ON A FAIR VALUE MODEL FOR PARTICIPATING LIFE INSURANCE POLICIES

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Abstract

The aim of this paper is to analyze both the term structure of interest and mortality rates role for evaluating a fair value of a life insurance business. In particular, a fair value accounting impact on reserve evaluations is discussed comparing a traditional deterministic model based on local rules for an Italian balance sheet calculation and a stochastic one based on a diffusion process for both mortality and financial risks. As proposed by IAS Board we will separate the embedded derivatives from their host contracts, so the fair value of a traditional life insurance contract would be expressed as the value of four components: the basic contract, the participation option, the option to annuitise and the surrender option. A numerical application to a traditional Italian life insurance policy is discussed.

Key words: Fair pricing, participation option, surrender option, guaranteed annuity option, Black&Scholes-CIR framework, Longstaff-Schwartz Least-Squares Approach.
JEL Classification: C15, G13, G22.

1. Introduction

Literature on International Accounting Standards in the last three years has been improved, not only in financial area, by authoritative scientific and professional contributions. The expectation for a final version of an accounting standards guide line for insurance companies by IASB organisation has given rise to an intense methodological debate focused on technical solutions for accounting rules application within insurance sector.

The meaning of "Fair Value" extended to insurance contracts involves, by a side, an adoption of new strategic choices for managing resources and structures and, on the other side, represents a way in which actuarial theory could assume a primarily role in methodological paths definition.

More in general, when a liquid market of insurance contracts is available, Fair Value of an insurance contract is equal to its market value; otherwise it is necessary to obtain an estimate of the market value using a theoretical model consistent with economic operators' behaviour in a particular risk condition. The forthcoming IAS guide line proposes that insurance liabilities should be valued as if they are traded among well-informed, independent investors in a liquid marketplace. IAS guide line allows using of stochastic methods to estimate future cash flows (liabilities including embedded options) arising from the contract.

In this context, insurance companies should adequate their internal procedures to characterize and estimate all the policies' components at fair value. As an example, traditional Italian participating life insurance contracts, also known as with-profits, entitle the policyholder of a certain part of the profits generated by the assets associated with the contract; these profits are credited to the mathematical reserve increasing the insurer's liabilities. According to the recent literature (Grosen and Jørgensen (2000), Bacinello (2001)), the value of the previous contract could be split into two components: a basic contract and a participation or bonus option; a bonus option is a participating European-style option where the benefit is annually adjusted according to the performance of a reference fund. Furthermore, if a contract provides that the policyholder can surrender the contract before maturity, this component is called surrender option. According to the recent insurance literature on this argument, as in the regular option literature, a contract with a surrender option is called American, a contract without a surrender option is called European (Bacinello (2003), Andreatta and Corradin (2003)). At last, some policies enable the policyholder to convert cash benefit at maturity into a guaranteed annuity payable throughout the remaining lifetime, calculated at a guaranteed rate. A guaranteed annuity option is a contract that provides the policyholder with the right to receive at maturity either a cash payment or an annuity, depending on which has the greater value (Milevsky and Promislow (2001), Ballotta and Haberman (2003)).

In reference to embedded derivatives, IFRS 4 clarifies that "an insurer needs not account for an embedded derivative separately at fair value if embedded derivative meets the definition of an insurance contract". Moreover "insurers will not need to separate surrender options within discretionary participatory features (DPF) contracts, irrespective of whether they transfer significant insurance risk or not". Then, the above directions establish that insurance companies have to evaluate the fair value of the embedded derivatives but not necessarily account them separately from the host contract.

References on the application of accounting rules based on Fair Value are easily founded in literature; De Angelis (2001) reviews international guide line for Fair Valuation of insurance companies; De Felice and Moriconi (2001) introduce a mark to market model for a fair pricing of life insurance participating policies with a minimum interest rate guaranteed.

In actuarial literature most papers deal with models for Fair Value of life insurance liabilities with embedded options; in particular Milevsky and Promislow (2001) propose a stochastic approach to model the future mortality hazard rate in insurance contract with option to annuitise; Bacinello (2003) deals with the problem of pricing a guaranteed life insurance participating policy which embeds a surrender option; Andreatta and Corradin (2003) propose a similar approach to price the embedded options via Monte Carlo simulation; Ballotta and Haberman (2003) propose a theoretical model for evaluating guaranteed annuity conversion options.

