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Long-memory in asset returns and volatility: evidence from West Africa

Abstract

This paper measures the degree of long-memory or long-range dependence in asset returns and volatility of two stock indices in Ghana and Nigeria. The presence of long-memory opens up opportunities for abnormal returns to be made by analyzing price history of a particular market.

The authors employ the Hurst exponent to measure the degree of long-memory which is evaluated by a semiparametric method, the Local Whittle estimator.

The findings show strong evidence of the presence of long-memory in both returns and volatility of the indices studied, suggesting that neither of the markets in Ghana and Nigeria is weak-form efficient.

Keywords: long-memory, Hurst exponent, Local Whittle, market efficiency.

JEL Classification: C12, G14.

Introduction and literature review

A financial time series is said to exhibit long-memory when its autocorrelation function (ACF) declines slowly and at an infinite spectrum at zero frequency (Ding et al., 1993). The Efficient Market Hypothesis (EMH) also states that prices fully and instantaneously reflect all the available information on the market and, according to the weak form efficiency of the EMH, one cannot make abnormal returns by analyzing past price information in predicting future prices of a market (Fama, 1970). In other words, if a market is efficient, there should be no long-memory in asset returns and volatility else the weak-form market efficiency definition is violated. Moreover, Henry (2002) states that evidence of persistence in equity returns means that stock returns will be predictable, while Barkoulas et al. (2000) suggest that, if a market is predictable, then, speculators can exploit for profit.

The motivation for this paper is based on two schools of thought in the available literature. There are researchers who believe long-memory exists and, as such, should be investigated to find the degree of its existence, so, market participants will be well informed about a particular market and those who believe long-memory is an illusion. For example, researchers such as Baillie (1996), Chan and Hammed (2006) explain the presence of long memory in emerging markets. They believe long-memory in emerging markets is caused by lack of flow of firm-specific information to investors. There are other explanations for the presence of long-memory in emerging markets by Kim and Wu

(2008), Rajan and Zingales (2003), Harvey (1994), Tolvi (2003), and Kim and Shamsuddin (2008). Other works on long-memory in returns and volatilities of stock markets by Barkoulas et al. (1997), Crato and Ray (2000), Panas (2001), Chen et al. (2006), Elder and Jin (2007), Lien and Yang (2010) also explain the presence of long-memory.

However, others believe long-memory is an illusion. For example, Aydogan and Booth (1988) re-examined results of earlier papers and concluded that evidence of long-memory processes in American stock returns was spurious. They concluded that it arose from the existence of pre asymptotic behavior in statistical estimates. Also, Cheung and Lai (1995) found little evidence for long-memory processes in a variety of international stock returns. Chow et al. (1995) found no compelling evidence to support long-memory in the equity returns they examined. Barkoulas and Baum (1996) failed to find any significant evidence of long-memory in the American stock market. Grau-Carles (2005) failed to find evidence of long-memory processes from the log return series from either the S&P 500 or the Dow Jones Industrial Average using a wide variety of statistical techniques. Zhuang et al. (2000) could find little evidence of long-memory processes in UK stock returns.

On the African continent, David G. McMillan and Pako Thupayagale (2008) examines long-memory in equity returns and volatility for the South African stock market using the ARFIMA-FIGARCH model in order to assess the efficiency of the market. The results show that long-memory exists in volatility, but not in returns. Also Morris, Q et al. (2009) extended the work of Jefferies, K. and Thupayagale, P. (2008). They tested for efficiency of the South African stock market with Wavelet and Markov Switching Regime analyses. They observed that the

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Wavelet analysis showed that most of the individual share prices and the share index time series were mean reverting over the long run and follow a long-memory process, giving evidence against the weak-form efficient market hypothesis (EMH).

On the basis of the divergent views about the existence or, otherwise, of long-memory in returns and volatility and the fact that most long-memory studies on the African continent concentrated on the South African stock market, this paper investigates the presence of long-memory in returns and volatility on the Ghanaian and Nigerian stock markets. We employ the Hurst exponent to measure the degree of long-memory in the markets under study. The Hurst exponent will be evaluated by the Local Whittle estimator (Robinson, 1995). The estimator is constructed based on the likelihood principle and it is robust to conditional heteroscedasticity, unlike the GARCH processes, which are common, but inefficient models, because they exhibit incorrect empirical long-term dependence (Los, 2003). The paper is organized as follows:

In Section one, we describe the data used in this study. In Section two, we describe the Local Whittle Estimator. Empirical results are discussed in Section three. Final Section gives the concluding remarks.

