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Genuine savings measurement under uncertainty and its implications for depletable resource management

Abstract

The concept of *genuine savings* has in recent years become widely accepted as a dynamic welfare indicator, which first appeared in Weitzman (1976) and then "formalized" by Pearce and Atkinson (1993). This paper attempts to generalize this concept in a stochastic setting using the Dasgupta-Heal-Solow growth model under the Merton (1975) type of population growth uncertainty. It is shown that the formula for genuine savings under uncertainty also involves a variance component reflecting the welfare loss from risk aversion (cf. Li and Lofgren, 2012). Moreover, the welfare implications of the risk-adjusted genuine savings on depletable resource management are explored.

Keywords: genuine savings, uncertainty, depletable resource, welfare measurement.
JEL Classification: D60, O40.

Introduction

It has been known for quite a while that genuine savings are a welfare indicator in a comprehensive deterministic dynamic growth model of the Ramsey type. More precisely, growth in the aggregate value of net investments of all relevant capital stocks indicates welfare improvement. The concept shows up in Weitzman (1976) for the first time in the proof of a main theorem on the proportionality between the Hamiltonian and the present value of future utilities. Later on, its implications for sustainability are explored by Arrow, Dasgupta and Maler (2003), Asheim (1994), Heal and Kristrom (2005) and Pearce and Atkinson (1993), among others. The measure has been popularized by Hamilton (1994), and used in practice by, among many others, Hamilton and Clements (1999) and Atkinson and Hamilton (2007). The purpose of this paper is to generalize this welfare measure in a stochastic context and explore its implications for depletable resources management. We will use a stochastic dynamic growth model with capital goods, a man-made capital and an exhaustible resource (c.f. Dasgupta and Heal, 1974; Hartwick, 1977; Solow, 1974; and Li and Lofgren, 2012) to show how the standard genuine savings formula from a deterministic setting should be completed by a variance component. Although the model is simple, the derivations are enough to understand how the result generalizes to a multi-sector version of the model under uncertainty.

The remaining parts of the paper is structured as follows. Section 1 presents the basic concept of genuine savings in a deterministic setting and discusses its welfare significance. Section 2 derives the main result on the risk adjusted concept of genuine savings in a stochastic growth model framework, and shows how the Weitzman foundation can be generalized. Section 3 explores the welfare implications of the result for depletable resource management, and the final section concludes.

1. The concept of genuine savings

To derive the concept of genuine savings in its most general form, we take advantage of the standard multi-sector dynamic general equilibrium growth model. Let $C(t)$ denote a vector of comprehensive consumption goods at time t, including environmental services and other externalities, and $U(C(t))$ the utility derived from consumption. Assume that the utility function U satisfies the usual regularity conditions, and let $K(t)$ denote a vector of all capital stocks at time t, including natural and environmental assets. The vector of net investments is denoted by $I(t) = \dot{K}(t)$, i.e. the change in capital stocks over time. The society's objective is to maximize intertemporal welfare (the present discounted value of today's and future utilities) i.e.

$$\max_{\{C(t),I(t)\}} \int_0^\infty U(C(t)) \exp(-\theta t)\, dt \tag{1}$$

subject to the initial condition $K(0) = K_0$, stock dynamics $\dot{K}(t) = I(t)$, the terminal stocks $\lim_{t\to\infty} K(t) \geq 0$, and the feasibility constraint $(C(t), I(t), K(t)) \in A(a)$, where $A(a)$ is a convex attainable possibility set subject to certain institutional constraints. The pure rate of time preference is assumed to be positive i.e. $\theta > 0$. Suppose that $\{C^*(t), I^*(t), K^*(t)\}_0^\infty$ is the unique solution to problem (1). Then, by the maximum principle, the pair $\{C^*(t), I^*(t)\}$ maximizes the current value Hamiltonian $H(C,I,K) = U(C(t)) + \Psi(t)I(t)$ conditional on the capital stock $K^*(t)$ at each time t, i.e. subject to the initial condition $K(0) = K_0$, stock dynamics $\dot{K}(t) = I(t)$, the terminal stocks $\lim_{t\to\infty} K(t) \geq 0$, and the feasibility constraint $(C(t), I(t), K(t)) \in A(\alpha)$, where $A(\alpha)$ is a convex attainable possibility set subject to certain institu-