The aim of this paper is to analyze an actuarial model to compare reserves evaluated on the basis of local rules with reserves calculated on a Fair Value basis, considering a stochastic approach for both mortality and financial risk. We focus, in particular, on the fair valuation of a surrendable participating contract with minimum return guaranteed and option to annuitise via Monte Carlo simulation. We implement the recent contributions of Bacinello (2003), Andreatta and Corradin (2003), considering the case of a guaranteed annuity option as in Ballotta and Haberman (2003). Moreover, our approach describes expected cash flows of the fair value liabilities and embedded derivates between inception and term.

Section 2 describes theoretical model used for the comparison described above. Section 3 presents an application of the model discussed in Section 2 for an evaluation of a surrendable participating policy with minimum return guaranteed and option to annuitise.

2. A Fair Value of the embedded options in a guaranteed life insurance participating policy

2.1. It does not exist a unique definition of Fair Value for whole insurance contracts. However definition proposed by Financial Accounting Standards Board (FASB) for financial transaction declares *"Fair Value is an estimate of the price an entity would have realized if it had sold an asset or paid if it had been relieved of a liability on the reporting date in an arm's-length exchange motivated by normal business considerations. That is, it is an estimate of an exit price determined by market interactions"*.

A similar definition of Fair Value is proposed by International Accounting Standards Committee (IASC): *"The amount for which an asset could be exchanged or liability settled, between knowledgeable, willing parties in an arm's length transaction"*; therefore, in the traditional conditions of market efficiency, Fair Value of an insurance policy could be equalized to its equilibrium-price. In absence of an efficient market, Fair Value could be estimated through a consistent theoretical bid/ask model joined with similar assets and liabilities.

Valuation techniques include using recent arm's length market transactions between knowledgeable, willing parties, if available, referring to the current fair value of another instrument that is substantially the same, discounted cash flow analysis and option pricing models.

The chosen valuation technique makes maximum use of market inputs and relies as little as possible on entity-specific inputs. It incorporates all factors that market participants would consider in setting a price and is consistent with accepted economic methodologies for pricing financial instruments. The fair value is based on:

- observable current market transactions in the same instrument;
- a valuation technique whose variables include primarily observable market data and that is calibrated periodically to observable current market transactions in the same instrument or to other observable current market data;
- a valuation technique that is commonly used by market participants to price the instrument and has been demonstrated to provide realistic estimates of prices obtained in actual market transactions (see IASB (2004)).

In order to present an actuarial model for a fair value estimation of a traditional life insurance contract, we suppose to operate in a traditional efficient market. We assume, in fact, that financial and insurance markets are perfectly competitive, frictionless, and free of arbitrage opportunities. Moreover, all the agents are supposed to be rational and non-satiated, and to share the same information.

Consider $\{r_t; t = 1,2,....\}$ and $\{\mu_{x+t}; t = 1,2,....\}$ as two diffusion processes driven the instantaneous interest rate and the intensity of mortality (referred to an insured aged x at issue), by the filtrations F^r and F^μ respectively; with reference to a generic insurance contract pay-out, the two stochastic processes are defined on a probability space $(\Omega, F^{r,\mu}, \mathrm{P})$ such that $F^{r,\mu} = F^r \cup F^\mu$.

Fair Value of a generic life insurance contract in $t \in [0, s]$ is expressed as follows:

$$FV({}_t\mathbf{V}_x) = \hat{\mathrm{E}}\left[\left(\sum_{\tau\in(t,s)} \mathbf{v}(t,\tau)\mathbf{CFL}_\tau - \sum_{\tau\in(t,s)} \mathbf{v}(t,\tau)\mathbf{CF}A_\tau\right)\middle| F_t^{r,\mu}\right], \tag{2.1}$$

where $\hat{\mathrm{E}}$ denotes the usual expectation under the risk-neutral probability measure; $\mathbf{v}(t,\tau)$ is the stochastic discount factor dependent on the spot-rate dynamic between t and τ; $\mathbf{CFL}_\tau$ and $\mathbf{CFA}_\tau$, are the annual random cash flows of insurance company and insured/policyholder respectively, jointly dependent on the spot rate and intensity of mortality dynamics; F_t^r, F_t^μ and $F_t^{r,\mu}$ are the σ-algebras associated with the above defined filtrations.

2.2. As stated in Section 1, to compute the fair value of a surrendable participating endowment policy with option to annuitise, we separate the whole contract in its components considering a basic contract, a participation option, an option to annuitise and a surrender option.

