“Loss portfolio transfer treaties within Solvency II capital system: a reinsurer’s point of view”

AUTHORS	Nicolino Ettore D’Ortona Gabriella Marcarelli Giuseppe Melisi

ARTICLE INFO	Nicolino Ettore D’Ortona, Gabriella Marcarelli and Giuseppe Melisi (2020). Loss portfolio transfer treaties within Solvency II capital system: a reinsurer’s point of view. *Insurance Markets and Companies*, *11*(1), 1-10. doi:10.21511/ins.11(1).2020.01
DOI	http://dx.doi.org/10.21511/ins.11(1).2020.01
RELEASED ON	Thursday, 26 March 2020
RECEIVED ON	Wednesday, 08 January 2020
ACCEPTED ON	Friday, 20 March 2020

JOURNAL	"Insurance Markets and Companies"
ISSN PRINT	2616-3551
ISSN ONLINE	2522-9591
PUBLISHER	LLC “Consulting Publishing Company “Business Perspectives”
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
10	2	8

BUSINESS PERSPECTIVES

LLC "CPC "Business Perspectives"
Hryhorii Skovoroda lane, 10,
Sumy, 40022, Ukraine

www.businessperspectives.org

Nicolino Ettore D'Ortona (Italy), **Gabriella Marcarelli** (Italy), **Giuseppe Melisi** (Italy)

LOSS PORTFOLIO TRANSFER TREATIES WITHIN SOLVENCY II CAPITAL SYSTEM: A REINSURER'S POINT OF VIEW

Abstract

Loss portfolio transfer (LPT) is a reinsurance treaty in which an insurer cedes the policies that have already incurred losses to a reinsurer. This operation can be carried out by an insurance company in order to reduce reserving risk and consequently reduce its capital requirement calculated, according to Solvency II. From the viewpoint of the reinsurance company, being a very complex operation, importance must be given to the methodology used to determine the price of the treaty.

Following the collective risk approach, the paper examines the risk profiles and the reinsurance pricing of LPT treaties, taking into account the insurance capital requirements established by European law. For this purpose, it is essential to calculate the capital need for the risk deriving from the LPT transaction. In the case analyzed, this requirement is calculated under Solvency II legislation, considering the measure of variability determined via simulation. This quantification was also carried out for different levels of the cost of capital rate, providing a range of possible loadings to be applied to the premium.

In the case of the Cost of Capital (CoC) approach, the results obtained provide a lower level of premium compared to the percentile-based method with a range between 2.69% and 1.88%. Besides, the CoC approach also provides the advantage of having an explicit parameter, the CoC rate whose specific level can be chosen by the reinsurance company based on the risk appetite.

Keywords best estimate liabilities, reserving risk, market risk, insurance pricing, Cost of Capital

JEL Classification G22, C58

Received on: 8th of January, 2020
Accepted on: 20th of March, 2020
Published on: 26th of March,2020

Nicolino Ettore D'Ortona, Ph.D., Associate Professor, Department of Law, Economics, Management and Quantitative Methods, University of Sannio, Italy.

Gabriella Marcarelli, Ph.D., Assistant Professor, Department of Law, Economics, Management and Quantitative Methods, University of Sannio, Italy.

Giuseppe Melisi, Ph.D., Adjunct Professor, Department of Law, Economics, Management and Quantitative Methods, University of Sannio, Italy.

Conflict of interest statement:
Author(s) reported no conflict of interest

INTRODUCTION

In a loss portfolio transfer, a reinsurer assumes and accepts an insurer's existing open and future claim liabilities through the transfer of the insurer's loss reserves. The liabilities may already exist, such as claims that have been processed but not yet paid, or may soon appear, such as incurred but not reported claims (IBNR).

In relation to the commitments undertaken, the reinsurer receives a reinsurance premium whose value is commensurate with the average current value of the transferred liabilities (discounted claims reserve)[1].

For this reason, the pricing process for this operation cannot be separated from the evaluation of the insurance company's technical reserves and must take into account all the risks associated with the settlement of future claims.

1 These transactions generally reduce the solvency capital requirement (SCR) lower than the reduction in own funds due to the payment of a transfer premium, i.e., the price of the LPT. Furthermore, if the forecasts on the timing distribution of compensation and the amount of the payments are correct, the reinsurer can obtain a profit from this operation.

http://dx.doi.org/10.21511/ins.11(1).2020.01

In this paper, the quantitative evaluations are obtained through a stochastic model and from a reinsurer's point of view. Besides, different approaches have been proposed and compared to identify a method for determining the premium consistent with the risk profile of the operation and with the risk appetite of the reinsurance company.

Besides, by dividing insurance risk and financial risk, the cost of capital is calculated considering reserving risk and market risk separately.

