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AUTHORS	Chuan-Zhong Li Karl-Gustaf Löfgren
ARTICLE INFO	Chuan-Zhong Li and Karl-Gustaf Löfgren (2012). Growth, pollution, and money-metric welfare in imperfect markets. *Environmental Economics*, *3*(4)
RELEASED ON	Tuesday, 11 December 2012
JOURNAL	"Environmental Economics"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

Chuan-Zhong Li (Sweden), Karl-Gustaf Löfgren (Sweden)

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Abstract

This paper shows how utility-based welfare measures in dynamic general equilibrium under imperfect markets can be transferred into a money metrics. In order to do this, we need to price forward looking components measured in units of utility. The typical comprehensive (green or inclusive) quasi-static welfare measure contains a core that looks like a comprehensive NNP component, as well as additional consumer surplus terms for both consumption goods and the externality. In addition, it contains a forward looking component with the discounted value of the marginal externality as a function to be integrated over time. To accomplish this, we need a price index that is independent of the market basket, or to assume that the marginal utility of income is constant over time. With respect to local welfare measures it turns out that growth in traditional NNP will surprisingly work, provided that we condition on a positive average marginal rate of return of investment, and use an augmented genuine saving concept.

Keywords: welfare measurement, comprehensive NNP, imperfect markets, money metrics.
JEL Classification: D61, D91, Q01.

Introduction

A series of recent papers cover welfare measurement in dynamic general equilibrium and, in particular, welfare measurement in imperfect market economies[1]. It turns out that all imperfections result in welfare measures that, in relation to the corresponding welfare measures in perfect market economies, generate extra forward looking terms which contain entities that are not properly priced, or not priced at all. Working exclusively in a utility metrics, Aronsson et al. (2004) measure the relative welfare losses (in comparison to first best) resulting from different market imperfections.

However, empirically meaningful measures are important if one attempts to do practical green accounting. Since the measurement of utility is not practically feasible, a money metrics is required. However, there are at least three complications: Firstly, externalities in consumption add an autonomous time dependence that makes the utility from a given consumption vector a function of the magnitude of the externality[2]. Secondly, the marginal utility of income will change over time, implying that the relationship between monetary and utility measures changes over time through a changed yardstick. This makes exact money metrics comparisons over time difficult. Finally, the imperfections are typically not priced or incorrectly priced.

The second problem is solved by an index idea in Weitzman (2001), which is slightly modified in Li and Löfgren (2002) to cover comparisons over time. The solution entails an empirically demanding price index. The third problem can be solved partly by measures of willingness to pay[3], and partly by estimates of marginal losses in production. This still leaves the first problem, but as we will show it can be handled by assuming that the instantaneous utility function can be separated into two components, one containing the externality and the other containing consumption.

In an attempt to investigate in what sense growth in NNP works as a local welfare indicator under imperfections, we introduce an exact local welfare indicator for an imperfect market economy. The measure shows what the time derivative of the value functions looks like under market imperfections. This "genuine saving" measure[4] is useful not only in itself, but also because it can help us to develop a criterion that tells us when growth in NNP (nota bene, not Green NNP) will work as a local welfare criterion.

To obtain a simple, but rich enough model, we will, like e.g. Aronsson et al. (2004), work with the Brock (1977) model. To keep the exposition as brief as possible, we will to a large extent draw on results from previous work, sometimes without introducing rigorous proofs.

1. The Brock model

The model used here is, with the exception for the separability property of the utility function, identical to a growth model introduced by Brock (1977). As in the Ramsey model, there is a single homogeneous good used for consumption[5] and investment. In order to introduce an externality, production is assumed to cause pollution, which generates an externality in consumption. Natural resources, as poten-

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The authors acknowledge comments from Dr. Peichen Gong, Swedish University of Agricultural Sciences. Chuan-Zhong Li also acknowledges financial support from the Swedish Research Council Formas.

[1] See e.g., Aronsson and Löfgren (1993, 1997, 1999), Arrow et. al., (2003), Dasgupta (2001), and Dasgupta and Mäler (2000).

[2] Under first best this problem will disappear, but also there is a need for green accounting.

[3] See Aronsson and Löfgren (1999).

[4] Possibly first developed in Aronsson and Löfgren (1998).