The basic contract is a standard endowment policy with benefit C_0, net constant annual premium[1] P, technical rate i. The Fair Value is expressed as

$$FV({}_t\mathbf{V}_x^B) = \hat{\mathrm{E}}\left\{\left(C_0\left[\sum_{\tau=1}^{s-t} \mathbf{v}(t,t+\tau)\ {}_{\tau-1/1}q_{x+t} + \mathbf{v}(t,s)_{s-t}\,p_{x+t}\right] - P\sum_{\tau=1}^{s-t}\mathbf{v}(t,t+\tau-1)_{\tau-1}\,p_{x+t}\right)\middle| F_t^{r,\mu}\right\}. \tag{2.2}$$

In order to value the participation option it is necessary to compute the fair value of the non-surrendable participating contract. In accordance with this contract the insurer pays a benefit C_τ if the insured dies within τ and $\tau+1$; otherwise C_s if the insured is alive at maturity s. Then, the fair value of the non-surrendable participating contract is given by:

$$FV({}_t\mathbf{V}_x^P) = \hat{\mathrm{E}}\left\{\left(\left[\sum_{\tau=1}^{s-t} C_{t+\tau-1}\mathbf{v}(t,t+\tau)\ {}_{\tau-1/1}q_{x+t} + C_s\mathbf{v}(t,s)_{s-t}\,p_{x+t}\right] - P\sum_{\tau=1}^{s-t}\mathbf{v}(t,t+\tau-1)_{\tau-1}\,p_{x+t}\right)\middle| F_t^{r,\mu}\right\}, \tag{2.3}$$

where the participation rule is

[1] The annual premium is computed using first order technical basis.

$$C_\tau = C_{\tau-1}(1+\phi_\tau) - C_0\left(1-\frac{\tau}{s}\right)\phi_\tau \tag{2.4}$$

and

$$\phi_\tau = \max\left(\frac{\eta I_\tau - i}{1-i}, i_{\min}\right). \tag{2.5}$$

$\eta \in [0,1]$ is the participation coefficient, I_t represents the rate of return of the reference portfolio during *t*-th anniversary of policy (see Section 3), $i_{\min}$ is the minimum readjustment measure.

The fair value of the participation option could be easily computed as the difference between $FV\left({}_t\mathbf{V}_x^P\right)$ and $FV\left({}_t\mathbf{V}_x^B\right)$.

Some traditional Italian policies enable the policyholder to convert cash benefit at maturity into a guaranteed annuity payable throughout the remaining lifetime, calculated at a guaranteed rate G.

A guaranteed annuity option is a contract that provides the policyholder with the right to receive at maturity either a cash payment or an annuity, depending on which has the greater value.

The guaranteed annuity option pay-out at maturity is expressed as (Ballotta and Haberman (2003))

$$\max\left(G\,C_s a_{x+s}, C_s\right) = C_s + G\,C_s \max\left(0, a_{x+s} - K\right) = C_s + OTA_s, \tag{2.6}$$

where $K = 1/G$, $OTA_s = G\,C_s \max\left(0, a_{x+s} - K\right)$ and

$$a_{x+s} = \hat{\mathrm{E}}\left(\sum_{\tau=1}^{\omega-(s+x)} \mathbf{v}(s, s+\tau)\,{}_\tau p_{x+s} \middle| F_s^{r,\mu}\right). \tag{2.7}$$

In that case we have to express the fair value of a non-surrendable participating contract with option to annuitise as

$$FV\left({}_t\mathbf{V}_x^{OTA}\right) = FV\left({}_t\mathbf{V}_x^P\right) + \hat{\mathrm{E}}\left[OTA_s\,\mathbf{v}(t,s)\,{}_{s-t}p_{x+t}\middle| F_t^{r,\mu}\right]. \tag{2.8}$$

The fair value of OTA could be easily computed as the difference between $FV\left({}_t\mathbf{V}_x^{OTA}\right)$ and $FV\left({}_t\mathbf{V}_x^P\right)$.

At last, our aim is the computation of the surrender option's fair value as the difference between the fair value of the whole contract and the fair value of the non-surrendable participating contract with option to annuitise.

According to Grosen and Jørgensen (2000) and Bacinello (2003) the whole contract is an American-style contract that embeds a surrender option. A surrender option is an American-style option that enables the policyholder to surrender the policy and receive the so called surrender value.

