“Audit fees prediction using fuzzy models”

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Audit fees prediction using fuzzy models

Abstract

The current study aims to predict the optimal amount of independent audit fees based on the factors influencing audit fees. To identify the factors influencing audit fees, the stakeholders of 30 auditing firms, members of the Iranian Association of Certified Public Accountants in Tehran selected randomly, were interviewed. Finally, the linear programming model for audit fees and its determinants is defined and sum of squared error is used to solve the function with minimum. Also, given that the data are quantitative and comparative and normally distributed, Pearson’s correlation coefficient is used to test the research hypotheses.

The results show that a positive significant correlation exists between the variables of expected time to perform audit procedures, the number of accounting documents, audit operation risk, complexity of operations, existence of specific rules and regulations governing the activities of the entity.

Keywords: audit, independent auditor, internal auditor, audit fees, fuzzy logic.

JEL Classification: H83, M4, M42.

Introduction

In recent years, mathematical formulas have contributed in this regard and the process is still expanding. In this study, we seek to convert human knowledge to mathematical models. What a fuzzy system does is the conversion of human knowledge into mathematical formulas.

The term “Fuzzy” is sometimes used along with the sets. Sets include conventional and fuzzy sets. In conventional (crisp) sets, a member either belongs to a set or not. It means that it is two-valued (0 or 1). However, fuzzy sets are multi-valued and include ranges between 0 and 1. The term “Fuzzy” in the Oxford Dictionary is defined as “ambiguous, vague, non-precise, puzzled, confused, and uncertain”, and fuzzy systems are knowledge-based or rule-based systems. The heart of a fuzzy system is a knowledge base formed by if-then rule. For example, consider the following fuzzy phase: if the vehicle speed is high, then, impose a small force to the gas pedal. We can, thus, construct a fuzzy system based on the rules (Li Wang, 1962).

In this paper, the fuzzy method is used for predicting the optimum amount of audit fees in cases where determining audit rate is vague and uncertain.

1. Literature review

Defond (2002) concluded that fees paid for non-audit services have no significant effect on auditor independence and, as a result, his audit fees.

Boo and Koh (2004) examined the correlation between audit firm reputation and characteristics of the audit team with audit fees. The results indicated that there is a positive correlation between reputation of audit firm and the audit fee.

Nikkinen and Sahlstorm (2005) indicated that the audit fee is directly correlated with the risk aspects (financial risk, operational risk and business risk). According to the results of this study, representation costs, size of company operations, audit complexity are able to describe the audit services fees.

Stewart and Kent (2006) studied the correlation between the audit costs, an effective audit committee and internal audit in Australia. The results showed that there is a positive correlation between the levels of audit fees, an inspection committee, the frequency of audit committee meetings and using internal audit.

Hogan & Wilkins (2008) refer to the fact that the possibility of inherent risk is positively related with whole audit fee, as well as audit working hours, but it has a negative correlation with the independent auditor reliance on internal audit function.

Gramling (2011) declared their view about the effect of decreasing the audit fee on independent auditor freedom to the AICPA and expressed concern on the reduction of the fee that can influence on the auditor incentive to reduce working hours and lowering the audit test level. They stated that the primary motivation of independent auditors to rely on internal audit is low cost of auditing.

Walid (2012) studied the factors affecting the audit fees and concluded that the most important factor affecting the determination of the amount of audit fees is whether the auditing company is one of the four largest firms or not, and the least important factor is the size of the auditing company, according to the number of its employees.

Khani & Yazdani (2012) identified the factors contributing to determine the cost of the audit in

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Iran, and concluded that the quality of audit reports has a significant positive impact on the cost of audit; quality is the most important factor in audit costs, and other factor affecting the audit costs is the audit company’s industrial expertise.

Baldacchino et al. (2014) reviewed the factors affecting the external audit fees in Malta and concluded that the costs of external audits are significantly affected by the size of audit client, complexity, line, control of the ownership and situation of the company.

In Iran, several studies were conducted about the subject which was briefly mentioned below:

Daroughe Hazrati and Pahlavan (2012) investigated the correlation between profit management, the provisions provided in the audit report, the type of auditor and composition of Board of Directors with audit fees. The results showed that the four factors have significant positive correlation with the audit fees.

Vaez et al. (2013) examined the impact of audit quality on audit fees. In this research, the audit quality standards are the size of auditing institute, audit expertise in the industry and continuation of auditor choice. The findings indicate that a negative significant correlation exists between the audit firm’s expertise and audit fees. The results also suggested that the factors of continuation of the selection of the auditor and the audit firm size have a positive significant correlation with audit fees.

2. Methodology

The time scope of the study is 6 months of early November 2014 to late April 2015. The research statistical population is the Audit Institutions, Member of Iranian Association of Certified Public Accountants in Tehran, which randomly and during the investigation, it was possible to interview 30 partners of institution of the selected firms.

2.1. Variables. Independent variables of the research are:

1. Predicted time to perform audit operation based on the operations size of the entity.
2. The number of the entity’s accounting records.
3. The initial audit or audits carried out in previous years.
4. Turnover of accounts related to the entity’s operating cycle, including sales, earnings and receipts – sale, payrolls – salary and wage, fixed assets and inventories cycles.
5. Audit operation risk.
6. Integrity and reputation of the senior management of the entity.
7. The complexity of the entity’s operations.
8. Presence of certain rules and regulations governing the activities of the entity.
9. The internal control system status for operating cycles.
10. The audit fee for each audit team job category, according to the time budget expected.
11. Overhead costs predicted by the auditing firm to perform an audit operation and the audit firm’s expected profit by auditing.

Dependent variable is the audit fees.

2.2. Hypotheses.

1. Hypothesis 1: There is a correlation between the variable of predicted time to perform auditing practices based on the size of the entity and audit fees.
2. Hypothesis 2: There is a correlation between the variable of number of the entity’s accounting records and audit fees.
3. Hypothesis 3: There is a correlation between the variable of initial audit or audits carried out in previous years and audit fees.
4. Hypothesis 4: There is a correlation between the variable of turnover of accounts associated with the entity’s operating cycle, including sales, revenues and receipts – sale, payrolls – salary and wage, fixed assets and inventories and audit fees.
5. Hypothesis 5: There is a correlation between the variable of risk of audit and audit fees.

2.3. The model used in the study. In this research, to provide independent audit optimal fee prediction model, TSK method is used. The method was introduced by Takagi & Sugeno that uses linear functions in the input and shows the input-output correlations that, in fact, are the substitution of the fuzzy sets used in fuzzy rules, such as:

If \( x \) is \( A \) and \( y \) is \( B \), then \( z = ax + by + c \).

Here \( a, b \) and \( c \) are linear function parameters that are in output, and are called output parameters. Linear equation, in the outcome, is called a linearized equation of the outcome (Tanaka, 2001).

3. Data analysis

3.1. Demographic data. The variables for the provision of demographic data of the research include field of study, level of education, work experience and organizational category that are shown in Table 1.

