

# “Regulation of a duopoly and environmental R&D”

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## Regulation of a duopoly and environmental R&D

### Abstract

The authors develop a three stage game model composed of a regulator and two firms. These firms compete on the same market where they offer the same homogeneous good and can invest in R&D to lower their emission/output ratio. By means of a tax per-unit of pollution and a subsidy per-unit of R&D level, the regulator can induce the first-best outcome. Interestingly, the investment in R&D is actually taxed when the marginal damage cost of pollution is high enough, because firms are tempted to overinvest in research.

**Keywords:** duopoly, emission tax, R&D subsidy, first-best.

**JEL Classifications:** C72, D62, H21, O32.

### Introduction

The environment suffers from a degradation which is more and more visible. Since the market cannot internalize these damages caused to the environment, the intervention of the state (regulator) is necessary. Many regulatory instruments are used among which we cite emission taxes, standards, emission permits, and research and development (R&D) subsidies. The strategy consisting in encouraging the development and the diffusion of cleaner technologies plays an important role because it enables to pollute less without compromising economic growth.

Milliman and Prince (1989) considered identical firms in a competitive industry, and evaluated the incentive effects of five environmental policy instruments, which are direct controls, emission subsidies, emission taxes, free marketable permits, and auctioned marketable permits, to promote technological changes in pollution control. They showed that emission taxes and auctioned permits provided the highest firm incentives to promote technological changes. Jung, Krutilla and Boyd (1996) extended this comparative study to a heterogeneous industry. Stranlund (1997) considered public aid to encourage the adoption of superior emission-control technologies combined with monitoring. This strategy is interesting when monitoring is not easy because the sources of pollution are widely dispersed or when emissions are not easily measured as in non-point pollution problems. Technological aid reduces the direct enforcement effort necessary for firms to reach the compliance goal. Consequently, firms adopt better pollution control technologies, which may serve to promote further innovative activity. Requate and Unold (2003) investigated incentives given by environmental regulatory instruments to get firms adopting advanced abatement technologies.

Farzin and Kort (2000) studied the regulation of a competitive firm and examined the effect of a higher

pollution tax rate on abatement investment, both under full certainty and when the timing or the size of the tax increase is not certain. They established the possibility that a higher pollution tax rate induces more pollution, and that a credible threat to accelerate the tax increase can induce more abatement investment. Fischer and Newell (2008) assessed how the nature of technological progress through learning and R&D, and the degree of knowledge spillovers, affected the desirability of different regulatory policies. Because of knowledge spillovers, the optimal policy involves a portfolio of different instruments targeted at emissions, learning, and R&D. Brêchet and Jouvet (2008) showed that environmental innovation does not reduce necessarily the marginal cost of abatement.

Using a two-stage game, D'Aspremont and Jacquemin (1988, 1990) examined the effects of the behavior of a duopoly in the cases of non-cooperation and cooperation in R&D in presence of positive and free R&D externalities. Ben Youssef (2009) considered a non-cooperative and symmetric three stage game played by two regulator-firm hierarchies. He showed that free R&D spillovers and the competition of firms on the common market help non-cooperating countries to better internalize transboundary pollution. Interestingly, international competition increases the per-unit emission tax and decreases the per-unit R&D subsidy.

This model differs from that of Ben Youssef (2009) by considering only one regulator and a duopoly, there is no transboundary pollution and, to simplify computations, there are no R&D externalities between firms.

We consider a three stage game consisting of a regulator and two identical firms competing in quantity and producing the same homogeneous good. The production process generates pollution and firms can invest in R&D to lower their emission/output ratio. Since firms constitute a duopoly and pollute the environment, the intervention of the regulator is necessary. The latter uses two regulatory instruments that

he announces at the first stage of the game: a per-unit emission tax and a per-unit R&D subsidy. Firms react by investing in R&D at the second stage, and by producing at the third stage. This game is solved backward to get a sub-game perfect Nash equilibrium.

We show that the regulator can induce firms to reach the socially-optimal levels of production and R&D by means of the two regulatory instruments, which are a tax per-unit of emission and a subsidy per-unit of R&D level.

Also, we establish the following. If, the marginal damage cost of pollution is sufficiently low, then the regulator really subsidizes pollution to correct the duopolistic distortion. Moreover, if the marginal damage of pollution is high enough, then the regulator actually taxes the investment in R&D because firms are tempted to overinvest in research.