1. Data

We use two indices: The Nigeria All Share Index (NIGALSH) and the Ghana Composite index (GSEALSH) with observations from 4/January/2011 to 9/September/2015. The indices used are daily closing values obtained from DataStream denominated in their respective local currency units. Daily data were used, because they improve the accuracy of the serial dependence estimate (Bollerslev and Wright, 2000). We estimate the Hurst exponent for each of the series (returns and volatilities) for the two indices. Each series was split into smaller values each having n observations.

We also adopt Perron and Qu (2010) convention of accounting for zero returns by eliminating returns with absolute magnitudes below 1.0×10^{-6} .

We follow, especially, the work of Anderson et al. (2001) and researchers such as Taylor (1986), Crato and Lima (1994), Starica and Granger (2005) and Bentes et al. (2008) who used the squared of log returns as the best approximation for volatility.

In using the Hurst exponent in determining the degree of long-memory, we let X_t be the price of an index at time t and r_t is the logarithmic return denoted by:

$$r_t = \ln \left(\frac{X_t}{X_{t-1}} \right). \quad (1)$$

Volatility is, therefore, denoted by:

$$v_t = r_t^2. \quad (2)$$

2. The Local Whittle estimator

The Local Whittle estimator proposed by Robinson (1995) is used for analysis of long-memory in the frequency domain. Since our interest is to know the degree of long-memory in a given index, the Hurst exponent which measures the degree of long-memory will be evaluated by the semiparametric Local Whittle estimator.

First, it is required to specify the parametric form of the spectral density, when the frequency λ degenerates to zero.

$$f(\lambda) \sim G(H) |\lambda|^{1-2H} \text{ as } \lambda \rightarrow 0, \quad (3)$$

where $G(H)$ is a constant. The computation involves an additional parameter m , an integer less than $N/2$, where N is the size of the series and such that as $N \rightarrow \infty$

$$\frac{1}{m} + \frac{m}{N} \rightarrow 0. \quad (4)$$

This means that as N gets large, m gets large as well, although slower. For a spectral density of the form of equation 1, the Whittle approximation of the Gaussian likelihood function is obtained by minimizing

$$Q(G, H) = \frac{1}{m} \sum_{j=1}^m \left(\frac{I(\lambda_j)}{G \lambda_j^{1-2H}} + \log(G \lambda_j^{1-2H}) \right), \quad (5)$$

where $\lambda_j = \frac{2\pi j}{N}$ and $I(\lambda_j)$ is the periodogram of the time series. Therefore, this estimator sums the frequencies up to $2\pi m/N$. When G above is replaced by its estimate \hat{G} , we get

$$\hat{G} = \frac{1}{m} \sum_{j=1}^m \left(\frac{I(\lambda_j)}{\lambda_j^{1-2H}} \right). \quad (6)$$

$R(H)$ may be defined as

$$\begin{aligned} R(H) &= Q(\hat{G}, H) - 1 = \\ &= \log \left(\frac{1}{m} \sum_{j=1}^m \frac{I(\lambda_j)}{\lambda_j^{1-2H}} - \frac{2H-1}{m} \sum_{j=1}^m \log \lambda_j \right). \end{aligned}$$

Under fitness of the fourth moment and other assumptions, Robinson (1995) showed that

$$\hat{H} = \text{agri min } R(H). \quad (7)$$

Converges in probability to actual value H , i.e.,

$$m^{1/2}(\hat{H} - H) \rightarrow d \text{ Normal}(0, 1/4). \tag{8}$$

Hence, choosing m is important. As m gets larger, \hat{H} converges to H faster. On the other hand, m should be small, if the series presents short-memory. This paper makes use of a limiting value of $m = (N/2) - 1$ to ensure \hat{H} converges to H faster.