Table 1. Basic information of interviewees

<table>
<thead>
<tr>
<th>Row</th>
<th>Description</th>
<th>Frequency</th>
<th>Frequency percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Field of study: Accounting</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Sum</td>
<td>30</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 1 (cont.). Basic information of interviewees

<table>
<thead>
<tr>
<th>Row</th>
<th>Description</th>
<th>Frequency</th>
<th>Frequency percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Education level:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Graduate</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>M.A.</td>
<td>16</td>
<td>53.3</td>
</tr>
<tr>
<td></td>
<td>Ph.D.</td>
<td>8</td>
<td>26.7</td>
</tr>
<tr>
<td></td>
<td>Sum</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>Work experience:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 to 10 years</td>
<td>4</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>10 to 15 years</td>
<td>8</td>
<td>26.7</td>
</tr>
</tbody>
</table>

3.2. Responses obtained from interviews. The results are presented in Table 2.

Table 2. Responses of audit institutions to the research question

<table>
<thead>
<tr>
<th>Row</th>
<th>Responses auditing</th>
<th>Frequency</th>
<th>Frequency percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial audit or audit conducted in previous years</td>
<td>22</td>
<td>73.33</td>
</tr>
<tr>
<td>2</td>
<td>Turnover of accounts associated with the entity’s operating cycle, including:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. Sales, revenues and receipts</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Purchase, costs and disbursements</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Payrolls</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Fixed assets</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5. Inventories</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Number, combination, and experience of financial personnel of the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Existence and the variety of the accounting systems and software of the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Existence and the quality of internal audit work in the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Changes in management and senior staff during the period of consideration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Integrity and reputation of senior management (taking into account the distortions found in the audit of previous years)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>The number of the entity’s accounting records</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>Existence of the entity’s audit committee</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>The situation of entity’s internal control system on the entity’s operating cycle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>How to manage the entity (management absolute supremacy or delegation of the appropriate authorities)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>The complexity of the entity’s operations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>The problem of activity continuity of the entity</td>
<td>12</td>
<td>40</td>
</tr>
<tr>
<td>14</td>
<td>Existence of certain rules and regulations governing the activities of the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Number of employees of the entity</td>
<td>7</td>
<td>23.33</td>
</tr>
<tr>
<td>16</td>
<td>Auditor expertise in the field of activity of the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Legal personality of the entity (service, commercial or productive)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Fee of audit staff given the time budget of the audited, number and combination of audit forces</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Correlation of previous auditor with the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Season of auditing work</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Auditing firm reputation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>Audit risk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Time budget required to perform audit procedures regarding the operation size of the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Volume of the work under performing in auditing firm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Overhead costs incurred by the auditing firm to perform audit operation of the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Expected profit of auditing organization by performing the audit of the entity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Financial position of auditing firm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.3. Effective factors in determining audit optimal fees. As shown in Table 2, some answers provided by audit firms are more frequent, which indicates emphasis and attention of audit firms to these factors. In this study, responses with frequencies more than the average (more than 15) will be introduced and used as factors determining the audit fees that are presented in Table 3.

Table 3. Factors influencing the prediction of audit fees

<table>
<thead>
<tr>
<th>Row</th>
<th>Effective factors in the prediction of audit fees</th>
<th>Frequency</th>
<th>Frequency percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initial audit or audit conducted in previous years</td>
<td>22</td>
<td>73.33</td>
</tr>
<tr>
<td>2</td>
<td>Turnover of accounts associated with the entity’s operating cycle, including: (Committee on Revision of Audit Guideline, 2002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. Sales, revenues and receipts</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>2. Purchase, costs and disbursements</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Payrolls</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Fixed assets</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5. Inventories</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3 (cont.). Factors influencing the prediction of audit fees

<table>
<thead>
<tr>
<th>Row</th>
<th>Effective factors in the prediction of audit fees</th>
<th>Frequency</th>
<th>Frequency percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Integrity and reputation of senior management (taking into account the distortions found in the audit of previous years)</td>
<td>19</td>
<td>63.33</td>
</tr>
<tr>
<td>4</td>
<td>The number of the entity’s accounting records</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>The entity’s internal control system of the entity’s operating cycle</td>
<td>27</td>
<td>90</td>
</tr>
<tr>
<td>6</td>
<td>The complexity of the entity’s operations</td>
<td>27</td>
<td>90</td>
</tr>
<tr>
<td>7</td>
<td>Existence of certain rules and regulations governing the activities of the entity</td>
<td>25</td>
<td>83.33</td>
</tr>
<tr>
<td>8</td>
<td>Fee of auditing staff given the time budget of the entity, number and combination of audit forces</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
<td>Audit risk</td>
<td>28</td>
<td>93.33</td>
</tr>
<tr>
<td>10</td>
<td>Time budget required to perform audit procedures regarding the operations of the entity</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>11</td>
<td>Overhead costs incurred by the auditing firm to perform auditing operation of the entity</td>
<td>21</td>
<td>70</td>
</tr>
<tr>
<td>12</td>
<td>Expected profit of auditing organization from performing the audit of the entity</td>
<td>30</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3 noticed that there is a correlation between effective factors in predicting the audit fee. According to the above descriptions, factors affecting the fee can be classified in Table 4.

Table 4. Correlations between factors affecting the prediction of audit fees

<table>
<thead>
<tr>
<th>Row</th>
<th>Correlations between effective factors in the prediction of audit fees</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Predicted time to perform audit procedures regarding the operation size of the entity:</td>
</tr>
<tr>
<td></td>
<td>1-1) The number of the entity’s accounting records</td>
</tr>
<tr>
<td></td>
<td>1-2) Initial audit or audit conducted in previous years</td>
</tr>
<tr>
<td></td>
<td>1-3) Turnover of accounts related to the operating cycle of the entity:</td>
</tr>
<tr>
<td></td>
<td>1-3-1) Sales, revenues and receipts cycle</td>
</tr>
<tr>
<td></td>
<td>1-3-2) Purchase, costs and disbursements cycle</td>
</tr>
<tr>
<td></td>
<td>1-3-3) Payroll cycle</td>
</tr>
<tr>
<td></td>
<td>1-3-4) Fixed assets cycle</td>
</tr>
<tr>
<td></td>
<td>1-3-5) Inventories cycle</td>
</tr>
<tr>
<td></td>
<td>1-4) Audit risk:</td>
</tr>
<tr>
<td></td>
<td>1-4-1) Integrity and reputation of the senior management</td>
</tr>
<tr>
<td></td>
<td>1-4-2) Complexity of the entity’s operations</td>
</tr>
<tr>
<td></td>
<td>1-4-3) Rules and regulations governing the activities of the entity</td>
</tr>
<tr>
<td>2</td>
<td>Fees of different levels of audit staff given the predicted time</td>
</tr>
<tr>
<td>3</td>
<td>Overhead costs predicted by the auditing firm to perform auditing</td>
</tr>
<tr>
<td>4</td>
<td>Expected profit of audit institution of performing auditing</td>
</tr>
</tbody>
</table>

3.4. Data normalization. The collected data related to the factors effective in prediction of the audit fee are qualitative that should be convert into quantitative data to be used in this study. Therefore, based on the information collected during the interviews, the best quantitative range for each variable is defined and, then, normalized. The result of normalizing the data is presented in Table 5.