[5] A vector of consumption and investment goods would change nothing essential in the analysis.

tial inputs in production, are suppressed, as they do not add to our principle findings. The instantaneous utility function at time t is written as:

$$u = u^0(c(t), x(t)) = u(c(t)) + \varepsilon(x(t)), \qquad (1)$$

where $u^0(\cdot)$ is a strictly concave (in both arguments), twice continuously differentiable function, which is increasing in consumption, $c(t)$, and decreasing in the stock of pollution, $x(t)$. Labor is normalized to one, which means that all entities presented above are represented on a per capita basis. The reason we introduce a separable utility function is that the externality will mess up the index problem which we will introduce below. Alternatively, we may regard such a utility function as a logarithmic transformation of a more general Cobb-Douglas utility function with interdependence.

Goods are produced by capital $k(t)$, and energy $g(t)$, per unit of labor. The production function can be written as follows:

$$y(t) = f(k(t), g(t)). \qquad (2)$$

It is assumed to be strictly concave, twice continuously differentiable and increasing in energy[1].

The accumulation of pollution obeys the following differential equation

$$\dot{x}(t) = g(t) - \gamma x(t), \qquad (3)$$

$$x(0) = x^0,$$

where $0 < \gamma < 1$ is a parameter reflecting the assimilative capacity of the environment. The emissions are treated in a simple way, with little loss of generality, by assuming that emissions equal the input of energy, $g(t)$. The accumulation of capital follows the equation:

$$\dot{k}(t) = f(k(t), g(t)) - c(t),$$

$$k(0) = k^0. \qquad (4)$$

Using a theorem in Weitzman (1976) one can conclude that the optimal value function in the social optimum can be written[2]

$$H^*(t)\theta^{-1} = \int_t^{\infty} u^a(c^*(s), x^*(s)) e^{-\theta(s-t)} ds, \qquad (5)$$

where

$$H^*(t) = u^a(c^*(t), x^*(t)) + \lambda^*(t)\dot{k}^*(t) + \mu^c(t)\dot{x}^*(t)$$

is the current value Hamiltonian. The top index denotes that the value of the Hamiltonian is measured along the optimal path. Here $\lambda^*(t)$ is a co-state variable that measures the future current utility value of one unit of capital invested today measured along the optimal growth path. $\mu^c(t) < 0$ is the corresponding co-state variable for the stock of pollution. Equation (5) tells us that the current value Hamiltonian at time t is directly proportional to the optimal value function measured in utility. The factor of proportionality equals the utility discount rate[3], θ. As shown in, for example, Aronsson et al.[4] (2004) the corresponding expression when the externality is not internalized takes the form

$$H^0(t) + \int_t^{\infty} \varepsilon_x(x^0(s))\dot{x}^0(s) e^{-\theta(s-t)} ds = \qquad (6)$$
$$= \theta \int_t^{\infty} u^a(c^0(s), x^0(s)) e^{-\theta(s-t)} ds,$$

where the integral expression on the left hand side measures the current value of the marginal externality along the future path of the market economy. In other words $\varepsilon_x(x^0(t))$ denotes the marginal disutility of additions to the stock of pollution. Everything in equation (6) is measured in utilities, and the agenda is to move equation (6) from a utility metrics into a money metrics.

2. A money metrics version in the case of imperfect markets

At each instant in time, the optimal consumption path can be reproduced by letting the consumer solve the following optimization problem

$$\max_{c,z} H(t) = \max_{c,z}[u(c(t)) + \lambda(t)z(t) + \varepsilon(x(t)] \qquad (7)$$

subject to

$$y^0(t) = p^0(t)c(t) + z(t), \qquad (8)$$

where $H(t)$ is the Hamiltonian maximized by the consumer. If we consider the Hamiltonian as a dynamic utility function, then it is a quasilinear utility function being nonlinear with respect to consumption and pollution but linear in the value of net investment (see Weitzman, 2001). Here $\lambda(t)$ is the marginal utility of money, $z(t) = q^0(t)\dot{k}(t)$ is the value of net investment, $q^0(t)$ is the market clearing price of the investment good at time t, and $p^0(t)$, here equal to

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[1] Note that the production function is net of capital depreciation, so that the marginal productivity of capital becomes negative for large capital stocks.

[2] More exactly, the solution to $\max_{c,g} \int_t^{\infty} u^0(c,x) e^{-\theta(s-t)} ds$, subject to the two differential equations (3) and (4) and the conditions $\lim_{t\to\infty} k(t) \geq 0$ and $\lim_{t\to\infty} x(t) \geq 0$, and the corresponding transversality conditions are also satisfied along the optimal path.

