"The volatility target effect in investment-linked products with embedded American-type derivatives"

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THE VOLATILITY TARGET EFFECT IN INVESTMENT-LINKED PRODUCTS WITH EMBEDDED AMERICAN-TYPE DERIVATIVES

Abstract

Volatility Target (VolTarget) strategies as underlying assets for options embedded in investment-linked products have been widely used by practitioners in recent years. Available research mainly focuses on European-type options linked to VolTarget strategies. In this paper, VolTarget-linked options of American type are investigated. Within the Heston stochastic volatility model, a numerical study of American put options, as well as American lookback options linked to VolTarget strategies, is performed. These are compared with traditional American-type derivatives linked to an equity index. The authors demonstrate that using a Volatility Target strategy as a basis for an embedded American-type derivative may make any protection fees significantly less dependent of changing market volatilities. Replacing an equity index with the VolTarget strategy may also result in reducing guarantee fees of the corresponding protection features in a highly volatile market environment.

Keywords	Investment-linked products, Volatility Target (VolTarget) strategy, American options
JEL Classification	G11, G12, G13, G17

INTRODUCTION

In the recent years, providers of structured products, in particular investment-linked products, face various challenges that result from extreme market environments. These market environments are characterized by very low interest rates, as well as by quickly changing market volatilities. In such market conditions, very often the key parameters of investment-linked products become unstable and change significantly with time. This makes investment-linked products less attractive to potential investors.

One way to address the above problem is to modify an investment concept (risky asset), which is used as a basis for an investment-linked product (such as S&P 500 or other equity index). The modified investment concept is a Volatility Target (VolTarget) strategy linked to the basic risky asset. A VolTarget strategy is a combination of a basic risky asset (e.g. S&P 500 index) and a riskless asset (e.g., a government bond) taken in certain proportions and dynamically rebalanced so that the overall portfolio volatility stays under control.

Between 2015 and 2018, one could observe an increase in interest rate levels on the US market. This increase allowed for higher risk budgets in investment-linked products, which some product provid-

ers used to differentiate their solutions from the competition by introducing new and innovative features within their capital-protected investment solutions. As one of the results of this process, new investment-linked products that contain embedded American-type derivatives linked to VolTarget strategies emerged on the market. The present paper is one of the first academic reviews studying the VolTarget effect in investment-linked products with embedded American-type options.

The paper is organized as follows. Section 1 describes the literature review. Section 2, we describe typical investment-linked products with embedded American options and provide motivation for studying VolTarget strategy-linked options of American type. In section 3, we briefly describe the VolTarget mechanism. In section 4, we recall the basic properties of the Heston market model with stochastic volatility and describe the way the model is used for our numerical simulations. In section 5, we review the least-squares method of Longstaff and Schwartz for pricing American-type options. The results of our numerical simulations are presented in the following section, before we give concluding remarks in the final section.

1. LITERATURE REVIEW

In existing literature, the VolTarget approach has been mostly considered either as an asset allocation method or as an underlying strategy for European-type derivatives. One of the first reviews of this allocation logic can be found in Krein and Fernandez (2012) or Benson et al. (2014). Also, Zakamulin (2014) considered volatility driven asset allocation concepts in more details. In Kim and Enke (2018), artificial neural networks are used in order to predict market volatility. This approach is applied in order to improve risk-return profiles of investment products.

In Albeverio et al. (2013), the VolTarget mechanism is considered from an academic perspective for the first time both as an investment instrument and as an underlying asset for a European option embedded in an investment product with guarantee. Albeverio et al. (2018) investigate an extension, which allows to choose the volatility target level depending on interest rate levels. Advantages and limitations of using a VolTarget strategy in such products are presented.

2. INVESTMENT PRODUCTS WITH EMBEDDED AMERICAN OPTIONS

American options are used by practitioners as embedded instruments in structured investment products with capital protection, mutual funds, as well as in investment-linked contracts in order to offer minimum guaranteed benefits. Motivated by this new use of American options, we decided to have a deeper look into investment-linked contracts with protection elements. We will make some simplifying assumptions (such as no running fees and a guarantee fee is paid in addition to the contract) in order to demonstrate the essential idea without going into a lot of technical details.

Let us consider the following version of an investment-linked contract: At time $t = 0$, a policyholder pays an amount M and chooses an index fund or a mutual fund in which the premium will be invested. Let us denote the fund value at time t by F_t. Then, $F_0 = M$.

Additionally, at time $t = 0$, the policyholder decides whether an annual guarantee will be included into the contract. Suppose the policyholder wants to add this feature to his/her investment-linked contract. Let P stand for a percentage protection level. Typically, the protection level is offered at 80%, 90% or 100% of the investment (this corresponds to $P = 0.8$, $P = 0.9$ and $P = 1$, respectively). This means that at any time τ during the contract life, the account value of the policyholder equals the fund value F_τ, but at least $P \cdot 100\%$ of the investment amount at the beginning of the year: $P \cdot F_{k-1}$, where $\tau \in [k-1, k)$ is the moment of time during the k-th year of the contract. Let us denote by A_τ the account value at time τ. Then, we have for any $\tau \in [k-1, k)$:

$$A_\tau = \max\left(F_\tau, P \cdot F_{k-1}\right). \tag{1}$$

In order to include the minimum benefit guarantee into the investment-linked contract, the policyholder has to pay an additional amount of money (a guarantee fee) at the beginning of each year. The guarantee fee G_{k-1} paid at the beginning of the k-th year is determined at time $k-1$ as a fair price of a certain American option issued at that time and expiring in one year. Let us explain this part more in detail.