Table 5. Normalized amounts of data

<table>
<thead>
<tr>
<th>Row</th>
<th>Factors affecting the prediction of the audit optimal fee</th>
<th>Fuzzy amount</th>
<th>Normalized value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Predicted time to perform audit procedures regarding the operations size of the entity:</td>
<td>[10.000 and 1.000]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-1) The number of the entity’s accounting records</td>
<td>[100.000 and 10.000]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-2) Initial audit or audit conducted in previous years</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-3) Turnover of accounts related to the operating cycle of the entity:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-3-1) Sales, revenues and receipts cycle</td>
<td>[100/000/000/000 and 1/000/000/000/000]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-3-2) Purchase, costs and disbursements cycle</td>
<td>[80/000/000/000 and 60/000/000/000/000]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-3-3) Payroll cycle</td>
<td>[16/000/000/000 and 16/000/000/000/000]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-3-4) Fixed assets cycle</td>
<td>[40/000/000/000 and 40/000/000/000/000]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-3-5) Inventories cycle</td>
<td>[3/000/000/000 and 30/000/000/000/000]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td>1-4</td>
<td>Audit risk:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-4-1) Integrity and reputation of the entity senior management</td>
<td></td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-4-2) Complexity of the entity’s operations</td>
<td></td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-4-3) Rules and regulations governing the activities of the entity</td>
<td></td>
<td>[1 and 0.1]</td>
</tr>
</tbody>
</table>
Table 5 (cont.). Normalized amounts of data

<table>
<thead>
<tr>
<th>Row</th>
<th>Factors affecting the prediction of the audit optimal fee</th>
<th>Fuzzy amount</th>
<th>Normalized value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-4-4) Internal control system of the operational cycles:</td>
<td>–</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-4-4-1) Internal control system on the sales, revenues and receipts cycle</td>
<td>–</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-4-4-2) Internal control system on the purchase, costs and disbursements cycle</td>
<td>–</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-4-4-3) Internal control system on payroll cycle</td>
<td>–</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-4-4-4) Internal control system on fixed assets cycle</td>
<td>–</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td></td>
<td>1-4-4-5) Internal control system on Inventories cycle</td>
<td>–</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td>2</td>
<td>Fees of different levels of audit staff given the predicted time</td>
<td>[96.296.525 and 962.965.250]</td>
<td>[1 and 0.1]</td>
</tr>
<tr>
<td>3</td>
<td>Expected profit and overhead costs predicted by the auditing firm to perform audit operation</td>
<td>30% of audit employees fees</td>
<td>3.0</td>
</tr>
</tbody>
</table>

3.5. Fuzzy reasoning using linear functions (Sugeno). Following functions are defined for audit fee, as well as factors effective in predicting audit fees and Table 4 factors:

1. Linear function of independent audit fees optimal amount \((U)\):

\[
U = F(S_1, S_2) = e_0 + e_1S_1 + e_2S_2,
\]

where:

\(S_1\): audit employees fees due to predicted time;

\(S_2\): overhead costs incurred and expected profit of audit firm;

\(e_0, e_1, e_2\): linear function parameters (fixed coefficients).

2. Linear function of predicted time to perform audits based on operations volume of the entity \((V_1)\):

\[
V_1 = F(W_1, W_2, W_3, W_4) = d_0 + d_1W_1 + d_2W_2 + d_3W_3 + d_4W_4,
\]

where:

\(W_1\): the number of the entity’s accounting records;

\(W_2\): turnover of accounts related to the entity’s operating cycle;

\(W_3\): risk of audit;

\(W_4\): initial audit or audits carried out in previous years;

\(d_0, d_1, d_2, d_3\) and \(d_4\): parameters of the linear function (constant coefficients).

3. Linear function of turnover of accounts related to the entity’s operating cycle \((W_2)\):

\[
W_2 = F(X_1, X_2, X_3, X_4, X_5) = C_0 + C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + C_5X_5,
\]

where:

\(X_1\): turnover of sales, revenues and earnings cycle;

\(X_2\): turnover of purchase, costs and disbursements cycle;

\(X_3\): turnover of payroll cycle;

\(X_4\): turnover of fixed assets cycle;

\(X_5\): turnover of inventories cycle;

\(C_0, C_1, C_2, C_3, C_4\) and \(C_5\): linear function parameters (fixed coefficients).

4. Linear function of audit risk \((W_3)\):

\[
W_3 = F(Z_1, Z_2, Z_3, Z_4) = b_0 + b_1Z_1 + b_2Z_2 + b_3Z_3 + b_4Z_4,
\]

where:

\(Z_1\): integrity and reputation of the senior management of the entity;

\(Z_2\): complexity of the entity’s operations;

\(Z_3\): presence of specific rules and regulations governing the activities of the entity;

\(Z_4\): entity’s internal control system in respect of operating cycles of the entity;

\(b_0, b_1, b_2, b_3\) and \(b_4\): linear function parameters (fixed coefficients).

5. Linear function of the internal control system status of the entity’s operating cycle \((Z_4)\):

\[
Z_4 = F(Y_1, Y_2, Y_3, Y_4, Y_5) = a_0 + a_1Y_1 + a_2Y_2 + a_3Y_3 + a_4Y_4 + a_5Y_5,
\]

where:

\(Y_1\): internal control system of sales, revenues and earnings cycle;

\(Y_2\): internal control system of purchase, costs and disbursements cycle;

\(Y_3\): internal control system of payroll cycle;

\(Y_4\): internal control system of fixed assets cycle;

\(Y_5\): internal control system of inventories cycle;

\(a_0, a_1, a_2, a_3\) and \(a_4\): linear function parameters (fixed coefficients).
Following, to solve the aforementioned linear functions with minimum error, the least squares will be used in Matlab. Therefore, in this section, the data of 10 sample companies collected during interviews with the audit firms, as well as data provided in Tables 6 to 12, will be used.

Table 6. The entity’s internal control system in 10 sample companies

<table>
<thead>
<tr>
<th>Sample company row</th>
<th>Internal control system of sales, revenues and earnings cycle</th>
<th>Internal control system of purchase cycle, costs and disbursement</th>
<th>Internal control system of payroll cycle</th>
<th>Internal control system of fixed assets cycle</th>
<th>Internal control system of inventories cycle</th>
<th>Overall condition of the entity’s internal control system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>0.9</td>
<td>0.7</td>
<td>0.75</td>
<td>0.85</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>0.85</td>
<td>0.8</td>
<td>0.65</td>
<td>0.6</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>0.6</td>
<td>0.7</td>
<td>0.6</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td>0.55</td>
<td>0.6</td>
<td>0.5</td>
<td>0.8</td>
<td>0.62</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>0.45</td>
<td>0.98</td>
<td>0.2</td>
<td>0.75</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>0.55</td>
<td>0.28</td>
<td>0.6</td>
<td>0.85</td>
<td>0.45</td>
<td>0.55</td>
</tr>
<tr>
<td>7</td>
<td>0.48</td>
<td>0.4</td>
<td>0.37</td>
<td>0.65</td>
<td>0.36</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>0.37</td>
<td>0.8</td>
<td>0.43</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>0.44</td>
<td>0.38</td>
<td>0.25</td>
<td>0.35</td>
<td>0.58</td>
<td>0.4</td>
</tr>
<tr>
<td>10</td>
<td>0.35</td>
<td>0.35</td>
<td>0.25</td>
<td>0.3</td>
<td>0.4</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 7. Risk of audit operations in 10 sample companies