[3] A proof of the theorem under global differentiability of the Hamiltonian with respect to t uses that $\frac{dH^*}{dt} = \frac{\partial H^*}{\partial t}$ along an optimal path. Integrating forwards and taking limits when the time horizon goes to infinity yields the result. See e.g. Aronsson et al. (2004) chapter 2.

[4] That is, the maximization ignores the differential equation for the stock of pollution. The result was first derived by Kemp and Long (1982). See also Löfgren (1992).

$q^0(t)$, is the market clearing price of consumption goods. Both are measured along the imperfect market growth path of the economy. The quasilinear form of the Hamiltonian implies that the demand functions will contain no income effects. The solution of the consumer's optimization problem can be summarized as:

$c^0(t) = c^\lambda(\lambda(t)p^0(t)) = c^0(p^0(t))$, and

$z^0(t) = q^0(t)\dot{k}^0(t) = y^0(t) - p^0(t)c^0(t)$.

The marginal utility of income will, however, not be constant over time. It will be governed by the differential equation for the co-state variable for the capital stock, $\lambda^c(t)$. From the conditions for an optimal consumption path, we know that

$$\dot{\lambda}^c(t) = [\theta - r(t)]\lambda^c(t), \qquad (9)$$

where $r(t) = f_k(t)$ is the real interest rate. By using that the current value shadow price along the market solution, which can be broken down into $\lambda^c(t) = \lambda(t)p^0(t)$, it is straightforward to show that[1]

$$\dot{\lambda}(t) = [\theta - (r + \frac{\dot{p}^0(t)}{p^0(t)}]\lambda(t) = [\theta - R(t)]\lambda(t), \qquad (10)$$

where $R(t)$ is the nominal interest rate. This means that the consumer's marginal utility of income will typically change over time, and make the transfer from the utility metrics to a money metrics more difficult. The additively separable instantaneous utility function is helpful, since it means that demand for goods will not depend on the stock of pollution, and the disutility of pollution will not depend on consumption.

Now define for each t, $\lambda(t)p_\varepsilon(t) = -\varepsilon_x(x(t))$. The interpretation of $p_\varepsilon(t)$ is the marginal willingness to pay to get rid of one unit of pollution.

From the strict concavity of the utility function, we know that the right hand side is monotone in $x^0(t)$, and we can invert to get

$x^0(t) = \varepsilon_x^{-1}[-\lambda(t)p_\varepsilon(t)] = x^0(p_\varepsilon(t))$.

The resulting relationship tells us how the stock of pollution is connected to the marginal willingness to get rid of pollution and it cannot, in all respects, be interpreted as a traditional demand function.

Now the utility function can, utilizing that

$dc = \dfrac{dc(p)}{dp}dp$ and $dx = \dfrac{dx(p_\varepsilon)}{dp_\varepsilon}dp_\varepsilon$,

after partial integration be written

$$u^a(c^0(t), x^0(t)) = \int_0^{c^0} u_c(c)dc + \int_0^{x^0} \varepsilon_x(x)dx =$$
$$= \lambda(t)[p^0(t)c(p^0(t)) - p_\varepsilon^0(t)x^0(t) +$$
$$+ \int_{p^0(t)}^{\bar{p}(t)} c(p)dp - \int_{p_\varepsilon^0(t)}^{\bar{p}_\varepsilon(t)} x(p_\varepsilon)dp_\varepsilon], \qquad (11)$$

where the last two terms are consumer surpluses and the upper integration bounds are the choke of prices. The integrals are always well-defined if we can normalize the reference utility function evaluated at zero consumption and zero pollution to be zero. Even without a normalization, the theory developed here is still applicable for welfare analysis since the difference between such integrals evaluated at different consumption and pollution levels always exists. In particular, $\bar{p}_\varepsilon(t)$, is the marginal willingness to pay to get rid of an extra unit of pollution at $x = 0$, while $p_\varepsilon^0(t)$ is the corresponding willingness to pay at the actual level of pollution at time t. The last equality follows from changing the variable of integration and partial integration using that $\lambda(t)p_\varepsilon^0 = -\varepsilon_x(x^0(t))$[2].

