“Comparing riskiness of exchange rate volatility using the Value at Risk and Expected Shortfall methods”

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COMPARING RISKINESS OF EXCHANGE RATE VOLATILITY USING THE VALUE AT RISK AND EXPECTED SHORTFALL METHODS

Abstract

This paper uses theValue at Risk (VaR) and the Expected Shortfall (ES) to compare the riskiness of the two currency exchange rate volatility, namely BitCoin against the US dollar (BTC/USD) and the South African Rand against the US dollar (ZAR/USD). The risks calculated are tail-related measures, so the Extreme Value Theory is used to capture extreme risk more accurately. The Generalized Pareto distribution (GPD) is assumed under Extreme Value Theory (EVT). The family of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models was used to model the volatility-clustering feature. The Maximum Likelihood Estimation (MLE) method was used in parameter estimation. Results obtained from the GPD are compared using two underlying distributions for the errors, namely: the Normal and the Student-t distributions. The findings show that the tail VaR on the BitCoin averaging 1.6 and 2.8 is riskier than on South Africa's Rand that averages 1.5 and 2.3 at 95% and 99%, respectively. The same conclusion is made about tail ES, the BitCoin average of 2.3 and 3.6 is higher (riskier) than the South African Rand averages at 2.1 and 2.9 at 95% and 99%, respectively. The backtesting results confirm the model adequacy of the GARCH-GPD in the estimation of VaR and ES, since all p-values are above 0.05.

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INTRODUCTION

Since its introduction in 2008, BitCoin is the number one traded cryptocurrency in the world in terms of volume. This decentralized currency can transact without the involvement of the central bank or any financial intermediaries. The transactions using BitCoin are done using a BitCoin network where transactions are authenticated on a blockchain. Due to the lack of backing from a central bank or any regulation, BitCoin users and traders are generally exposed or assumed to be at higher risk (volatility). Like all countries in the world, BitCoin trading in South Africa has gained momentum. This means that there are growing movements of people's investments between the South African Rand, which is the currency used to transact in the Republic of South Africa, and the BitCoin, and investors are thus exposed to market risk.

To measure market risk associated with any financial asset, the Basel Committee on Banking Supervision (BCBS) is responsible for developing supervisory guidelines for banks and financial trading desks. BCBS has recommended that Value at Risk (VaR) be computed and reported. VaR is a statistic that measures the riskiness of financial en-

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tities or portfolios of assets. It is defined as the maximum monetary (say dollars) amount expected to be lost over a given time horizon, at a pre-defined statistical confidence level. This statistic can be used to determine capital requirements. Though VaR is widely accepted and is popular, its weaknesses are well documented. The BCBS stated that "a number of weaknesses have been identified when using VaR for determining regulatory capital requirements, including the inability to capture tail risk" (BCBS, 2013). BCBS (2019) went further to recommend that banks use the Expected Shortfall (ES) instead of VaR for calculating market, credit, and operational risks. The ES is calculated by averaging all of the returns in the distribution that are worse than the VAR of the portfolio at a given level of confidence. For instance, for a 95% confidence level, the Expected Shortfall is calculated by taking the average of returns in the worst 5% of cases (Risk Glossary, 2020).

According to Danielsson (2011), the behavior of the financial time series shows leptokurtic behavior. Leptokurtosis is the tendency for financial asset returns to have distributions that exhibit fat tails and excess peaks from the mean. It is responsible for extreme returns, hence the use of extreme value theory (EVT) models is recommended in efforts to correctly capture the financial risk of financial assets.

It is from this perspective that this study seeks to provide a detailed comparison of risks associated with each of the two currencies so as to provide insight to the foreign currency traders in the Republic of South Africa and member countries of the Rand Union. In this paper, the Generalized Pareto Distribution (GPD) model will be used to capture returns in the tails of currencies and to calculate VaR and ES under the Normal and Student-t distributed errors in comparing the two financial assets, BTC/USD and ZAR/USD.

1. LITERATURE REVIEW

Regardless of the popularity of VaR, its limitations are well documented by many researchers. Rockafellar and Uryasev (2002) showed that VaR is not only incoherent but also fails to precisely estimate the risk of loss when the loss distributions have'fat tails'. "This significantly discreditsthe accuracy of this risk measure"(Chen, 2018). Nonetheless, VaR remains a popular risk measure as it is very simple to calculate and understand. Artzneret al. (1999) not only showed the incoherence of VaR but also introduced the Expected Shortfall, and called it a perfect risk measure. Pflug (2000) showed that ES is a coherent risk measure, based on coherent risk measure theory.ESisarisk measure sensitive to the shape of the tail of the distribution of returns on a portfolio, unlike the more commonly used VaR.

