“Optimal control of continuous life insurance model”

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OPTIMAL CONTROL OF CONTINUOUS LIFE INSURANCE MODEL

Abstract

The problems of mixed life insurance and insurance in the case of death are considered in the article. The actuarial present value of life insurance is found by solving a system of differential equations. The cases of both constant effective interest rates and variables, depending on the time interval, are examined. The authors used the Pontryagin maximum principle method as the most efficient one, in order to solve the problem of optimal control of the mixed life insurance value. The variable effective interest rate is considered as the control parameter. Some numerical results were given.

Keywords actuarial value, control, insurance, function, life insurance, model

JEL Classification C58, C61, G20, G22

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INTRODUCTION

It is difficult to overestimate the importance of life insurance in our time. As in the sphere of monetary policy with risks due to uncertainty, and so in life insurance, there are such factors that are difficult to predict (Jung & Mongelli, 2016). Therefore, there is an increase in the number of approaches, models, theories for the optimal choice of an insurance product.

Despite a 4.3% reduction in gross insurance premiums in January-March 2017 in Ukraine in comparison with the same period in 2016, life insurance issues are becoming more relevant every year. Even though Ukrainian insurance companies pay more attention to the corporate identification and marketing communications, the growth of popularity of insurance companies is not observed. Trynchuk (2017) highlights the importance of integrating corporate identity into the strategy of a company. The overall economic situation in the country, the populations, income the effectiveness of the legal system, and the lack of reliable investment tools affect the development of the life insurance market in Ukraine, nevertheless researchers pay more attention to the life insurance models, and finding optimum control of continuous life insurance model becomes vital.

In countries with a stable economy, long-term life insurance is not only a way to protect yourself against unforeseen events, but also the ability to accumulate funds. The peculiarity of models of long-term life insurance is that these models are based on the methods of financial mathematics, taking into account the change in the value of money over time. Therefore, it is very important to calculate the moment and sum of insurance compensation and to forecast possible risks.

1. LITERATURE REVIEW

Kurylo et al. (2017) conducted in-depth analysis of the Ukrainian insurance market development and its connection with the global trends in insurance, confirming statistical information that the Ukrainian insurance market in fact is underdeveloped.

Porrini (2017) pays attention to the effects of digitization on the distribution at the insurance market, and puts emphasis on the assessment of the individual's risk profile using Big Data.

Gerber (1997) considers concise introduction to life contingencies, the theory behind the actuarial work around life insurance and pension funds. In addition to the model of life contingencies, the theory of compound interest is explained and it is shown how mortality and other rates can be estimated from the observations.

Russo et al. (2017) analyzed an intensity-based framework for surrender modeling in life insurance and defined factors such as mortality rates, interest rates and surrendering.

The way to minimize risks in life insurance with dependent mortality risk was considered by Biagini et al. (2017). The performed calculations for minimizing risks in the strategy proposed by the authors are based on the assessment of financial assets and a family of longevity bonds.

Li et al. (2007) examined the determinants of life insurance consumption in OECD countries. Consistent with the previous results, a significant positive income elasticity of life insurance demand was found. The demand also increases with the number of dependents and level of education, and decreases with the life expectancy and social security expenditure. The country's level of financial development and its insurance market's degree of competition appear to stimulate life insurance sales, whereas the high inflation and real interest rates tend to decrease consumption. In overall, the life insurance demand is better explained when the product market and socioeconomic factors are jointly considered. In addition, the use of GMM estimates helps reconcile the findings with the previous puzzling results based on the inconsistent OLS estimates of given heteroscedasticity problems in the data.

Financial management in the American insurance companies was considered by Hampton (1993). The problems of solvency of insurance companies and the methods of its measurement are analyzed. The basic principles of planning of profit for the new activity of insurance company are shown as well, the assessment of risk degree and forecasting of losses is given.