1. LITERATURE REVIEW

The definition and analysis of the methodologies of the insurance companies' technical reserves have always been at the center of studies and researches in the actuarial literature due to the importance of this balance sheet item and the difficulty of obtaining an objective assessment by the assessor. In particular, the problems faced are many: the determination of a stochastic method to provide a measure of the variability of the distribution of the claims reserve; the quantification of the reserving risk; the study of the goodness of the applied methodologies.

Regarding the stochastic methods and simulation approaches, it is worth mentioning the monograph by Daykin, Pentikainen, and Pesonen (1993), which provides innovative elements with respect to the classical theory, introducing stochastic models and simulation techniques for insurance.

Further fundamental studies in the actuarial literature on this topic are represented by Mack (1993) and England and Verrall (2006), which provide two of the most commonly used stochastic methods for the determination of the variability of the distribution of the claims reserve and repeatedly repeated in subsequent works by other authors.

These methods are often used in actuarial professional practice to quantify the risk of reservation, in relation to its component linked to the ultimate claim cost. As for the financial risk linked to technical provisions, the papers of Butsic (1994), Parker (1994), and Wilkie (1984) have been considered, which provide different approaches to quantify this risk, also through simulations.

Simulation procedures for the comparison of reservation methods have been studied by Peintikainen and Rantala (1992), providing an overview of different approaches that can be used for these evaluations. In addition, the same authors in another paper (1986) define the runoff risk relating to the claims reserve, also used by D'Ortona and Melisi (2014) to compare between the main methods used in actuarial practice.

As for the LPT reinsurance treaties, there are not many studies in the literature, except for the work of Mcnair et al. (2002) who analyze different types of LPT from many points of view: regulatory authorities, auditors, tax authorities, as well as those who do the pricing.

In this paper, through the aforementioned stochastic approaches, and also considering the financial component as a risk element, the LTP is analyzed from the reinsurer's point of view, using a pricing model that also takes into account the capital need of the operation and defines a safety loading related to that cost.

2. METHOD

Before analyzing the methodology of the study used for the determination of the reinsurance premium, it is necessary to classify and define all the risks assumed by the reinsurance company through the LTP.

Indeed, the evaluation convenience of the treaty must take into account:

- the ultimate claim cost risk;
- the timing risk;
- the interest rate risk;
- the expense risk.

The ultimate claim cost risk concerns the total value of the compensation required for the liquidation of the individual claims generations subject

http://dx.doi.org/10.21511/ins.11(1).2020.01

to the treaty. The value of the price of the damaged goods has an impact on this value, and, for claims relating to personal injury, the evolution of the regulatory provisions concerning biological damage. A greater (lower) level of the ultimate cost than the expected one represents a disadvantageous (advantageous) situation for the reinsurer and vice versus advantageous (disadvantageous) for the insurer.

The timing risk concerns the timing distribution of compensation and consequently relates to both the amount of deferred payments and the length of the period within which the claims of a generation are definitively liquidated. An acceleration of the claims settlement process with respect to the expected time development of payments constitutes a disadvantageous situation for the reinsurer and, vice versa, for the insurer.

The interest rate risk concerns the difference between the return made by the reinsurer in the investments of the reserve, and the discount rate applied in the calculation of the premium.

The expense risk concerns the differences between the present costs of administration and settlement of the object of the treaty claims and the relative loads present in the premium.

2.1. Claim costs

One assumes that the portfolio subject to reinsurance is composed of a homogeneous risk class and that in each of accident year i (i = 0, 1, …, t), the random claim settlement is given by the sum of a random number of claims, each one subject to a single claim settlement, as follows:

$$\tilde{X}(i)=\sum_{k=0}^{\tilde{N}(i)}\tilde{Y}_k(i), \tag{1}$$

where i = 0, 1, …, t; $\tilde{N}(i)$ represents the total number of claims incurred in the year generation i, each subject of a random settlement $\tilde{Y}_k(i)$. It should be noted that the uncertainty of the number of claims at the end of the accident year depends mainly on the IBNR claims, while the uncertainty of individual compensation covers all claims, both those not yet reported and those reported but not yet paid at the end of the year.

As compensation for each claim can be made in the same year of occurrence or in subsequent years, with a single payment or multiple payments, the aggregated claims cost can be represented by this formula:

$$\tilde{X}(i)=\sum_{j=0}^{t}\tilde{X}(i,j), \tag{2}$$

where i = 0, 1, …, t; $\tilde{X}(i,j)$ represents the amount paid for settlements regarding claims incurred during the accident year i and settled after j years; so i is the accident year and t measures the maximum duration of deferral of the final claim compensation.