In order for risk practitioners to fully captureextreme or'tail'risk (very big uncertainty or fluctuation in returns)as mentioned in BCBS (2019), Extreme Value Theory (EVT) has been developed. The field of EVT was pioneered by Fisher and Tippett (1928) and Pickands (1975). Fisher and Tippett (1928) obtained three asymptotic limits describing the distributions of extremes assuming variables are independent. The arguments leading to modelling extremes using the generalized Pareto distribution are attributable to Pickands (1975). EVT is the theory of measuring and modeling extreme events (large fluctuations) (tails of statistical distributions), i.e. it is well suited to financial assets with extreme returns (very large fluctuations in returns). The EVT assumes independent and identically distributed (iid) observations. This iid assumption does not always hold for financial time series data. To correct this, McNeil and Frey (2000) proposed a two-stage methodology in the form of a GARCH-EVT model using five index returns in his illustrations. The first step is to capture the heteroscedasticity (non-constant variation or fluctuations) features by fitting a GARCH model. The second step is to apply the EVT to residuals extracted from a selected GARCH model using the Generalized Pareto Distribution (GPD) or the Generalized Extreme Value Distribution (GEVD). The second part of this modelling process allows one to capture or describe the large fluctuations in prices and returns. The merits of the GARCH-EVT hybrid model lie in its ability to capture conditional heteroscedasticity (changing variation) in the data through the GARCH framework, while at the same time modelling the extreme tail (large fluctuations) behavior through the EVT method. Byström (2004) applied the GARCH-EVT model to the Swedish and American stock markets and compared different EVT-AMS methods and the Peak over Threshold (POT) method, which in general perform similarly. Murenzi et al.(2015) used the hybrid GARCH models with the EVT model to estimate VaR for the Rwandese currency exchange rate. Their findings indicated that the filtered EVT model performed better than ARMA-GARCH models. Chebbi and Hedhli (2014) applied the GARCH-EVT in managing portfolios with time-varying copula in the Tunisian, American, French, and Moroccan stocks. The copula is able to quantify the interdependence betweenthe different countries' stocks.

Koliai (2016) presented a GARCH-EVT model with an R-vine model to manage portfolio risks that consisted of equity, currency indices, and commodities. Chinhamu et al. (2017) investigated the performance of the Generalized Lambda Distribution (GLD), the Generalized Pareto Distribution (GPD), and the Generalized Extreme Value Distribution (GEVD) in modeling daily VaR and ES in platinum, gold, and silver price log-returns. Their findings showed that GPD and GLD generally outperform GEVD for VaR and ES estimation for negative precious metal returns.

This study seeks to apply this hybrid model (GARCH-GPD) in the calculation of the VaR and ES using data from exchange rates BTC/USD and ZAR/USD and to compare their riskiness.

2. METHODOLOGY

This section describes the three steps that will be taken to fit the model.

The conditional approach to estimating VaR proposed by McNeil and Frey (2000) follows a three-step chronology. Firstly, an Autoregressive Moving Average (ARMA) or similar model with GARCH errors is fitted to the returns data $\{Y_t\}$ using a pseudo maximum likelihood estimation approach. The standardized residuals from this model are extracted. The residuals series $\{X_t\}$ are realizations of the unobserved *iid* noise variables.

Secondly, extreme value theory (EVT) is then used to model the tail behavior of the data. The $VaR_q(X)$ is then calculated using the Generalized Pareto Distribution (GPD) tail estimation procedure.

Finally, the *VaR* of the asset is computed using the following formula:

$$VaR_q(Y) = \mu_{t+1} + \sigma_{t+1} \cdot VaR_q(X), \quad (1)$$

$$ES_q(Y) = \mu_{t+1} + \sigma_{t+1} \cdot ES_q(X), \quad (2)$$

where is μ_{t+1} is the forecasts from the mean equation and σ_{t+1} is estimated from the volatility prediction model. $VaR_q(X)$ and $ES_q(X)$ are the *VaR* and *ES* of the standardized residuals.

2.1. ARMA – GARCH

GARCH models allow one to explain the varying, up and down movements (volatility), in asset prices,e.g. Bitcoin prices, and ZAR/USD exchange rates. The GARCH family volatility models to be considered are GARCH (1,1), EGARCH(1,1), GJRGARCH(1,1), and APARCH(1,1). All four volatility models are to be fitted to the exchange rates and their residuals are used in modelling the tail behavior of the series.

The GARCH(p, q) volatility model is mathematically defined as:

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{P} \beta_j \sigma_{t-1}^2, \quad (3)$$

where α_i and β_j are respectively ARCH and GARCH terms, and w is a constant; σ_t^2 and σ_{t-1}^2 are respectively the fitted conditional volatility from the model and its previous value. ε_t^2 are the squared error terms in the model. The simplest model, the random walk with GARCH(1,1) variance/volatility model, is of the form:

$$\begin{cases} X_t = \mu + \varepsilon_t \\ \sigma_t^2 = w + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1,}^2 \end{cases}. \quad (4)$$

Exchange rates are unpredictable, and it makes sense to assume a random walk model.

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2.2. Exponential GARCH (EGARCH) model

The EGARCH model allows efficient capturing of volatility clustering and asymmetric effects. An EGARCH(1, 1) volatility model can be expressed as:

$$\ln\left(\sigma_t^2\right) = w + \alpha_1\left(\frac{|\varepsilon_{t-1}|}{|\sigma_{t-1}|}\right) + \\ +\gamma\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + \beta_1 \ln\left(\sigma_{t-1}^2\right), \quad (5)$$

where w, α_1, β_1 are model coefficients. γ is the leverage effect.

2.3. GJR-GARCH model

The GJR-GARCH volatility model was proposed by Glosten et al. (1993). This model takes into account the asymmetry property of financial data in obtaining the conditionals.

