typical Gaussian distribution, 'k' remains in and around close vicinity of '3'. Higher 'k' indicates more mass in the tails. Hence, for all type of risk management purposes, it becomes quite essential to track. It would be interesting to note that 'extreme value theory' is linked to very high values of 'k'. Under such circumstances, an extreme deviation from the mean has been observed. This indicates to an asymptotic distribution of extreme order statistics. However, this remains in questions only with independent, identically distributed (iid) continuous random variables only. $V_n = max\{x_1, x_2, \dots \dots, x_n\}$ denotes maximum number of '*n*' sample observations of iid variables (Fisher & Tippett, 1928). Three non-degenerate limiting distributions for proper rescaled sample maxima V_n will exist within the limits of $n \to \infty$, according to this seminal work. They are denoted as Generalized Extreme Value distributions or 'GEV'.

A famous volatility finder (Wiggins, 1992) has tested two famous models for extreme value volatility namely 'high and low of local volatility of a geometric random walk' by Parkinson (1980) and 'open-close volatility measure' by Garman and Klass (1980). Wiggins has found that these models are far more reliable compared to their 'close to close' counterparts. As a final conclusion, Wiggins noted that for a specific value of single historical estimator has to be considered then extreme value method would be efficient, whereas close to close will work in a more sophisticated mathematical model. Since the current study, a single historical estimator is used (read as financial Reynolds number), thus extreme value calculations to find the decay component alpha become apt. Hence, the current study remains valid and finds a strong theoretical foothold from Wiggins's famous work.

Mathematically speaking:

1st Gumbel (GEV Type I): $G_I(x) = \exp\{-e^{-x}\}$, when $x \in R$,

2nd Fréchet (GEV Type II): $G_{II,\alpha}(x) = \exp\{-x^{-\alpha}\}$, when $\|x \ge 0$,

3rd Weibull (GEV Type III):
$G_{III,\alpha}(x) = \exp\{-(-x)^{\alpha}\}$, when $|x \le 0 + |x > 0$.

$|x$ is an indicator function and α is a positive parameter referred to as '*Tail Index*'.

If the Weibull and Gumbel hypotheses are observed to be completely rejected, but the Fréchet hypothesis is not rejected, then there could well be sound evidence for a power law distribution.

$\dot{F}(x) = P(X > x)$ of any stochastic variable X whose maxima could be defined by certain specific distribution functions $G(x)$. $G(x)$ stands for "Generalized Extreme Value" distribution.

$\dot{F}(x) = -\ln G(x)$, when $\ln \ln G(x) > -1$.

Hence, extending this concept to trail probabilities for the specified stochastic variable x:

1st Category: Medium Tail: $\dot{F}(x) = \exp(-x)$, when $|x \ge 0$,

2nd Category: Fat Tail: $\dot{F}(x) = x^{-\alpha}$, when $|x \ge 1$,

3rd Category: Thin Tail: $\dot{F}(x) = (-x)^{\alpha}$, when $\|-1 \le x \le 0$, where $\Vert(x)$ is an indicator function. Ideally all these functions indicate the patterns of decay for $\dot{F}(x)$. Asymptotic decay of the tails of any distribution ideally follows any of these three categories. Fat tail decay exponent or α generally follow a hyperbolic pattern.

Generalized Pareto Distribution comes up with a combination equation of all the three categories. It shows:

$$\dot{F}_{\xi}(x) = (1+\xi x)^{-\frac{1}{\xi}}.$$

In this case, the sign ξ acts as a classifier with following conditions:

Condition 1 $\xi \to 0$, hinting at medium tail (the distribution is moderate and it carries moderate chances of any major catastrophic event).

Condition 2 $\xi > 0$ hinting at fat tail (the distribution is riskier and more chances of any major catastrophic event).

Condition 3 $\xi < 0$ hinting at thin tail (the distribution is safer and less chances of any major catastrophic event).

Tail exponent or α too is related to the classifier ξ:

$$\alpha = \frac{1}{|\xi|}.$$

A 1975 paper from the University of Michigan changed the feat of calculation of tail probabilities (Hill, 1975). Hill came up with a maximum likelihood estimator for ξ.

$$\hat{\xi} = \frac{1}{k}\sum_{i=1}^{k}\{\ln x_{(n-i+1)} - \ln x_{(n-k)}\},$$

where $x(i)$ stands for the i order statistics and k denotes the number of n sample observations for which the asymptotic decay is calculated.