At the time of stipulation of the treaty t, the information already registered is as follows:

$$\left\{\tilde{X}(i,j): i=0,1,...,t; j=0,1,...,t-i\right\}.$$

While the components to be estimated:

$$\left\{\tilde{X}(i,j): i=0,1,...,t; j=t-i+1,...,t\right\},$$

so the formula for the current random value of future liquidations relating to open claims (or IBNR) is as follows:

$$\tilde{Z}(i)=\sum_{j=t-i+1}^{t}\tilde{X}(i,j)\tilde{v}(t,i+j), \tag{3}$$

where $\left\{\tilde{y}(t,t+k),\ k=1,2...,t\right\}$ is the term structure of price of an k-period zero-coupon bond.

For all the generations, the following formula is obtained:

$$\tilde{Z}=\sum_{i=1}^{t}\tilde{Z}(i). \tag{4}$$

2.2. Best estimate of liabilities and reinsurance premium

From the previous, the formula for the determination of the best estimate of liabilities (BEL) is as follows:

$$E\left[\tilde{Z}(i)\right]=\sum_{j=t-i+1}^{t}E\left[\tilde{X}(i,j)\tilde{v}(t,i+j)\right], \tag{5}$$

i = 0, 1, ..., t,

$$E\left[\tilde{Z}\right]=\sum_{i=1}^{t}E\left[\tilde{Z}(i)\right]. \quad (6)$$

The value of BEL of the generations or of the portfolio subject to the treaty can be considered as the basis for calculating the reinsurance premium. However, to determine the reinsurance premium, safety loads must be defined based on an analysis of the risks associated with the timing and extent of future compensation, and the effects of the disturbances affecting the yield structure used for the calculation of BEL.

The uncertainty of the timing distribution of the flows and the rates of return set the insurance risk and the investment risk for the reinsurer. To quantify these risks, one refers to this decomposition of the variance of Z:

$$V\left[\tilde{Z}(i)\right]=E\left[V\left[\tilde{Z}(i)\,|\,\tilde{v}\right]\right]+ \\ +V\left[E\left[\tilde{Z}(i)\,|\,\tilde{v}\right]\right], \quad (7)$$

where the first component $E\left[V\left[\tilde{Z}(i)\,|\,\tilde{v}\right]\right]$ represents a measure of insurance risk representing an average, on the possible profiles of the term structure of the discount factors, of the variability of the current random value $\tilde{Z}(i)$ caused by the uncertainty of future compensation flows; the second component $V\left[E\left[\tilde{Z}(i)\,|\,\tilde{v}\right]\right]$ represents a measure of investment risk, evaluating the variability of the current random value $\tilde{Z}(i)$ caused by the uncertainty of the term structure of rates, in a context in which the effect of the uncertainty of the compensation flows is considered on average.

2.3. Assumptions and simulation method

The aggregate cost follows a Poisson composed distribution.

The number of claims $n(i)=nI_n(i)$, where $I_n(i)$ is the growth annual index of portfolio and n is the average number of claims.

The claims number occurred in year *i* and paid after *j* years is obtained from:

$\tilde{n}(i,j)=n(i)\tilde{q}(i,j)\tilde{g}_n(j)$, where $\tilde{g}_n(j)$ is the temporal distribution function of the number of claims and indicates the probability that a claim occurred in year *i* is liquidated in year *j*, while $\tilde{q}(i,j)$ is the structure function that modifies the average frequency of annual claims paid. Assuming the same structure function throughout the generation, in particular by assuming an autoregressive process for the structure function, the amounts of compensation in the subsequent years of development are correlated.

The single claim cost for generation *i* paid for after *j* years:

$$\tilde{Y}(i,j)=m\tilde{I}_m(i)\tilde{g}_m(j), \quad (8)$$

where $i = 0, 1, \ldots, t, j = 0, 1, \ldots, t$; m is the average cost of the generation compensation for the base year $i = 0$ and paid in the same year of occurrence; $\tilde{I}_m(i)$ is a growth index of the average cost of claims due to inflation or other potential factors from the base year to year *i*; $\tilde{g}_m(j)$ is a differentiating factor of the average cost of claims according to the year of development:

$$\tilde{g}_m(j)=g_m(0)\frac{\tilde{I}_m(i+j)}{\tilde{I}_m(i)}, \quad (9)$$

where $i = 0, 1, \ldots, t, j = 0, 1, \ldots, t$.

Total amount of claims occurring in *i* and paid after *j* years:

$$\tilde{X}(i,j)=nm\tilde{I}_n(i)\tilde{I}_m(i)\tilde{g}_X(j)\tilde{q}(i,j), \quad (10)$$

where $\tilde{g}_X(j)=g_n(j)\tilde{g}_m(j)$ governs the aggregated cost of claim distribution within each generation.