<table>
<thead>
<tr>
<th>Row of sample company</th>
<th>Integrity and reputation of the entity’s senior management</th>
<th>Complexity of the entity’s operations</th>
<th>Rules and regulations governing the activities of the entity</th>
<th>Entity’s internal control system</th>
<th>Audit operation risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.45</td>
<td>0.9</td>
<td>0.75</td>
<td>0.62</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>0.55</td>
<td>0.8</td>
<td>0.75</td>
<td>0.65</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
<td>0.6</td>
<td>0.65</td>
<td>0.8</td>
<td>0.65</td>
</tr>
<tr>
<td>5</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
<td>0.5</td>
<td>0.55</td>
</tr>
<tr>
<td>6</td>
<td>0.7</td>
<td>0.5</td>
<td>0.5</td>
<td>0.55</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.55</td>
<td>0.5</td>
<td>0.4</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>8</td>
<td>0.6</td>
<td>0.4</td>
<td>0.3</td>
<td>0.33</td>
<td>0.4</td>
</tr>
<tr>
<td>9</td>
<td>0.75</td>
<td>0.3</td>
<td>0.2</td>
<td>0.4</td>
<td>0.35</td>
</tr>
<tr>
<td>10</td>
<td>0.8</td>
<td>0.2</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 8. Turnover of accounts in respect of operating cycles at 10 sample companies

<table>
<thead>
<tr>
<th>Row</th>
<th>Turnover of sales accounts, revenues and earnings</th>
<th>Turnover of buying accounts, costs and payoffs</th>
<th>Turnover of payroll accounts</th>
<th>Turnover of fixed assets account</th>
<th>Turnover of inventories accounts</th>
<th>Turnover of accounts for operating cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amount Normalized value</td>
<td>Amount Normalized value</td>
<td>Amount Normalized value</td>
<td>Amount Normalized value</td>
<td>Amount Normalized value</td>
<td>Amount Normalized value</td>
</tr>
<tr>
<td>1</td>
<td>950/000/000/000 0.95</td>
<td>540/000/000/000 0.9</td>
<td>137/000/000/000 0.86</td>
<td>232/000/000/000 0.73</td>
<td>19/800/000/000 0.66</td>
<td>1/939/400/000/000 0.89</td>
</tr>
<tr>
<td>2</td>
<td>840/000/000/000 0.84</td>
<td>390/000/000/000 0.65</td>
<td>156/000/000/000 0.98</td>
<td>200/000/000/000 0.75</td>
<td>23/100/000/000 0.77</td>
<td>1/609/900/000/000 0.74</td>
</tr>
<tr>
<td>3</td>
<td>450/000/000/000 0.45</td>
<td>348/000/000/000 0.58</td>
<td>88/000/000/000 0.55</td>
<td>380/000/000/000 0.95</td>
<td>11/400/000/000 0.38</td>
<td>1/277/400/000/000 0.58</td>
</tr>
<tr>
<td>4</td>
<td>300/000/000/000 0.6</td>
<td>600/000/000/000 0.1</td>
<td>51/200/000/000 0.32</td>
<td>172/000/000/000 0.43</td>
<td>16/200/000/000 0.54</td>
<td>999/400/000/000 0.27</td>
</tr>
<tr>
<td>5</td>
<td>860/000/000/000 0.86</td>
<td>216/000/000/000 0.36</td>
<td>128/000/000/000 0.8</td>
<td>146/000/000/000 0.37</td>
<td>10/500/000/000 0.35</td>
<td>1/363/500/000/000 0.62</td>
</tr>
<tr>
<td>6</td>
<td>600/000/000/000 0.8</td>
<td>510/000/000/000 0.85</td>
<td>107/000/000/000 0.67</td>
<td>220/000/000/000 0.55</td>
<td>27/000/000/000 0.9</td>
<td>1/464/200/000/000 0.67</td>
</tr>
<tr>
<td>7</td>
<td>720/000/000/000 0.72</td>
<td>258/000/000/000 0.43</td>
<td>97/000/000/000 0.61</td>
<td>720/000/000/000 0.18</td>
<td>21/900/000/000 0.73</td>
<td>1/169/500/000/000 0.53</td>
</tr>
<tr>
<td>8</td>
<td>350/000/000/000 0.35</td>
<td>900/000/000/000 0.15</td>
<td>32/000/000/000 0.2</td>
<td>200/000/000/000 0.5</td>
<td>6/900/000/000 0.23</td>
<td>678/900/000/000 0.31</td>
</tr>
<tr>
<td>9</td>
<td>660/000/000/000 0.68</td>
<td>480/000/000/000 0.8</td>
<td>64/000/000/000 0.4</td>
<td>252/000/000/000 0.63</td>
<td>24/900/000/000 0.83</td>
<td>1/480/900/000/000 0.68</td>
</tr>
<tr>
<td>10</td>
<td>900/000/000/000 0.9</td>
<td>432/000/000/000 0.72</td>
<td>97/000/000/600 0.61</td>
<td>320/000/000/000 0.8</td>
<td>14/400/000/000 0.48</td>
<td>1/764/000/000/000 0.81</td>
</tr>
</tbody>
</table>
Table 9. Predicted time to perform audit procedures in 10 sample companies

<table>
<thead>
<tr>
<th>Company row sample</th>
<th>The number of the entity’s accounting records</th>
<th>Turnover of accounts about the operating cycles</th>
<th>Audit operation risk</th>
<th>Initial audit or audits carried out in previous years</th>
<th>Time budget</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amount</td>
<td>Normalized value</td>
<td>Hours</td>
<td>Normalized value</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>95,000</td>
<td>0.95</td>
<td>8.9</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>85,000</td>
<td>0.85</td>
<td>7.4</td>
<td>0.45</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>80,000</td>
<td>0.8</td>
<td>5.8</td>
<td>0.35</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>76,000</td>
<td>0.76</td>
<td>2.7</td>
<td>0.65</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>72,000</td>
<td>0.72</td>
<td>6.2</td>
<td>0.55</td>
<td>0.1</td>
</tr>
<tr>
<td>6</td>
<td>65,000</td>
<td>0.65</td>
<td>6.7</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>7</td>
<td>50,000</td>
<td>0.5</td>
<td>5.3</td>
<td>0.4</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>40,000</td>
<td>0.4</td>
<td>3.1</td>
<td>0.75</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>34,000</td>
<td>0.34</td>
<td>0.6</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>25,000</td>
<td>0.25</td>
<td>0.81</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

In connection with the column of audit team employees’ fees in Table 10, it is necessary to note the following descriptions.

Calculation of the audit staff fees is so that first time predicted for audit is divided among various levels of audit staff, then, it is multiplied in fee rate, that is: Fee rate of each level of audit staff × time allocated to different levels of audit staff = audit employees fees.

The result of this rate increase from 2010 to 2014 is summarized in Table 11. In the study, calculated rates for 2013 have been used.

Table 11. Fee rate of each audit job category

<table>
<thead>
<tr>
<th>Year</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of increase Organizational category</td>
<td>-</td>
<td>15%</td>
<td>9%</td>
<td>18%</td>
<td>25%</td>
</tr>
<tr>
<td>Manager</td>
<td>91,500</td>
<td>105,225</td>
<td>114,695</td>
<td>135,340</td>
<td>169,175</td>
</tr>
<tr>
<td>Supervisor</td>
<td>87,000</td>
<td>100,050</td>
<td>109,655</td>
<td>128,685</td>
<td>160,856</td>
</tr>
<tr>
<td>Senior auditor</td>
<td>77,000</td>
<td>88,550</td>
<td>96,520</td>
<td>113,894</td>
<td>142,368</td>
</tr>
<tr>
<td>Auditor</td>
<td>60,000</td>
<td>69,000</td>
<td>75,210</td>
<td>88,748</td>
<td>110,935</td>
</tr>
<tr>
<td>Auditor assistant</td>
<td>19,500</td>
<td>22,425</td>
<td>24,443</td>
<td>28,843</td>
<td>36,054</td>
</tr>
</tbody>
</table>

Table 12 presents the audit team employees fee calculation for 10 samples companies.

