

# “Can spatial dependence enhance industry sustainability? The case of pasture-based beef”

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## Can spatial dependence enhance industry sustainability? The case of pasture-based beef

### Abstract

Can sustainability be enhanced by maximizing the sum of private and social benefits from an industry? This might take place, for example, by identifying production options that increase profitability side-by-side with societal goals such as renewable energy production and carbon sequestration, healthier communities, environmental quality, and economic development. The authors explore this issue for pasture-based beef (PBB), a nascent industry where industry profitability, community development, and quality of life can be enhanced by explicitly linking the PBB supply chain spatially and intertemporally, thereby increasing the sum of private and social benefits.

The paper develops a framework based on optimal control theory that integrates a spatial component in which the production of PBB and alternative energy production as well as greenhouse gas emission reduction enhances private as well as social wealth. This model provides a basic foundation for developing agglomeration economies in a spatially dependent industry in which other locations are able to supply resources to given locations as a way of improving regional economic and environmental conditions.

The framework is subsequently employed to identify possible industry conditions and configurations that demonstrate how profits, economic development, and environmental improvement can be created through increased pasture-beef production in a region where economic activities across locations play a crucial role across the spatial domain. Of course, the intensification of benefits derived from the agglomeration economies requires coordination and cooperation among the key players within the impacted region.

**Keywords:** spatial optimal control, pasture-based beef, on-farm energy, sustainability.

**JEL Classification:** Q20, Q40.

### Introduction

The increased use of pasture as the primary diet in the beef industry has been attributed to positive effects not only in terms of animal welfare but also to human health, the land resource, the ecosystem and economic development. In fact, raising cows on pasture improves water quality and decreases soil erosion while enhancing green space (Paine et al., 2009). Pasture-based land use is generally recognized as reducing soil erosion compared to row-crop production. In addition, the waste produced from livestock can be used as a natural fertilizer as well as a source of alternative energy which eventually maintains land quality and provides renewable fuels to farmers, reducing dependency on products derived from fossil fuels (Fulhage et al., 1993; Pimentel and Pimentel, 2008).

The combination of these attributes makes it appealing to analyze the issue of whether and how spatial dependence in an industry can be exploited to meet the private goal of increased profit side-by-side with the societal goal of improved quality of life. We explore this issue with respect to the growing pasture-based beef (PBB) industry, where beef production is based almost exclusively on the pasture resource. The framework proposed can be used to identify possible industry conditions and configurations that demonstrate how profits, economic development, environmental improvement and other so-

cial benefits can arise through increased pasture-beef production in a region employing a clustering system whereby economic activities across locations play a crucial role across the spatial domain. The framework can be applied to Appalachia or other similar regions where industry profitability, community development and quality of life are linked spatially and intertemporally.

The economic value of the cattle sector in general and the PBB sector in particular are clearly large and well documented (Rodríguez, 2012). What is less obvious is the ecosystem value of transitioning to more production, and its associated contribution (or detraction) to the multi-attribute functions increasingly expected by society and policymakers from the land resource.

In order to intensify the benefits derived from this industry, spatial effects or influences associated with production within the region are explicitly taken into account. This approach allows visualizing the development of clustering systems that eventually strengthen the economic activities and social benefits within an area. The incorporation of spatial effects in our model is intended to meet the policy goal of sustainability through enhanced private and social benefits.

Our objective is to develop a conceptual framework based on optimal control theory that integrates a spatial component in which the production of PBB and alternative energy as well as GHG emission

reduction enhances profitability and social welfare within a region. By linking the pasture resource to income opportunities as well as to climate change mitigation, this study can provide a framework to better achieve sustainability in regions where variations in topography combined with fluctuations in seasonal conditions combine in ways that potentially increase production risk and reduce profitability.

## 1. Theoretical background

Optimal control (OC) provides the framework to illustrate the integrated PBB concept proposed as a way of optimizing farm resources in a dynamic environment. Specifically, OC allows us to maximize farm-level profitability while enhancing social welfare when sustainable practices are taken into consideration.

Chiang (2000) describes the fundamental components of an OC model. A control variable can be seen as a policy tool that is able to impact state variables which means that any selected control path involves a linked state path (Chiang, 2000). On the other hand, Perman et al. (2003) establish that an optimal control model does not necessarily need to have the state and control variables present in the objective functions. In addition, the latter state that what makes dynamic optimization important is to obtain the values of these variables at each point in time up to the planning horizon as the solution to the problem. The initial values of state variables and their evolution over time are based on some physical, economic and biological system that is captured through a set of differential equations or state equations. Moreover, control variables represent instruments in which their values can be chosen by the decision maker with the purpose of steering the evolution of the state variables intertemporally. Another essential variable in the OC model is the co-state variable which is commonly known as the shadow price. This variable basically denotes the marginal valuation of the state variable at each point in time which varies over time (Perman et al., 2003).

Cacho (1998) employs an OC model using a meat production function in which grass is the primary input while stocking rate and fertilizer applications have an indirect control over production. Four state variables including soil depth and animal weight, and control variables such as the stocking rate capture annual seasonal variations. Saliba (1985) explores the interactions among management choices, soil loss through erosion, and farmland productivity. The author analyzes four models developed by other researchers and concludes that none of them directly addresses the relationship between soil erosion and

soil productivity. In addition, tradeoffs among intensity of crop rotation, soil conservation practices and production inputs are not sufficiently explained, and are limitations that the author seeks to overcome. The optimization model developed considers a profit maximizing farmer in which the contributions and costs of soil among other inputs in crop yield are analyzed when making decisions with regard to input use and conservation methods. The objective function takes into account crop rotation, output price and other variables in which the marginal value of soil depth is categorized as the costate variable. Similarly, McConnell (1983) develops an economic model where the use of soil can be optimized from a social and private point of view. He proposes a production function in which explanatory variables such as technological change, soil loss, and soil depth determine output. The model also establishes that farmers' behavior toward soil is influenced by the soil's effect on profits in which the farmer makes use of the land in order to maximize the value of the farm plus the present value of the profit stream at the end of the planning horizon. This implies setting up an objective function as well as the Hamiltonian equation and derives the Pontryagin necessary conditions (first order conditions of each variable) to find the optimal path of each variable considered (McConnell, 1983).