The paper has the following structure. In section 1, we introduce the model. Section 2 studies the reaction of firms, in section 3 we derive the socially-optimal emission tax and R&D subsidy, and the last section concludes.

### 1. The model

We consider an industry composed of two firms producing the same homogeneous good sold on the market having the following inverse demand function  $p = a - (q_i + q_j)$ , where  $q_i$  is the quantity produced by firm  $i$  and  $a > 0$  is the maximum willingness to pay for the good by consumers. One reason for the market structure we adopt is that the markets of industries engaging in important R&D investments are oligopolistic. This is the case of the energy production industry.

Since firms constitute a polluting duopoly, they are regulated. The regulator maximizes the social welfare and uses two regulatory instruments that he announces at the first stage of the game:<sup>1</sup> a per-unit emission tax  $t_i$  inducing the socially-optimal levels of production and pollution, and a per-unit R&D subsidy  $r_i$  inducing the socially-optimal levels of R&D and emission/production ratio. Firms react by investing in research at the second stage, and by offering their production on the market at the third stage. This three stage game is solved backward to get a sub-game perfect Nash equilibrium.

The production activity of firms generates pollution and these latter can invest in R&D in order to lower their fixed emission/output ratio. The level  $x_i$  of

R&D costs  $kx_i^2$ , where  $k > 0$  is an investment cost parameter. Thus, the marginal cost of investment in R&D is increasing. This assumption ensures the concavity of the objective functions of firms and the regulator.

By normalizing the emission per-unit of production to one without innovation, the emission/output ratio of firm  $i$  is  $e_i = 1 - x_i$ , and its emission of pollution is  $E_i = (1 - x_i)q_i$ . Therefore, we suppose that there are no positive R&D spillovers between firms.

The damage cost caused by firm  $i$  is  $D_i = \alpha E_i$ , where  $\alpha > 0$  is the marginal damage cost of pollution.

The cost of producing the quantity  $q_i$  by firm  $i$  is  $q_i^2$ . Thus, the marginal cost of production is increasing<sup>2</sup>. The profit of firm  $i$  is  $\Pi_i = p(q_i, q_j)q_i - q_i^2 - kx_i^2$ , and its profit net of taxes and subsidies is:

$$V_i(x_i, q_i, q_j) = \Pi_i - t_i E_i + r_i x_i.$$

The consumer surplus engendered by the consumption of  $Q = q_i + q_j$  is:

$$CS = \int_0^{q_i+q_j} p(u)du - p(q_i, q_j)(q_i + q_j) = \frac{1}{2}(q_i + q_j)^2.$$

The social welfare is equal to the consumer surplus, minus damages and subsidies, plus taxes and the net profits of firms. After simplifications, it becomes equal to the consumer surplus minus damages plus the profits of firms:

$$S(q_i, q_j, x_i, x_j) = CS - D_i - D_j + \Pi_i + \Pi_j. \quad (1)$$

Notice that taxes and subsidies do not appear in the social welfare function because the taxes diminished from the firms profits are added to the consumer welfare, and the subsidies added to the firms profits are diminished from the consumer welfare.

### 2. The reaction of firms

Given the emission taxes and the R&D subsidies,  $(t_i, r_i), i = 1, 2$ , announced by the regulator at the first stage, each firm reacts by choosing its optimal innovation and production levels in the second and third stages, respectively. By backward induction, at the third stage, each firm maximizes its net profit with respect to its production level, and at the second stage, it maximizes its net profit with respect to its R&D level.

<sup>1</sup> These two instruments are necessary in this model. Indeed, even if the socially-optimal level of pollution can be implemented by only one instrument, such as pollution permits, there is no incentive for firms to reach the socially-optimal levels of production and R&D.

<sup>2</sup> If we use a linear production cost function, then the socially-optimal production levels will be given by their sum, and we would not be able to determine the socially-optimal levels of research while assuming that the two firms are active.

The third stage first-order conditions of firms are:

$$\frac{\partial V_i}{\partial q_i} = \frac{\partial V_j}{\partial q_j} = 0. \quad (2)$$

The resolution of system (2) gives:

$$q_i^* = \frac{1}{15} [3a - 4t_i(1 - x_i) + t_j(1 - x_j)]. \quad (3)$$

We deduce the following:

$$\frac{\partial q_i^*}{\partial x_i} = \frac{4}{15} t_i, \quad \frac{\partial q_i^*}{\partial x_j} = -\frac{t_j}{15}.$$

When the emission tax is positive; if firm  $i$  increases its R&D level, then its emission ratio diminishes enabling it to expand its production, thus obliging the competing firm to diminish its production.