The remaining terms in equation (6) above can be rewritten as

$$\lambda^c(t)\dot{k}^0(t) + \int_t^\infty \varepsilon_x(x^0(s))\dot{x}^0(s)e^{-\theta(s-t)}ds =$$
$$= \lambda(t)q^0(t)\dot{k}^0(t) - \int_t^\infty \lambda(s)p_\varepsilon^0(s)\dot{x}^0(s)d(s-t)ds, \qquad (12)$$

where $d(s-t) = e^{-\theta(s-t)}$. Equation (12) is a generalized genuine saving measure for an imperfect market economy. As will be shown below, it tells us that welfare will locally increase (decrease) if the utility value of net investment minus the discounted sum of the marginal externality along the future path of the economy is positive (negative). Because of the forward looking components we have, so far, to stay within a utility metrics. Substituting (11) and (12) into (6) yields

$$\lambda(t)\{p^0(t)c(p^0(t)) - p_\varepsilon^0(t)x^0(t) + \int_{p^0(t)}^{\bar{p}(t)} c(p)dp - \int_{p_\varepsilon^0(t)}^{\bar{p}_\varepsilon(t)} x(p_\varepsilon)dp_\varepsilon +$$
$$+ q^0(t)\dot{k}^0(t)\} - \int_t^\infty \lambda(s)p_\varepsilon^0(s)\dot{x}^0(s)d(s-t)ds =$$
$$= \theta\int_t^\infty [\lambda(s)[p^0(s)c(p^0(s)) - p_\varepsilon^0(s)x^0(s) + \int_{p^0(s)}^{\bar{p}(s)} c(p)dp - \int_{p_\varepsilon^0(s)}^{\bar{p}_\varepsilon(s)} x(p_\varepsilon)dp_\varepsilon]ds. \qquad (13)$$

The expression is monstrous, and we have accomplished very little of practical relevance. The left hand side is the static welfare measure in a utility metrics, and the right hand side is the value function scaled by θ, still embedded in a utility metrics. To make progress, we have to introduce a device that

[1] To see the technicalities see Li and Löfgren (2004).

[2] For technical details see Weitzman (2001) and Li and Löfgren (2002).

enables us to move the marginal utility of income outside the integrals. One way to do this is to assume that the marginal utility of income is constant over time. This is implicitly done in all practical compensatory index formulas. The omission is hidden, since the Konüs-Allen compensatory price index is static. This means, however, that the index is incomplete, since the asset position – saving/investment decisions – are neglected. The time dependence of the marginal utility of income is typically assumed away both in static and dynamic index theory[1]. In a purely theoretical context, this assumption is not satisfactory. Our solution is empirically demanding, but is theoretically more sound.

To solve the index problem, we introduce a price index that is independent of the market basket in the economy. It was invented in Weitzman (2001), and we modify it to handle index comparisons over time by defining the ideal consumer price index (CPI)

$$\pi(t) = \frac{p(t;c_0)c_0}{p(t_0;c_0)c_0} \tag{14}$$

as a measure of the price level at time t relative to that at time t_0. In the definition (14), $p(t_0;c_0)$ and $p(t;c_0)$ denote the imputed market-clearing prices for consuming the market basket c_0 at the two points in time. Since the utility function is "stationary", i.e. does not change its functional form over time, we have from utility maximization that

$$u_c(c_0) = \lambda\ (t_0)p(t_0;c_0) = \lambda(t)p(t;c_0),$$

which implies that

$$\lambda\ (t_0)p(t_0;c_0)c_0 = \lambda(t)p(t;c_0)c_0,$$

meaning that

$$\lambda\ (t_0) = \pi(t)\lambda\ (t) \tag{15}$$

is a constant[2]. One way to view the index is to regard it as a PPP-type of index that connects the economy at two points in time. In practice, the construction of the index seems to be a very difficult task. In theory it is easy to show that[3]

$$\pi(t) = \frac{\lambda(t_0)}{\lambda(t)} = \exp[\int_0^t R(s)ds - \theta t] \tag{15a}$$

which means that the ideal price index depends on the relative strength of the money rate of interest and the pure time preference. Its value will increase when the interest rate is higher than the time preference. This is, however, as far as we can see, the only way to end up in a money metrics version of the welfare measure that is currently available. Just to show how the idea works, we re-scale the right hand side of equation (13) by the index formula to obtain