Torell, Lyon and Godfrey (1991) construct a dynamic OC model in which the stocking rate is the instrumental variable while the average herbage production represents the state variable with the purpose of maximizing the discounted NPV from grazing over future years specifically applied in eastern Colorado. The stocking rate model developed employs a deterministic approach where forage conditions, costs and prices are foreseen at the time the stocking rate choice is made (Torell et al., 1991). On the other hand, Standiford and Howitt (1992) utilize the stocks of livestock and oak trees as state variables while the amount of oak firewood cut and livestock density are included as control variables. The objective is to maximize the NPV of profits based on firewood, hunting and livestock revenues. Under these circumstances, the farm manager has to make decisions on a yearly basis since oak trees negatively impact livestock revenue but positively impact hunting returns. Thus, ranch managers select optimal hunting levels by controlling livestock density and firewood harvesting. The authors evaluate the optimal trajectory for each control variable under different scenarios for a policy analysis, specifically in the Californian hardwood rangeland region due to the dynamic interaction among the resources available in the area (Standiford and Howitt, 1992).

Only one known study integrates a spatial component into the OC framework. Brock and Xepapadeas (2009) propose an OC model in which spatial effects of accumulated state variables in other locations are considered as influencing given sites in an abstract format in which specific locations are not specified, allowing for broad applications. They establish that the integration of the model kernel expressions is an appropriate tool for dynamic situation when spatial effects are taken into account.

**2. The model**

An OC model is developed to examine whether and how a niche product such as PBB can benefit the farmer and society by integrating consumer preferences for a leaner beef product against a backdrop of energy, climate and environmental objectives. The model allows decision variables to respond over time to accrued influences of previous control management choices on state variables and crop production, and is intended to capture the dynamic effects in three interconnected production functions that eventually determine farm-level profitability. Management-intensive grazing practices (such as rotational or buffer grazing) allow farmers to identify the optimal choice between using pasture in the production of beef versus stockpiling grass for hay wherein benefits (and costs) are dispersed across locations.

This model integrates the OC approaches proposed by McConnell (1983), Saliba (1985) and Cacho (1998) as well as incorporates a spatial component based on Brock and Xepapadeas (2009). In addition to the explicit integration of a spatial component, this study is unique in that it also includes potential ecosystem benefits of the PBB industry, vis-à-vis electricity production, digested manure as well as GHG emission reductions. Beyond mere farm-level profitability, this model also provides the basis for agglomeration economies to enhance economic and environmental development within a region. This can be achieved when the optimal private path overlaps the socially optimal path.

**2.1. Entrepreneur’s perspective.** As a starting point, we developed equation (1) with the main purpose of illustrating the objective function without considering the spatial component in contrast to equation (4) which captures the spatial influences. However, it is essential to point out that our conceptual model is derived from equation (2) to equation (28). Assuming that the value of the land at the end of the planning horizon  $T$  is not considered (Standiford and Howitt, 1992; Cacho, 1998) since the resale of the business is not an argument, the objective function in which the entrepreneur maximizes the present value

of the profit stream or discounted accumulated profits over the planning horizon (McConnell, 1983; Saliba, 1985) is:

$$\text{Max}_{\gamma} J = \int_0^T e^{-rt} [p_{\alpha} f_{\alpha}(\gamma_{\gamma}) + p_{\xi} f_{\xi}(\omega_t(\gamma) + p_{\psi} f_{\psi}(\xi_t)) - c_{\alpha} \gamma - c_{\xi} \omega - cs] dt. \tag{1}$$

Table 1. Definition of variables

Variable type/ Function	Variable symbol and unit description
Control	$\gamma$ is the Stocking rate cow-calf units (acre)
State	$\rho$ is the pasture mass (lbs./acre) $\eta$ is the soil organic matter (lbs./acre)
Prices	$p_{\alpha}$ is the price of beef (\$/lbs.) $p_{\xi}$ is the price of electricity (\$/kWh) $p_{\psi}$ is the carbon price (\$/CO <sub>2e</sub> ton)
Costs	$c_{\alpha}$ are the beef production costs (\$/lbs.) $c_{\xi}$ are the electricity production costs (\$/kWh) $cs$ are the fixed costs (\$)
Others	$\alpha$ is the beef production (lbs./acre) $\xi$ is the electricity production (kwh/head) $\psi$ is the GHG emission reduction function (\$/co <sub>2e</sub> ton) $\mu$ is the harvested forage by stocking (lbs./acre) $\beta$ is the digested manure application (lbs./acre) $\sigma$ is the forage growth (lbs./acre) $\theta$ is the hay for winter feed (lbs./acre) $\zeta$ is the nutrients accumulation (lbs./acre) $\omega$ is the amount of manure collected (lbs./head) $v$ are the precipitation inches $\rho_{t+1}$ is the pasture mass at the end of the feeding season (%) $e^{-rt}$ is the continuous time discount factor $e^{-\delta t}$ is the continuous time welfare factor $\delta$ is the welfare value of future generations $r$ is the private discount rate $t$ is the specific time period $T$ is the end of the planning horizon
Spatial	$z$ are the given locations $z'$ are the other locations $Z$ are the entire spatial domain $P$ are the concentrations of pasture mass from $z'$ (lbs.) $N$ are the accumulated soil organic matter from $z'$ (lbs.)

Equation (1) represents the objective function of the farmer which is to maximize the discounted accumulated profits over the planning horizon  $T$  within a non-spatial context. Notice that equation (1) is only used to illustrate our starting point; but our main objective function is presented in equation (4) since it is the one integrating the spatial component.

As part of the integration of the spatial component in our OC model, we need to outline a set of assumptions. Since the farm of interest might be surrounded by a diverse group of businesses throughout the spatial domain, their spatial influences toward its production functions might differ depending on the operational nature of every nearby farm. This implies that besides the farm of interest, other businesses in the surrounding area might be producers of beef and hay among other agricultural products. Therefore, we need to consider the spatial in-

fluences in our objective function which is represented in equation (4). This spatial diversity leads us to the following assumption.

**Assumption 1:** Locations  $z'$  are adjacent forage-based farms in which the spatial effects are heterogeneous across locations.

The slope of the pastureland available in an area has an impact on land use, especially for grazing as well as fertilizer applications. In fact, the steeper the slope the less pasture at the site is consumed by cattle since animals tend to gather and graze more in flat or less steep slopes. This might have a negative effect on the grazable land area available for beef production (Laca, 2000, Holechek, 1988). This leads us to the following assumption.

**Assumption 2:** The slope of farms in location  $z$  is flat while land slope in location  $z'$  might be steeper which is a limiting factor for machinery use as well as grazing.

$P$  and  $N$  represent inputs (Brock and Xepapadeas, 2009) in the production functions presented in the objective function (4). In our model, these quantities can be captured in the amount of undigested manure available and in hay production. This is true not only because the change in state variables is influenced by these variables in some way but also due to the fact that they play an essential role in energy and beef production as well as, eventually, in GHG emission reductions. Since these variables are mobile across locations, this allows for clustering among locations as a strategy of optimizing resources available in the entire spatial domain. The development of interconnected businesses and suppliers in a geographic region enhances the ability of firms to cluster together in a way that creates economic activity as well as concentration of knowledge (Dearlove, 2001).