A GJR-GARCH (1,1) model is given as:

$$\sigma_t^2 = w + (\alpha_1 + \lambda_1 I_{t-1})\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2, \quad (6)$$

where w, α_1, β_1 are model coefficients, the conditions for non-negativity are: $w > 0$, $\alpha_1 > 0$, $\beta_1 \geq 0$, and $\alpha_1 + \lambda_1 \geq 0$. That is, the model is still admissible, even if $\lambda_1 < 0$ provided that, $\alpha_1 + \lambda_1 \geq 0$.

2.4. APARCH model

The APARCH (Asymmetric Power ARCH) volatility model is used to model better the leverage effect than the standard GARCH. The volatility equation of the APARCH(p, q) can be written as:

$$\sigma_t^\delta = w + \sum_{i=1}^{p}\alpha_i\left(|a_{t-1}| - \gamma_i a_{t-1}\right)^\delta + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^\delta, \quad (7)$$

where $\delta > 0$ and $-1 < \gamma_j < 1$, $j = 1, ..., p$. Note that $\delta = 2$ and $\gamma_1 = ,..., = \gamma_p = 0$ the APARCH volatility model is reduced to GARCH volatility model.

2.5. EVT – The Generalized Pareto Distribution (GPD)

The peak over threshold (POT) approach in fitting the Generalized Pareto Distribution (GPD) is used to model the standardized residuals from the selected GARCH family model. Balkema and deHaan (1974) and Pickands (1975) showed that for threshold u that is large enough, the POT approach leads to the use of the GPD. The GPD is defined as follows:

$$G_{\xi,\beta}(x) = \begin{cases} 1 - \left(1 + \dfrac{\xi(x-u)}{\beta}\right)^{-1/\xi} & if\ \xi \neq 0 \\ 1 - \exp\left(-\left(\dfrac{x-u}{\beta}\right)\right) & if\ \xi = 0 \end{cases}, \quad (8)$$

where $x > 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$. When $\xi < 0$ and $\beta > 0$, $G_{\xi,\beta}(x)$ is a GPD with the shape parameter or tail index ξ, a scale parameter β and a threshold u The value of ξ shows how heavy the tail is, with a bigger value indicating a heavy tail.

2.6. Parameter estimation of GPD

Let u be a sufficiently high threshold, assuming n observations y such that $y_i - u \geq 0$, the subsample $\{y_1 - u, ..., y_n - u\}$ has an underlying distribution of a GPD, where $y_i - u \geq 0$ for $\xi \geq 0$, $0 \leq y_i - u \leq -\beta/\xi$ for $\xi < 0$, then the logarithm of the probability density function of $y_i - u$ is:

$$\ln\left(f\left(y_i - u\right)\right) = \\ = \begin{cases} -\ln(\beta) - \dfrac{1+\xi}{\xi}\ln\left(1 + \xi\left(\dfrac{y_i - u}{\beta}\right)\right) & if\ \xi \neq 0 \\ -\ln(\beta) - \dfrac{1}{\beta}\left(y_i - u\right) & if\ \xi = 0 \end{cases}. \quad (9)$$

Then the log-likelihood $L(\xi, \beta | y_i - u)$ for the model is the logarithm of the joint density of the n observations, i.e.

$$L(\xi, \beta | y_i - u) = \\ = \begin{cases} -n\ln(\beta) - \dfrac{1+\xi}{\xi}\sum_{i=1}^{n}\ln\left(1 + \xi\left(\dfrac{y_i - u}{\beta}\right)\right) & if\ \xi \neq 0 \\ -n\ln(\beta) - \dfrac{1}{\beta}\sum_{i=1}^{n}\left(y_i - u\right) & if\ \xi = 0 \end{cases}. \quad (10)$$

The parameters (ξ, β) are obtained by maximizing the log-likelihood function of the subsample under a suitable threshold u.

2.7. Conditional VaR

If F is an extreme distribution above some threshold u, then the $F_u(x) = G_{\xi,\beta}(x)$, where $0 \le x < x_F - u$ and $\xi \in \mathbb{R}$ and $\beta > 0$, if $x \ge u$ then:

$$\begin{aligned}&\bar{F}(x) = P(X > u)P(X > x \setminus X > u) = \\ &\bar{F}(u)P(X-u > x-u \setminus X > u) = \\ &= \bar{F}(u)\bar{F}(x-u) = \bar{F}(u)\left(1+\xi\frac{x-u}{\beta}\right).\end{aligned} \tag{11}$$

Given $\bar{F}(u)$, $\bar{F}(x)$ is the formula for survival tail probabilities, it is inverse, gives the highest quantile of the distribution which represent the Value at Risk and is given by:

$$VaR_\alpha = q_\alpha(F) = u + \frac{\beta}{\xi}\left(\left(\frac{1-\alpha}{\bar{F}(u)}\right)^{-\xi} - 1\right), \tag{12}$$

and the Expected Shortfall is given as:

$$CVaR_\alpha = \frac{VaR_\alpha}{1-\xi} + \frac{\beta - \xi u}{1-\xi}. \tag{13}$$

2.8. Backtesting

To assess model adequacy and effectiveness in the computation of VaR, two backtesting methodologies are used. The Kupiec unconditional coverage test (Kupiec, 1995) and Christoffersen conditional coverage test (Christoffersen, 1998).