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tional constraints. The pure rate of time preference is assumed to be positive i.e. $\theta > 0$. Suppose that $\{C^*(t), I^*(t), K^*(t)\}_0^\infty$ is the unique solution to problem (1). Then, by the maximum principle, the pair $\{C^*(t), I^*(t)\}$ maximizes the current value Hamiltonian $H(C, I, K) = U(C(t)) + \Psi(t)I(t)$, conditional on the capital stock $K^*(t)$ at each time t, i.e.

$$H^*(t) = U(C^*(t)) + \Psi(t)I^*(t) = \\ = \max_{(C(t), I(t), K(t)) \in A} U(C(t)) + \Psi(t)I(t), \quad (2)$$

where $\Psi(t)$ is a vector of shadow prices of capital, satisfying the Euler equation $\dot{\Psi}(t) - \theta\Psi(t) = -\partial H^*(t)/\partial K$. By advocating the dynamic envelope theorem, Weitzman (1976) derived the following result

$$\dot{H}^*(t) = \theta\Psi(t)I(t) \quad (3)$$

which together with equation (2) yields $\dot{H}^*(t) = \theta[H^*(t) - U(C^*(t))]$. The solution to this differential equation reads

$$H^*(t) = \theta W^*(t), \quad (4)$$

where

$$W^*(t) = \int_t^\infty U(C^*(t)) \exp(-\theta(s-t))ds \quad (5)$$

denotes the maximal intertemporal welfare at time t, i.e. the comprehensive wealth. The relationship in (4) is the well-known Weitzman foundation, namely *the maximized Hamiltonian in (2) corresponds to "the interest on wealth" or the constancy-equivalent of future utilities*. Since the rate of time preference θ is assumed to be positive, the equation is (4) implies growth in the flow value of the Hamiltonian $H^*(t)$ and that in the stock value of $W^*(t)$ is also proportional to each other, and therefore growth in the Hamiltonian value over time $\dot{H}^*(t)$ indicates welfare improvement/sustainability at time t such that $\dot{W}^*(t) > 0$. It is worth mentioning that while the main result in and Weitzman (1976) and Weitzman (2003) was the correspondence theorem in (4), the welfare significance of (3) was left aside as an intermediate step in the proof of the theorem. In another influential paper by Pearce and Atkinson (1993), the aggregated value of net investments in all relevant capital stocks i.e. $\Psi(t)I(t)$ in (3) was formalized as *genuine savings*. Since the pure rate of time preference $\theta > 0$, a positive value of genuine savings implies growth in the maximized Hamiltonian value $\dot{H}^*(t) > 0$, indicating welfare improvement/sustainability such as $\dot{W}^*(t) > 0$. Loosely speaking, this means that the future prospect as seen from tomorrow is better than that of today and in an intergenerational context this can be interpreted as that development from this to the next generation is sustainable.

2. Genuine savings in a stochastic context

In this section, we analyze the genuine savings issue under uncertainty using the Dasgupta-Heal-Solow growth model (Dasgupta and Heal, 1974; Solow, 1974) with a homogenous capital good and an exhaustible natural resource. In the same vein as in Merton (1975), we consider a stochastic population growth and explore the degree of such uncertainty on capital formation, resource depletion and dynamic welfare. We assume that the production function (net of depreciation) $F(K(t), E(t), L(t))$ is homogenous of degree one, with $K(t)$ as the capital stock, $E(t)$ as the input of extracted natural resource, and $L(t)$ labor input at time t. As shown in Solow (1974), in a deterministic setting, the capital stock evolves according to

$$\dot{K}(t) = F(K(t), E(t), L(t)) - C(t) = \\ = L(t)F(k(t), e(t), 1) - C(t), \quad (6)$$

with $K(0) = K_0 > 0$, where $\dot{K}(t) = dK(t)/dt$, and $C(t)$ denotes consumption. The last equality follows from homogeneity of the production function with $k(t) = K(t)/L(t)$, $e(t) = E(t)/L(t)$, $c(t) = C(t)/L(t)$ defined as the per capita value of capital, resource input and consumption, respectively. The dynamics equation for the exhaustible resource is simply

$$\dot{X}(t) = -E(t), \quad (7)$$

with $X(0) = X_0 > 0$. Let the population at time t be $L(t) = L(0)\exp(nt)$ with an initial size $L(0) > 0$ and a growth rate of n. Then, the per capita capital and resource dynamics equations can be readily derived as

$$\begin{aligned} \dot{k}(t) &= f(k(t), x(t)) - nk(t) - c(t) \\ \dot{x}(t) &= -e(t) - nx(t) \end{aligned} \quad (8)$$