**Assumption 3:** Manure is collected during the winter season in the barn.

Deals with the collection of manure during winter (when animals are more concentrated) and transported from adjacent farms to the farm of interest.

**Assumption 4:** Undigested manure and hay are completely mobile.

Since hay is also transported from nearby hay farms to the farm of interest, we define it as a mobile input.

**Assumption 5:** Production functions are differentiable and concave which presents diminishing returns over time.

Due to the fact that spatial distributions are not uniform across locations or are spatially heterogeneous,

this allows for the emergence of agglomeration economies or clustering through resource optimization which could turn out to be persistent in a heterogeneous steady state among locations (Brock and Xepapadeas, 2009). In other words, state variables are optimized when management decisions are manifested through sustainable practices considering the entire space domain. However, for simplicity it is assumed that the land endowment for each enterprise in the entire spatial domain is constant, which implies that every farm has the same number of acres on average. This provides the basis for assumption 6.

**Assumption 6:** Pastureland in the PBB industry is predetermined.

Furthermore, mathematical expressions have been designed to illustrate the effects of variables developed in adjacent locations on the production functions in a given location. In order to integrate the spatial effects in locations  $Z$  (the given locations) caused by the accumulated state variables in other locations identified as  $z'$ , it is essential to consider the kernel formulation which basically measures the influences of sites  $z'$  on location  $z$  developed by Brock and Xepapadeas (2009). For instance, variables such as pasture mass and soil organic matter (our state variables) identified in nearby locations can be expressed as part of the production functions of the farm of interest by integrating the kernel function. Following Brock and Xepapadeas (2009), the spatial influences of the concentrated state variables  $\rho(t, z')$  and  $\eta(t, z')$  in locations  $z'$  (adjacent locations) on the state variables  $\rho(t, z)$  and  $\eta(t, z)$  in locations  $z$  (locations or areas of interest) are represented in equations (2) and (3), respectively:

$$P(t, z) = \int_{z' \in Z} w(z - z') \rho(t, z') dz, \quad (2)$$

$$N(t, z) = \int_{z' \in Z} w(z - z') \eta(t, z') dz. \quad (3)$$

The integration of these state variables into the production functions at locations of interest is an approach to illustrate the spatial interaction when the kernel function is employed. In fact, the application of the kernel influence function,  $w(\cdot)$ , as described by Brock and Xepapadeas (2009) allows us to describe explicitly the impact of state variables located at spatial locations  $z'$  on state variables at particular sites  $z$  in which the entire spatial domain is represented as  $Z$  ( $z, z' \in Z$ ). In other words,  $P(\cdot)$  (accumulated pasture mass) and  $N(\cdot)$  (soil organic matter) from locations  $z'$  (adjacent locations) reflect spatial spillovers on the beef,  $f_\alpha$  and electricity,  $f_\xi$ , production functions

on  $z$  locations. The integration of these adjacent state variables into the objective function on the entrepreneurs in the given locations allows the

development of “dynamic system forces” that leads to agglomeration economies in the region (Brock and Xepapadeas, 2009).

$$\text{Max } J = \int_{\gamma} \int_{z=Z}^T e^{-r} [p_{\alpha} f_{\alpha}(\gamma_{t,z}, P_{t,z}, N_{t,z}) + p_{\xi} f_{\xi}(\omega_{t,z}(y), P_{t,z}, N_{t,z}) + p_{\psi} f_{\psi}(\xi_{t,z}) - c_{\alpha} \gamma_{t,z} - c_{\xi} \omega_{t,z} - c_{\psi} \xi_{t,z}] dz dt. \quad (4)$$

Equation (4) denotes our intended objective function that maximizes the discounted accumulated profits over the planning horizon  $T$  when spatial spillovers are internalized while the value of the land at the end of the planning horizon  $T$  is not considered since it is not an argument. Figure 1 provides a simplified overview of the path of the state variables when decision variables are taken into account.

Conceptually, the objective function is subject to changes in pasture mass available and soil organic matter accumulation per acre and their corresponding initial amounts at the beginning of the feeding season in locations  $z$  in which spatial effects are taken into consideration:

$$\Delta p_{t,z} = p_{t+1,z} - p_{t,z} = f(\gamma_{t,z}, \eta_{t,z}, \rho_{t,z}, \beta_{t,z}, v_{t,z}, P_{t,z}, N_{t,z}). \quad (5)$$