The unconditional coverage test by Kupiec assumes that the proportion of violations of VaR estimates must be close to the corresponding tail probability level if the model is adequate.

Let x^p, be the number of violations observed at level p, i.e., $r_t < VaR_p$ (for long positions) or $r_t > VaR_p$, (for short positions). The test procedure involves comparing the corresponding proportion of violation $[x^p / N]$ to p.

The H_0:E$[x^p / N = p]$ i.e. (the expected proportion of violations is equal to p).

Under H_0, the Kupiec statistic, which is given by

$$\begin{aligned}LR_{UC} &= 2\ln\left(\left(\frac{x^p}{N}\right)^{x^p}\left(1-\frac{x^p}{N}\right)^{N-x^p}\right) - \\ &-2\ln\left(p^{x^p}(1-p)^{N-x^p}\right),\end{aligned} \tag{14}$$

and it follows a chi-square distribution with one degree of freedom.

The Christoffersen test extends the unconditional coverage test by taking into account the independence of violations (i.e., clustering of extremes). The Christoffersen test statistic is given by

$$\begin{aligned}LR_{CC} &= LR_{UC} + \\ &+2\ln\frac{\left[(1-\pi_0)^{\Phi_{00}}\pi_0^{\Phi_{01}}(1-\pi_1)^{\Phi_{10}}\pi_1^{\Phi_{11}}\right]}{\left[(1-\pi)^{(\Phi_{00}+\Phi_{10})}\pi^{(\Phi_{01}+\Phi_{11})}\right]},\end{aligned} \tag{15}$$

where Φ_{ij} is the number of returns in state i who have been in state j previously (state 1 indicates that the VaR estimate is violated and state 0 indicates that it is not) and π_i is the probability of having an exception that is conditional on state i the previous day. This statistic follows a chi-square distribution with two degrees of freedom.

3. RESULTS

Quantitative exchange rates data was collected and modelled so as to achieve the set objectives. Data was obtained from the finance sector website (Investing.com). The currencies considered are the South African Rand (ZAR) against the US dollar (USD) and the BitCoin (BTC) against the US dollar. The data was analyzed in an R-programming environment. The daily exchange rates considered were from 1/1/2015 to 30 /06/2021. The log returns were calculated and used to do the modelling.

The formula used is:

$$y_t = \log\left[\frac{P_t}{P_{t-1}}\right], \tag{16}$$

whereP_t and $P_{t-1}$are today and yesterday's closing values of daily prices (exchange rates), respectively.

In Figures 1 and 2, the time series plots reveal several trends in the mean and variance, confirming non-stationarity of the exchange rates prices. The log returns are stationary, around the zero-mean, although volatility is non-constant and clustered, indicating heteroscedasticity, and is common with financial data. Isolated extreme returns are visible and are caused by shocks in the financial markets.

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Source: Authors' own work.

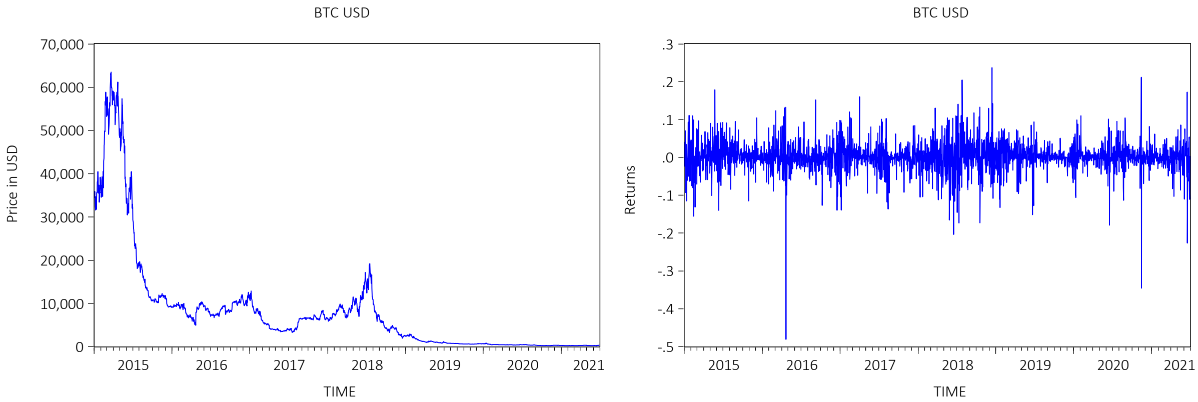

Figure 1. Time series plot of one-day BTC/USD prices (left) and one-day log returns (right)

Source: Authors' own work.