$$\theta\int_t^\infty u^0(c^*(s), x^*(s))e^{-\theta(s-t)}ds = \tag{16}$$

$$= \theta\int_t^\infty [\lambda(s)\pi(s)[p_r^0(s)c(p_r^0(s)) - p_{\varepsilon r}^0(s)x^0(s) +$$

$$+ \int_{p_r^0(s)}^{\bar{p}_r(s)} c(p_r)dp_r - \int_{p_{\varepsilon r}^0(s)}^{\bar{p}_{\varepsilon r}(s)} x(p_\varepsilon)dp_{\varepsilon r}]ds,$$

where $p_r^0 = \frac{p^O(t)}{\pi(t)}$ and $p_{\varepsilon r} = \frac{p_\varepsilon(t)}{\pi(t)}$ are the deflated real prices. We now use equation (15) to substitute λ_0 for $\pi(s)\lambda(s)$. As λ_0 is a constant, we move everything into a money metrics by dividing both sides by λ_0.

It is now obvious that the same exercise can be carried out on the left hand side of (13). We can write the money metrics version of the value function in the following manner

$$\begin{aligned} \theta^{-1}GCGNNP \ &= \theta^{-1}\{p_r^0(t)c(p_r^0(t)) + q_r^0(t)\dot{k}^*(t) + \\ &+ \int_{p_{\varepsilon r}^0(t)}^{\bar{p}_{\varsigma r}(t)} c(p)dp_r - [p_{\varepsilon r}^0(t)x^0(t) + \int_{p_{\varepsilon r}^0(t)}^{\bar{p}_\varepsilon^r(t)} x(p_\varepsilon)dp_\varepsilon^r + \\ &+ \int_t^\infty p_{\varepsilon r}(s)\dot{x}(s)d(s-t)ds]\}, \end{aligned} \tag{17}$$

where *GCGNNP* stands for Generalized Comprehensive Green Net National Product. The first two terms are the real NNP plus a real consumer surplus term connected to consumption goods. These terms are the only ones that will be present in a perfect market economy. Li and Löfgren (2002) name the corresponding measure in a perfect market economy as the Generalized Comprehensive Net National Product. Comprehensive stands for the fact that all relevant consumption goods are represented, as well as all investment goods that contribute to production. In this context, this statement may seem empty, since we only include one consumption good and one investment good. However, as we noted from the start, vectors of consumption and investment goods would not change the preceding analysis. The world Green is skipped in Li and Löfgren (2002), since we deal with a first best allocation.

[1] See e.g. Pollak (1989) and Klevmarken (2005). The only place where both omissions are handled differently is a recent paper by Li and Löfgren (2004a).

[2] As the reader might note, we have "reproduced" the proof of benchmark independence in Weitzman (2001). Note also that if the marginal utility of consumption is a function of the stock externality we cannot index on the externality, since there are no prices at time zero for the externality.

[3] Solve the differential equation (10).

The second line in equation (17) consists of the green parts of *GCGNNP*. The first two components take care of the current utility loss from the externality. This consist of the money metrics version of the externality component in the separable utility function. Following Smith (1776) and Dupuit (1844), we can call $p_{r\varepsilon}(t)x^0(t)$ value in disuse[1]. The term, $\int_{p^0_{\varepsilon r}(t)}^{\bar{p}_\varepsilon(t)} x(p_\varepsilon)dp_\varepsilon$, is the "consumer loss" from the negative externality, and, finally, the last term, $\int_t^\infty p_{\varepsilon r}(s)\dot{x}(s)d(s-t)ds$, is the current value of the future negative consequences of the externality. This term was not invented when Smith and, for that matter, Dupuit wrote their treatises. Note that if the stock of pollution is constant over time, the future negative consequences from it will vanish from *GCGNNP*. Hence, in steady state Weitzman's welfare measure in equation (5) will hold even under externalities.

In the above example, the externality is not internalized. It is well known that the externality can be internalized by introducing a dynamic Pigouvian tax coinciding with the marginal externality along the optimal path. This task is empirically very demanding, since the dynamic Pigouvian tax requires that we know how the marginal externality develops along the future optimal path, and not the actual market path, of the economy. In this case, the last term in the expression (17) will disappear. Note, however, that it will remain in the expression as long as the externality is not fully internalized[2].

It is of course a trivial exercise to obtain a complete welfare expression for uninternalized positive externalities in consumption. The only visible difference will be that the expression in the second line will be preceded by a positive sign.