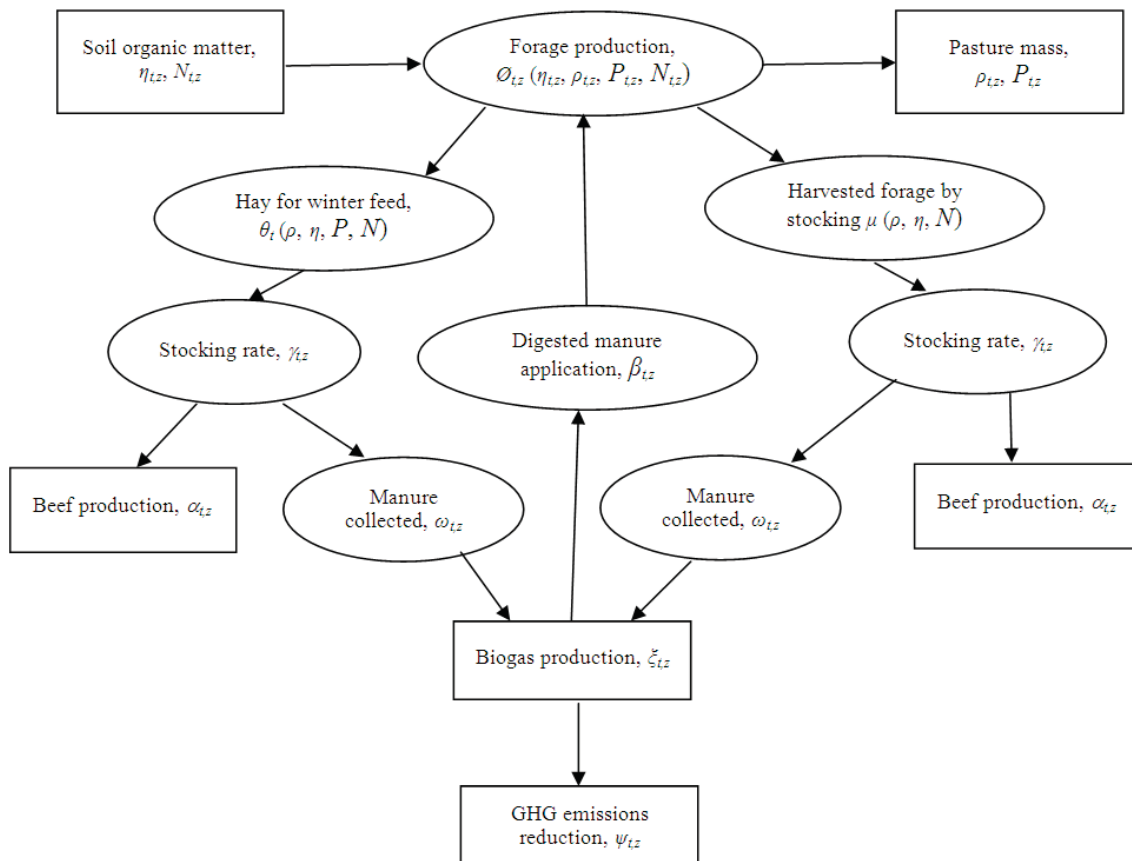


Fig. 1. Paths of soil organic matter and pasture mass in locations  $z$

$$\phi_{t,z} = f(\gamma_{t,z}, \eta_{t,z}, \rho_{t,z}, \beta_{t,z}, v_{t,z}, P_{t,z}, N_{t,z}). \quad (6)$$

All these influences imply (again, conceptually): forage growth would impact beef production as well as energy production. Thus, the contribution of hay for winter and harvested forage by stocking would positively impact beef production,  $\frac{\partial \alpha}{\partial \phi} > 0$ , shown in equation (11) and alternative energy production,  $\frac{\partial \xi}{\partial \phi} > 0$ , or equation (12) since forage is the primary diet in this beef industry which eventually would be transformed into manure, the primary input in the bio-gas production process. Therefore, the GHG emission

reduction function,  $\frac{\partial \psi}{\partial \phi} > 0$ , presented in equation (13) would be positively impacted by forage growth since it contributes to carbon offsets and, in addition, forage growth would also impact the GHG emission function in a positive manner  $\frac{\partial \psi}{\partial \phi} > 0$ , through carbon sequestration since pasture lands would sequester CO<sub>2</sub>.

Equation (6) defines the forage growth function which is basically dependent on stocking rate,  $\gamma_{t,z}$ , soil organic matter,  $\eta_{t,z}$ , pasture mass at the beginning of the feeding season,  $\rho_{t,z}$ , digested manure or natural

nutrients application,  $\beta_{t,z}$ , the average precipitation, a weather condition,  $v_{t,z}$  and the accumulated pasture mass,  $P_{t,z}$ , as well as concentration of soil organic matter,  $N_{t,z}$ , from locations  $z'$ . Most of these are implicitly affected by the amount of carbon available in the soil. The impacts of each variable on this function are the following (notice that subscripts  $t$  and  $z$  have been dropped for simplification):

The stocking rate negatively influences forage growth, i.e.  $\frac{\partial \phi}{\partial \gamma} < 0$ . However, digested manure or nutrient application as well as soil organic matter can be used

to counteract this negative effect, i.e.  $\frac{\partial \phi}{\partial \beta} > 0$  and

$\frac{\partial \phi}{\partial \eta} > 0$ , since they both increase nutrient availability

which enhances forage growth per acre. In addition, this function is positively affected by the pasture mass available at the beginning of the feeding season,

$\frac{\partial \phi}{\partial \rho_1} > 0$  and precipitation influences forage growth

positively,  $\frac{\partial \phi}{\partial v} > 0$ . Moreover, forage growth is influenced by the spatial effects from locations  $z'$  through

the accumulated pasture mass,  $\frac{\partial \phi}{\partial P} > 0$ , in the form of

hay and accumulated soil organic matter,  $\frac{\partial \phi}{\partial N} > 0$ , in

the form of undigested manure from locations  $z'$  to be used in locations  $z$ .

**Steady state condition 1:** As previously mentioned, the change of pasture mass available per acre is influenced by the stocking rate, the soil organic matter accumulation rate, the pasture mass at the beginning of the feeding season, the nutrient application rate, the accumulated pasture mass as well as soil organic matter concentrations from locations  $z'$  and precipitation. In other words, pasture mass is in a steady state condition or reaches equilibrium due to the influences of each variable on forage growth,  $\phi_{t,z} = f(\eta, \rho, \beta, v, P, N, \gamma)$ , in which management decisions and clustering among locations are considered. This means that the change in pasture mass is optimized when these strategies are employed since generally recognized sustainable practices (such as pasture-based systems and rotational grazing) are taken into account in the entire spatial domain, optimizing stocking rate in the process. This, in turn, optimizes beef and energy production as well as GHG emission reduction through a carbon offset in location  $z$ . The relationship between pasture mass, soil organic matter and beef yield is presented in Figure 2.

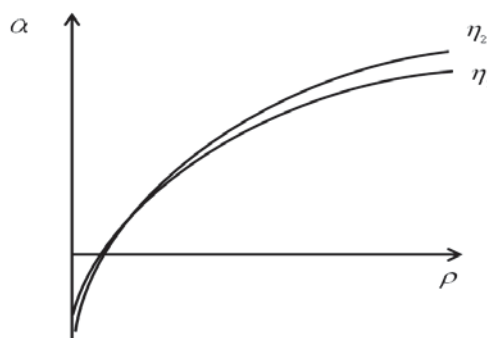


Fig. 2. Effects of soil organic matter on pasture mass and beef production

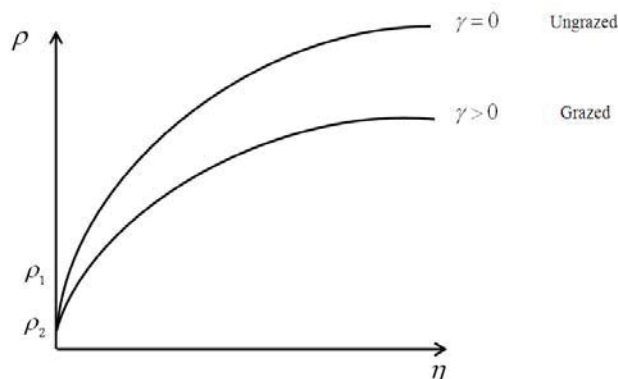


Fig. 3. Effects of stocking rate on forage growth and their relationship with soil organic matter

Figure 2 shows that both soil organic matter ( $\eta_1$ ) and additional nutrients ( $\eta_2$ ) influence pasture mass positively which, in turn, increases beef production. On the other hand, stocking rate is assumed to negatively influence both pasture mass as well as soil organic matter availability (Figure 3). Idling pasture land ( $\gamma = 0$ ) allows pasture mass to grow since more nutrients are available. However, stocking cattle ( $\gamma > 0$ ) decreases pasture mass through consumption as well as nutrient availability.