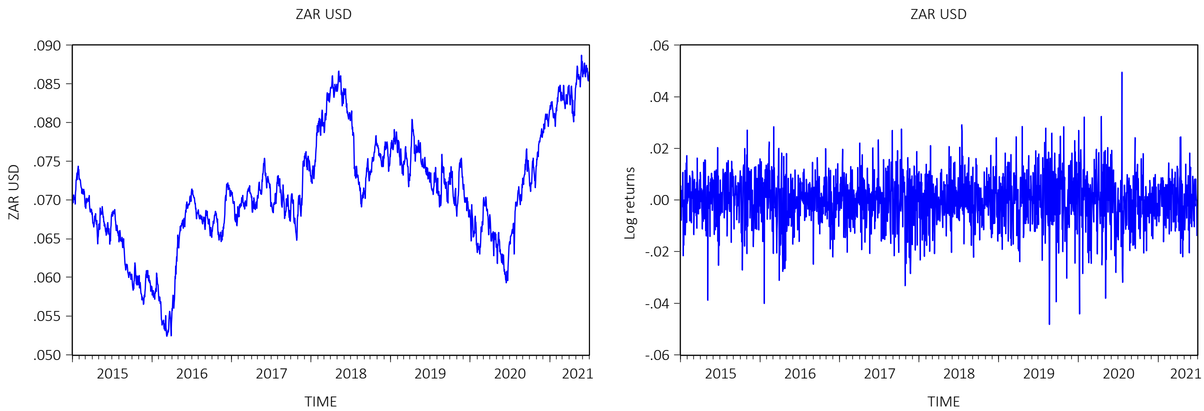

Figure 2. Time series plot of ZAR/USD prices (left) and one-day log returns (right)

3.1. Descriptive statistics

Table 1 gives the descriptive statistics of the two exchange rates.

A positive mean for BTC/USD indicates a small increasing trend over time, whereas the opposite is true with a negative mean for the ZAR/USD, indicating a slight decreasing trend over time for the return series.

The Jarque-Bera test rejects the null hypothesis of Normality at the 5% level of significance, suggesting that extreme value theory distributions could be useful in capturing any heavy tails.

Table 1. Descriptive statistics of exchange rate returns

Source: Authors' own work.

	Observations	Mean	Median	Maximum	Minimum	Skewness	Kurtosis
BTC/USD	2370	0.001990	0.001757	0.237220	–0.480904	–0.994382	16.15451
ZAR/USD	1694	–0.000125	0.000000	0.049546	–0.048252	–0.264130	4.121644

TEST	BTC/USD		ZAR/USD	
	Statistic	p-value	Statistic	p-value
Jarque-Bera	17478.40	0.000000	108.4967	0.000000
Ljung-Box	11.7	0.0006249	0.40504	0.5245
ARCH LM Test	52.87	4.345e–07	70.789	2.28e-10

Unit Root Test	BTC/USD		ZAR/USD	
	Statistic	p-value	Statistic	p-value
ADF Test	–52.20130	0.0001	–40.47263	0.0000
PP Test	–52.10963	0.0001	–40.47011	0.0000
KPSS Test	0.092067	0.347000	0.090747	0.347000

The *p*-value of the Ljung-Box test for ZAR/USD returns confirms non-rejection of the null hypothesis of no autocorrelation. The fitting of a statistical distribution usually assumes homoscedasticity and no autocorrelation. However, autocorrelation is confirmed for the BTC/USD since*p*-value = 0.0006249 < 0.05. The two step approach will therefore be used to help deal with the autocorrelation.

The null hypothesis of no ARCH effects is rejected at the 5% level of significance using the ARCH LM test, suggesting the use of GARCH family models should be considered when analyzing the above-mentioned return series

The unit root test and stationary tests show that, at the 5% level of significance, the null hypothesis of a unit root is rejected, and it can be concluded that both exchange rate return series are stationary. The KPSS test results shows that all returns are stationary, since all *p*-values are greater than 0.05, therefore the null hypothesis of stationarity is accepted

3.2. GARCH-EVT models

Based on literature and the financial characteristics presented above, using GARCH-EVT model could lead to a better measure of risk in the tail.

Table 2. Estimated GARCH parameters for both BTC/USD and ZAR/USD

Source: Authors' own work.