The term structure of the discount values is represented by

$$\left\{\tilde{v}(t,t+k)=e^{-\tilde{y}(t,t+k)},k=01,2,...,t\right\}$$
$$\text{with } \tilde{y}(t,t+k)=\int_0^k\delta(t,t+u)\,du$$

is the cumulative function of the instantaneous intensity of interest $\delta(t,s)$, and it is represented by a deterministic component δk and a stochastic component $\tilde{U}(k)$, so:

$$\tilde{y}(k)=\delta k+\tilde{U}(k). \quad (11)$$

http://dx.doi.org/10.21511/ins.11(1).2020.01

Independence between $\tilde{X}(i,j)$ and $\tilde{v}(t,t+k)$.

The simulation has been applied to a hypothetical portfolio of an insurer.

First, it was arbitrarily chosen n, the average number of claims, and the three central moments of the single claim cost distribution. So, it is simulated the claims number incurred in the year, with the inverse of Anscombe transformation, and the total amount of claims, with Wilson-Hilferty formula for a Poisson composed distribution.

For each generation, the average number of claims occurring during the year was determined using the formula $n(i) = nI_n(i)$ and placing $I_n(i) = (1+i_n)^i$ i = 1, 2, ...,t.

Furthermore, an autoregressive process for the structure function was used

$$\tilde{q}(i,j) = a_q + b_q \tilde{q}(i,j-1) + \tilde{\varepsilon}_q$$

with $\tilde{q}(i,0) = 1$ and $\tilde{\varepsilon}_q = N(0,\sigma_q)$ (12)

and one simulated r = 4,000 trajectories and so, for each generation, the temporal distribution of the number of claims.

From the r trajectories that one indicates with

$$\{\tilde{q}(i,j;h):\ \mathrm{i} = 1,2,...,t;\ j = t-i+1,...,t\},$$
$$\mathrm{h} = 1,2,...,r,$$

assigning the following values to the probabilities

$$\overline{g}_n = \{0,22;\ 0,18;\ 0,15;\ 0,12;\ 0,10;\ 0,08;$$
$$0,06;\ 0,04;\ 0,027;\ 0,016;\ 0,007\},$$

hypothesized independent of the year of claims generation, the temporal distribution of the number of claims is obtained through the following relation: $n(i,j;h) = n(i)q(i,j;h)g_n(j)$.

For the cost of the individual claims that increase the inflation, an autoregressive process is used to derive the trajectories of the rate of inflation.

$$\tilde{i}(\tau+1) = \\ = \max\{i_m + b_m[\tilde{i}(\tau) - i_m] + \tilde{\varepsilon}_{\inf}; i_{\min}\}, \quad (13)$$

where $i_{\min}$ is the minimum inflation rate, i_m is the average inflation rate, $\tilde{i}(0) = i_m$ and $\tilde{\varepsilon}_{\inf} = N(0;\sigma_{\inf})$.

Therefore, from this relation

$$n(i,j;h) = n(i)q(i,j;h)g_n(j).,$$

the flows relating to future compensation for each generation and for each trajectory have been obtained:

$$\{X(i,j;h):\ \mathrm{i} = 1,\ 2,...,t;\ j = t-i+1,...,t\},$$
$$\mathrm{h} = 1,2,...,r$$

As previously mentioned, the trajectories of the term structure of interest rate have been simulated assuming that the random component of the cumulative function of the instantaneous intensity of interest is a process of Ornstein-Uhlenbeck:

$$d\tilde{U}(k) = -\alpha\tilde{U}(k)dk + \sigma dW_k$$

with the initial condition $\tilde{U}(0) = 0$ and where W_k is a Wiener process.

Obtained the cash flows and the trajectories for the discount rates, the present value of compensation for each generation and for each trajectory have been calculated.

$$Z(i;h;u) = \sum_{j=t-i+1}^{t} X(i,j;h)v(t,i+j;u), \quad (14)$$
$$\mathrm{i} = 1,\ 2,\ ...,\ \mathrm{t}.$$

Therefore, the expected value (namely BEL), the variance, the values of the distribution function, and the different components of the variance have been determined.

$$E[Z(i;u)] = \frac{1}{r}\sum_{h=1}^{r} Z(i;h;u), \quad (15)$$
$$\mathrm{i} = 1,\ 2,\ ...,\ \mathrm{t};\ \mathrm{u} = 1,\ 2,...,s,$$

$$V[Z(i;u)] = \frac{1}{r}\sum_{h=1}^{r} Z^2(i;h;u) - \left[\frac{1}{r}\sum_{h=1}^{r} Z(i;h;u)\right]^2, \quad (16)$$
$$\mathrm{i} = 1,\ 2,\ ...,\ \mathrm{t};\ \mathrm{u} = 1,\ 2,...,s,$$

$$E\left[V\left[\tilde{Z}(i)\,|\,\tilde{v}\right]\right]= \\ =\frac{1}{s}\sum_{u=1}^{s}\left\{\frac{1}{r}\sum_{h=1}^{r}Z^2(i;h;u)-\left[\frac{1}{r}\sum_{h=1}^{r}Z(i;h;u)\right]^2\right\}, \quad (17)$$