3. DISCUSSION

It has been noticed from Table 1 that the calculation of tail index or α has been carried out on a substantial big data for financial Reynolds number (Re) in high frequency domain compared to regular day closing basis. Thus, positive tail exponent or tail index is closer to 3 in the first case; as the observations are lesser the tail exponent value inches towards 4. This observation echoes another research using log returns of stock markets in regular and HFT domain (Pan & Sinha, 2008). Ample amount of work has established the fact of log returns across the globe follow power law (inverse cubic law to be more precise). Stock returns, especially abnormal returns both in the positive and the negative directions, were tested in time and again across large geographical boundaries. However, the same study on risk was rather unusually absent. Thus, the only differencing factor from other studies till date would be the econophysical volatility proxy under consideration, instead of log returns of daily close. However, this study depicts power law connection in risk proxy (volatility) as well. Risk and return were found to follow similar power law connection. Stock markets are usually a specific form of long memory process having strong correlations with its lags. Such high degree of autocorrelation indicates higher degree of predictability. In finance, long memory in price volatility has been observed both for stocks (Ding at al., 1993) and exchange rates and in trading volume (Velasco, 2000). Hence, they behave in a similar way to the physical systems. Hyperbolic discounting in the form of asymptotic corrections in the tails does indicate fair traces of psychological decision making (Farmer, Doyne, & Geanakoplos, 2011).

Table 1. Comparison of the power law exponent α of the cumulative distribution function for various index based volatility proxy (financial Reynolds number)

Volatility representatives	Δt	Positive tail exponent, α
Re regular Nifty	1 day	3.98
Re high frequency Nifty	1 minute	3.55

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CONCLUSION

According to Table 1, financial Reynolds number observations were substantially higher in case of ReHFT, resulting in a more accurate positive tail exponent or α. The observation points were substantially lower for regular Re in comparison to ReHFT. This shows significant difference in tail exponent or α value. This unique observation echoes studies (Pan & Sinha, 2008) on Indian indices in the past. However, that study was focused on log returns generated from index observations, hence it would be distinctly different compared to the current study. Financial Reynolds number (both Re and ReHFT) represents "Risk". Hence it would be appropriate to quote that both risk and return follow power law in their tail exponent. Interestingly both risk and return follow inverse cubic law as well. Hence, the fundamental attributes in both risk and return remain similar in the extreme events.

IMPLICATIONS AND CONCLUDING NOTE

'Risk' in financial time series has a direct connection with the decay of asset returns, as well as volatility movement. Returns of various assets were found to decay in an asymptotic manner in the fat tail of its respective distribution. This study confirms risk of assets (bourse as underlying asset) to follow the same. Power law driven decay in fat tails of both risk and return prove that the cardinal traits of both follow similar rationale. Empirical testing for power laws is quite difficult due to the very fact that a power law is an asymptotic property. Thus the probability of a large regular real-time data set entirely inside the asymptotic regime could well be a far cry. It has been observed in the past that certain power law converge very quickly hence for most of the regime the power law is a good approximation. However in many instances the power law converge very slowly. It may produce a pseudo accurate result unless there is a very large sample of data. Recent studies have achieved this precision of prediction (asymptotic decay) by studying high frequency data, rather welcomingly involving millions of observations (Farmer, Doyne, & Geanakoplos, 2011).

Every research is supposed to extend the body of existing knowledge. This humble piece of research empirically proves that 'asset returns' and 'risk proxy' on the same underlying (read as 'CNX Nifty High Frequency Trading') have similar fat tail exponent or 'α' in the tails of their distribution. The first one has 3.33 (Pan & Sinha, 2008) and the second one has 3.55. This could provide enough impetus to the policymakers in the variable income market.