Variables	BTC/USD							
	GARCH (1,1)		gjrGARCH (1,1)		APARCH (1,1)		eGARCH (1,1)	
	Norm	STD	Norm	STD	Norm	STD	Norm	STD
Variance Equation	Estim p--	Estimp-	Estimp-	Estim p-	Estimp-	Estimp-	Estimp-	Estim p-
w	0.0001	0.0001	0.0001	0.0001	0.0037	0.0049	−0.2929	−0.1270
	(0.001)	(0.001)	(0.001)	(0.001)	(0.007)	(0.018)	*(0.001)*	(0.0026)
α_1	0.2012	0.1487	0.2427	0.1620	0.1790	0.114	0.0293	−0.0218
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.096)	(0.3128)
β_1	0.7972	0.850	0.8007	0.8500	0.8312	0.901	0.9515	0.9794
	(0.001)	(0.001)	(0.001)	(0.001)	*(0.001)*	*(0.001)*	*(0.001)*	*(0.001)*
λ1	–	–	−0.099	−0.0253	−0.0634	0.328	0.3245	0.2834
	–	–	(0.001)	(0.5053)	(0.345)	(0.004)	*(0.001)*	*(0.001)*
Shape	–	3.376	–	3.365	–	3.019	–	2.9764
	–	(0.001)	–	(0.001)	–	*(0.001)*	–	*(0.001)*
Delta	–	–	–	–	0.7512	0.399	–	–
	–	–	–	–	*(0.001)*	*(0.001)*	–	–
Goodness of fit								
AIC	−3.8097	−3.9761	−3.8121	−3.9754	−3.8481	−4.0219	−3.8449	−4.0068
BIC	−3.8024	−3.9664	−3.8023	−3.9633	−3.8359	−4.0073	−3.8352	−3.9946
Variables	**ZAR/USD**							
	GARCH (1,1)		**gjrGARCH (1,1)**		**APARCH (1,1)**		**eGARCH (1,1)**	
	Norm	**STD**	**Norm**	**STD**	**Norm**	**STD**	**Norm**	**STD**
Variance Equation	Estim p-	Estimp-	Estimp-	Estimp-	Estim p-	Estimp-	Estim p-	Estimp-
w	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−0.0371	−0.0606
	(0.1018)	(0.1194)	(0.407)	(0.8195)	(0.7047)	(0.6802)	*(0.0001)*	*(0.0001)*
α_1	0.0496	0.0524	0.0626	0.0675	0.02295	0.0205	0.0548	0.0541
	(0.0001)	(0.0001)	*(0.0001)*	(0.0067)	*(0.0001)*	(0.0034)	*(0.0001)*	*(0.0001)*
β_1	0.9271	0.9216	0.9697	0.9609	0.9559	0.9568	0.9958	0.9934
	(0.0001)	(0.0001)	*(0.0001)*	*(0.0001)*	*(0.0001)*	*(0.0001)*	*(0.0001)*	*(0.0001)*
λ1	–	–	−0.0673	−0.0653	−0.4627	−0.4940	0.0505	0.0645
	–	–	*(0.0001)*	*(0.0001)*	(0.0092)	(0.0535)	*(0.0001)*	*(0.0001)*
Shape	–	12.905	–	16.322	–	12.843	–	16.64
	–	(0.0001)	–	(0.2131)	–	*(0.0001)*	–	*(0.0013)*
Delta	–	–	–	–	2.5844	2.5880	–	–
	–	–	–	–	*(0.0001)*	*(0.0001)*	–	–
Goodness of fit								
AIC	−6.3641	−6.3756	−6.3770	−6.3832	−6.3700	−6.3781	−6.3786	−6.3843
BIC	−6.3545	−6.3628	−6.3641	−6.3672	−6.3539	−6.3588	−6.3657	−6.3683

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Source: Authors' own work.

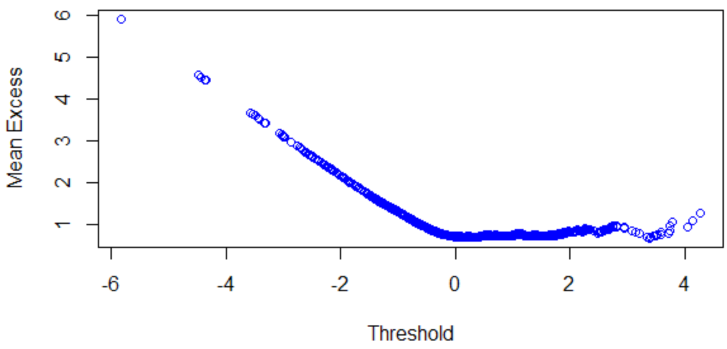

Figure 3. Mean excess function for BTC/USD returns

As the return series are non-Normal and have a heteroscedasticity feature, as suggested in other research work, using EVT and GARCH model to capture some features of the data is necessary.

All the four GARCH(1,1) models were estimated with both Normal distributed errors and Student's t-distributed errors. Their residuals were extracted, standardized and used to fit the Generalized Pareto distribution models. The aim is to fit a statistical distribution (the GPD) to the extreme residuals and use the estimated parameters and formulas to calculate VaR and ES.

To estimate the GPD model, a threshold$u$ must be selected. This threshold determines the number of observations above the threshold, N_u. As the rule of thumb also suggests that it is ideal to choose the threshold that gives about 100 observations for fitting the Pareto distribution when the data set is large enough (McNeil & Frey, 2000).

The mean excess plots determine a suitable threshold, which is necessary for fitting the GPDmodel. The choice of a threshold should be depicted by linear increases in the mean excess plot. Figure 3 presents the mean excess function of BTC/USD returns. By observing the meanexcess function in Figure 3, a threshold of between 0.8 and 1.5 seems to be a reasonable choice. The 90th percentile was selected. It provided a reasonable choice as it yielded enough data points for analyses and it falls within the above range.

The parameters of the GPD were estimated using the Maximum Likelihood method and are presented in Table 3.

n and t represent the assumption of Normal and Students't-distributed errors, respectively.Most of the ξ are positive, suggesting the presence of heavy tails, except for the APARCH-GPD-N and eGARCH-GPD-N models.

After observing the mean excess function for ZAR/USD in Figure 4, a threshold of between 0.8 and 1.8 seems to be a reasonable choice for ZAR/USD returns. Again, the threshold at 90th percentile was selected. This is a reasonable choice as it yields enough data points for analyses and it falls within the above range. The pa-

Table 3. Maximum likelihood estimates (MLE) of GARCH (1,1) residuals from BTC/USD to the Generalized Pareto Distribution

Source: Authors' own work.