with $(k(0), x(0)) = (k_0, x_0)$, where $f(k(t), x(t)) = = F(K(t)/L(t), X(t)/L(t), 1)$ denotes the per capita production function (net of depreciation) at time t. The per capita production function is assumed to satisfy the Inada conditions $f_k(.) > 0$, $f_x > 0$, $f_{kk} < 0$, $f_{xx} < 0$ and $f_{kk}f_{xx} - f_{kx}^2 > 0$, where subscripts denote partial derivatives. To introduce uncertainty in the model, we now assume that the growth of the labor force follows a geometric Browning motion[1] of the following form (Merton, 1975):

[1] Geometric Browning motion is used to guarantee that the labor force remains positive. Note, however, that this does not result in an equation for the capital and resource stocks per capita that is Geometric Brownian motion.

$$dL(t) = nL(t)dt + \sigma L(t)dz(t), \tag{9}$$

where $dz(t)$ is the stochastic differential of a simple Wiener process. The drift of the process in (9) is governed by the expected rate of labor growth n. In other words, over a short interval of time dt, the proportionate change of the labor force (dL/L) is normally distributed with mean $(n)dt$ and variance $\sigma^2 dt$. We can now use Ito's lemma to transform the uncertainty of growth in the labor force into uncertainty about the growth of the per capita capital and resource stock. By a straightforward derivation, we obtain

$$\begin{aligned} dk(t) &= [f(k(t),x(t)) - c(t) - (n-\sigma^2)k(t)]dt - k(t)\sigma dz(t) \\ dx(t) &= [-e(t) - (n-\sigma^2)x(t)]dt - x(t)\sigma dz(t) \end{aligned} \tag{10}$$

with $(k(0),x(0)) = (k_0, x_0)$. In other words, we have translated uncertainty with respect to the growth rate of the labor force into uncertainty with respect to the capital and the resource per unit of labor and, indirectly, to uncertainty with respect to output per unit of labor, $y(t) = f(k(t),x(t))$. The optimization problem as of date t is to find an optimal consumption policy, and the stochastic Ramsey problem is typically written as

$$\max_{c(s),e(t)} E_t \int_t^T u(c(s)) \exp(-\theta(s-t))ds \tag{11}$$

subject to the initial conditions $k(t) = k_t$ and $x(t) = x_t$ and the dynamics equations in (10) for all $s \geq t$, where E_t denotes the mathematical expectation taken at time t. The function $u(c(s))$ is the instantaneous utility function at time s which is assumed to be twice continuously differentiable, and $\theta > 0$ is the pure rate of time preference. The upper integration limit T is the first exit time from the solvency set G i.e. $T = \inf\{s \geq t; k_s(\omega), x_s(\omega) \notin G\}$ with $G = \{k_s(\omega), x_s(\omega); k_s > 0, x_s > 0\}$. In other words, the process is stopped when the capital stock becomes non-positive (when bankruptcy occurs). In most contexts it is realistic to assume that the optimal control process $c^*(s)$ for $s \geq t$ is conditioned solely on past observed values of the state process $k(s)$ and $x(s)$. In such a case, mathematicians would say that the control process is adapted to the state process. Here, it is assumed that the optimal control function is a time autonomous Markov control of the following type $c^*(s) = c(k(s), x(s))$ meaning that the control at time s only depends on the state of the system at this time. In particular, it does not depend on the starting point or time as a separate argument. Then, the optimal value function

$$V(t,k(t),x(t)) = E_t \int_t^T u(c^*(s)) \exp(-\theta(s-t))ds, \tag{12}$$

will also be time-autonomous as stated in the following lemma.

Lemma. $V(t,k(t),x(t)) = V(0,k(0),x(0))$ for $k(t)=k(0)$ and $x(t) = x(0)$ where the endogenous time spent in the solvency set G is $T_G = T - t$, q.e.d.