Cox and Hogan (1995) used an option pricing framework to estimate the life insurer risk-based capital. Stock market data and statutory asset and liability data are used to calculate the implied level of statutory risk-based capital for each of 18 insurers. They calculate the level of risk-based capital required to avoid subsidy from the guaranty fund. Their results suggest that less capital is required than that required under the New York actuarial risk-based capital formula. Firm rankings, however, are similar under both methods, although the methods are not directly comparable. They also determine the level of capital required, if the subsidy provided to the sample of insurers by a guaranty fund is the same as that provided by the Federal Deposit Insurance Corporation (FDIC) to the U.S. banks. This level of capital is chosen because of the dominance of investment products for life insurers. When the results are compared with those found from a similar study of the U.S. banks, it appears that the sample life insurers hold relatively greater capital than do the sample banks.

Gaillardetz and Lakhmiri (2011) introduced a premium principle for equity-indexed annuities (EIAs). The traditional actuarial loadings that protect insurance companies against risks cannot be extended to the valuation of EIAs since these products are embedded with the various financial guarantees. They proposed a loaded premium that protects the issuers against the financial and mortality risks. They firstly obtain the fair premium based on a fair value of the equity-linked contract using the arbitrage-free theory. Assuming a specific risk level for hedging errors, a new participation rate based on a security loading was obtained. A detailed numerical analysis is performed for the point-to-point EIA.

Campbell (1980) emphasizes that bequest motives and risk aversion should not be confounded although they may have similar effects. To clarify the issue, Lewis (1989) analyzes the problem from the perspective of the insurance beneficiaries rather the perspective of the wage earner, on whose life the insurance contract is written.

Chen et al. (2001) provide evidence of a gender effect, combined with a life cycle effect.

Huang et al. (2008) solved a portfolio choice problem that includes life insurance and labor income under the constant relative risk aversion (CRRA) preferences. They focus on the correlation between the dynamics of human capital and financial capital and model the utility of the family as opposed to separating consumption and bequest. They simplify the underlying Hamilton–Jacobi–Bellman equation using a similarity reduction technique that leads to an efficient numerical solution. Households for whom shocks to human capital are negatively correlated with shocks to financial capital should own more life insurance with greater equity/stock exposure. Life insurance hedges human capital and is insensitive to the family's risk aversion, consistent with the practitioner guidance.

One of the activities of the insurance company is managing of its assets. Bazilevich (2008) proposes a model of asset management of insurance company.

Engsner et al. (2017) focused on multi-period cost-of-capital approach, and highlighted computational aspects of the cost-of-capital margin and its influence on life insurance products.

According to D'Ortona and Staffa (2016), a surrender value calculation method should be based on the profit recovery concept, it can affect the formation of an effective assessment of insurance policies and the insurance products.

The effective management of insurance company activity is considered by Kozmenko et al. (2014).

Almost any economic-financial system changes during the time. For a management of the system it is necessary to have a management function. Changing the parameters of this function, it is possible to get motion of the system on an optimal trajectory.

One of the most efficient methods of solving the management problem is the method of the Pontryagin maximum principle (Pontryagin at al., 1983; Arutyunov et al., 2006; Shell, 1969).

2. RESEARCH METHODOLOGY

Let us consider the mixed life insurance for a period of n years for a person aged x. We assume a uniform distribution of the moments of death within the one-year age intervals. This actuarial value of benefits for the case of mixed life insurance $\overline{A}_{x:n|}$ in the amount of the unit at the time of the insured event is subject to the following relationship:

$$\overline{A}_{x:n|} = \overline{A}^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1}, \quad (1)$$

where $\overline{A}^{1}_{x:\overline{n}|} = \frac{i}{\delta} \cdot A^{1}_{x:\overline{n}|}$ is the actuarial present value of insurance for a period of n years, with payment of a unit size at the time of death of a person (x); $A^{1}_{x:\overline{n}|}$ is the actuarial present value of insurance for a period of n years, with payment of a unit size at the end of the year of death of the person (x); $A_{x:\overline{n}|}^{\ \ 1}$ is the actuarial present value of endowment insurance for a period of n years; $\delta = \ln(1+i)$ is interest intensity; i is effective interest rate.