$$V\left[E\left[\tilde{Z}(i)\,|\,\tilde{v}\right]\right]= \\ =\frac{1}{s}\sum_{u=1}^{s}\left(\frac{1}{r}\sum_{h=1}^{r}Z(i;h;u)\right)^2- \\ -\left[\frac{1}{s}\sum_{u=1}^{s}\left(\frac{1}{r}\sum_{h=1}^{r}Z(i;h;u)\right)\right]^2, \quad (18)$$

$$F_{Z(i)}(z)=\frac{C(i;z)}{r+s}, \quad (19)$$

where

$$C(i;z)=\sum_{h}\sum_{u}y^{(h;u)}y^{(h;u)}= \\ =\begin{cases}1, Z(\mathrm{i;h;u})\le z\\ 0, \text{otherwise}\end{cases}. \quad (20)$$

Here we show the parameters of models relating to the insurance and financial components:

Table 1. Base scenario: hypothesis[2]

Claims number	N	10,000.00
Growth annual index of the portfolio	i_n	0.0100
Claims cost	α_1	0.0060
	α_2	0.0010
	α_3	0.0001
Sector inflation	i_{min}	0.0250
	i_m	0.0400
	a_m	0.0500
	b_m	0.0700
Structure function	a_q	0.4000
	b_q	0.6000

3. RESULTS

The simulative procedure described was applied to a hypothetical enterprise. In particular, based on the previous assumptions, 4,000 square matrices $\|X(i,j;h)\|$ of order t+1 = 11 were generated. For each trajectory, h, the components

$$X(i,j;h)=X(i,j;1):\ \mathrm{i}=0,\ 1,\ 2,...,t; \\ j=0,\ ...,t-i,\ \mathrm{h}=1,\ 2,...,r.$$

represent the information recorded by the insurance company at time t, and have been identified with the corresponding values of the matrix obtained in the first simulation (h = 1); while the components

$$\{X(i,j;h)=X(i,j;1):\ \mathrm{i}=1,\ 2,...,t;\}, \\ j=t-i+1,...t\},\ \mathrm{h}=1,\ 2,...,r$$

represent the values of the elements of the lower triangle of the matrix.

S = 1,000 term structures of discount rates were then generated to estimate the values of the distribution function of the random variable Z and $Z(i)$ and its moments.

The first result is the main statistics on the current value of compensation for each generation.

Table 2. Statistics on the current value of compensation

i	E[Z(i)]	V[Z(i)]	V[E[Z(i)|v]]	E[V[Z(i)|v]]
1	7,360	147,964	5,382	142,581
2	24,326	1,388,490	61,563	1,326,927
3	53,162	5,737,390	314,222	5,423,167
4	95,975	18,260,295	1,070,246	17,190,049
5	161,248	52,902,709	3,060,080	49,842,630
6	248,581	116,472,694	7,450,415	109,022,279
7	358,328	249,157,336	15,920,166	233,237,170
8	492,238	426,482,192	30,906,814	395,575,378
9	660,624	765,466,903	56,258,771	709,208,132
10	860,448	1,397,536,849	96,662,761	1,300,874,088
Total	2,962,289	5,666,136,326	1,078,972,788	4,587,163,538

Figures 1 and 2 show the probability density function and the cumulative distribution function of future compensation for the whole of generation Z.

2 In the present work, the hypotheses assumed in Peintikainen and Rantala (1992) are used.

http://dx.doi.org/10.21511/ins.11(1).2020.01

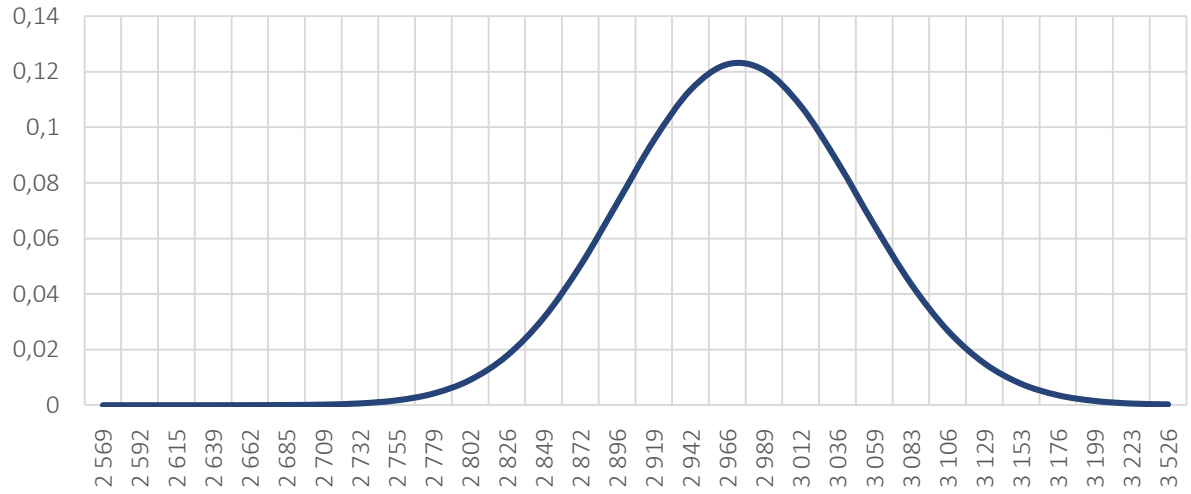

Figure 1. Probability density function of current random value of future compensation for the whole of generation *Z*