It is straightforward to see that formula (1) may be rewritten as follows:

$$A_\tau = F_\tau + \max\left(P \cdot F_{k-1} - F_\tau, 0\right). \quad (2)$$

The second term in the above expression represents the payoff at time $\tau \in [k-1,k)$ of a one year American put option on the underlying fund F issued at time $t = k-1$ with the strike price $K = P \cdot F_{k-1}$. In order to receive the amount (2) at time τ, the investor should purchase the American put option described above for its fair price at time $t = k-1$ in addition to his/her current investment F_{k-1}. Let us denote the fair price of the option by P_{k-1}. This amount represents the guarantee fee paid at time $t = k-1$:

$$G_{k-1} = P_{k-1}. \quad (3)$$

Let us consider the following example that illustrates this discussion.

Suppose a 45-year old client is interested in an investment-linked solution with an investment horizon of 15 years. Suppose the client's initial investment into a chosen fund is $F_0 = 100,000$. The investor chooses to have his/her investment protected with the protection level $P = 0.8$. So he/she needs to purchase a one year American put option on the underlying fund with the strike price $P \cdot F_0 = 0.8 \cdot 100,000 = 80,000$. Suppose that the fair price of that American put option at time $t = 0$ equals 1% of the investment amount: $P_0 = 0.01 \; F_0 = 1,000$. So the guarantee fee paid by the client at time $t = 0$ is $G_0 = P_0 = 1,000$.

Suppose that after the first year, the underlying fund has grown by 10% (after all fees). This means that the American put option issued at time $t = 0$ expires worth-

less. The investment amount at time $t = 1$ is $F_1 = (1.1) \cdot F_0 = 110,000$. At this point, the client decides that hi/she wants to continue into the next year with the same protection level of 80%. So he/she buys a new one year American put option on the underlying fund with the strike price $P \cdot F_1 = 0.8 \cdot 110,000 = 88,000$. Suppose that the fund volatility went down compared to $t = 0$, but the market interest rate stayed the same. So the option price went down compared to $t = 0$ and it is now 0.8% of the investment amount: $P_1 = 0.008 \cdot F_1 = 880$. The guarantee fee paid by the client at time $t = 1$ is $G_1 = P_1 = 880$.

Further, suppose after the second year, the underlying fund has lost 30% of its value, i.e. $F_2 = (0.7) \; F_1 = 77,000$. However, since the investor has purchased a minimum benefit guarantee, his/her portfolio only loses 20% of its value. Indeed, since the underlying fund's value at time $t = 2$ is below the strike price of 88,000, the American put option at time $t = 2$ pays $88,000 - 77,000 = 11,000$. So the account value as of time $t = 2$ is $A_2 = F_2 + (P \cdot F_1 - F_2) = P \cdot F_1 = 88,000$. This is equivalent to a 20% loss.

A structured product with a so-called high-watermark lock-in, which has an embedded American lookback option, is one way to design this profile. In that case, at any time τ, the account value is the current fund value F_τ, but at least $P \cdot 100\%$ of the highest achieved account value since the beginning of the year: $P \cdot \max_{k-1 \le S \le \tau} F_S$, where $\tau \in [k-1,k)$ is the moment of time during the k-th year of the contract. In this case, the account value A_τ for any $\tau \in [k-1,k)$ takes the following form:

$$A_\tau = \max\left(F_\tau, P \cdot \max_{k-1 \le s \le \tau} F_S\right). \quad (4)$$

Similar to the case of a minimum benefit guarantee, in order to include the high-watermark lock-in protection into the investment-linked contract, the investor has to pay a guarantee fee at the beginning of each year. The guarantee fee G_{k-1} paid at the beginning of the k-th year is determined at time $k-1$ as a fair price of an American floating-strike lookback option issued at that time and expiring in one year. Let us explain this part more in detail.

Let us rewrite formula (4) in the following way:

$$A_\tau = F_\tau + \max\left(P \cdot \max_{k-1 \le s \le \tau} F_S - F_\tau, 0\right). \qquad (5)$$