The differential equation of mixed life insurance is considered by Bowers et al. (1997):

$$\frac{d\overline{A}_{x:\overline{n}|}}{dx} = \overline{A}_{x:\overline{n}|}\left[\delta(x) + \mu(x)\right] - \mu(x), \quad (2)$$

where $\mu(x)$ is the intensity of mortality upon the attainment of the age of x.

In the case of insurance with payments made at the time of death for the perpetual insurance contract concluded with the person under the age of x, we have the following relationship:

$$\frac{d\overline{A}_x}{dx} = \overline{A}_x[\delta(x) + \mu(x)] - \mu(x), \quad (3)$$

where $\overline{A}_x$ is the present actuarial value of benefits for the perpetual insurance contract concluded with the person (x).

For a constant interest accrual intensity δ and a constant mortality intensity μ, the actuarial present value of perpetual life insurance with payment of the unit value is equal to:

$$\overline{A}_x = \frac{\mu}{\delta + \mu}. \quad (4)$$

Let us consider the whole life insurance for the term of n years for the person (x). It is assumed that the insurance benefit is paid only, if the policyholder dies during the n years from the date of the insurance contract conclusion.

The differential equation of insurance in the case of death is given by Bowers et al. (1997):

$$\frac{d\overline{A}^{1}_{x:\overline{n}|}}{dx} = \overline{A}^{1}_{x:\overline{n}|}[\delta(x) + \mu(x)] + +\mu(x+n)\cdot A_{x:\overline{n}|}^{\ \ 1} - \mu(x). \quad (5)$$

Let us consider the insurance annuities. The insurance annuities are the series of payments made continuously or at regular intervals, while the person is alive. Payments may be temporary, that is, made within a certain number of years, or perpetual. Let us consider the annuities with continuous payments (continuous annuities) in the amount of a unit per year. The perpetual (lifetime) insurance annuity provides payments until death.

For the insurance annuity with a term of payment of n years for a person (x) $\overline{a}_{x:\overline{n}|}$ we have the following relationship:

$$1 = \delta \cdot \overline{a}_{x:\overline{n}|} + \overline{A}_{x:\overline{n}|}. \quad (6)$$

In the case of perpetual annuity insurance $\overline{a}_x$ we have:

$$1 = \delta \cdot \overline{a}_x + \overline{A}_x. \quad (7)$$

The actuarial present value of the perpetual insurance annuity $\overline{a}_x$ with the continuous payments is subject to the following differential equation:

$$\frac{d\overline{a}_x}{dx} = \overline{a}_x[\mu(x) + \delta(x)] - 1. \quad (8)$$

The equation (8) shows that the rate of change in the actuarial present value with age is equal to a component $\overline{a}_x \cdot \mu(x)$ associated with mortality, plus a component $\overline{a}_x \cdot \delta(x)$ associated with the accrued interest, minus one.

Let us consider the concept of net premium. One of the conditions for finding the net premium $\overline{P}(\overline{A}_x)$ for the perpetual insurance contract is the fulfillment of the following equation:

$$\overline{P}(\overline{A}_x) = \frac{\overline{A}_x}{\overline{a}_x}. \quad (9)$$

Net premium for the endowment insurance for a period of n years is as follows:

$$\overline{P}(\overline{A}_{x:\overline{n}|}) = \frac{\overline{A}_{x:\overline{n}|}}{\overline{a}_{x:\overline{n}|}}. \quad (10)$$

Let us consider the problem of determining the present actuarial value of payments both for the case of mixed life insurance for a period of n years for a person (x) and for the payments in the case of death. We will consider options of both the constant interest intensity and according to the effective interest rate on the time interval.