Typically, the surrender value of a constant periodical premiums policy is given by

$$R_t = C_0\frac{t}{s} + \left(C_t - C_0\right)\left(1+i_{sur}\right)^{-(s-t)}, \tag{2.9}$$

where i_{sur} is an annually compounded discount rate.

The mechanism underlying a surrender option is the following: at any time *t=1,2,..,s-1*, the policyholder compares the surrender value with the expected payoff from the continuation value, and exercises the option if the surrender value is higher.

In order to price the surrender option, Grosen and Jørgensen (2000) and Bacinello (2003) propose a binomial tree model à la Cox, Ross and Rubinstein (1979): the fair price of the whole contract and the continuation price can be computed by means of a backward recursive procedure operating from time *s-1* to time *0*. Instead of a binomial tree model Andreatta and Corradin (2003) use the Least Squares Monte Carlo Approach following Longstaff and Schwartz (2001).

In these articles authors value the surrender option for a surrendable participating contract without option to annuitise. So the continuation value at time *s-1* for a given path *j* is expressed as (see Andreatta and Corradin (2003) p. 18):

$$W_{s-1}^{(j)} = v^{(j)}(s-1,s)C_s^{(j)} - P\,. \tag{2.10}$$

To distinguish our model from those described above, we have introduced in the continuation value of a surrendable participating policy a guaranteed annuity option, remembering Italian policy features. If we suppose to be at time *s-1,* we have to compute a different continuation value. To continue means receive, at time *s*, the benefit C_{s-1}, if the insured dies within *s-1* and *s*, or to be entitled of a contract whose total random value equals $\max\left(G\,C_s a_{x+s}, C_s\right)$, if the insured is alive. Therefore, the continuation value at time *s-1* for a given path *j* is expressed by:

$$W_{s-1}^{(j)} = v^{(j)}(s-1,s)\left[C_{s-1}^{(j)}\, q_{x+s-1} + \max\left(G\,C_s^{(j)} a_{x+s}^{(j)}, C_s^{(j)}\right) p_{x+s-1}\right] - P\,. \tag{2.11}$$

The fair value of the whole contract $FV\left({}_{s-1}V_x^T\right) = F_{s-1}^{(j)}$ is therefore the maximum between the continuation value and the surrender value $R_{s-1}^{(j)}$:

$$F_{s-1}^{(j)} = \max\left(W_{s-1}^{(j)}, R_{s-1}^{(j)}\right). \tag{2.12}$$

Assume now to be at time *t < s – 1*. As in the Fackler-Fourer's recursive formula, to continue means to immediately pay the premium P and to receive, at time *t+1*, the benefit C_t, if the insured dies within one year, or to be entitled of a contract whose total random value (including the option of surrendering it in the future), equals F_{t+1} if the insured is alive. Therefore the continuation value at time *t* is given by the following expression:

$$W_t = \left[C_t\, q_{x+t} v(t,t+1) + p_{x+t} \hat{E}\left(v(t,t+1)F_{t+1}\right)\right] - P. \tag{2.13}$$

Longstaff and Schwartz (2001) propose that the conditional expected value of the future option value $\hat{E}\left(v(t,t+1)F_{t+1}\right)$ can be estimated from the cross-sectional information in the simulation by using least squares, that is by regressing the discounted realized payoffs from continuation on functions of the values of the state variables[1].

Finally, the fair value of the surrender option could be evaluated as the difference between $FV\left({}_t\mathbf{V}_x^T\right)$ and $FV\left({}_t\mathbf{V}_x^{OTA}\right)$.

3. An application of the fair value actuarial model

3.1. The demographic stochastic process $\{\mu_{x+t}; t = 1,2,....\}$ is described by a Mean-Reverting Brownian Gompertz (MRBG) model; in particular, we take into account a traditional actuarial approach, where T_x is a random variable representing the remaining lifetime of a policy-

[1] Dealing with Andreatta and Corradin (2003) we choose to base the regression on two state variables, the cash benefit and the reference fund and regress according to a third-order polynomial model
$\hat{E}\left(v(t,t+1)F_{t+1}^j\right) \approx a_1 + a_2 C_t + a_3 C_t^2 + a_4 C_t^3 + a_5 S_t + a_6 S_t^2 + a_7 S_t^3 + a_8 C_t S_t + a_9 C_t^2 S_t + a_{10} C_t S_t^2$.