REFERENCES

1. Abhyankar, A., Copeland, L. S., & Wong, W. (1995). Moment Condition Failure in High Frequency Financial Data: Evidence from the S&P 500. *Applied Economics Letters, 2*(8), 288-290. https://doi.org/10.1080/135048595357258
2. Arms, Richard W. (1996). *Trading without Fear* (1st ed.). John Wiley and Sons.
3. Bachelier, L. (1900). Théorie de La Sp' Eculation. *Annales scientifiques de l'École Normale Supérieure, 17,* 21-86. Retrieved from http://eudml.org/doc/81146
4. Botta, F., Moat, H. S., Stanley, H. E., & Preis, T. (2015). Quantifying Stock Return Distributions in Financial Markets. *PLoS ONE, 10*(9), 1-10. https://doi.org/10.1371/journal.pone.0135600
5. Bree, D. S., & Joseph, N. L. (2013). Fitting the Log Periodic Power Law to Financial Crashes: A Critical Analysis. *ArXiv:1002.1010v2*, 1-30. Retrieved from https://www.researchgate.net/publication/45899349_Fitting_the_Log_Periodic_Power_Law_to_financial_crashes_a_critical_analysis
6. Ding, Z., Granger, C. W. J., & Engle, R. F. (1993). A Long Memory Property of Stock Returns and a New Model. *Journal of Empirical Finance, 1*(1), 83-106. https://doi.org/10.1016/0927-5398(93)90006-D
7. Dorsey, D. (1993). *Technical Analysis of Stocks and Commodities.* New York.
8. Fama, E. F. (1965). The Behavior of Stock-Market Prices. *The Journal of Business, 38*(1), 34-105. Retrieved from http://www.jstor.org/stable/2350752
9. Farmer, D., & Geanakoplos, J. (2011). Power Laws in Economics and Finance. *SSRN Electronic Journal,* 1-57. https://doi.org/10.1146/annurev.economics.050708.142940
10. Fisher, R. A., & Tippett, L. H. C. (1928). Limiting Forms of the Fre Quency Distribution of the Largest or Smallest Member of a Sample. *Proceedings of the Cambridge Philosophical Society, 24*(2), 180-290. https://doi.org/10.1017/S0305004100015681

11. Gabaix, X. (2016). Power Laws in Economics: An Introduction. *Journal of Economic Perspective, 30*(1), 185-206. Retrieved from https://pubs.aeaweb.org/doi/pdf/10.1257/jep.30.1.185

12. Gardes, L., & Girard, S. (2008). A Moving Window Approach for Nonparametric Estimation of the Conditional Tail Index. *Journal of Multivariate Analysis, 99*(10), 2368-2388. https://doi.org/10.1016/j.jmva.2008.02.023

13. Gardes, L., & Girard, S. (2010). Conditional Extremes from Heavy-Tailed Distributions: An Application to the Estimation of Extreme Rainfall Return Levels. *Extremes, 13*(2), 177-204. Retrieved from https://link.springer.com/article/10.1007/s10687-010-0100-z

14. Garman, M. B., & Klass, M. J. (1980). On the Estimation of Security Price Volatilities from Historical Data. *Journal of Business, 53*(1), 67-78. Retrieved from https://www.jstor.org/stable/2352358?seq=1#page_scan_tab_contents

15. Gibrat, R. (1931). *Les Inégalités Économiques; Ap- Plications: Aux Inégalités Des Richesses, à La Con- Centration Des Entreprises, Aux Populations Des Villes, Aux Statistiques Des Familles, Etc., d'une Loi Nouvelle, La Loi de l'effet Proportionnel. Libraire Du Recueil Sirey*. Paris.

16. Hill, B. (1975). A Simple General Approach to Inference about the Tail of a Distribution. *The Annals of Statistics, 3*(5), 1163-1174. https://doi.org/10.1214/aos/1176343247

17. Jansen, D. W., & De Vries, C. G. (1991). On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective. *The Review of Economics and Statistics, 73*(1), 18-24. Retrieved from http://www.jstor.org/stable/2109682

18. Kelly, B. (2014). The Dynamic Power Law Model. *Extremes, 17*(4), 557-583. https://doi.org/10.1007/s10687-014-0193-x

19. Kyle, A., & Obizhaeva, A. (2013). *Large Bets and Stock Market Crashes* (AFA 2013 San Diego Meetings Paper). Retrieved from https://pages.nes.ru/aobizhaeva/Kyle_Obizhaeva_crashes.pdf