Empirically, the GCGNNP concept is very demanding. One needs not only a measure of the consumer surplus for (all) goods that are priced in markets, but also a consumer loss (surplus) measure for goods that are not priced in markets. It is not unreasonable to assume that one can come up with an acceptable measure of the marginal willingness to pay for getting rid of one unit of pollution today. However, in the future looking component, we need a marginal willingness to pay measure for all future periods. In Aronsson and Löfgren (1999), there is a numerical example where it is assumed that today's current marginal willingness to pay is used in the future, but updated after a certain time span. Using a "relative utility metrics", it is shown that the measurement error is not overwhelmingly large[3].

Another complication we have not dealt with is the one that emerges if the utility function is not separable. The problem that surfaces is that, as the externality is not a part of the consumer's optimization problem, we cannot derive a full set of demand functions containing consumption goods as well as (net) externalities that supports the equilibrium path of the economy. If we could find the current marginal willingness to pay for pollution along an optimal path, we would be able to introduce Pigouvian taxes, but then we are back to the first best analysis in Li and Löfgren (2002). Hence introducing additivity seems convenient, since otherwise the demand functions for consumption goods would contain the stock of pollution as an extra argument.

In the analysis above, we have omitted possible externalities in production. The reason is not that it would further complicate the analysis, but rather that we want to reduce notational clutter. Let us, for example, assume that pollution also affects production so that the production function reads

$$y = f(g(t), k(t), x(t)), \tag{18}$$

where $f_x(g,k,x) < 0$, i.e. pollution hampers production at the margin. This would add the following extra term to the static utility welfare measure in equation (6).

$$\begin{aligned}
&\int_t^\infty \lambda^c(t) f_x(g^0(t), k^0(t), x^0(t)\dot{x}^0(t)d(s-t)dt = \\
&= \int_t^\infty \lambda(t)p^0(t) f_x(g^0(t), k^0(t), x^0(t))\dot{x}^0(t)d(s-t)ds = \\
&= \int_t^\infty \lambda(t)\pi(t)p_r^0(t) f_x(\cdot)\dot{x}^0(t)d(s-t)ds = \\
&= \lambda_0(t)\int_t^\infty p_r^0(t) f_x(\cdot)\dot{x}^0(s)d(s-t)ds, \qquad (19)
\end{aligned}$$

and although we need future deflated prices, they are, at least in this case, formed in the market.

3. GCGNNP and NNP growth as local welfare indicators

Today it is well known that growth in comprehensive NNP is an incomplete welfare indicator in the sense that it does not necessarily indicate a local welfare improvement in a market economy[4].

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[1] Adam Smith, in attempt to explain the water and diamond paradox, introduced the terms value in exchange and value in use. Value in exchange is price times quantity and value in use is, as explained by Dupuit, value in exchange plus the consumer surplus.

[2] For more details, see Aronsson et al. (1997, 2004).

[3] It is typically less than 3%.

[4] For recent work on the topic see Dasgupta and Mäler (2000), Asheim and Weitzman (2001) and Li and Löfgren (2004b).

However, if one conditions on certain variables in the economy, growth in comprehensive NNP will work as a local welfare indicator in a perfect intertemporal market economy. Asheim and Weitzman (2001) show that, deflated by a Divisia consumption price index, growth in real NNP indicates a local welfare improvement, provided that the real interest rate is positive. Li and Löfgren (2006) reveal that growth in comprehensive NNP indicates a local welfare improvement, independent of the consumer price index, provided that the "overall rate of return" (the average rate of return) from investment in the economy is positive[1].

Under imperfect markets, one would like to know whether similar results can be accomplished. This turns, out to be the case. If we measure the comprehensive NNP in the special "ideal" price index, using an extended rate of return concept measured in constant real prices, we are able to reproduce a weaker version of the Li and Löfgren result[2]. To accomplish this, we reproduce and use a generalized genuine saving concept[3].