Proof. The optimal control is a Markov control, i.e., it depends only on the initial stock $(k(t), x(t))$ at time t. Let $\tau = s - t$, then we can express the optimal consumption stream by $c^*(s) = c^*(\tau + t)$ with $\tau = 0$ for $s = t$ and $\tau = T_G$ for $s = T$. The time spent in the solvency set $T_G = T - t$ for a given experiment $\omega_\tau = \omega_{s-t}$ is endogenous and the solvency set does not change due to the rescaling. Therefore, we have

$$\begin{aligned} V(t,k,x) &= E_t \int_t^T u^*(k(s),x(s)) \exp(-\theta(s-t))ds = \\ &= E_0 \int_0^{T_G} u^*(k(\tau),x(\tau)) \exp(-\theta\tau)d\tau = V(0,k,x). \end{aligned} \tag{13}$$

The second equality follows since substituting $k(s) = k(\tau + t)$ and $x(s) = x(\tau + t)$ into the time autonomous stochastic differential equation (10), we obtain a process that starts at $(0,k,x)$ with the same probability law on an equivalent solvency set as the process that starts at (t,k,x), and the optimal control is Markov. The third equality follows from the definition of a value function.

Since the value function does not explicitly depend on the initial calendar date t, we redefine it by

$$W(k(t),x(t)) = V(t,k(t),x(t)). \tag{14}$$

According to the principle of optimality, the value function which should satisfy the following Bellman equation

$$0 = \max_c \{u(c) - \theta W(k,x) + A^c W(k,x)\}, \tag{15}$$

with A^c as the backward operator for a given c such that:

$$A^c W(k,x) = \frac{1}{dt} E_t \left\{ [W_k, W_x] \begin{bmatrix} dk \\ dx \end{bmatrix} + \frac{1}{2} [dk, dx] \begin{bmatrix} W_{kk} & W_{kx} \\ W_{xk} & W_{xx} \end{bmatrix} \begin{bmatrix} dk \\ dx \end{bmatrix} \right\} \tag{16}$$

where subscripts denote partial derivatives, i.e. $W_i = \partial W / \partial i$ for $i = k, x$, and $W_{ij} = \partial^2 W / \partial i \partial j$ for $i = k, x$ and $j = k, x$ are the first and second-order partial derivatives of the value function to the capital and resource stocks, respectively. Given the

optimal consumption policy $c^*(t)$, a light rearrangement of equation (15) leads to the following proposition.

Proposition 1. Along the optimal growth path $c^*(t)$, the interest on intertemporal welfare is equal to a risk-adjusted value of the current value Hamiltonian

$$\theta W(k,x) = u(c^*) + A^c W(k,x), \tag{17}$$

i.e. the maximum expected sustainable utility over time.

The proposition is a generalized version of Weitzman foundation (4) with an extra variance component, the last term in (16), being added to the deterministic Hamiltonian function (cf. Aronsson and Löfgren, 1995). Note that for this particular model, the shadow price vector $\Psi(t)$ in (4) is given by (W_k, W_x). To derive a dynamic welfare measure like the genuine saving, we follow Weitzman (1976) and Arrow, Dasgupta and Mäler (2003) by differentiating the value function $W(k(t),x(t))$ with respect to time using the Leibniz rule to obtain

$$\dot{W}(k(t),x(t)) = -u(c(t)) + \theta W(k(t),x(t)). \tag{18}$$

Now, using the differential equation in (18), we obtain after substituting for $\theta W(k,x)$ in (17) the following proposition on the generalized genuine saving.

Proposition 2. The risk-adjusted genuine savings i.e. the expected rate of change in the value function at concurrent time t can be expressed as

$$\dot{W}(k(t),x(t)) = A^c W(k,x) =$$
$$= \frac{1}{dt} E_t \left\{ [W_k, W_x] \begin{bmatrix} dk \\ dx \end{bmatrix} + \frac{1}{2}[dk, dx] \begin{bmatrix} W_{kk} & W_{kx} \\ W_{xk} & W_{xx} \end{bmatrix} \begin{bmatrix} dk \\ dx \end{bmatrix} \right\}. \tag{19}$$

Note that the row vector $[W_k, W_x]$ are the accounting prices per unit of capital and resource stock, and thus the first term after the second equality sign corresponds to the conventional genuine savings under certainty i.e.