The symmetric expression of (3) is:

$$q^* = \frac{1}{5} [a + t(x - 1)]. \quad (4)$$

The second stage first-order condition of firm  $i$  is<sup>1</sup>:

$$\frac{dV_i}{dx_i} = \frac{\partial q_i^*}{\partial x_i} \frac{\partial V_i}{\partial q_i} + \frac{\partial q_j^*}{\partial x_i} \frac{\partial V_i}{\partial q_j} + \frac{\partial V_i}{\partial x_i} = 0. \quad (5)$$

At the equilibrium, by using (2), equation (5) is simplified, and using (3) for the partial derivatives, then (4), the symmetric<sup>2</sup> solution of (5) is:

$$x^* = \frac{16t(a - t) + 75r}{150k - 16t^2}. \quad (6)$$

The optimal R&D level for firms depends on both the emission tax and the research subsidy, which confirms the fact that the two regulatory instruments are necessary.

### 3. The socially-optimal emission tax and R&D subsidy

At the first stage, by using the expressions of the optimal production and R&D levels for firms determined at the third and second stages, the regulator maximizes its social welfare given by (1) with respect to  $t_i$ ,  $t_j$ ,  $r_i$  and  $r_j$ . However, this direct method is not easy to do. Therefore, we will use a simpler method. Indeed, the regulator will choose the socially-optimal production and R&D levels at third

and second stages, respectively. Then, by equalizing the obtained socially-optimal quantities to those optimal for firms, he determines the socially-optimal emission tax and R&D subsidy. The model is resolved as if it was a two-stage game<sup>3</sup>.

The third stage first-order conditions of the regulator are:

$$\frac{\partial S}{\partial q_i} = \frac{\partial S}{\partial q_j} = 0. \quad (7)$$

The resolution of system (7) gives:

$$\hat{q}_i = \frac{1}{8} [2a + \alpha(3x_i - x_j - 2)]. \quad (8)$$

The symmetric expression of (8) is:

$$\hat{q} = \frac{1}{4} [a + \alpha(x - 1)]. \quad (9)$$

The second stage first-order conditions of the regulator:

$$\frac{dS}{dx_i} = \frac{\partial \hat{q}_i}{\partial x_i} \frac{\partial S}{\partial q_i} + \frac{\partial \hat{q}_j}{\partial x_i} \frac{\partial S}{\partial q_j} + \frac{\partial S}{\partial x_i} = 0, \quad (10)$$

$$\frac{dS}{dx_j} = \frac{\partial \hat{q}_i}{\partial x_j} \frac{\partial S}{\partial q_i} + \frac{\partial \hat{q}_j}{\partial x_j} \frac{\partial S}{\partial q_j} + \frac{\partial S}{\partial x_j} = 0. \quad (11)$$

The second-order conditions are verified when  $k$  is sufficiently high with respect to  $a$  and  $\alpha$ . Thus, the investment cost parameter should be sufficiently high to ensure the concavity of the social welfare function.

By using (7), systems (10)-(11) are simplified. Using (8) for the partial derivatives, and then (9), the symmetric solution of (10)-(11) is:

$$\hat{x} = \frac{\alpha(a - \alpha)}{8k - \alpha^2}. \quad (12)$$

The socially-optimal R&D level is positive when  $k$  is sufficiently high and the following condition is verified:

$$\alpha < a. \quad (13)$$

<sup>1</sup> The second-order condition is verified for  $k$  sufficiently high with respect to  $a$  and  $\alpha$ .

<sup>2</sup> We look for the symmetric equilibria because the model is symmetric and computations are easier. As it will be explained in the following section, the backward resolution of the game is stopped at the second stage. For this reason, we have the right to look for the symmetric equilibria at this second stage.

<sup>3</sup> We remind that the regulator maximizes his social welfare function. The optimal values obtained are said to be socially-optimal. The socially-optimal levels of production and R&D are decentralized by the use of the emission tax and R&D subsidy: the regulator determines the socially-optimal emission tax and R&D subsidy to induce firms to reach the socially-optimal levels of production and R&D. The values of the emission tax and R&D subsidy are not important in their selves. Indeed, expression (1) shows that what determines the social welfare level are production and R&D levels. Therefore, if we find an emission tax and R&D subsidy such that the production and the R&D levels chosen by firms are equal to the socially-optimal ones, then these emission tax and R&D subsidy are socially-optimal. This is what we do: we do not solve directly the first stage, but we derive an equivalent and symmetric solution.