We start by introducing a variation of a well known result on the shape of the time derivative of the optimal value function. Under externalities, it equals net investment in the capital stock as well as the future externality. To see this, we write the value function in equation (6) in the following manner:

$$W^0(t)=\int_t^\infty u(c^0(s),x^0(s))d(s-t)ds=$$

$$=[H^0(t)+\int_t^\infty \varepsilon_x(x(s))\dot{x}^0(s)d(s-t)ds)]\theta^{-1}, \tag{20}$$

where W^0 (t) is total welfare along the imperfect path of the market economy at time t. Differentiating the integral of the instantaneous utility function with respect to t, and using the relationship in equation (20) result in the following neat expression, where the right hand side was introduced already in equation (12).

$$\dot{W}^0(t)=\lambda^c(t)i^0(t)+\int_t^\infty \varepsilon_x(x^0(s))\dot{x}^0(s)d(s-t)ds). \tag{21}$$

In other words, welfare increases at time t (locally) if net investment plus the current value of the negative future externality is positive. The first term in equation (21) is, transferred into a money metrics, known as genuine saving[4].

The current externality nets out since the time derivative of the forward looking term with respect to the lower integration bound equals the time derivative of the utility function with respect to the stock of pollution, except for the minus sign.

From the Euler equation for the consumer's optimization problem it follows that

$$\frac{\partial H^0}{\partial k}=\theta\lambda^c(t)-\dot{\lambda}^c(t). \tag{22}$$

Putting $\dot{k}=i$ we introduce the product

$$i^0(t)\frac{\partial H^0}{\partial k}=i^0(t)(\theta\lambda^c(t)-\dot{\lambda}^c(t)). \tag{23}$$

Now from equations (20) and (21) it follows that:

$$\theta\dot{W}^0(t)=u_c(c^0(t))\dot{c}^0(t)+\lambda^c(t)\dot{i}^0(t)+\dot{\lambda}^c(t)i^0(t)+$$
$$+\theta\int_t^\infty \varepsilon_x(x^0(s))\dot{x}^0(s)d(s-t)ds= \tag{24}$$
$$=\theta[\lambda^c(t)i^0(t)+\int_t^\infty \varepsilon_x(x^0(s))\dot{x}^0(s)d(s-t)ds]=\theta GGS,$$

where the integral times θ measures the annuity equivalent of the future externality. The first equality follows by differentiating the right hand side of equation (20) with respect to time. It is the time derivative of GNNP measured in a utility metrics. The second equality follows from equation (21) and is the annuity equivalent of generalized genuine saving, GGS.

Hence, by combining equation (23) and (24) we can write

$$i^0(t)\frac{\partial H^0(t)}{\partial k}=\theta[\dot{W}^o(t)-\int_t^\infty \varepsilon(x(s))\dot{x}(s)d(s-t)ds]-\dot{\lambda}^0(t)i^0(t)=$$
$$=\lambda(t)[p^0(t)\dot{c}^0(t)+q^0(t)\dot{i}^0(t)]=\lambda(t)\dot{y}_p^0(t), \tag{25}$$

where $\dot{y}_p^0(t)$ is the growth in comprehensive NNP at constant prices.

Finally, dividing both members by the welfare indicator (generalized genuine saving) in equation (21) we obtain

$$i^0(t)\frac{\partial H^0}{\partial k}[\lambda(t)q^0(t)i^0(t)-$$
$$-\int_t^\infty \lambda(s)p_\varepsilon(x^0(s))\dot{x}^0(s)d(s-t)ds]^{-1}=\lambda(t)\dot{y}_p^0(t)[GGS]^{-1}. \tag{25a}$$

In a perfect market economy, the integrals in the denominators vanish and we have

$$\rho(t)=i^0(t)\frac{\partial H^0}{\partial k}(\lambda(t)q^0(t)i^0(t))^{-1}=$$
$$=\frac{\lambda(t)[p^0(t)\dot{c}^0(t)+q^0(t)\dot{i}(t)]}{\lambda(t)q^0(t)i^0(t)}=\frac{\dot{y}_p^0}{q^0(t)i^0(t)}, \tag{26}$$

[1] The overall rate of return can be interpreted as the net-investment weighted own rate of interest.

[2] Probably also the Asheim and Weitzman result, but the proof is left to the reader.

[3] Introduced by Aronsson and Löfgren (1998).