To do this, let us consider the solution of the following system of equations:

$$\begin{cases} \frac{d\overline{A}_{x:\overline{n}|}}{dx} = \overline{A}_{x:\overline{n}|}[\delta(x) + \mu(x)] - \mu(x) \\ \frac{d\overline{A}^{1}_{x:\overline{n}|}}{dx} = \overline{A}^{1}_{x:\overline{n}|}[\delta(x) + \mu(x)] + \\ +\mu(x+n)\cdot A_{x:\overline{n}|}^{\ \ 1} - \mu(x) \\ \overline{A}_{x:\overline{n}|} = \overline{A}^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1} \end{cases} \quad (11)$$

Let us consider the case where the mortality function $\mu(x)$ obeys the Makeham law (Bowers et al., 1997):

$$1000 \cdot \mu(x) = 0.7 + 0.05 \cdot (10^{0.04})^x. \quad (12)$$

For this distribution the following relation is valid:

$$\mu \cdot (x+n) = \mu(x) \cdot (10^{0.04})^n + + \frac{0.7 \cdot [1 - (10^{0.04})^n]}{1000}. \quad (13)$$

Let us consider the time interval $[x_0;\ T]$. To solve the system (11), it is necessary to assume the pre-initial conditions in the form of:

$$\overline{A}^{1}_{x:\overline{n|}} = \frac{i}{\delta} \cdot A^{1}_{x:\overline{n|}} = \frac{i}{\delta} \cdot \frac{(M_x - M_{x+n})}{D_x}. \qquad (14)$$

$$A_{x:\overline{n|}}^{\ \ 1} = \frac{D_{x+n}}{D_x}, \qquad (15)$$

where D_x, M_x – switching functions which are determined from tables.

The adjusted value of the present actuarial value of payment unit of the mixed life insurance and payments in the case of death is determined by the numerical solution of equations (11) and the initial conditions (14), (15).

Using the relations (6) and (10) for the person (x), we can obtain the distribution of insurance annuity and net premium for the mixed life insurance for a period of n years.

Setting the optimal control problem

Let us consider the solution of the optimal control problem of the actuarial present value of benefits for the case of mixed life insurance for a period of n years for the person aged x. The differential equation of this type of insurance has the form (2).

It is known the actuarial insurance value at the initial time:

$$\overline{A}_{x:\overline{n|}}(x_0) = A_0. \qquad (16)$$

Let us introduce a control function on which some restrictions are imposed:

$$U(x) \in U. \qquad (17)$$

Let us also consider the objective function in the form of:

$$J(U) = g \cdot (\overline{A}_{x:\overline{n|}}(T)). \qquad (18)$$

Using the control function $U(x)$, it is necessary to obtain the extremum of the objective function.

Problem statement

Let us consider the problem statement in the time interval $[x_0 = x;\ T = x_0 + n]$.

1. Differential equation:

$$\frac{d\overline{A}_{x:\overline{n|}}}{dx} = \overline{A}_{x:\overline{n|}}[\delta(x) + \mu(x)] - \mu(x) + U(x). \qquad (19)$$

2. Initial conditions:

$$\overline{A}_{x:\overline{n|}}(x) = A_0. \qquad (20)$$

3. Control function:

$$U(x) = \alpha(x) \cdot \delta(x) \cdot \overline{A}_{x:\overline{n|}}. \qquad (21)$$

where $0 \le \alpha(x) \le 1$.

4. Objective function:

$$J = \int_{x_0}^{T} \{(1-\alpha(t)) \cdot \mu(t) + \alpha(t) \cdot \delta(t)\} \cdot \overline{A}_{x:\overline{n|}} dt \to \max. \qquad (22)$$

The problem solution

Solution of the optimal control problem.

To solve this problem (19) – (22), let us apply the Pontryagin maximum method (Pontryagin et. al., 1983).

To simplify further expressions, let us denote $A(x) = \overline{A}_{x:\overline{n|}}$.