Table 12. How to calculate the auditing team employees in 10 sample companies

<table>
<thead>
<tr>
<th>Sample company row</th>
<th>Manager</th>
<th>Supervisor</th>
<th>Senior auditor</th>
<th>Auditor</th>
<th>Auditor assistant</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocated time</td>
<td>332</td>
<td>903</td>
<td>1,900</td>
<td>2,850</td>
<td>3,515</td>
<td>9,500</td>
</tr>
<tr>
<td>Fee rate</td>
<td>169.175</td>
<td>160.856</td>
<td>142.368</td>
<td>110.935</td>
<td>36.054</td>
<td></td>
</tr>
<tr>
<td>Fee</td>
<td>56,166.100</td>
<td>145,252.968</td>
<td>270,499.200</td>
<td>316,164.750</td>
<td>126,729.810</td>
<td>914,812.828</td>
</tr>
<tr>
<td>Sample company 2</td>
<td>Allocated time</td>
<td>297</td>
<td>808</td>
<td>1,700</td>
<td>2,550</td>
<td>3,145</td>
</tr>
<tr>
<td>Fee rate</td>
<td>169.175</td>
<td>160.856</td>
<td>142.368</td>
<td>110.935</td>
<td>36.054</td>
<td></td>
</tr>
<tr>
<td>Fee</td>
<td>50,244.975</td>
<td>129,971.648</td>
<td>242,025.600</td>
<td>282,884.250</td>
<td>113,389.830</td>
<td>818,516.303</td>
</tr>
</tbody>
</table>
1. Least squares function in Matlab, it is necessary to use the information of 10 sample companies using programming model will be as follows:

According to data presented in Table 4, it is obvious that the status of internal control system of a company depends on the following 5 basic factors:

- Internal control system of sales, revenues and earnings cycle;
- Internal control system of purchase, costs and disbursements cycle;
- Internal control system of payroll cycle;
- Internal control system of fixed assets cycle;
- Internal control system of inventories cycle.

Which mathematical expression (linear programming model) will be as follows:

\[ Z_4 = F(Y_1, Y_2, Y_3, Y_4, Y_5) = a_0 + a_1Y_1 + a_2Y_2 + a_3Y_3 + a_4Y_4 + a_5Y_5. \]

According to the information of internal control system presented in Table 6, the linear programming model for 10 samples is as follows:

1. \[ 0.8 = a_0 + 0.9a_1 + 0.9a_2 + 0.7a_3 + 0.75a_4 + 0.85a_5 \]
2. \[ 0.7 = a_0 + 0.85a_1 + 0.8a_2 + 0.65a_3 + 0.6a_4 + 0.7a_5 \]
3. \[ 0.65 = a_0 + 0.7a_1 + 0.6a_2 + 0.7a_3 + 0.6a_4 + 0.65a_5 \]
4. \[ 0.62 = a_0 + 0.65a_1 + 0.55a_2 + 0.6a_3 + 0.5a_4 + 0.8a_5 \]
5. \[ 0.6 = a_0 + 0.6a_1 + 0.45a_2 + 0.98a_3 + 0.2a_4 + 0.75a_5 \]
6. \[ 0.55 = a_0 + 0.55a_1 + 0.28a_2 + 0.6a_3 + 0.85a_4 + 0.45a_5 \]
7. \[ 0.45 = a_0 + 0.48a_1 + 0.4a_2 + 0.37a_3 + 0.65a_4 + 0.36a_5 \]
8. \[ 0.5 = a_0 + 0.37a_1 + 0.8a_2 + 0.43a_3 + 0.4a_4 + 0.5a_5 \]
9. \[ 0.4 = a_0 + 0.44a_1 + 0.38a_2 + 0.25a_3 + 0.35a_4 + 0.58a_5 \]
10. \[ 0.33 = a_0 + 0.35a_1 + 0.35a_2 + 0.25a_3 + 0.3a_4 + 0.4a_5. \]

Following, to solve aforementioned linear functions using the information of 10 sample companies using least squares function in Matlab, it is necessary to insert the value of all 10 firms in the intended functions which result is presented below:

1. Linear function of the internal control system of the entity in operating cycles (Z4):

According to data presented in Table 4, it is obvious that the status of internal control system of a company depends on the following 5 basic factors:

- Internal control system of sales, revenues and earnings cycle;
- Internal control system of purchase, costs and disbursements cycle;
- Internal control system of payroll cycle;
- Internal control system of fixed assets cycle;
- Internal control system of inventories cycle.

Which mathematical expression (linear programming model) will be as follows:

\[ Z_4 = F(Y_1, Y_2, Y_3, Y_4, Y_5) = a_0 + a_1Y_1 + a_2Y_2 + a_3Y_3 + a_4Y_4 + a_5Y_5. \]

According to the information of internal control system presented in Table 6, the linear programming model for 10 samples is as follows:

1. \[ 0.8 = a_0 + 0.9a_1 + 0.9a_2 + 0.7a_3 + 0.75a_4 + 0.85a_5 \]
2. \[ 0.7 = a_0 + 0.85a_1 + 0.8a_2 + 0.65a_3 + 0.6a_4 + 0.7a_5 \]
3. \[ 0.65 = a_0 + 0.7a_1 + 0.6a_2 + 0.7a_3 + 0.6a_4 + 0.65a_5 \]
4. \[ 0.62 = a_0 + 0.65a_1 + 0.55a_2 + 0.6a_3 + 0.5a_4 + 0.8a_5 \]
5. \[ 0.6 = a_0 + 0.6a_1 + 0.45a_2 + 0.98a_3 + 0.2a_4 + 0.75a_5 \]
6. \[ 0.55 = a_0 + 0.55a_1 + 0.28a_2 + 0.6a_3 + 0.85a_4 + 0.45a_5 \]
7. \[ 0.45 = a_0 + 0.48a_1 + 0.4a_2 + 0.37a_3 + 0.65a_4 + 0.36a_5 \]
8. \[ 0.5 = a_0 + 0.37a_1 + 0.8a_2 + 0.43a_3 + 0.4a_4 + 0.5a_5 \]
9. \[ 0.4 = a_0 + 0.44a_1 + 0.38a_2 + 0.25a_3 + 0.35a_4 + 0.58a_5 \]
10. \[ 0.33 = a_0 + 0.35a_1 + 0.35a_2 + 0.25a_3 + 0.3a_4 + 0.4a_5. \]