[4] See Hamilton (1994). Weitzman (1976) is, to our knowledge, the first to understand that genuine saving is a local welfare indicator in a perfect market economy. This knowledge seems to be a kind of Folk-Theorem. It pops up everywhere in dynamic growth theory. Hamilton should, however, have the lion's share of the credit, since he also uses the concept empirically.

where $\rho(t)$ can be interpreted as the average rate of return on investment. Clearly, if the overall rate of return of investment is positive (negative), and there is growth in comprehensive NNP at fixed prices, then the welfare indicator, $q^0(t)i^0(t)$, is positive (negative) and welfare increases (decreases) locally. This is the result presented in Li and Löfgren (2006). Conditional on a positive rate of return on investment, NNP growth at constant prices indicates a welfare improvement independent of the price index. More exactly, using equations (26) and (10), it is straightforward to show that

$$\rho(t) = \theta - \dot{\lambda}^c i^0(t)[\lambda^c(t)i^0(t)]^{-1} =$$

$$= R(t) - \dot{q}^0(t)i^0(t)[q^0(t)i^0(t)]^{-1}. \quad (26a)$$

In other words, $\rho(t)$ is the nominal interest rate minus the average inflation rate for investment goods.

Under imperfect markets, the corresponding expression contains forward looking terms that have to be handled to obtain a money metrics version of a corresponding result. Using the price index in equation (15), we can re-scale equation (25a) to obtain

$$\rho_r = i^0(t)\frac{\partial H_r^0}{\partial k}[q_r^0(t)i^0(t) - \int_t^\infty p_{\varepsilon r}^0(x(s))\dot{x}^0(s)d(s-t)ds]^{-1} =$$

$$= \dot{y}_{pr}^0(t)[q_r^0(t)i^0(t) - \int_t^\infty p_{\varepsilon r}^0(x(s))\dot{x}^0(s)d(s-t)ds]^{-1} = \quad (26b)$$

$$= \dot{y}_{pr}(t)[GGS_r]^{-1}.$$

Here prices and income have been deflated by the ideal price index, and the constant, λ_0 has been put equal to one. The rate of return concept is also augmented and imbedded in a money metrics. It has to be measured in the ideal price index, and it is also different in the sense that it is relative to another "capital base"; net investment deducted by the change in the money value of the externality along the future path of the imperfect market economy. Hence consumer preferences enter the rate of return measure in a very direct manner. However, conditional on these conditions, the previous result stands; growth in comprehensive NNP at constant prices indicates a local welfare improvement conditional on a "positive rate of return". Note also that in a steady state, $\dot{x}^0(t) \equiv 0$, equation (26b) collapses into (26).

Clearly, if we, as in all practical consumer price index computations, are willing to assume that the marginal utility of money is constant, we do not have to rescale all prices with the ideal price index, we can use any scaler in equation (25b). However, we still have to assume something about the future willingness to pay for getting rid of a marginal unit of pollution. Finally, if we are willing to assume that the stock of pollution will remain constant over time $p(t)$ can be measured by the difference between the nominal interest rate and the rate of inflation of investment goods, an entity that is easily observable in practice.

Conclusions

This paper has shown how utility-based welfare measures in dynamic general equilibrium under imperfect markets can be transferred into a money metrics. The sufficient conditions are, however, rather demanding. To start with, we need a price index that is independent of the market basket, or we have to assume that the marginal utility of income is constant over time. The latter assumption is implicit in all practical applications of index theory, but nevertheless dubious. It can be remedied theoretically by using the index approach presented in this paper. Nevertheless, it is not easy to see how this can be applied in practice.

Secondly, we need to price forward looking components measured in units of utility. It is difficult to see how this can be avoided. Under perfect market conditions and perfect foresight, the forward looking information is buried in the current market prices of consumer and investment goods. The reason is that the perfect market economy supports the optimal growth path. Under imperfect market conditions, corresponding current shadow prices are not available; either for externalities in consumption or for externalities in production. However, as shown by Aronsson et al. (2004), in numerical examples, current willingness to pay or current prices may be good approximations. A more radical way out, is to assume that the economy is in a steady state.

Thirdly, the typical comprehensive quasi-static welfare measure (GCGNNP) will contain a core that looks like an extended (green) NNP component, as well as consumer surplus terms for both consumption goods and the externality and, in addition, a forward looking component with the discounted marginal externality as the function to be integrated over time.

Finally, with respect to local welfare measures, growth in traditional NNP will surprisingly work, provided that one conditions on a positive average marginal return of investment. However, unlike a previous result in Li and Löfgren (2006), the rate of return concept has to be augmented with the current value of the future marginal externality, and growth in NNP at current prices has, in general, to be deflated by the benchmark independent price index. In a steady state the two results coincide.

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