Model	u	ξ	Se(ξ)	$\hat{\sigma}$	Se($\hat{\sigma}$)
GARCH(n)-GPD	1.1598	0.00582	0.06067	0.76030	0.06758
GARCH(t)-GPD	1.1360	0.00806	0.05983	0.74643	0.06592
gjrGARCH(n)-GPD	1.1742	0.00198	0.06119	0.75760	0.06761
gjrGARCH(t)-GPD	1.1387	0.00877	0.06007	0.74071	0.06553
APARCH(n)-GPD	1.1556	–0.00541	0.06002	0.75179	0.06649
APARCH(t)-GPD	1.0446	0.08214	0.06814	0.57907	0.05443
eGARCH(n)-GPD	1.1674	–0.00301	0.06058	0.74487	0.06617
eGARCH(t)-GPD	1.0178	0.02036	0.06038	0.64500	0.05720

Source: Authors' own work.

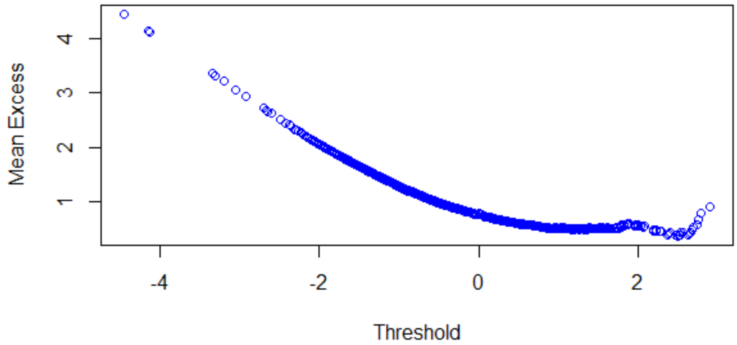

Figure 4. Mean excess function for ZAR/USD returns

Table 4. Maximum likelihood estimates (MLE) of GARCH (1,1) residuals from ZAR/USD to the Generalized Pareto Distribution

Source: Authors' own work.

Model	u	$\hat{\xi}$	Se($\hat{\xi}$)	$\hat{\sigma}$	Se($\hat{\sigma}$)
GARCH(n)-GPD	1.2031	0.04059	0.08097	0.47385	0.05283
GARCH(t)-GPD	1.2027	0.04018	0.08033	0.47536	0.05277
gjrGARCH(n)-GPD	1.2182	0.02741	0.07788	0.47203	0.05159
gjrGARCH(t)-GPD	1.2166	0.02857	0.07527	0.47564	0.05110
APARCH(n)-GPD	1.2014	0.02049	0.07397	0.48851	0.05205
APARCH(t)-GPD	1.2067	0.02305	0.074497	0.48626	0.05198
eGARCH(n)-GPD	1.2243	0.04838	0.08199	0.45048	0.050549
eGARCH(t)-GPD	1.2100	0.02193	0.07155	0.48073	0.05042

rameters of the GPD were estimated using the Maximum Likelihood method and are presented in Table 4.

Again, *n* and *t* represent the assumption of Normal and Student's-distributed errors.

All the ξ are positive suggesting the presence of heavy tails. The ξ for the ZAR/USD are bigger and positive, suggesting that it may be a riskier asset; however, the standard errors of the estimates of ξ are bigger as well. The VaR and ES will give a better picture of the risk.

3.3. Value at Risk and Expected Shortfall

The Value at Risk and Expected Shortfall values are calculated at 95% and 99% levels for both exchange rates understudy. Tables 5 and 6 show the estimated values of each VaR and ES.

At both the 95% and 99% levels of significance, daily VaR data suggests that Bit Coin is riskier than the South African rand, as the VaR statistic is higher. The higher VaR statistics mean that one loses more at a specified level of significance.

Table 5. VaR estimates using fitted hybrid GARCH(1,1)-GPD

Source: Authors' own work.

Model	BTC/USD		ZAR/USD	
	95%	99%	95%	99%
GARCH(n)-GPD	1.68783	2.922216	1.53794	2.348641
GARCH(t)-GPD	1.654816	2.870768	1.538596	2.351382
gjrGARCH(n)-GPD	1.699733	2.922682	1.55025	2.34193
gjrGARCH(t)-GPD	1.653669	2.861568	1.551253	2.350387
APARCH(n)-GPD	1.675701	2.875891	1.544158	2.354978
APARCH(t)-GPD	1.457608	2.512381	1.548177	2.358383
eGARCH(n)-GPD	1.683199	2.876632	1.543495	2.323335
eGARCH(t)-GPD	1.468054	2.538347	1.547495	2.347134

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Table 6. ES estimates using fitted hybrid GARCH(1,1)-GPD

Source: Authors' own work.

Model	BTC/USD		ZAR/USD	
	95%	99%	95%	99%
GARCH(n)-GPD	2.45568	3.697297	2.046006	2.891008
GARCH(t)-GPD	2.411539	3.637375	2.04792	2.894734
gjrGARCH(n)-GPD	2.459889	3.685268	2.044937	2.858924
gjrGARCH(t)-GPD	2.405488	3.624072	2.050724	2.873361
APARCH(n)-GPD	2.420641	3.614369	2.050053	2.877833
APARCH(t)-GPD	2.125469	3.274639	2.053965	2.883285
eGARCH(n)-GPD	2.424288	3.614144	2.033105	2.852593
eGARCH(t)-GPD	2.135825	3.228368	2.046567	2.864137

At both the 95% and 99% significance levels, daily ES data suggests that BitCoin (BTC) is riskier than the South African rand (ZAR), as the ES statistic is higher. The higher ES statistic implies that the average loses are greater above the given threshold (VaR statistic).