Let us write the Hamiltonian function:

$$H(x) = \Psi(x)\begin{Bmatrix} -\mu(x) + [\delta(x) + \mu(x)] \times \\ \times A(x) + \alpha(x) \cdot \delta(x) \cdot A(x) \end{Bmatrix} + \qquad (23)$$
$$+ \{1 - \alpha(x)\} \cdot \mu(x) \cdot A(x) + \alpha(x) \cdot \delta(x) \cdot A(x),$$

where $\Psi(t)$ is auxiliary function that satisfies the equation

$$\frac{d\Psi(x)}{dx} = -\frac{\partial H(x)}{\partial A(x)} = \qquad (24)$$
$$= -\{\Psi(x) \cdot [\delta(x) + \mu(x) - \alpha(x) \cdot \delta(x)] +$$
$$+ (1 - \alpha(x)) \cdot \mu(x) + \alpha(x) \cdot \delta(x)\}.$$

For the auxiliary function the transversality condition will be carried out

$$\Psi(T) = 0. \qquad (25)$$

By analyzing the Hamiltonian function (23) and bearing in mind that at each point of the optimal trajectory this function is maximized with respect to the control parameters, we obtain the optimal management strategy:

$$U(x)=\begin{cases}\delta(x)\cdot A(x), & x_0\le x\le x_*\\ 0\,, & x_*<x\le T\end{cases} \quad (26)$$

where x_* is time of switching for the control function, which is found from condition

$$\Psi(x_*)-\frac{\delta(x_*)-\mu(x_*)}{\delta(x)_*}=0. \quad (27)$$

The auxiliary variable $\Psi(t)$ on the interval $\left[x_0;\ x_*\right]$ at the control $\alpha(x)=1$ is determined by solving the boundary problem:

$$\begin{cases}\dfrac{d\Psi(x)}{dx}=-\Psi(x)[2\delta(x)+\mu(x)]-\delta(x)\\ \Psi(x_*)=\dfrac{\mu(x_*)-\delta(x_*)}{\delta(x_*)}\end{cases} \quad (28)$$

On the interval $\left[x_*;\ T\right]$ at the control $\alpha(x)=0$ the function $\Psi(t)$ is determined by solving the problem:

$$\begin{cases}\dfrac{d\Psi(x)}{dx}=-\Psi(x)[\delta(x)+\mu(x)]-\mu(x)\\ \Psi(T)=0\end{cases} \quad (29)$$

3. RESULTS AND DISCUSSION

As a numerical implementation of the proposed algorithm we will use the software product of AnyLogic Company (http://www.anylogic.com/about-us). Let us consider the distribution of the present actuarial value of benefits for the case of mixed life insurance for a period of n years for a person (x) and for the case of death.

Initial data: $x_0=35;\ T=x_0+n=65.$ Let us consider some options for distribution of the effective interest rate over the time interval $[x_0;\ T]$.

Option A

Effective interest rate is constant:

$$i=\{0.01;0.03;0.06;0.1\}. \quad (30)$$

Option B

Effective interest rate increases according to the law:

$$i=0.003\cdot(t-35)+0.01. \quad (31)$$

Option C

Effective interest rate decreases according to the law:

$$i=-0.003\cdot(t-35)+0.1. \quad (32)$$

Option D

Effective interest rate varies according to the law:

$$i=-0.0004\cdot(t-50)^2+0.1. \quad (33)$$

Figure 1 shows the distribution of the present actuarial value of benefits for the case of mixed life insurance for the permanent effective interest rates (Option A).

Figure 2 shows the distribution of the present actuarial value of benefits for the case of death for the permanent effective interest rates (Option A).

Figure 3 shows the distribution of the present actuarial value of benefits for the case of mixed life insurance in the case of variable effective interest rates (Options B, C, D).

Figure 4 shows the distribution of the present actuarial value of benefits for the case of death (Options B, C, D).

As a numerical experiment of the resulting optimization control problem, let us consider the following initial data $x_0=x=35;\ T=x_0+n=65.$ Mortality function $\mu(x)$ obeys the Makeham law (12).