The Hurst exponent H which measures the size and direction of persistence in a time series is a bounded real number; $H \in [0,1]$. the value of H implies persistence or anti-persistence.

- ◆ If $H = 0.5$, it means that all autocorrelations tend rapidly to zero and the time series is a random walk. Hence, we conclude no long-memory in time series, thus, market is efficient.
- ◆ If $H > 0.5$, stronger memory effect which means persistence or mean aversion in the time series.

This means, an increase (decrease) of asset price is likely to follow another increase (decrease).

- ◆ If $H < 0.5$, it suggests anti-persistence or mean reversion which means an increase (decrease) of asset price is likely to follow a decrease (increase).

3. Empirical results

The results from Table 1 show that the return series are not normally distributed. The kurtosis coefficients are high ($k > 3$) and very high for GSEALSH. The return series are positively and negatively skewed. The null hypothesis of normality by the use of the Jarque-Bera statistic at the 1% level of significance is failed to be accepted. The result from the Augmented Dickey-Fuller tests shows that the return series are stationary. These results give proof that the series are not weak-form efficient, a violation of the Efficient Market Hypothesis (EMH).

Table 1. Summary statistics for daily return series

	NIGALSH	GSEALSH
Number of observations	1222	1222
Minimum value	-0.0428	-0.0832
Maximum value	0.07985	0.0763
Median	0.0000	0.0002
Mean	0.0001	0.0006
Standard deviation	0.0094	0.0063
Skewness	0.4469	-0.2481
Kurtosis	7.3200	42.8817
Jarque-Bera	2781.6	93969
ADF	-11.1760	-7.0932