holder aged x ; as a consequence the probability of survival to time s , conditional on being alive at age x , is equal to

$$ {}_{s}p_{x} = \mathrm{Prob}\left(T_{x} > \mathrm{s} \middle| F_{t}^{\mu}\right). \tag{3.1} $$

We define $\mu_{x+t:t}$ to be the hazard rate for an individual aged $x+t$, at calendar year t , it follows that 0 can be arranged as

$$ {}_{s}p_{x} = \hat{E}\left[e^{-\int_{0}^{s}\mu_{x+t:t}dt} \middle| F_{t}^{\mu} \right]. \tag{3.2} $$

The time evolution of the hazard rate $\mu_{x+t:t}$ is expressed by an exponential form as follows $\mu_{x+t:t} = \mu_{x:0}e^{g_{x,s}t+\sigma_{x}Y_{t}}$, with $g_{x,s}, \sigma_{x}, \mu_{x:0} > 0$, where $g_{x,s}$ resumes on time s the deterministic correction due to age x and the effect of *longevity risk;* $\{Y_{t}\}$ is a stochastic process introduced to model random variations in the forecast trends; σ_{x} represents the standard deviation of the process $\{\mu_{x+t:t}; t = 1,2,....\}$; in particular the stochastic process $\{Y_{t}\}$ is described by a *mean reverting* diffusion process

$$ dY_{t} = -bY_{t}dt + dW_{t}, \quad Y_{0} = 0, \quad b \geq 0, \tag{3.3} $$

where b is the *mean reversion* coefficient and $\{W_{t}\}$ is a standard Brownian motion.

The time dynamic of the instantaneous interest rate (spot rate) $\{r_{t}; t = 1,2,....\}$ is modelled by a mean reverting square root diffusion equation as in Cox, Ingersoll and Ross model (CIR); therefore, we assume the following stochastic equation:

$$ dr_{t} = k(\theta - r_{t})dt + \sigma_{r}\sqrt{r_{t}}\,dZ_{t}^{r}, \tag{3.4} $$

where k is the *mean reversion* coefficient, θ is the long term rate, σ_{r} is the volatility parameter and $\{Z_{t}^{r}\}$ is a standard Brownian motion.

Fair pricing of an insurance participating policy also depends on reference portfolio's dynamic; in particular we assume to work in a Black-Scholes economy where the reference portfolio is compounded mainly by a bond index and a minority by a stock index. The two components are described by the following equation

$$ dS_{t}^{(i)} = r_{t}\,S_{t}^{(i)}dt + \sigma^{(i)}\,S_{t}^{(i)}dZ_{t}^{(i)}, \qquad i = \begin{cases} 1: \text{stock index} \\ 2: \text{bond index} \end{cases} \tag{3.5} $$

where $S_{t}^{(i)}, \sigma^{(i)}$ e $\{Z_{t}^{(i)}\}$, are, for each reference portfolio's component, market price, volatility parameter and a *Wiener* process. At last, the three sources of financial uncertainty are correlated:

$$ dZ_{t}^{(k)}dZ_{t}^{(j)} = \rho_{k,j}dt \qquad k, j = 1,2,r, \tag{3.6} $$

hence, reference portfolio could be expressed as a combination of the random variables introduced above

$$ S_{t} = (1-\alpha)S_{t}^{(1)} + \alpha S_{t}^{(2)}. \tag{3.7} $$

As a result, the annual rate of return of the reference fund at time t is defined as:

$$I_t = \frac{S_t}{S_{t-1}} - 1. \tag{3.8}$$

3.2. Since a closed form solution of expression 0 is not available, a policy's Fair Value can be obtained via Monte Carlo simulation. 0 reports some contract features used for numerical analysis.

Table 1

Contract Features

Sex	Male
Age	55
Duration	10
Technical rate	1.00%
Mortality Table	SIM 92
Sum Assured	100 €
Participation coefficient	87.5%
Minimum Adjustment Measure	0.00%
Guaranteed rate for annuity	5.78%
Annually compounded surrender discount rate	3.00%
Reference portfolio participation coefficient: Stock index Bond Index	 10% 90%

0 shows parameters estimated on market data and used in mortality and financial risk diffusion processes. In particular, with reference to CIR model, risk-adjusted parameters are calibrated on market value of euro swap interest rates through Brown and Dybvig (1986) framework. Parameters for MRGB model are calibrated on mortality hazard rates derived from an Italian projected life table called "RG48". At last, reference fund parameters are estimated on daily market value of Emu-Bond Index and MSCI World Index observed between 2001 and 2003.