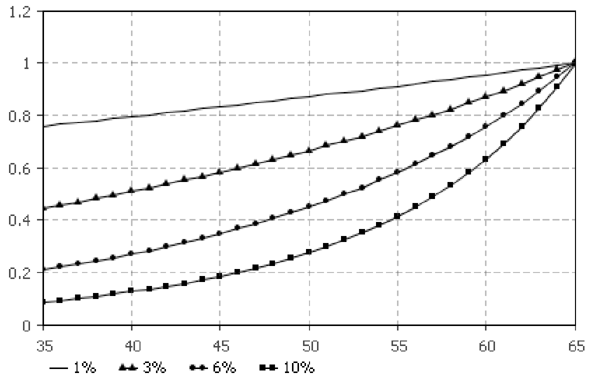

Figure 1. Distribution of the present actuarial value of benefits for the case of mixed life insurance

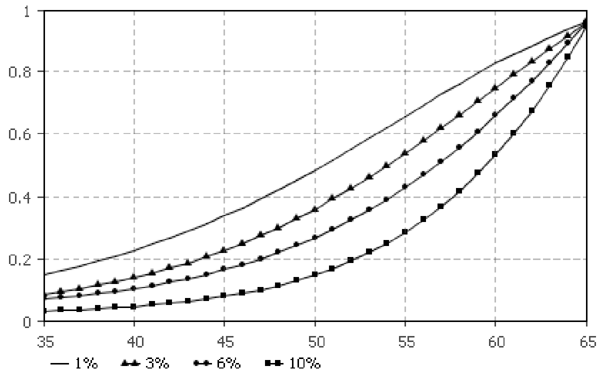

Figure 2. Distribution of the present actuarial value of benefits for the case of death

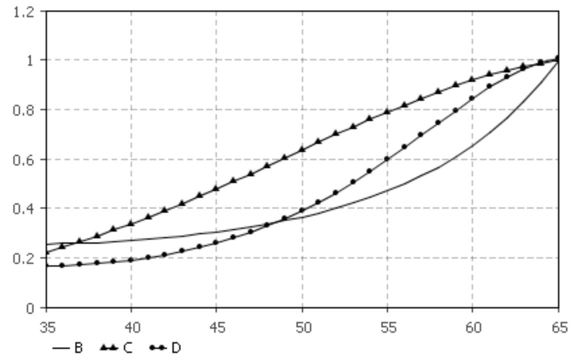

Figure 3. Distribution of the present actuarial value of benefits for the case of mixed life insurance.

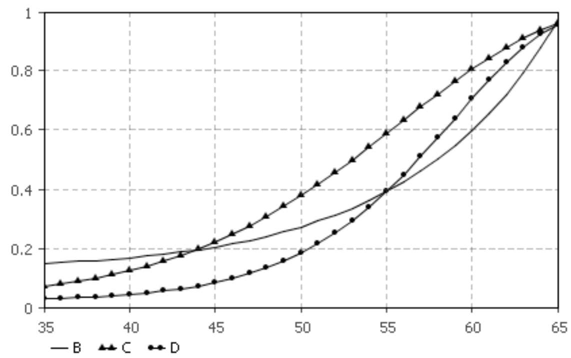

Figure 4. Distribution of the present actuarial value of benefits for the case of death