Since the aim is finding values of \(a_0, a_1, a_2, a_3, a_4\) and \(a_5\), so that the sum of squared error \((a_0 + a_1Y_1 + a_2Y_2 + a_3Y_3 + a_4Y_4 + a_5Y_5)^2\) is least, so we have:

sum of squared error: \(e^2 = (a_0 + 0.9a_1 + 0.9a_2 + 0.7a_3 + 0.75a_4 + 0.85a_5 - 0.8)^2 + (a_0 + 0.85a_1 + 0.8a_2 + 0.65a_3 + 0.6a_4 + 0.7a_5 - 0.7)^2 + (a_0 + 0.7a_1 + 0.6a_2 + 0.7a_3 + 0.6a_4 + 0.65a_5 - 0.62)^2 + (a_0 + 0.65a_1 + 0.55a_2 + 0.6a_3 + 0.5a_4 + 0.8a_5 - 0.6)^2 + (a_0 + 0.6a_1 + 0.45a_2 + 0.98a_3 + 0.2a_4 + 0.75a_5 - 0.5^2) + (a_0 + 0.55a_1 + 0.28a_2 + 0.6a_3 + 0.85a_4 + 0.45a_5 - 0.4^2) + (a_0 + 0.48a_1 + 0.4a_2 + 0.37a_3 + 0.65a_4 + 0.36a_5 - 0.3^2) + (a_0 + 0.4a_1 + 0.38a_2 + 0.25a_3 + 0.35a_4 + 0.58a_5 - 0.2^2) + (a_0 + 0.35a_1 + 0.35a_2 + 0.25a_3 + 0.3a_4 + 0.4a_5). \)
2. linear function of audit risk ($W_3$):
According to data presented in Table 4, it is obvious that audit risk in a company under investigation depends on 4 basic factors:

- integrity and reputation of the senior management of the entity;
- complexity of the entity’s operations;
- presence of certain rules and regulations governing the activities of the entity;
- status of internal control system of the entity’s operating cycle.

Which mathematical expression (linear programming model) will be as follows:

$$W_3 = F \left( Z_1, Z_2, Z_3, Z_4 \right) = b_0 + b_1Z_1 + b_2Z_2 + b_3Z_3 + b_4Z_4.$$  

According to the information of audit risk provided in Table 7, linear programming model for 10 sample companies is as follows:

1. $0.8 = b_0 + 0.45b_1 + 0.9b_2 + 0.9b_3 + 0.62b_4$
2. $0.75 = b_0 + 0.55b_1 + 0.88b_2 + 0.75b_3 + 0.65b_4$
3. $0.7 = b_0 + 0.6b_1 + 0.8b_2 + 0.75b_3 + 0.7b_4$
4. $0.65 = b_0 + 0.8b_1 + 0.6b_2 + 0.65b_3 + 0.8b_4$
5. $0.55 = b_0 + 0.6b_1 + 0.6b_2 + 0.6b_3 + 0.5b_4$
6. $0.5 = b_0 + 0.7b_1 + 0.5b_2 + 0.5b_3 + 0.55b_4$
7. $0.45 = b_0 + 0.55b_1 + 0.5b_2 + 0.4b_3 + 0.45b_4$
8. $0.4 = b_0 + 0.6b_1 + 0.4b_2 + 0.3b_3 + 0.33b_4$
9. $0.3 = b_0 + 0.75b_1 + 0.3b_2 + 0.2b_3 + 0.4b_4$
10. $0.3 = b_0 + 0.8b_1 + 0.2b_2 + 0.1b_3 + 0.6b_4$

Since the goal is to find the values of $b_0, b_1, b_2, b_3$ and $b_4$, so that the sum of the squared errors $(b_0 + + b_1Z_1 + b_2Z_2 + b_3Z_3 + b_4Z_4)^2$ is least, so we have:

The sum of squared error: $\varepsilon^2 = (b_0 + 0.45b_1 + 0.9b_2 + + 0.9b_3 + 0.62b_4 - 0.8)^2 + (b_0 + 0.55b_1 + 0.88b_2 + + 0.75b_3 + 0.65b_4 - 0.75)^2 + (b_0 + 0.6b_1 + 0.8b_2 + + 0.75b_3 + 0.75)^2 - (b_0 + 0.6b_1 + 0.6b_2 + 0.6b_3 + 0.5 - 0.55)^2 + (b_0 + 0.7b_1 + 0.5b_2 + 0.5b_3 + 0.55b_4 - 0.5)^2 + + (b_0 + 0.55b_1 + 0.5b_2 + 0.4b_3 + 0.45b_4 - 0.45)^2 + (b_0 + + 0.6b_1 + 0.4b_2 + 0.3b_3 + 0.33b_4 - 0.4)^2 + (b_0 + + 0.75b_1 + 0.3b_2 + 0.2b_3 + 0.4b_4 - 0.35)^2 + (b_0 + + 0.8b_1 + 0.2b_2 + 0.1b_3 + 0.6b_4 - 0.3)^2.$  

3. Linear function of turnover of accounts related to the entity’s operating cycle ($W_2$):
According to data presented in Table 4, it is obvious that turnover of accounts in a company under investigation depends on 5 basic factors:

- turnover of sales, revenues and earnings cycle;
- turnover of purchase, costs and disbursements cycle;
- turnover of payroll cycle;
- turnover of fixed assets cycle;
- turnover of inventories cycle.

Which mathematical expression (linear programming model) will be as follows:

$$W_2 = F \left( X_1, X_2, X_3, X_4, X_5 \right) = c_0 + c_1X_1 + + c_2X_2 + c_3X_3 + c_4X_4 + c_5X_5.$$  

According to the data of turnover presented in Table 8, linear programming model for 10 sample companies is as follows:

1. $0.89 = c_0 + 0.95c_1 + 0.9c_2 + 0.86c_3 + 0.73c_4 + 0.66c_5$
2. $0.74 = c_0 + 0.84c_1 + 0.65c_2 + 0.98c_3 + 0.5c_4 + 0.77c_5$
3. $0.58 = c_0 + 0.45c_1 + 0.58c_2 + 0.55c_3 + 0.95c_4 + 0.38c_5$
4. $0.27 = c_0 + 0.3c_1 + 0.1c_2 + 0.32c_3 + 0.43c_4 + 0.54c_5$
5. $0.62 = c_0 + 0.86c_1 + 0.36c_2 + 0.8c_3 + 0.37c_4 + 0.35c_5$
6. $0.67 = c_0 + 0.6c_1 + 0.85c_2 + 0.67c_3 + 0.55c_4 + 0.9c_5$
7. $0.53 = c_0 + 0.72c_1 + 0.43c_2 + 0.61c_3 + 0.18c_4 + 0.73c_5$
8. $0.31 = c_0 + 0.35c_1 + 0.15c_2 + 0.2c_3 + 0.5c_4 + 0.23c_5$
9. $0.68 = c_0 + 0.66c_1 + 0.8c_2 + 0.4c_3 + 0.63c_4 + 0.83c_5$
10. $0.81 = c_0 + 0.9c_1 + 0.9c_2 + 0.61c_3 + 0.8c_4 + 0.48c_5.$

Since the goal is to find the values of $c_0, c_1, c_2, c_3, c_4$ and $c_5$, so that the sum of the squared error $(c_0 + c_1X_1 + + c_2X_2 + c_3X_3 + c_4X_4 + c_5X_5)^2$ are least, so we have:

The sum of squared error: $\varepsilon^2 = (c_0 + 0.95c_1 + 0.9c_2 + + 0.86c_3 + 0.73c_4 + 0.66c_5 - 0.89)^2 + (c_0 + 0.84c_1 + + 0.65c_2 + 0.98c_3 + 0.5c_4 + 0.77c_5 - 0.74)^2 + (c_0 + + 0.45c_1 + 0.58c_2 + 0.55c_3 + 0.95c_4 + 0.38c_5 - 0.58)^2 + + (c_0 + 0.3c_1 + 0.1c_2 + 0.32c_3 + 0.43c_4 + 0.54c_5 - 0.27)^2 + (c_0 + 0.86c_1 + 0.36c_2 + 0.8c_3 + 0.37c_4 + + 0.35c_5 - 0.62)^2 + (c_0 + 0.6c_1 + 0.85c_2 + 0.67c_3 + + 0.55c_4 + 0.9c_5 - 0.67)^2 + (c_0 + 0.72c_1 + 0.43c_2 + + 0.61c_3 + 0.18c_4 + 0.73c_5 - 0.53)^2 + (c_0 + 0.35c_1 + + 0.15c_2 + 0.2c_3 + 0.5c_4 + 0.23c_5 - 0.31)^2 + (c_0 + + 0.66c_1 + 0.8c_2 + 0.4c_3 + 0.63c_4 + 0.83c_5 - 0.68)^2 + + (c_0 + 0.9c_1 + 0.9c_2 + 0.61c_3 + 0.8c_4 + 0.48c_5 - 0.81)^2.$

4. Linear function of predicted time to perform audits based on operations size of the entity ($V_1$):
According to the data presented in Table 4, it is obvious that predicted time required to perform an audit in a company depends on 4 basic factors:

- number of accounting records of the entity;
- turnover of accounts related to the entity’s operating cycle;
According to the anticipated audit fees, presented in Table 10, the linear programming model for 10 sample companies is as follows:

1. \( 1.235 = e_0 + 0.95e_1 + 0.285e_2 \)
2. \( 1.105 = e_0 + 0.85e_1 + 0.255e_2 \)
3. \( 1.001 = e_0 + 0.77e_1 + 0.231e_2 \)
4. \( 0.936 = e_0 + 0.72e_1 + 0.216e_2 \)
5. \( 0.845 = e_0 + 0.65e_1 + 0.195e_2 \)
6. \( 0.715 = e_0 + 0.55e_1 + 0.165e_2 \)
7. \( 0.65 = e_0 + 0.5e_1 + 0.15e_2 \)
8. \( 0.624 = e_0 + 0.48e_1 + 0.144e_2 \)
9. \( 0.455 = e_0 + 0.35e_1 + 0.105e_2 \)
10. \( 0.43 = e_0 + 0.33e_1 + 0.1e_2 \)

Since the aim is finding values of \( e_0, e_1 \) and \( e_2 \), so that the sum of the squared error \((e_0 + e_1S_1 + e_2S_2)^2\) is least, so we have:

sum of squared error: \( e^2 = (d_0 + 0.95d_1 + 0.89d_2 + + 0.8d_3 + 0.1d_4 - 0.95)^2 + (d_0 + 0.85d_1 + 0.74d_2 + + 0.45d_3 + 0.1d_4 - 0.85)^2 + (d_0 + 0.8d_1 + 0.58d_2 + + 0.35d_3 + 0.1d_4 - 0.74)^2 \) 

4. Results of linear functions

Having solved the linear functions using least squares in Matlab, the following results were obtained:

1. The results of solving a linear function of the entity’s internal control system in respect to operating cycles (Z_4):
   - \( a_0 = 0.0100 \)
   - \( a_1 = 0.1326 \)
   - \( a_2 = 0.1828 \)
   - \( a_3 = 0.2209 \)
   - \( a_4 = 0.2149 \)
   - \( a_5 = 0.2272 \)

So, we will have:

\[ Z_4 = a_0 + a_1Y_1 + a_2Y_2 + a_3Y_3 + a_4Y_4 + a_5Y_5 = \]
\[ = 0.0100 + 0.1326Y_1 + 0.1828Y_2 + 0.2209Y_3 + 0.2149Y_4 + 0.2272Y_5. \]

The results of solving a linear function of audit risk (W_3):

1. \( b_0 = 0 \)
2. \( b_1 = 0.1171 \)
3. \( b_2 = 0.5398 \)
4. \( b_3 = 0.1904 \)
5. \( b_4 = 0.1262 \)

So, we will have:
The results of solving a linear function of turnover of accounts related to the entity’s operating cycle ($W_2$):

1. $c_0 = 0$
2. $c_1 = 0.4593$
3. $c_2 = 0.2877$
4. $c_3 = 0.0688$
5. $c_4 = 0.1780$
6. $c_5 = 0.0077$

So, we will have:

$$W_2 = C_0 + C_1X_1 + C_2X_2 + C_3X_3 + C_4X_4 + C_5X_5 =$$
$$= 0 + 0.4593X_1 + 0.2877X_2 + 0.0688X_3 +$$
$$+ 0.1780X_4 + 0.0077X_5$$

3. The results of solving a linear function of the predicted time to perform an audit based on the operations of the entity ($V_1$):

1. $d_0 = 0$
2. $d_1 = 0.8169$

$$V_1 = d_0 + d_1W_1 + d_2W_2 + d_3W_3 + d_4W_4 =$$
$$= 0 + 0.8169W_1 + 0.0631W_2 + 0.0773W_3 +$$
$$+ 0.0442W_4$$

4. The results of solving a linear function of the independent audit fee optimum amount ($U$):

1. $e_0 = 0$
2. $e_1 = 1.000$
3. $e_2 = 1.000$

So, we will have:

$$U = e_0 + e_1S_1 + e_2S_2 = 1.000S_1 +$$
$$+ 1.000S_2$$

5. Hypotheses testing

The first hypothesis states that a significant correlation exists between the expected time to perform an audit based on the operations size of the entity and the audit fee.

Table 13. The first hypothesis test result (normality test and correlation analysis)

<table>
<thead>
<tr>
<th></th>
<th>$y_1$</th>
<th>$x_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Normal parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.79960</td>
<td>0.61500</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.270760</td>
<td>0.208393</td>
</tr>
<tr>
<td>Most extreme differences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Absolute</td>
<td>0.123</td>
<td>0.122</td>
</tr>
<tr>
<td>Positive</td>
<td>0.123</td>
<td>0.122</td>
</tr>
<tr>
<td>Negative</td>
<td>-0.093</td>
<td>-0.093</td>
</tr>
<tr>
<td>Kolmogorov-Smirnov Z</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asymp. sig. (2-tailed)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pearson’s correlation</td>
<td>1</td>
<td>1.000 **</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>N</td>
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<td>10</td>
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</table>

$y_1$ Pearson’s correlation

<table>
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<tr>
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</tr>
</thead>
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<td>10</td>
</tr>
<tr>
<td>Normal parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.79960</td>
<td>0.62200</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>0.270760</td>
<td>0.236352</td>
</tr>
<tr>
<td>Most extreme differences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Absolute</td>
<td>0.123</td>
<td>0.161</td>
</tr>
<tr>
<td>Positive</td>
<td>0.123</td>
<td>0.126</td>
</tr>
</tbody>
</table>

Notes: a – test distribution is normal, ** – correlation is significant at the 0.01 level (2-tailed).

According to the above Table, it can be seen that the correlation coefficient at the error level 1% is also significant which is marked with **.

The second hypothesis states that a significant correlation exists between the number of the entity’s accounting documents and audit fee.