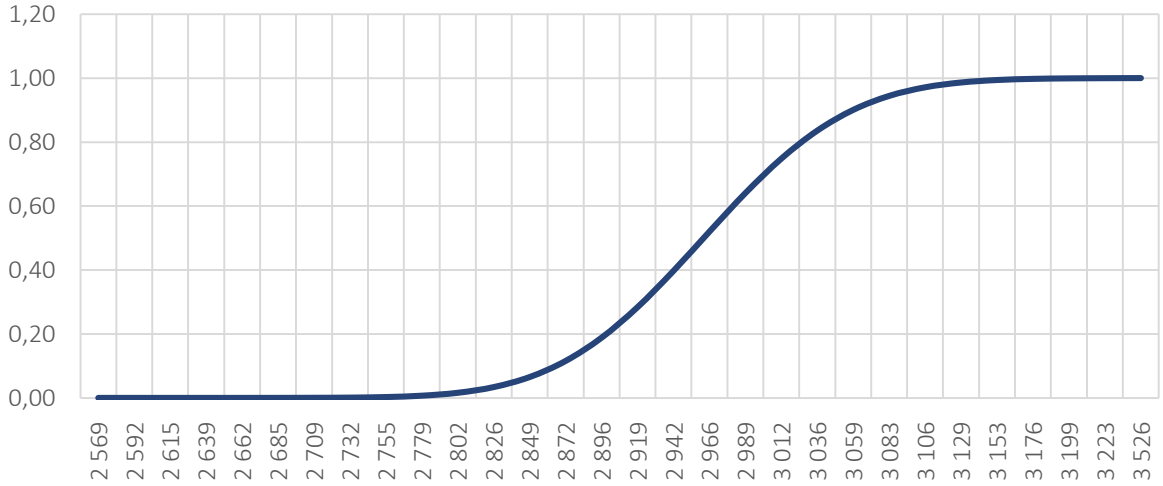

Figure 2. Cumulative distribution function of generation *Z*

Table 3. Cumulative distribution function of generation *Z* – *F(z)*

z	F(z)	z	F(z)
2,568,569	0.00000003	2,919,037	0.28378013
2,591,934	0.00000021	2,942,402	0.39731176
2,615,298	0.00000110	2,965,766	0.52002814
2,638,663	0.00000531	2,989,131	0.64058456
2,662,027	0.00002286	3,012,495	0.74829221
2,685,392	0.00008827	3,035,860	0.83585477
2,708,756	0.00030638	3,059,225	0.90066546
2,732,121	0.00095787	3,082,589	0.94436463
2,755,485	0.00270389	3,105,954	0.97122064
2,778,850	0.00690945	3,129,318	0.98627254
2,802,215	0.01602875	3,152,683	0.99397002
2,825,579	0.03385971	3,176,047	0.99756340
2,848,944	0.06534762	3,199,412	0.99909519
2,872,308	0.11563950	3,222,776	0.99969154
2,895,673	0.18838363	3,526,362	1.00000000

Table 4 shows the coefficient of variation and the relative levels of investment risk ("CoV Fin") and insurance risk ("CoV Ins") for each generation.

Table 4. Coefficient of Variation (CoV) of BEL (Best Estimate Liabilities)

i	CoV Tot	CoV Fin	CoV Ins
1	0.0523	0.0100	0.0513
2	0.0484	0.0102	0.0474
3	0.0451	0.0105	0.0438
4	0.0445	0.0108	0.0432
5	0.0451	0.0108	0.0438
6	0.0434	0.0110	0.0420
7	0.0441	0.0111	0.0426
8	0.0420	0.0113	0.0404
9	0.0419	0.0114	0.0403
10	0.0434	0.0114	0.0419
Total BEL	0.0254	0.0111	0.0229

3.1. Safety loading of premium

With the distribution function, it is possible to calculate the safety loading to be applied to the reinsurance premium. Table 5 shows the average value of BEL and 95th percentile that can represent the reinsurance premium with a safety loading equal to differences between the percentile and the average value.