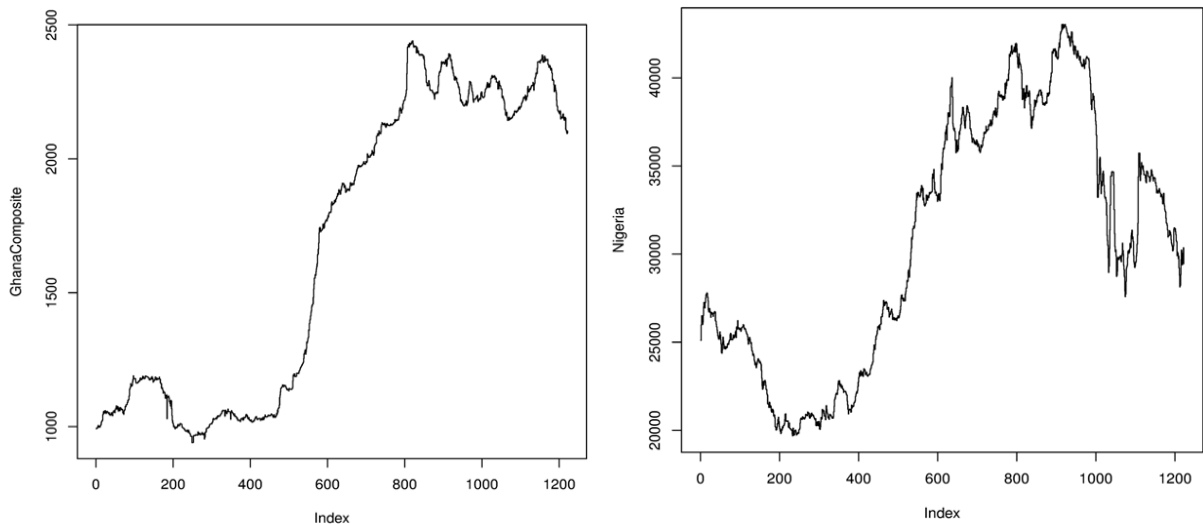


Fig. 1. Plot of daily price index for Ghana and Nigeria

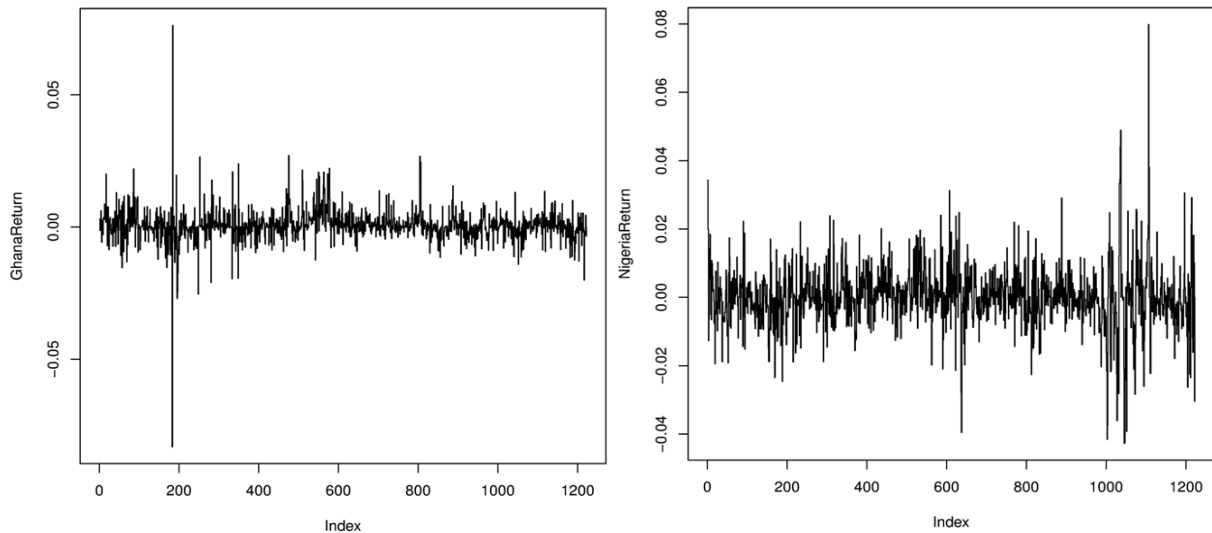


Fig. 2. Plot of daily return series for Ghana and Nigeria

Tables 2 and 3 present the main results of this study. The values for the Hurst exponent are stronger in Table 3 (volatility) than in Table 2 (returns). The Hurst exponent for the return and volatility series for Ghana are 0.6536 and 0.9996 and that of Nigeria are 0.9478 and 0.9990, respectively. Since the Hurst exponent obtained for both series is greater than 0.5, we conclude the presence of long-memory in both markets. This means memory effect is stronger and both indices exhibit mean reversion, implying an increase (decrease) of asset price is likely to follow another increase (decrease). Both indices are less liquid, and information does not flow well in setting current prices on the markets. This makes the markets inefficient in the weak-form.

The conclusion of this study being the existence of long-memory in the Ghanaian and Nigerian stock markets, thus, conforms to the studies in the literature by Baillie (1996), Chan and Hammed (2006), Kim and Wu (2008), Rajan and Zingales (2003), Harvey (1994), Tolvi (2003) and Kim and Shamsuddin (2008), Barkoulas et al. (1997), Crato and Ray (2000), Panas (2001), Chen et al. (2006), Elder and Jin (2007), Lien and Yang (2010), McMillan and Thupayagale (2008), Morris, Q. et al. (2009) and Jefferies and Thupayagale (2008) who believe long-memory exists and should be investigated. Also, our results and approach are consistent with the work of Cajueiro and Tabak (2006) who computed the Hurst exponent using the

Local Whittle estimator in measuring the presence of long-memory in the Shanghai and Shenzhen stock exchanges.

Table 2. The Hurst exponent of returns with a 95% confidence interval

Index	H
NIGALSH	0.9478 ± 0.0005
GSEALSH	0.6536 ± 0.0003

Table 3. The Hurst exponent of volatility with a 95% confidence interval

Index	H
NIGALSH	0.9990 ± 0.000016
GSEALSH	0.9996 ± 0.000015

Conclusion

In this paper, we investigated the presence of long-memory in two stock indices from West Africa: Ghana and Nigeria. We estimated the Hurst exponent of the returns and volatility series by using the Local Whittle estimator proposed by Robinson (1995). The results obtained show evidence of strong memory effect in both returns and volatility, as the Hurst exponent for both markets is greater than 0.5. These results provide evidence that none of the indices studied are weak-form efficient, providing opportunities for abnormal returns to be made in analyzing past prices.

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