Table 2

Estimated Parameters set

r_t		μ_{x+t}		S_t	
r_0	0.015268	b	0.5	$S_0^{(1)}$	123.57
k	0.245439	σ	0.18	$S_0^{(2)}$	1240.22
θ	0.058359	g	0.10	$\sigma^{(1)}$	0.01
σ_r	0.053524			$\sigma^{(2)}$	0.11

With reference to an insured aged 55 and a contractual term of ten years, 0 reports, the annual expected cash flow of mathematical reserve computed with local rules, fair value of basic contract, participating option, option to annuitise, surrender option and whole contract respectively; we simulate 10,000 paths for the mortality hazard rate, the spot rate and the reference fund, described in 0, 0 and 0.

Table 3

Local Reserve and Fair Value of a contract with embedded options

Year	Local Reserve	Basic Contract	Participation Option	Participating contract / Local Reserve	Option to annuities	Surrender Option	Whole Contract
0	-	-17.37	16.01		-	0.70	-0.66
1	9.19	-7.89	16.44	92.99%	-	0.72	9.27
2	18.97	1.81	17.00	99.18%	-	0.75	19.57
3	29.40	11.87	17.69	100.56%	-	0.79	30.35
4	40.54	22.39	18.49	100.83%	-	0.84	41.72
5	52.46	33.46	19.40	100.78%	-	0.89	53.76
6	65.17	45.17	20.42	100.64%	-	0.96	66.55
7	78.78	57.59	21.55	100.45%	-	1.03	80.16
8	93.37	70.81	22.78	100.23%	-	1.10	94.68
9	108.93	84.91	24.10	100.07%	-	1.17	110.18
10	125.51	100.00	25.51	100.00%	-	-	125.51

The introduction of a fair value accounting system produces a reduction of insurance liabilities and so a different profits distribution over time until the policy's maturity. In particular, for a 10-year contract analyzed, 0 shows that:

- the liability increase is an average about of 1.91%;
- the participating option is the most relevant embedded contract's component, performing in average about of 34.76% over the whole contract value;
- a high performance of the surrender option component, in average about of 1.44% over the whole contract value, is explained by the implicit option annuity component's value equals zero, in reference to a guaranteed annuity coefficient used in the traditional Italian life insurance policies.

0 shows second moments, variation coefficient and skewness of contract value components. These parameters offer a measure of riskness in reference to each contracts component, useful to calculate adequate margins for risk under a Fair Value account system.

Table 4

Second Moments, Variation Coefficient and Skewness

Year	Fair Value Whole contract			Fair Value Surrender Option		
	σ	$\frac{\sigma}{\mu}$	$\frac{\mu_3}{\sigma^3}$	σ	$\frac{\sigma}{\mu}$	$\frac{\mu_3}{\sigma^3}$
0	5.04	1.07	2.81	7.57	12.67	-1.03
1	5.20	0.36	2.76	7.74	12.55	-1.02
2	5.43	0.22	2.64	7.92	12.29	-1.00
3	5.76	0.16	2.45	8.07	11.87	-0.97
4	6.20	0.13	2.20	8.17	11.30	-0.96
5	6.80	0.12	1.92	8.15	10.54	-0.93
6	7.59	0.11	1.64	7.99	9.60	-0.91
7	8.62	0.10	1.42	7.58	8.48	-0.89
8	9.89	0.10	1.27	6.75	7.03	-0.87
9	11.49	0.10	1.16	1.59	1.55	1.79
10	12.81	0.10	1.14	-	-	-

To analyse value's sensitivity of each component, we have derived regions of Fair Value in correspondence to the most relevant parameters: K, i_{sur} and t.

In particular in $t = 0$:

- Figure 1 presents the whole contract Fair Value behaviour in relation to the two parameters K and i_{sur}, showing a non increasing monotone trend, concave in increasing the two parameters;
- Figure 2 shows the surrender option Fair Value behaviour in relation to the two parameters K and i_{sur}; it highlights a non decreasing monotone trend, convex in increasing the K parameter, while it is concave in increasing the i_{sur} parameter.