Equation (7) represents the initial pasture mass available per acre at the beginning of the feeding season in location  $z$ :

$$\rho(t = 0, z) = \rho_{0,z}. \tag{7}$$

Equation (8) represents the change in soil organic matter accumulated per acre in location  $z$  which depends on the soil organic matter at the start of the feeding season,  $\eta_{t,z}$ , and the amount of soil organic matter available at the end of the feeding season,  $\eta_{t+1,z}$ , in location  $z$ . The change on soil organic matter is essentially the nutrient accumulation function,  $\zeta_{t,z}$ .

$$\Delta \eta_{t,z} = \eta_{t+1,z} - \eta_{t,z} = f(\gamma_{t,z}, \beta_{t,z}, \kappa p_{t,z}, \eta_{t,z}, N_{t,z}, P_{t,z}) \tag{8}$$

Equation (9) defines the nutrient accumulation function which is a function of the stocking rate,  $\gamma(t, z)$ ,

the digested manure application,  $\beta(t, z)$ , the percentage of the remaining pasture mass at the end of the feeding season,  $\kappa\rho_{t+1,z}$ , in which  $\kappa$  is a constant term with values  $0 < \kappa < 1$ , the soil organic matter available at the beginning of the feeding season,  $\eta(t, z)$ , the concentration of soil organic matter,  $N(t, z)$ , as well as accumulated pasture mass from locations  $z'$ . The influences of each variable on this function are shown as follows (after dropping subscripts  $t$  and  $z$  for simplicity):

$$\zeta = f(\beta, \kappa\rho_{t,z}, \eta, N, P, \gamma). \tag{9}$$

Stocking rate negatively affects the nutrient accumulation function,  $\frac{\partial \zeta}{\partial \gamma} < 0$ , since it is extracted

from the soil through harvested forage by livestock and hay production for winter feed. On the other hand, the percentage of the remaining pasture mass at the end of the feeding season,  $\frac{\partial \zeta}{\partial \kappa\rho_{t+1}} > 0$ , and

the digested manure application,  $\frac{\partial \zeta}{\partial \beta} > 0$ , contribute

in counteracting this negative impact. In addition, the soil organic matter at the beginning of the feeding season would influence this function positively, i.e.,  $\frac{\partial \zeta}{\partial \eta_t} > 0$ . Furthermore, nutrient accumulation is

positively influenced by the concentration of soil organic matter,  $\frac{\partial \zeta}{\partial N} > 0$ , and accumulated pasture

mass,  $\frac{\partial \zeta}{\partial P} > 0$ , from locations  $z'$  in a form of undigested manure and hay respectively to be used in location  $z$ .

Under this scenario, these influences suggest that: the fact that the availability of nutrients enhances forage growth for stocking implies that nutrient accumulation would positively influence beef production,  $\frac{\partial \alpha}{\partial \zeta} > 0$ , through the increase of pasture

available for grazing and the winter season which eventually would increase the animal's weight. Likewise, nutrients would impact energy production in a positive manner,  $\frac{\partial \xi}{\partial \zeta} > 0$ , through the contribu-

tion of pasture growth and spatial influences ( $N$ ). This occurs due to the fact that the forage harvested by the stocking rate and hay for winter feeding is positively influenced by nutrient accumulation in

location  $z$  which would eventually be transformed into manure and utilized as an input for electricity production. Since alternative energy production enhances carbon offsets, GHG emission reduction

function,  $\frac{\partial \psi}{\partial \zeta} > 0$ , is positively influenced which

progressively increases GHG emission reduction in location  $z$ .

**Steady state condition 2:** The change of soil organic matter per acre is explained by the influences of the stocking rate, pastureland for carbon sequestration, digested manure or nutrient application, the percentage of the remaining pasture mass, the soil organic matter at the beginning of the feeding season, the concentration of soil organic matter, and pasture mass from location  $z'$  on the nutrient accumulation function. In other words, the soil organic matter is in a steady state condition or reaches equilibrium due to the impact of each variable on nutrient accumulation,  $\zeta = f(\beta, \kappa\rho_{t,z}, \eta, N, P, \gamma)$ , in which sustainable management decisions are considered. This would contribute to the levels of beef and energy production and eventually GHG emission reductions through a carbon offset. This occurs because the resources available are efficiently utilized when the soil organic matter system is at a stable stage during a given period of time. The relationship between the stocking rate and soil organic matter and renewable energy production is illustrated in Figure 4. Stocking rate enhances soil organic matter through manure which influences energy production positively.

Equation (10) represents the initial soil organic matter available per acre in location  $z$  at the beginning of the feeding season.

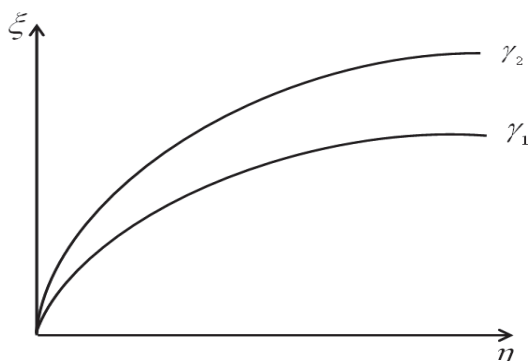
$$\eta = (t = 0, z) = \eta_{0,z}. \tag{10}$$

Equation (11) represents beef production explicitly represented in the objective function which depends on stocking rate,  $\gamma_{t,z}$ , concentration of pasture mass,  $P_{t,z}$ , and soil organic matter,  $N_{t,z}$  as depicted in equation (4).

$$\alpha_{t,z} = f(\gamma_{t,z}, P_{t,z}, N_{t,z}). \tag{11}$$

Equation (12) represents the electricity production explicitly incorporated in the objective function that depends on the amount of manure collected,  $\omega_{t,z}$ , which is a function of the stocking rate,  $\gamma(t, z)$ , and spatial effects of the state variables from locations  $z'$ .

$$\zeta_{t,z} = f(\omega_{t,z}, (\gamma_{t,z}), P_{t,z}, N_{t,z}). \tag{12}$$



**Fig. 4. Effects of stocking rate on soil organic matter and energy production**

Equation (13) represents the GHG emission reduction function explicitly incorporated in the objective function that depends on the amount of energy produced,  $\xi_{t,z}$ . The relationship between GHG emission reduction and (alternative) energy production is illustrated in Figure 5. Energy production from biogas enhances GHG emission reduction or decreases CO<sub>2</sub> emissions through methane capture known as the “carbon offset” technique.

$$\psi_{t,z} = f(\xi_{t,z}). \tag{13}$$

Due to the fact that  $P$  and  $N$  are inputs (Brock and Xepapadeas, 2009) in the production functions (as manure and hay) represented in equations (11) and (12) respectively, this provides the basis for stimulating regional economic development through clustering systems within a diversified, spatially distributed industry in the region.