3.4. Back testing results for Value at Risk

The estimated VaR are back tested using the Kupiec unconditional coverage test and Christoffersen conditional coverage. The p-values of each test are presented in Table 7.

Based on the *p*-values from both Kupiec likelihood ratio test and Christoffersen's test likelihood ratio test, the fitted GARCH-GPD models are well suited to the returns series understudy, since the observed*p*-values are greater than 0.05. Hence, the null hypothesis of model adequacy is accepted.

4. DISCUSSION

Four GARCH models are applied to the two data sets understudy, namely: the BTC/USD and the ZAR/USD. All four models were considered under two commonly used error distributions, that is normally and Student's t distributed residuals. The residuals were extracted and used to fit Generalized Pareto Distribution, and the estimated parameters were used to estimate risk statistics.

Under the BTC/USD time series, the extreme value index (ξ) seems to be inconclusive as to whether the tails are heavy or short. The GARCH models with Normally distributed errors would mean the tails are short, while the Student's t distributed errors suggest that the tails are heavier. In the case of the ZAR/USD currency series, the EVI suggests that the tails are heavy, hence the use of the EVT model is deemed more appropriate.

The computed values of both VaR and Conditional VaR indicate that there is a higher amount expected to be lost in the BTC/USD than in the ZAR/USD at both the 95% and 99% confidence levels. This leads to the conclusion that the BTC is riskier than the ZAR. This could be due to the fact that the BTC is not backed by any central bank and is unregulated hence making it highly risky to keep as a savings tool. This information is useful to

Table 7. Backtest results for VaR estimates using fitted GARCH(1,1)-GPD

Source: Authors' own work.

Model	Kupiec test (p-values)				Christoffersen's test (p-values)			
	BTC/USD		ZAR/USD		BTC/USD		ZAR/USD	
	95%	99%	95%	99%	95%	99%	95%	99%
GARCH(n)-GPD	0.8878	0.8845	0.7622	0.2375	0.3483	0.7897	0.3307	0.2838
GARCH(t)-GPD	0.7403	0.5698	0.7622	0.2375	0.7208	0.7051	0.3307	0.2838
gjrGARCH(n)-GPD	0.9624	0.5698	0.6779	0.2375	0.3714	0.7051	0.7768	0.3726
gjrGARCH(t)-GPD	0.9624	0.5698	0.6779	0.6217	0.3921	0.7051	0.7768	0.7136
APARCH(n)-GPD	0.8873	0.7224	0.5970	0.7978	0.1658	0.7639	0.4684	0.7975
APARCH(t)-GPD	0.8878	0.7902	0.6779	0.6217	0.8849	0.7393	0.5247	0.7136
eGARCH(n)-GPD	0.7426	0.7224	0.9377	0.6217	0.1099	0.7639	0.9139	0.7136
eGARCH(t)-GPD	0.5365	0.8845	0.7622	0.6217	0.8186	0.7897	0.5784	0.7136

South African and global investors who need to understand how much risk they take when converting their savings or investments to BitCoin instead of the South African currency, the Rand (ZAR).

The backtest results gave p-values that are way above 0.05. The high p-values indicate that the hybrid model used in the study isgoodand fitsthe currency data set used. The Kupiec test suggests that the GARCH(1,1)-GPD with Normally distributed errors is the best fitting model for BTC/USD, while eGARCH(1,1)-GPD with Normally distributed errors is the best-fitted model for ZAR/USD, as they both have higher p-values at both 95 % and 99% significance levels.

Christoffersen's test likelihood ratio test enables one to ascertain whether the output model violations are independent.A violation is when the actual loss exceeds the VaR estimate. In all the models, the p-values are greater than 0.05, leading to a conclusion that indeed the violations are independent of each other.

CONCLUSION

The purpose of this study is to use VaR and ES to compare the riskiness of the daily returns of the BitCoin (BTC) and South African Rand (ZAR), both currenciesbeingagainst the US Dollar. Both VaR and ES conclude that BTC is riskier than ZAR. This could be of great help to forex market risk managers in South Africa, particularly in choosing whether to keep their savings in the local currency or consider the cryptocurrency, BitCoin.

The hybrid model did capture fat tails and improved the computation of the VaR and ES as shown by the positive EVI (ξ) and backtesting procedures. The high p-values above 0.5 suggest that the GARCH (1,1)-GPD is a very good fit for the two currencies.

These results do not imply that GARCH (1,1)-GPD will always give good fits for every currency data set. For further research, out-of-sample backtests and comparisons with generalized POT models are recommended, such as DIPOT (Dynamic Intensity Peaks Over Threshold) and PORT (Peak Over Random Threshold).

AUTHOR CONTRIBUTIONS

Conceptualization: Thabani Ndlovu, Delson Chikobvu.
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Formal analysis: Thabani Ndlovu, Delson Chikobvu.
Investigation: Thabani Ndlovu.
Methodology: Thabani Ndlovu.
Project administration: Delson Chikobvu.
Resources: Delson Chikobvu.
Software: Thabani Ndlovu.
Supervision: Delson Chikobvu.
Validation: Thabani Ndlovu.
Writing – original draft: Thabani Ndlovu.
Writing – review & editing: Thabani Ndlovu, Delson Chikobvu.

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