Table 14. The second hypothesis test result (normality test and correlation analysis)

<table>
<thead>
<tr>
<th></th>
<th>$y_1$</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Normal parameters</td>
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<tr>
<td>Mean</td>
<td>0.79960</td>
<td>0.62200</td>
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<tr>
<td>Std. deviation</td>
<td>0.270760</td>
<td>0.236352</td>
</tr>
<tr>
<td>Most extreme differences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Absolute</td>
<td>0.123</td>
<td>0.161</td>
</tr>
<tr>
<td>Positive</td>
<td>0.123</td>
<td>0.126</td>
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</table>
Table 14 (cont.). The second hypothesis test result (normality test and correlation analysis)

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<tbody>
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<td>10</td>
</tr>
<tr>
<td>Most extreme differences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative</td>
<td>-0.093</td>
<td>-0.161</td>
</tr>
<tr>
<td>Kolmogorov-Smirnov Z</td>
<td>0.388</td>
<td>0.508</td>
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<tr>
<td>Asymp. sig. (2-tailed)</td>
<td>0.998</td>
<td>0.958</td>
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<tr>
<td>$y_i$</td>
<td>Pearson’s correlation</td>
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</tr>
<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>10</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Pearson’s correlation</td>
<td>0.974 *</td>
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<td></td>
<td>Sig. (2-tailed)</td>
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</tr>
<tr>
<td></td>
<td>N</td>
<td>10</td>
</tr>
</tbody>
</table>

Notes: a – test distribution is normal, ** – correlation is significant at the 0.01 level (2-tailed).

According to the above index, it is also observed that the correlation coefficient between the independent variable of the number of accounting records of the entity and the dependent variable of audit fee is equal to 0.974 and the probability value for examining its significance $H_0: \rho = 0$ is 0.000, which is smaller than 0.50, so, the $H_0$ is not confirmed. So, with 95% confidence, the presence of a positive significant correlation is confirmed.

Table 15. The third hypothesis test result (normality test and correlation analysis)

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
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<td>10</td>
</tr>
<tr>
<td>Normal parameters *</td>
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<td></td>
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<tr>
<td>Mean</td>
<td>0.79960</td>
<td>0.64000</td>
</tr>
<tr>
<td>Std. deviation</td>
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<td>0.464758</td>
</tr>
<tr>
<td>Most extreme differences</td>
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<tr>
<td>Absolute</td>
<td>0.123</td>
<td>0.381</td>
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<tr>
<td>Positive</td>
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<td>Negative</td>
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<td>-0.381</td>
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<tr>
<td>Kolmogorov-Smirnov Z</td>
<td>0.388</td>
<td>1.204</td>
</tr>
<tr>
<td>Asymp. sig. (2-tailed)</td>
<td>0.998</td>
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<td>$y$</td>
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<tr>
<td></td>
<td>Sig. (2-tailed)</td>
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<tr>
<td></td>
<td>N</td>
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<tr>
<td>$x_i$</td>
<td>Pearson’s correlation</td>
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<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td>0.954</td>
</tr>
<tr>
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<td>N</td>
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</table>

Notes: a – test distribution is normal.

It is also observed that the correlation coefficient between the independent variable of the initial audit and the dependent variable of audit fee is equal to –0.021, and the probability value for examining its significance $H_0: \rho = 0$ is 0.954, which is smaller than 0.05, so, the $H_0$ is confirmed. So, with 95% confidence, the presence of a negative significant correlation is not confirmed.

The fourth hypothesis states that a significant correlation exists between the variable of turnover of accounts related to the entity’s operating cycle, including sales, revenues and receipts-purchase, payrolls-salary and wage and fixed assets and inventories and audit fee.

This hypothesis is statistically expressed as follows: the symbol $\rho$ is the community correlation coefficient.

Table 16. The fourth hypothesis test result (normality test and correlation analysis)

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<tr>
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<td>Mean</td>
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<td></td>
</tr>
<tr>
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Table 16 (cont.). The fourth hypothesis test result (normality test and correlation analysis)

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<td>10</td>
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<tr>
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</tr>
<tr>
<td>Kolmogorov-Smirnov Z</td>
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<td>.455</td>
</tr>
<tr>
<td>Asymp. sig. (2-tailed)</td>
<td>.998</td>
<td>.986</td>
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<tr>
<td>Y Pearson’s correlation</td>
<td>1</td>
<td>.147</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.696</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>x4 Pearson’s correlation</td>
<td>.147</td>
<td>1</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.686</td>
<td></td>
</tr>
</tbody>
</table>

Notes: a – test distribution is normal.

It is observed that the correlation coefficient between the independent variable of the turnover of the accounts related to the entity’s operational cycles including sales, revenues and receipts cycles – purchase, payrolls-salary and wage, fixed assets and inventories and the dependent variable of audit fee is equal to 0.147 and the probability value for examining its significance $H_0: \rho = 0$ is 0.686, which is smaller than 0.05, so, the $H_0$ is confirmed. So, with 95% confidence, the presence of a positive significant correlation is not confirmed.

The fifth hypothesis suggests that a significant correlation exists between the risk of audit and audit fees. The symbol $\rho$ is the community correlation coefficient.

Table 17. The fifth hypothesis test result (normality test and correlation analysis)

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>x5</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>10</td>
<td>10</td>
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<tr>
<td>Normal parameters</td>
<td>Mean</td>
<td>.79960</td>
</tr>
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<td></td>
<td>Std. deviation</td>
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</tr>
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<td>Absolute</td>
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</tr>
<tr>
<td></td>
<td>Positive</td>
<td>.123</td>
</tr>
<tr>
<td></td>
<td>Negative</td>
<td>.093</td>
</tr>
<tr>
<td>Kolmogorov-Smirnov Z</td>
<td>.388</td>
<td>.402</td>
</tr>
<tr>
<td>Asymp. sig. (2-tailed)</td>
<td>.998</td>
<td>.997</td>
</tr>
<tr>
<td>Y Pearson’s correlation</td>
<td>1</td>
<td>.992 **</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>x5 Pearson’s correlation</td>
<td>.992 **</td>
<td>1</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Notes: a – test distribution is normal, ** – correlation is significant at the 0.01 level (2-tailed).

According to the above Table, it can be seen that the correlation coefficient at the error level 1% is also significant which is marked with **.

**Conclusion**

The study examined the prediction of the optimal amount of independent audit fee based on effective factors that the results of solving the function and a model to predict the optimal amount of the independent audit fee are provided in the following:

1. Function of internal control system on the entity’s operating cycle ($Z_4$):
   \[ Z_4 = 0.0100 + 0.1326 Y_1 + 0.1828 Y_2 + 0.2209 Y_3 + 0.2149 Y_4 + 0.2272 Y_5 \]

2. Function of audit operation risk ($W_3$):
   \[ W_3 = 0 + 0.1171 Y_4 + 0.5398 Z_2 + 0.0194 Z_3 + 0.1262 Z_4 \]

3. Function of turnover of financial accounts related to the entity’s operating cycle ($W_2$):
   \[ W_2 = 0.0056 + 0.6078 X_1 + 0.0453 X_2 + 0.0896 X_3 + 0.2297 X_4 + 0.0171 X_5 \]

4. Function of the time budget predicted based on the size of the entity operation ($V_1$):
   \[ V_1 = 0 + 0.8169 W_1 + 0.0631 W_2 + 0.0773 W_3 + 0.0442 W_4 \]

5. Function of optimal amount of the independent audit fees ($U$):
   \[ U = 0 + 1.000 S_1 + 1.000 S_2 \]
References