The objective function is composed of total revenue from beef,  $p_\alpha \alpha_{t,z}$ , electricity,  $p_\xi \xi_{t,z}$ , and carbon offset,  $p_\psi \psi_{t,z}$ , revenues minus the variables costs of

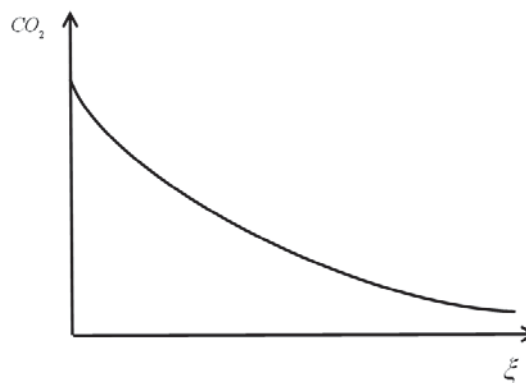
$$MaxH(\gamma, p, \eta, \lambda_p, \lambda_\eta, P, N) = p_\alpha f_\alpha(\gamma, P, N) + p_\xi f_\xi(\omega(\gamma), P, N) + p_\psi f_\psi(\xi) - c_\alpha \gamma - c_\xi \omega - cs + \lambda_p \Delta p + \lambda_\eta \Delta \eta. \tag{14}$$

The derivative of the Hamiltonian with respect to the control variable must be equal to zero according

$$\begin{aligned} \text{For } \gamma : \frac{\partial H(\cdot)}{\partial \gamma} = 0 \rightarrow & p_\alpha \frac{\partial \alpha}{\partial \gamma} - c_\alpha + p_\xi \frac{\partial \xi}{\partial \gamma} + p_\psi \frac{\partial \psi}{\partial \gamma} - c'_\xi \omega + \lambda_p \frac{\partial f_\omega(\gamma, \eta, p, \beta, v, P)}{\partial \gamma} + \lambda_\eta \frac{\partial f_\eta(\gamma, \beta, \kappa p_{t+1}, \eta, N)}{\partial \gamma} = 0 \\ \rightarrow & -\lambda_p \frac{\partial f_\phi(\cdot)}{\partial \gamma} - \lambda_\eta \frac{\partial f_\zeta(\cdot)}{\partial \gamma} = p_\alpha \frac{\partial f_\alpha}{\partial \gamma} + p_\xi \frac{\partial f_\xi}{\partial \gamma} + p_\psi \frac{\partial f_\psi}{\partial \gamma} - c_\alpha - c'_\xi \omega. \end{aligned} \tag{15}$$

The right hand side (RHS) of equation (15) shows the product of beef price and the influence of stocking rate on beef production plus the product of electricity price and the influence of stocking rate on the production of this renewable fuel plus the carbon price and the effects of this control variable on the GHG emission reduction function. The RHS also

production,  $c_\alpha \gamma_{t,z}$ , which depend on stocking rate, and energy production  $c_\xi \omega_{t,z}$ , which depends on the amount of manure collected. The carbon offset is captured through the reduction of methane emissions as part of the alternative energy production process in which variable costs are already embedded in the energy production. The total costs are also impacted by fixed costs associated with grass-based beef as well as energy production and carbon offset expressed as  $cs$ .



**Fig. 5. Effects of energy production on CO<sub>2</sub> emissions**

In keeping with the approaches of Cacho (1998) and Brock and Xepapadeas (2009), subscripts  $t$  and  $z$  have been dropped for simplification. For this optimal control problem, there are four types of necessary conditions that will be explained below (Saliba, 1985). The Hamiltonian is composed of the integrand function plus the product of the co-state variables and their corresponding equation of motion (Chiang, 2000).

Equation (14) presents the Hamiltonian for this problem:

to the maximum principle (Saliba, 1985). The optimal path of  $\gamma$  in a spatiotemporal scenario is:

captures the variable costs associated with the amount of animal units on the farm and the variables costs associated with manure collection. On the other hand, the left hand side (LHS) of this equation expresses the product of the pasture mass co-state variable and the influence of stocking rate on forage growth and the product of the soil organic

matter co-state variable and the effects of stocking rate on the nutrient accumulation function. In other words, equation (15) represents the benefits of higher stocking rate per acre in terms of profits from beef and energy production as well as carbon offsets shown on its RHS while the LHS implies the costs associated with the number of head per acre in terms of the marginal value of increasing one additional animal per acre to enhance beef and renewable energy production as well as to reduce GHG emissions through energy production.

Another important variable is the auxiliary variable also known as the co-state variable which is basically a valuation variable (i.e., its value changes at different time periods), named the shadow price of the related state variable. This variable is integrated into the optimal control model through the Hamiltonian function. This function is used to optimize the control variable before employing the maximum principle (Chiang, 2000). In this model, the shadow price represents the amount of money farmers would be willing to pay (WTP) for an additional pound of pasture mass produced per acre and an additional lb. of soil organic matter per acre. In fact, if the cost associated with any of these two state variables were less than the shadow price, the present value of the profit stream or the value of the objective function would increase. In contrast, if the associated costs were higher than the shadow price, then the value of the objective function would decrease while an equal cost would keep it unchanged. Every co-state equation presents the change rate of each co-state variable (Saliba, 1985). Thus, the optimal path of each co-state variable is represented through the marginal value (Cacho, 1998; Saliba, 1985) of  $\lambda_\rho$  and  $\lambda_\eta$ :

$$\frac{\partial H(.)}{\partial \rho} = r\lambda_\rho - \dot{\lambda}_\rho$$

$$\dot{\lambda}_\rho = r\lambda_\rho - \frac{\partial H(.)}{\partial \rho} \rightarrow \dot{\lambda}_\rho = r\lambda_\rho - p_\alpha \frac{\partial \alpha}{\partial \rho} - p_\xi \frac{\partial \xi}{\partial \rho} - p_\psi \frac{\partial \psi}{\partial \rho} \quad (16)$$