20. Kyle, A., & Obizhaeva, A. (2016). Market Microstructure Invariance: Empirical Hypotheses. *Econometrica, 84*(4), 1345-1404. https://doi.org/10.3982/ECTA10486

21. Laloux, L., Potters, M., Cont, R., Aguilar, J. P., & Bouchaud, J. P. (1998). Are Financial Crashes Predictable? *Europhysics Letters, 45*, 1-5. https://doi.org/10.1209/epl/i1999-00122-9

22. Loretan, M., & Philips, P. (1994). Testing the Covariance Stationarity of Heavy-Tailed Time Series: An Overview of the Theory with Applications to Several Financial Datasets. *Journal of Empirical Finance, 1*(2), 211-248. https://doi.org/10.1016/0927-5398(94)90004-3

23. Lux, T. (1996). The Stable Paretian Hypothesis and the Frequency of Large Returns. *Applied Financial Economics, 6*, 463-475. https://doi.org/10.1080/096031096333917

24. Mandelbrot, B. B. (1963). The Variation of Certain Speculative Prices. In Fractals and Scaling in Finance. *Journal of Business, 36*(4), 394-419. https://doi.org/10.1086/294632

25. Mantegna, R. N., & Stanley, H. E. (1995). Scaling Behaviour in the Dynamics of an Economic Index. *Nature, 376*, 46-49. https://doi.org/10.1038/376046a0

26. Nirei, M., Stachurski, J., Takaoka, K., & Watanabe, T. (2018). *Herding and Power Laws in Financial Markets Herding and Power Laws in Financial Markets* (CARF-F-434, F-434). Retrieved from https://www.carf.e.u-tokyo.ac.jp/admin/wp-content/uploads/2018/05/F434.pdf

27. Pan, R. K., & Sinha, S. (2008). Inverse-Cubic Law of Index Fluctuation Distribution in Indian Markets. *Physica A: Statistical Mechanics and Its Applications, 387*(8-9), 2055-2065. https://doi.org/10.1016/j.physa.2007.11.031

28. Pape, B. (2007). Asset Allocation and Multivariate Position Based Trading. *Journal of Economic Interaction and Coordination, 2*(2), 163-193. https://doi.org/10.1007/s11403-007-0021-3

29. Parkinson, M. (1980). The Extreme Value Method for Estimating the Variance of the Rate of Return. *Journal of Business, 53*(1), 61-65. Retrieved from https://www.jstor.org/stable/2352357

30. Plerou, V., Gopikrishnan, P., Gabaix, X., & Stanley, H. E. (2004). On the Origin of Power-Law Fluctuations in Stock Prices. *Quantitative Finance*, 11-15. Retrieved from https://www.researchgate.net/publication/1942178_On_the_Origin_of_Power-Law_Fluctuations_in_Stock_Prices

31. Reynolds, O. (1883). An Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and of the Law of Resistance in Parallel Channels. *Proceedings of the Royal Society of London*, 84-99.

32. Reynolds, O. (1901). Papers on Mechanical and Physical Subjects. *Reprinted from Various Transactions and Journals, 2*, 1881-1900. Retrieved from https://archive.org/details/papersonmechanic03reynrich/page/n8

33. Rosen, S. (1981). The Economics of Superstars. *The American Economic Review, 71*(5), 845-858. https://www.jstor.org/stable/1803469

34. Sornette, D. (2003). *Why Stock Markets Crash, Critical Events in Complex Financial Systems*. Princeton University Press.

35. Velasco, C., & Lobato, I. N. (2000). Long-Memory in Stock-Market Trading Volume. *Journal of Business & Economic Statistics, 18*(4), 410-427. https://doi.org/10.2307/1392223

36. Wiggins, J. B. (1992). Estimating the Volatility of S&P 500 Futures Prices Using the Extreme Value Method. *Journal of Futures Markets, 12*, 265-273. https://doi.org/10.1002/fut.3990120303

37. Wu, L. (2006). Dampened Power Law: Reconciling the Tail Behavior of Financial Security Returns. *Journal of Business, 79*(3), 1445-1473. Retrieved from http://faculty.baruch.cuny.edu/lwu/papers/dpl.pdf

http://dx.doi.org/10.21511/imfi.16(1).2019.23