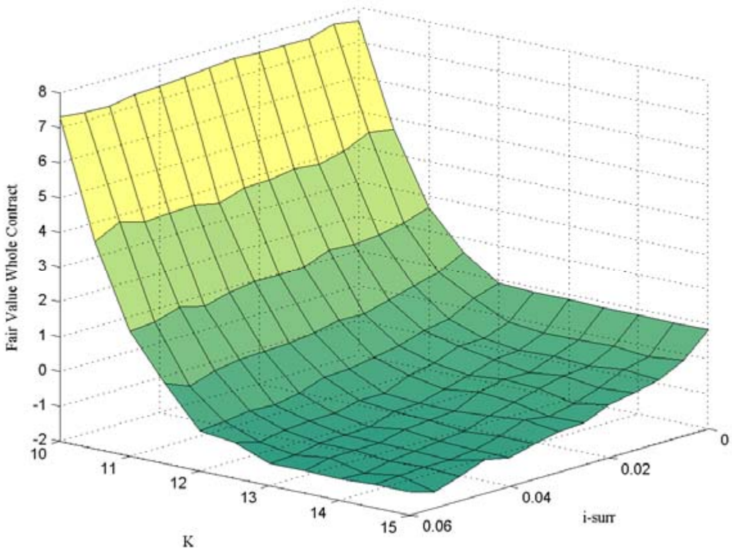

Fig. 1. Fair Value of the Whole contract

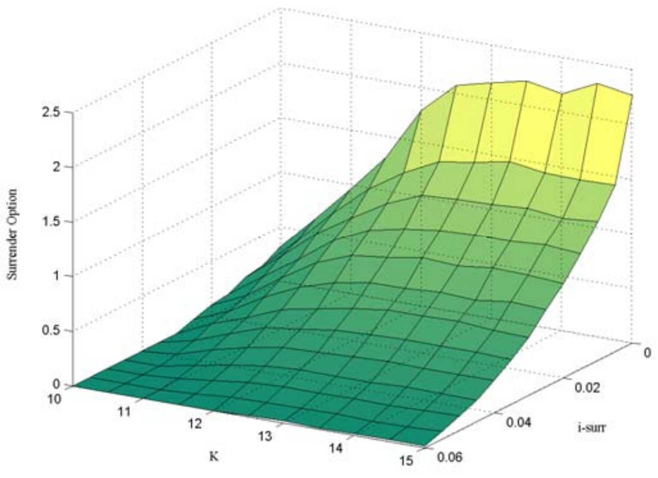

Fig. 2. Fair Value of the Surrender Option

At last, Figure 3 presents the annuity option Fair Value behaviour in relation to the two coordinates t and K; it can be observed that results found in Ballotta and Haberman (2003) are confirmed, as the annuity option Fair Value shows a non increasing monotone trend, concave in increasing the two parameters.

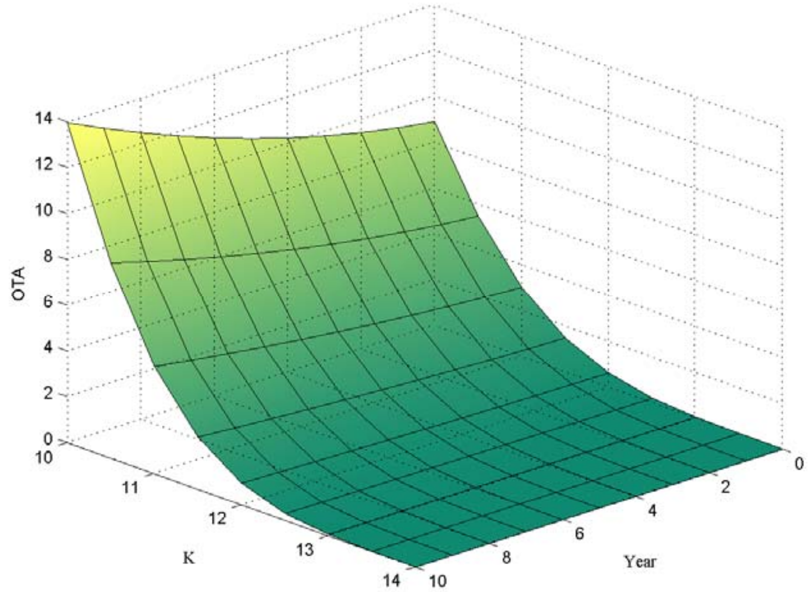

Fig. 3. Fair Value of the Option to Annuitise

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