Equation (16) denotes that changes in the marginal value of pasture mass available per acre at each point in time,  $\dot{\lambda}_\rho$  depends on the product of the discount rate,  $r$  and the current value of the co-state variable,  $\lambda_\rho$  less the product of beef price,  $p_\alpha$  and the influences of pasture mass on beef production function,  $\frac{\partial \alpha}{\partial \rho}$ ; less the product of the electricity price,  $p_\xi$  and the effects of pasture mass on the

energy production function,  $\frac{\partial f_\xi}{\partial \rho}$ ; less the product of

the carbon offset price,  $p_\psi$ , and the influences of pasture mass on the reduction of GHG emissions,  $\frac{\partial \psi}{\partial \rho}$ , in each time period. Thus, the implicit cost of

pasture mass produced per acre must grow at the rate of discount minus the contribution of the pasture mass available either for stocking through the harvested forage and hay per acre to the current returns from beef and energy production as well as GHG emission reductions through “carbon offsets”.

$$\rho(t=0, z) = \rho_{0,z} \quad (17)$$

$$\Delta p_{t,z} = p_{t+1,z} - p_{t,z} = f(\gamma, \eta, p, \beta, v, P, N). \quad (18)$$

Equations (17) and (18) present the initial pasture mass available per acre at the beginning of the grazing season and its change at location  $z$ , respectively.

$$\frac{\partial H(.)}{\partial \eta} = r\lambda_\eta - \dot{\lambda}_\eta$$

$$\dot{\lambda}_\eta = r\lambda_\eta - \frac{\partial H(.)}{\partial \eta} \rightarrow \dot{\lambda}_\eta = r\lambda_\eta - p_\alpha \frac{\partial \alpha}{\partial \eta} - p_\xi \frac{\partial \xi}{\partial \eta} - p_\psi \frac{\partial \psi}{\partial \eta}. \quad (19)$$

Equation (19) implies that the changes in the marginal value of soil organic matter per acre at each point in time,  $\dot{\lambda}_\eta$ , depends on the product of the discount rate,  $r$ , and the current value of the co-state variable,  $\lambda_\eta$ ; less the product of the beef price,

$p_\alpha$ , and the effects of soil organic matter on the beef production function,  $\frac{\partial \alpha}{\partial \eta}$ ; less the product of

the electricity price,  $p_\xi$ , and the influences of soil organic matter on the energy production function,  $\frac{\partial \xi}{\partial \eta}$ ; less the carbon offset price,  $p_\psi$ , and the im-

pacts of soil organic matter,  $\frac{\partial \psi}{\partial \eta}$ , on the reduction

of GHG emissions at each point of time. The implicit cost of soil organic matter per acre must grow at the rate of discount less its positive impact on forage production per acre that enhances current returns from beef and electricity production as well as methane emission or CO<sub>2</sub> emission reductions.

$$\eta(t=0, z) = \eta_{0,z} \quad (20)$$

$$\Delta \eta_{t,z} = \eta_{t+1,z} - \eta_{t,z} = f(\gamma, \beta, \kappa p_{t+1}, \eta, N, P). \quad (21)$$

Equation (20) and (21) represent the initial soil organic matter at the start of the feeding season per acre and its change in location  $z$ , respectively.

The state equations are:

$$\frac{\partial H}{\partial \lambda_p} \Delta p_{t,z} = p_{t+1,z} - p_{t,z} = f(\gamma, \eta, p, \beta, v, P, N), \quad (22)$$

$$\frac{\partial H}{\partial \lambda_\eta} \Delta \eta_{t,z} = \eta_{t+1,z} - \eta_{t,z} = f(\gamma, \beta, \kappa p_{t+1}, \eta, N, P), \quad (23)$$

Equation (22) represents the state equation for pasture mass while equation (23) denotes the state equation for soil organic matter. These two equations are subject to the initial conditions of each state variable in order to solve them intertemporally. These functional relationships are able to capture the effects of management decisions (control variables) on the state variables (Saliba, 1985).

The endpoint considers the initial conditions of every state variable as well as the transversality condition:

$$\rho(t = 0, z) = \rho_{0,z}, \quad (24)$$

$$\eta(t = 0, z) = \eta_{0,z}, \quad (25)$$

Equations (26) and (27) display the transversality conditions in the final period  $T$ . This is the last condition considered in an optimal control model. This condition essentially represents what would occur in the final period of time (Chiang, 2000). Following

$$\text{Max}_\gamma : V = \int_0^T \int_{z=0}^z e^{-\delta t} [p_\alpha f_\alpha(\gamma_{t,z}, P_{t,z}, N_{t,z}) + p_\xi f_\xi(\omega_{t,z}(y), P_{t,z}, N_{t,z}) + p_\psi f_\psi(\xi_{t,z}) - c_\alpha \gamma_{t,z} - c_\xi \omega_{t,z} - c_s s_{t,z}] dz dt. \quad (28)$$

As McConnell (1983) suggests, the socially efficient strategy would be equal to the private goal when the private discount rate,  $r$ , is equal to the value of the welfare of future generations,  $\delta$ . This value represents the implementation of sustainable practices in the present time period, and is reflected at the end of planning horizon  $T$ . When this interaction,  $\delta = r$ , takes place and the market works efficiently, society and the farmer would be efficiently interconnected and the path of the stocking rate would be socially optimal. This would eventually influence the paths of the pasture mass and the soil organic matter per acre. This also occurs due to the fact that clustering systems enhance competition within related industries in which the firms actively involved in the clustering benefit from the surrounding environment. Therefore, the implementation of sustainable practices in the PBB industry would benefit the farmer as well as surrounding communities. In addition, since the farmer is taking into con-

Saliba's approach, these equations establish that the marginal values of each state variable considered will influence the market price of its related product. This spatial optimal control (SOC) model also provides for tradeoffs between beef and energy production while abating GHG emissions by selecting stocking rate as the main decision variable in this model.

$$\lambda_\rho(T) = \frac{\partial J_\rho(T)}{\partial \rho(T)}, \quad (26)$$

$$\lambda_\eta(T) = \frac{\partial J_\eta(T)}{\partial \eta(T)}. \quad (27)$$

**2.2. Determining the optimal product mix from among beef, electricity and carbon offset.** During planning horizon  $T$ , the marginal value of pasture mass produced and soil organic matter per acre would have an impact on the market value of beef, energy and carbon prices. This occurs due to the fact that beef and energy production as well as GHG emission reductions are mutually dependent on state variables in locations  $z$  as well as the spatial influences of state variables from locations  $z'$  through the interaction between stocking rate, the feeding seasons based on the harvested forage by stocking, the hay for winter feed and undigested manure.