Table 5. BEL and 95th percentile

i	E(BEL)	95th percentile	95th percentile/ E(BEL) – 1
1	7,360	7,974	8.34%
2	24,326	26,306	8.14%
3	53,162	57,419	8.01%
4	95,975	103,570	7.91%
5	161,248	173,525	7.61%
6	248,581	266,910	7.37%
7	358,328	385,322	7.53%
8	492,238	527,432	7.15%
9	660,624	706,200	6.90%
10	860,448	923,055	7.28%
Total BEL	2,962,289	3,082,589	4.06%

3.2. Safety loading of premium with CoC approach

Table 6 shows the results obtained.

Table 6. Premium with CoC approach

CoC	6%	8%	10%
Safety loading	37,327	49,770	62,212
Premium	2,999,617	3,012,059	3,024,502
95th percentile	3,082,589		
95th percentile/ E(BEL) – 1	–2.69%	–2.29%	–1.88%

To analyze the behavior of the components of financial and insurance risk, the variance decomposition was applied with reference to some variations of the parameters that characterize the noise factors. The first scenario of Table 7 represents the base scenario ("Baseline" or "Scenario 1") of which the results have been reported in the previous tables. Scenario 2 eliminates the source of risk in the inflation rate process and allows the effects of the structure function $q(i, j)$ to be measured. Scenario 3 represents a situation in which there is an increase in the uncertainty of the financial habitat in which the insurance and reinsurance companies operate.

Table 7. Parameters in the scenarios

Scenario	a	s_{inf}
Baseline (1)	0.010	0.015
Scenario 2	0.010	0.000
Scenario 3	0.020	0.015

4. DISCUSSION

From the cumulative distribution function of $Z - F(z)$ shown in Table 3, it can be seen that the variability is small: for example, the values of 90th and 50th percentile are very similar.

Furthermore, analyzing the results obtained for the coefficient of variation, shown in Table 4, we note that, for any generation, the variability of financial risk is less than the insurance risk, so the level of variability in the present value of future compensations is mainly determined by the uncertainty of the flow of compensation. Moreover, the coefficient of variation for the whole of the generations is lower than the sum

http://dx.doi.org/10.21511/ins.11(1).2020.01

Table 8. Sensitivity analysis on investment risk and insurance risk

i	CoV Tot 1	CoV Tot 2	CoV Tot 3	CoV Fin 1	CoV Fin 2	CoV Fin 3	CoV Ins 1	CoV Ins 2	CoV Ins 3
1	0.0523	0.0522	0.0550	0.0100	0.0100	0.0199	0.0513	0.0512	0.0513
2	0.0484	0.0480	0.0517	0.0102	0.0102	0.0207	0.0474	0.0469	0.0474
3	0.0451	0.0444	0.0489	0.0105	0.0105	0.0217	0.0438	0.0431	0.0438
4	0.0445	0.0436	0.0487	0.0108	0.0108	0.0225	0.0432	0.0422	0.0432
5	0.0451	0.0431	0.0494	0.0108	0.0108	0.0229	0.0438	0.0417	0.0438
6	0.0434	0.0413	0.0481	0.0110	0.0110	0.0234	0.0420	0.0398	0.0420
7	0.0441	0.0418	0.0488	0.0111	0.0111	0.0238	0.0426	0.0403	0.0426
8	0.0420	0.0385	0.0471	0.0113	0.0113	0.0242	0.0404	0.0368	0.0404
9	0.0419	0.0391	0.0471	0.0114	0.0113	0.0244	0.0403	0.0374	0.0403
10	0.0434	0.0399	0.0486	0.0114	0.0114	0.0245	0.0419	0.0382	0.0419
Total BEL	0.0254	0.0204	0.0330	0.0111	0.0111	0.0238	0.0229	0.0171	0.0229

of the single generation, which indicates the existence of compensating effects produced by the noise factors.

Through the distribution, the percentage of safety loading determined by a percentile-based approach was obtained. In particular, in the third column of Table 5, there is the safety loading in percentage on the mean for any generation and for the total of generations. Again, the coefficient of variation for all generations is less than the sum of single generation. This shows the consistent and material effect of mitigation.

It can be noted that to obtain a positive technical result with a 95% probability, the net reinsurance premium must be charged with around 4%.

Table 6 instead shows the premium levels determined by the CoC approach. In all the scenarios that were considered, from six percent (6%) to ten percent (10%) of Cost of Capital rate, one has obtained that the pure premium is less than the 95th percentile of the distribution, even if it is very close.

Therefore, an approach based on the Cost of Capital determines a cheaper premium for the insurance company, even in the hypothesis of a request by the reinsurer of a 10% return rate on capital.