$$\frac{1}{dt} E_t [W_k, W_x] \begin{bmatrix} dk \\ dx \end{bmatrix} =$$
$$= W_k[f(k,x) - c - (n-\sigma^2)] + W_x[-e - (n-\sigma^2)], \tag{20}$$

and the second term

$$\frac{1}{2dt} E_t \left\{ [dk, dx] \begin{bmatrix} W_{kk} & W_{kx} \\ W_{xk} & W_{xx} \end{bmatrix} \begin{bmatrix} dk \\ dx \end{bmatrix} \right\} =$$
$$= \frac{1}{2} \left\{ W_{kk}\sigma^2 k^2 + 2W_{kx}\sigma^2 kx + W_{xx}\sigma^2 x^2 \right\}, \tag{21}$$

is the variance component originated from Ito calculus. It can be readily shown that the matrix the matrix W_{ij} for $i = k, x$ and $j = k, x$ is negative definite for "well-behaved" maximization problems with regular utility and production functional forms[1]. To satisfy this condition, it is sufficient for the utility function to be jointly concave in all consumption goods, and the production to be jointly concave in all relevant capital stocks. This means that we would under a stochastic growth problem expect that a positive net investment value would not be enough to indicate a local welfare improvement. The value of net investment has to be large enough to compensate for the risk aversion loss from the uncertainty in order for the dynamic welfare not to decline over time. Loosely speaking, if we regard the terms $\frac{1}{2}W_{kk}$, $\frac{1}{2}W_{xx}$ and W_{kx} as the "prices" of risk and $\sigma^2 k^2$, $\sigma^2 x^2$ and $\sigma^2 kx$, respectively, as the "quantities" of risk, then the whole expression on the right-hand-side of (19) can be interpreted as a generalized genuine savings measure. A reasonable economic interpretation of this result is that, under the presence of uncertainty, precautionary savings (cf. Leland, 1968; and Turnovsky and Smith, 2006) corresponding to the absolute value of the variance component are required in order to sustain the same dynamic welfare as in the deterministic case.

3. Welfare implications on depletable resource management

In the literature of natural resource economics, two important rules have been proposed for efficient resource utilization (Hotelling's rule) and sustainable development (Hartwick's rule). For a cake-eating economy with some fixed initial stock such as oil and minerals, Hotelling's rule says that along an optimal resource extraction path, the user cost per unit of the stock, W_x i.e. the net price of the extracted resource after the marginal extraction cost being accounted for should grow at the same rate of the interest rate. For the productive economy with a depletable resource described above with the Dasgupta-Heal-Solow model, Hartwick's rule indicates that when the deterministic genuine savings as in equation (20) are equal to zero i.e. $W_k dk/dt + W_x dx/dt = 0$, a constant level of consumption c and thereby utility $u(c)$ can be sustained. In other word, if the cost of resource depletion $W_x dx/dt < 0$ for $dx/dt < 0$ can be compensated by a corresponding increase in the value of the productive capital k namely $W_k dk/dt > 0$, then a constant level of consumption and utility can be sustained (c.f. Hartwick, 1977). Of course, the rule with zero genuine savings should be

[1] In case that $\sigma = 0$, the variance term would vanish and the result collapses to the deterministic genuine savings measure.

followed through the entire time path over the future (cf. Asheim et al., 2003). In the presence of uncertainty, however, the simple Hartwick investment rule with $W_k dk/dt + W_x dx/dt = 0$ is obviously not sufficient to sustain consumption of utility. With the variance component as in (21) being negative as touched upon above, the deterministic-equivalent part of the genuine savings amount has to be enhanced for sustainable development via for example slower resource extraction or faster capital accumulation. Concerning the economics of sustainable development, it is worth mentioning that the interest has been shifted from sustaining the narrow instantaneous utility $u(c)$ time to sustaining the more general intertemporal welfare $W(k(t), x(t))$ over time. A direct welfare implication of *Proposition 2* is the following local-in-time sustainability result.

Proposition 3. If the deterministic part of genuine savings given in (20) can, at least, compensate for the risk-aversion loss given in the variance component in (21) such that the sum of them as in (19) is non-negative, then dynamic welfare $W(k(t), x(t))$ can be sustained over an infinitesimal period dt from a concurrent date t i.e. the development at time t is sustainable.

Note that this proposition means that the prospect as of time $t + dt$ may look better (at least not worse) than that of time t in terms of the present value of current and future utilities if the generalized genuine savings in (19) at time t are non-negative. However, it does not imply that the instantaneous utility would follow the same trend. An increase in the wealth-like measure $W(k(t), x(t))$ at time may be perfectly consistent with a short-term sacrifice in consumption i.e. $c(t+dt) - c(t) < 0$ combined with some larger increase in $c(s)$ in some future dates $s > t + dt$. With a greater wealth $W(k(t), x(t))$, the future consumption set would be larger and if a resource allocation over time would be feasible, then in principle the instantaneous utilities would be larger. If the generalized genuine savings in (19) at time t are non-negative over the whole future, the development becomes globally sustainable over time.