**2.3. Society's perspective:** The value of the farmin location  $z$  to society when spatial influences are considered can be represented as:

sideration environmental improvement which allows reducing potential negative externalities from his/her operation, it contributes to achieving social efficiency.

### Conclusions

The spatial optimal control model developed here shows that the increased use of pasture as the primary diet for cattle in the beef industry would cause positive effects not only in terms of animal welfare but also to human health, the land resource, the ecosystem and economic development. The waste produced from livestock can be used as natural fertilizer as well as a source of alternative energy which would help entrepreneurs generate additional income. This model also implies that if affiliated businesses along the food supply chain within a region can leverage spatial influences, it enhance both the industry and society. In fact, the development of clustering systems plays a crucial role in

our model since the spatial effects permit the expansion of both private and social benefits from this nascent industry.

The model is built on the premise that sustainability is enhanced when an industry is structured to generate both private and social benefits. When the use of natural resources promises the highest private present value compared to conserving it in a natural state for the wellbeing of society, it is very likely to experience divergence between the two sectors (Krutilla, 1967). However, the industry configuration discussed portrays an alternative that would optimize resource use within a spatial domain in a sustainable way to meet present needs without compromising future ones. The combination of appropriate land use for sustainable production and proper waste management practices would

maintain the required nutrients for high quality soil as well as improved water and air quality, so firms are able to obtain a premium from their high quality products while enhancing the ecosystem which eventually has a positive effect on society. Of course, the intensification of benefits derived from the agglomeration economies requires cooperation and coordination among the key players within the impacted region, a managerial decision that is best left for further research consideration.

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### References

1. Brock, W., and Xepapadeas, A. (2009). The Emergence of Optimal Control Agglomeration in Dynamic Economics, University of Wisconsin-Department of Economics and Athens University of Economics-Department of International and European Economic Studies. Retrieved from [http://www.ssc.wisc.edu/~wbrock/Brock\\_Xepapadeas\\_Optimal\\_Agglomeration.pdf](http://www.ssc.wisc.edu/~wbrock/Brock_Xepapadeas_Optimal_Agglomeration.pdf).
2. Cacho, O.J. (1998). Solving Bioeconomic Optimal Control Models Numerically, In J. Gooday (Ed.), Proceedings of the Bioeconomic Workshop, Australian Agricultural and Resource Economics Society Conference, 22 January, Armidale, ABARE, Canberra. University of New England, Armidale, ABARE Bioeconomic Workshop, pp. 13-26.
3. Chiang, A.C. (2000). *Elements of Dynamic Optimization*, 2<sup>nd</sup> ed., Illinois: Vaveland Press, Inc. pp. 161-176.
4. Dearlove, D. (2001). The Cluster Effect: Can Europe Clone Silicon Valley? 3<sup>rd</sup> ed., Quarter New York, NY, Booz & Company.
5. Fulhage, C.D., D. Sievers and J.R. Fischer. (1993). Generating Methane Gas from Manure, University of Missouri Extension, Department of Agricultural Engineering. Available at <http://extension.missouri.edu/publications/DisplayPub.aspx?P=G1881>.
6. Holechek, J.L. (1988). An Approach for Setting the Stocking Rate, *Rangelands*, 10 (1), pp. 10-14.
7. Krueger, C.R. (1998). *Grazing in the Northeast: Assessing Current Technologies, Research Directions, and Education Needs*. NRAES-113, in Proceeding from the Grazing in the Northeast Workshop, March 25-26, Camp Hill, Pennsylvania.
8. Krutilla, J.V. (1967). Conservation Reconsidered, *The American Economic Review*, 57 (4), pp. 777-786.
9. Laca, E.C. (2000). Modelling Spatial Aspects of Plant Animal Interactions, *Grassland Ecophysiology and Grazing Ecology*, New York, CAB International, pp. 209-231.
10. McConnell, K.E. (1983). An Economic Model of Soil Conservation, *American Journal of Agricultural Economics*, 65 (1), pp. 83-89.
11. Paine, L., P. Reedy, R. Skora, and J. Swenson (2009). A Consumer's Guide to Grass-Fed Beef. UW Cooperative Extension, University of Wisconsin-Madison, pp. 1-12. Retrieved from <http://www.wisconsinlocalfood.com/recipes/CGGFB.pdf>.
12. Perman, R., Y. Ma, J. McGilvray, and M. Common (2003). *Natural Resource and Environmental Economics*, 3<sup>rd</sup> ed., Pearson Education Limited, Gosport, pp. 364-375.
13. Pimentel, D. and Pimentel, M. (2008). *Food, Energy and Society*, 3<sup>rd</sup> ed. CRC Press, Taylor and Francis Group, Boca Raton, FL, pp. 67-75, 277-310.
14. Portelli, J. (2008). Home, Home on the Feedlot: A Study of the Sustainability of Grass-Fed and Grain-Fed Beef Production. Working Paper for ENVS 664 Sustainable Design, pp. 1-11. Retrieved from [http://www.greendesignetc.net/GreenProducts\\_08\\_\(pdf\)/Portelli\\_Joe-Grass\\_fed\\_beef\(paper\).pdf](http://www.greendesignetc.net/GreenProducts_08_(pdf)/Portelli_Joe-Grass_fed_beef(paper).pdf).
15. Rodriguez, I. (2012). Land as a Renewable Resource: Integrating Climate, Energy, and Profitability Goals using an Agent-Based NetLogo Model. Ph.D. dissertation, West Virginia University. Available at [http://wvuscholar.wvu.edu:8881/R/DM8XH8GE946BBDV5AQV8S3DL543LY1K7LTSJCTF9FX1NYQSXX4-01059?func=results-jump-full&set\\_entry=000002&set\\_number=000022&base=GEN01-WVU01](http://wvuscholar.wvu.edu:8881/R/DM8XH8GE946BBDV5AQV8S3DL543LY1K7LTSJCTF9FX1NYQSXX4-01059?func=results-jump-full&set_entry=000002&set_number=000022&base=GEN01-WVU01).
16. Saliba, C. (1985). Soil Productivity and Farmer's Erosion Control Incentives – A Dynamic Modeling Approach, *Western Journal of Agricultural Economics*, 10 (2), pp. 354-364.
17. Standiford, R.B., and R.E. Howitt (1992). Solving Empirical Bioeconomic Models: A Rangeland Management Application, *American Agricultural Economics Association*, pp. 421-433.
18. Torell, L.A., Lyon, K.S. and Godfrey, E.B. (1991). *Long-Run Versus Short-Run Planning Horizons and the Rangeland Stocking Rate Decision*, American Agricultural Economics Association, pp.795-807.