The results obtained show that:

- the level of current variability is mainly determined by the uncertainty of the flow of compensation;
- long-term level that characterizes the processes of the inflation rate and the structure function;
- for each generation, the relative incidence of investment risk is, however, influenced by the variability of insurance parameters;
- the relative incidence of investment risk, for all generations, is roughly equal to the standard deviation of the noise factor assumed in the various scenarios;
- the relative incidence of insurance risk is decreasing with the stretch of the compensation flow, due to the strength of recall towards this because, in the variance decomposition, when considering the effect of one of the risk components, the variability of the other one is in any case considered, even if only on average.

CONCLUSION

In conclusion, in a framework of general hypotheses, the analysis of the sources of risk in loss portfolio transfer operations can be effectively conducted through the techniques of stochastic simulation.

In particular, the numerical application has allowed us to observe that the financial factor has, however, a limited weight in determining the variability of the reinsurer, while very important is the influence of

insurance factors (portfolio size, timing of settlements, increase of the average annual cost). Therefore, the influence of insurance factors (inflation, portfolio size, time distribution of compensation) is very significant and must be adequately modeled in LPT operations.

Anyway, the introduction of a stochastic financial model for the yield structure enables to evaluate the level of the variability of compensation obligations concerning the risks reinsured measuring the contribution due to the financial factor and insurance factor.

Furthermore, the use of an internal model enables to determine the safety loading to be added to the reinsurance premium through different methods that the reinsurance company can adopt also taking into account its model for calculating the solvency capital requirement or the capital need.

In fact, it can determine the safety loading by setting a percentile of the cumulative distribution function obtained via simulation, in line with its risk appetite, or it can adopt a method based on the cost of capital. Following this second approach, one can price a reinsurance contract by the quantification of the reserve risk and the interest rate risk by using different levels of cost of capital rate required in operation, ensuring a wider analysis and more alternative to fix the reinsurance price consistent with the methodologies and parameters set for its risk appetite.

AUTHOR CONTRIBUTIONS:

Conceptualization: Nicolino Ettore D'Ortona, Gabriella Marcarelli, Giuseppe Melisi.
Data curation: Giuseppe Melisi.
Investigation: Gabriella Marcarelli, Giuseppe Melisi.
Methodology: Nicolino Ettore D'Ortona, Gabriella Marcarelli, Giuseppe Melisi.
Software: Nicolino Ettore D'Ortona.
Supervision: Nicolino Ettore D'Ortona.
Writing – original draft: Nicolino Ettore D'Ortona.
Writing – review & editing: Gabriella Marcarelli, Giuseppe Melisi.

REFERENCES

1. Butsic, R. P. (1994). Determining the proper interest rate for loss reserve discounting: an economic approach. In J. D. Cummins & R. A. Derrig (Eds.), *Managing the Insolvency Risk of Insurance Companies* (pp. 249-261). https://doi.org/10.1007/978-94-011-3878-9_9
2. Daykin, C. D., Pentikainen, T., & Pesonen, M. (1993). *Practical risk theory for actuaries* (546 p.). Chapman & Hall.
3. D'Ortona, N. E., & Melisi, G. (2014). Run-off error in the outstanding claims reserves evaluation. In C. Perna & M. Sibillo (Eds.), *Mathematical and Statistical Methods for Actuarial Sciences and Finance* (pp. 95-98). Springer, Cham. https://doi.org/10.1007/978-3-319-05014-0_22
4. England, P. D., & Verrall, R. J. (2006). Predictive distributions of outstanding liabilities in general insurance. *Annals of Actuarial Science, 1*(2), 221-270. https://doi.org/10.1017/S1748499500000142
5. Mack, T. (1993). Distribution-free calculation of the standard error of chain ladder reserve estimates. *ASTIN Bulletin: The Journal of the IAA, 23*(2), 213-225. https://doi.org/10.2143/AST.23.2.2005092
6. Mcnair, A., Quane, A., Russell, C., Perry, G., Townley, L., Bruce N., & Shaw, R. (2002). *Loss portfolio transfers* (Giro Working Party Paper). Retrieved from https://www.actuaries.org.uk/system/files/documents/pdf/Macnair_0.pdf
7. Parker, G. (1994). Two stochastic approaches for discounting actuarial functions. *ASTIN Bulletin: The Journal of the IAA, 24*(2), 167-181. https://doi.org/10.2143/AST.24.2.2005063
8. Pentikainen, T., & Rantala, J. (1986). Run-off risk as a part of claims fluctuation. *ASTIN Bulletin: The Journal of the IAA, 16*(2), 113-147. https://doi.org/10.2143/AST.16.2.2015004
9. Pentikainen, T., & Rantala, J. (1992). A simulation procedure for comparing different claims reserving methods. *ASTIN Bulletin: The Journal of the IAA, 22*(2), 191-216. https://doi.org/10.2143/AST.22.2.2005115
10. Wilkie, A. D. (1984). *Using a stochastic model to estimate the distribution of real rates return on ordinary shares.* Transactions of 22 Congress of Actuaries.

http://dx.doi.org/10.21511/ins.11(1).2020.01