Conclusion

In this paper, we have attempted to generalize the concept of genuine savings in a stochastic growth framework and explore its welfare implications. This is accomplished by using the Dasgupta-Heal-Solow model with two capital stocks, a man-made capital good and a natural resource stock that is depletable. To simplify the analysis, we take advantage of the Merton stochastic population growth to introduce uncertainty in the per capita (man-made) capital and resource stocks. The derived results are, however, general for multi-sector growth models under uncertainty, provided the regularity conditions on utility and production functions are satisfied. If the value function, defined as the expected present value of future utilities, is jointly concave in all capital stocks, then the risk-related variance component associated with the generalized genuine savings would always be negative. Thus, to achieve sustainable development, more precautionary savings are needed to compensate for the welfare loss from risk aversion. Concerning the well-known Hartwick's rule, the result here means that it is not sufficient to reinvest the resource rent in the productive capital stocks to retain a constant utility level over time. As long as the uncertainty is present, the conventional genuine savings component has to be positive, i.e. the rate of capital accumulation should be faster than that in the deterministic case to compensate the loss in resource depletion. Since the variance component term depends on the degree of uncertainty, any measure that can reduce the fluctuation of the per-capita capital and resource stocks also improves welfare and promotes sustainability.

References

1. Aronsson, T. and Löfgren, K.G. (1995). National Product Related Welfare Measures in the Presence of Technological Change, Externalities and Uncertainty, *Environmental and Resource Economics*, 5, pp. 321-332.
2. Arrow, K. J., Dasgupta, P. and Mäler, K. (2003). Evaluating Projects and Assessing Sustainable Development in Imperfect Economies. *Environmental And Resource Economics*, 26, pp. 647-685.
3. Asheim, G.B. (1994). Net National Product as an Indicator of Sustainability, *Scandinavian Journal of Economics*, 96, pp. 257-65.
4. Asheim, G.B., Buchholz, W. and Withagen, C. (2003). The Hartwick Rule: Myths and Facts, *Environmental and Resource Economics*, 25, pp. 129-150.
5. Atkinson, G. and Hamilton, K. (2007). Progress along the path: evolving issues in the measurement of genuine saving, *Environmental and Resource Economics*, 37, pp. 43-61.
6. Dasgupta, P.S. and Heal, G.M. (1974). The Optimal Depletion of Exhaustible Resources, *Review of Economic Studies*, Symposium, pp. 3-28.
7. Hamilton, K. (1994). Green Adjustment to GDP, *Resource Policy*, 20, pp. 155-68.

8. Hamilton, K. and Clemens, M. (1999). Genuine Savings rates in Developing Countries, *World Bank Economic Review*, 13, pp. 333-56.
9. Hartwick, J.M. (1977). Intergenerational Equity and the Investing of Rents from Exhaustible Resources, *American Economic Review*, 67, pp. 972-974.
10. Heal, G. and Kriström, B. (2005). National Income and the Environment, *Handbook of Environmental Economics*, 3, pp. 1147-1217.
11. Leland H.E. (1968). Saving and uncertainty: the precautionary demand for saving, *Quarterly Journal of Economics*, 82, pp. 465-473.
12. Li C.Z. and Löfgren K.G. (2012). Genuine Saving under Stochastic Growth, *Letters in Spatial and Resource Science*, 5, pp. 167-174.
13. Merton, R. (1975). An Asymptotic Theory for Growth under Uncertainty, *Review of Economic Studies*, 42, pp. 375-93.
14. Pearce, D. and Atkinson, G.(1993). Capital Theory and the Measurement of Sustainable Development, An Indicator of Weak Sustainability, *Ecological Economics*, 8, pp. 103-108.
15. Solow, R.M. (1974). Intergenerational Equity and Exhaustible Resources, *Review of Economic Studies*, Symposium, pp. 29-45.
16. Turnovsky, S.J., and Smith, W.T. (2006). Equilibrium Consumption and Precautionary Savings in a Stochastically Growing Economy, *Journal of Economic Dynamics and Control*, 30, pp. 243-278.
17. Weitzman, M.L. (1976). On the Welfare Significance of National Product in a Dynamic Economy, *Quarterly Journal of Economics*, 99, pp. 1-13.
18. Weitzman, M.L. (2003). *Income, Wealth and the Maximum Principle*, Cambridge: Harvard University Press.