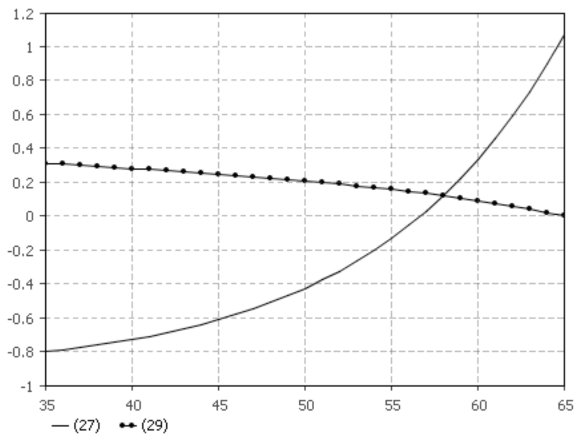

Figure 5. Finding the switching point of control problem $(i = 0.01)$

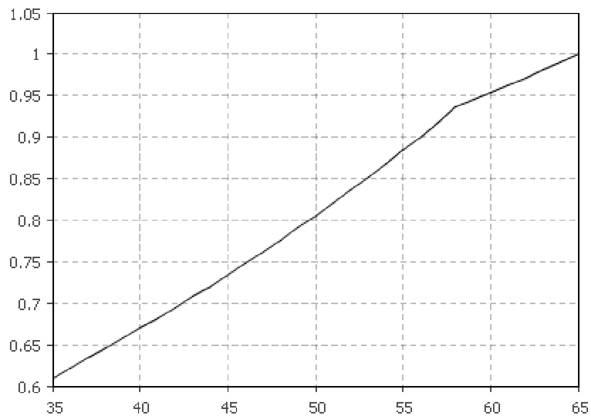

Figure 6. Distribution of the present actuarial value of benefits for the case of mixed life insurance $(i = 0.01)$

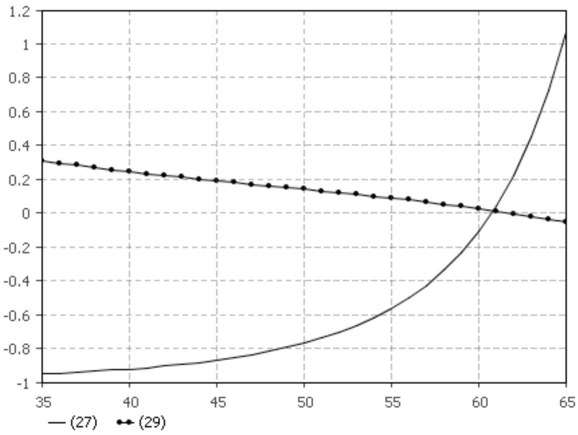

Figure 7. Finding the f switching point of control problem (Option B)

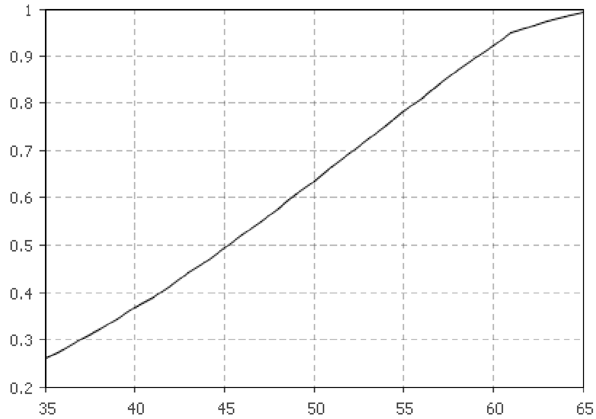

Figure 8. Distribution of the present actuarial value of benefits for the case of mixed life insurance (Option B)

Option A. Effective interest rate is constant

$$i = 0.01.$$

Figure 5 presents the solution of the differential equation (29) and equation (27). The switching point is equal to:

$$x_* = 58.$$

Figure 6 presents solution of the problem of optimal control for the actuarial present value of benefits for the case of mixed life insurance for a period of n years for the person aged x upon maximizing the objective function in the form (22).

Option B. Effective interest rate varies according to the law:

$$i = -0.001 \cdot (t - 35) + 0.04. \qquad (34)$$

Figure 7 presents the solution of differential equation (29) and equation (27). The switching point is equal to: $x_* = 61$.

Figure 8 presents solution of the problem of optimal control for the actuarial present value of benefits for the case of mixed life insurance for a period of n years for the person aged x upon maximizing the objective function in the form (22).

CONCLUSION

The problem of distribution of the present actuarial value of benefits for the case of mixed life insurance for a period of *n* years for a person (*x*) and for the case of death for different effective interest rates is considered. It was done with the help of the software product of AnyLogic Company.

The resulting control model of insurance payments enables to obtain the maximum profit of the insurance company. In accordance with the Pontryagin maximum principle the authors obtained the control function in the form of relay functions. The distribution of insurance payments for the case of mixed life insurance upon maximizing the objective function is demonstrated.

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