Therefore, the marginal damage cost should be lower than the maximum willingness to pay for the good by consumers.

By equalizing the production level chosen by firms given by (4) to the socially-optimal one given by (9), we determine the socially-optimal emission tax:

$$t = \frac{a - 5\hat{q}}{1 - \hat{x}}. \quad (14)$$

Then, by equalizing the R&D level chosen by firms given by (6) to the socially-optimal one given by (12), we get the socially-optimal subsidy:

$$r = \frac{1}{75} [2(75k - 8t^2)\hat{x} - 16t(a - t)]. \quad (15)$$

We can then state the following result.

*Proposition 1. The regulator can induce both firms to reach the socially-optimal levels of production and R&D by means of a tax per-unit of pollution and a subsidy per-unit of R&D level.*

By using (9), (12) and (14), we obtain:

$$\lim_{k \rightarrow +\infty} t = \frac{1}{4} (5\alpha - a). \quad (16)$$

We deduce that:

$$\lim_{k \rightarrow +\infty} t < 0 \Leftrightarrow \alpha < \frac{a}{5}. \quad (17)$$

Consider the case where  $k$  is high enough. Thus, when  $\alpha$  is sufficiently high, the emission tax is positive, and when it is sufficiently low, the emission tax is negative meaning that the regulator subsidizes production (or pollution because they are proportional). Indeed, when the marginal disutility of pollution is not very important, the regulator subsidizes production to deal with the duopolistic distortion.

By using (12), (15) and (16), we have:

$$\lim_{k \rightarrow +\infty} r = \frac{1}{60} (4a - 5\alpha)(a - \alpha). \quad (18)$$

We deduce that:

$$\lim_{k \rightarrow +\infty} r < 0 \Leftrightarrow \alpha > \frac{4}{5} a. \quad (19)$$

Consider the case where  $k$  is sufficiently high. When the marginal damage of pollution is high enough, the emission tax is positive, which may induce firms to overinvest in R&D with respect to what is socially-optimal; to correct this, the regulator will actually tax the investment in research. On the other hand, when the marginal disutility of pollution is sufficiently low, the emission tax is

negative meaning that the regulator subsidizes pollution, which may incite firms to underinvest in R&D; to remedy this, the regulator subsidizes the investment in R&D.

*Proposition 2. When the investment cost parameter is sufficiently high, then:*

1. *If the marginal damage cost of pollution is low enough, then pollution is really subsidized.*
2. *If the marginal damage cost of pollution is high enough, then the investment in R&D is actually taxed.*

Notice that condition (13) and  $k$  are sufficiently high with respect to  $a$  and  $\alpha$  ensure that the optimal quantities of production, R&D and pollution are strictly positive.

### Conclusion

We have developed a three-stage game model composed of a regulator and two firms. These firms compete on the same market where they offer the same homogeneous good, and can invest in R&D to decrease their emission/output ratio. Since firms constitute a duopoly and their production activity is polluting, the regulator imposes at the first stage two regulatory instruments which are: a tax per-unit of pollution and a subsidy per-unit of R&D level. Firms react by choosing the optimal levels of R&D and production respectively at the second and third stages. The game is solved backward to get a sub-game perfect Nash equilibrium.

We show that by means of a per-unit emission tax and a per-unit R&D subsidy, the regulator can realize the first-best outcome since he induces firms to implement the socially-optimal levels of production and R&D.

When the marginal damage cost of pollution is low enough, the regulator subsidizes pollution to correct the duopolistic distortion. Interestingly, when the marginal damage of pollution is high enough, the regulator actually taxes the investment in R&D because firms are tempted to overinvest in research.

Let us notice that we have supposed that there are no R&D externalities between firms. The introduction of free and costly research externalities is studied in Ben Youssef and Zaccour (2009). Costly research externality complicates enormously the resolution of the model and unable us to get explicit solutions.

Finally, it would be interesting to extend this model to the case of an oligopoly in Cournot or Bertrand competition, or to the case where firms hold private information concerning their cost of production and/or of R&D.

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