The second term in the above expression represents the payoff at time $\tau \in [k-1, k)$ of a one year American lookback option on the underlying fund F issued at time $t = k-1$ with the strike price $K = P \cdot \max_{k-1 \le s \le \tau} F_S$. In order to receive the amount (5) at time τ, the investor should purchase the American lookback option described above for its fair price at time $t = k-1$ in addition to his/her current investment F_{k-1}. Let us denote the fair price of the option by P^{LB}. This amount represents the P^{LB}_{k-1} guarantee fee paid at time $t = k-1$:

$$G_{k-1} = P^{LB}_{k-1}. \qquad (6)$$

Typical investment-linked contracts have a life time between five and up to twenty years depending on market practice and local tax regulations. As was explained earlier, every year a product provider determines a new guarantee fee depending on the market environment at the renewal date of the contract by evaluating the embedded American put option or American lookback option on the underlying risky asset. This may lead to significant changes in guarantee fees from year to year due to the changing market environments. In particular, market volatility, as well as market interest rate, may play a significant role in these fluctuations of the guarantee fee. Unstable and changing guarantee fees reduce the attractiveness of an investment-linked contract in the eyes of a potential investor. Additionally, higher market volatilities normally would lead to higher option prices and as a result to higher guarantee fees. Product providers are interested in new approaches that keep under control the guarantee fees of investment-linked contracts and would limit the influence of market environments on guarantee fees.

One way to address this problem is to replace the underlying risky asset of an investment-linked contract with a Volatility Target (VolTarget) strategy linked to the same risky asset as in the original contract. In this article, we focus on investment-linked products with guarantee levels smaller or equal 100%, which we saw more often as a result of a comparable low interest rate environment. Recently, such solutions were more often using embedded derivatives of American type. We illustrate the idea that using a Volatility Target strategy as underlying assets in such a product framework reduces the influence of market volatilities on American option prices. This in turn results in more stable guarantee fees of investment-linked contracts that have American options embedded in their design.

3. VOLATILITY TARGET MECHANISM

The Volatility Target (VolTarget) concept is based on the idea to dynamically adjust a portfolio aiming to control the volatility of the strategy. For a portfolio consisting of risky and riskless assets, the allocation of assets is decided on rebalancing dates, taking into account the ratio between the realized volatility of the chosen risky asset and a volatility target. In this paper, we assume a constant volatility target, an extended version allowing the volatility target to fluctuate over time is considered in Albeverio et al. (2018).

The VolTarget portfolio construction is outlined in the following steps. For further details, we refer to Albeverio et al. (2013).

An investor chooses the values of the following constants: the volatility target VT and a maximum portfolio leverage L. These constants are usually chosen in accordance with a particular application or in line with some levels observed for historical time series.

At each point in time t_k, $k = 1, 2, ...$, when the portfolio is rebalanced, one determines the realized volatility $V_r(t_k)$ of the underlying risky asset. Then, the portfolio is split between riskless and risky asset using the portfolio weights α_1 and α_2 following the equation below:

$$\alpha_1(t_k) = \min\left(\frac{VT}{V_r(t_k)}; L\right), \qquad (7)$$

$$\alpha_2(t_k) = 1 - \alpha_1(t_k). \qquad (8)$$

During the time period $[t_k, t_{k+1})$ the portfolio weights α_1 and α_2 are constant, before they are updated at the next portfolio rebalancing time t_{k+1}.

L is the maximum leverage factor of the strategy. This parameter should avoid an excessive exposure of the strategy to the risky asset in a market environment with low volatility. In practice, the factor L is between 1 and 2. The case $L = 1$ is called from practitioners Volatility Capped (VolCap) portfolios instead of VolTarget portfolios.

4. FINANCIAL MARKET MODEL

In this study, we assume that the risky asset dynamics are described by the Heston stochastic volatility model (see Heston, 1993), i.e. the risky asset dynamics follow the stochastic process $\{S(t)\}_{0 \leq t \leq T}$ satisfying the equation:

$$dS(t) = rS(t)dt + \sigma(t)S(t)dW_S(t), \qquad (9)$$

with r being the risk-free interest rate and $W_S(t)$ being a standard Wiener process. The dynamics of the stochastic volatility $\{\sigma(t)\}_{0 \leq t \leq T}$ in equation (9) can be described by:

$$d\sigma(t)^2 = v\left(\beta - \sigma(t)^2\right)dt + \qquad (10)$$
$$+ \sigma_\sigma \sigma(t)dW_\sigma(t),$$

where $\beta > 0$ is the mean long-term volatility, $v > 0$ is the rate at which the volatility reverts toward its long-term mean and the volatility of the volatility is denoted by $\sigma_\sigma > 0$. Further, $W_\sigma(t)$ is a standard Wiener process correlated with $dW_S(t)$ with the constant real-valued correlation ρ. In order to ensure that the process $\{\sigma^2(t)\}_{0 \leq t \leq T}$ is strictly positive, we will assume that the Feller condition $2v\beta > \sigma_\sigma^2$ holds (see Albrecher et al., 2007).

By making use of this description of the pure risky asset S, our core interest lies on a VolTarget portfolio linked to the risky asset S. To describe this portfolio in more detail, we consider a stochastic process $\{V(t)\}_{0 \leq t \leq T}$. As introduced before-hand, we assume that the VolTarget portfolio follows a rebalancing rule time instances $t_1 < t_2 < \ldots < t_n$, $0 < t_k < T$ $(k = 1, \ldots, n)$. On each interval $[t_k, t_{k+1}]$, $(k = 0, \ldots, n)$, the stochastic process $\{V(t)\}_{0 \leq t \leq T}$ is given by:

$$V(t) = \beta_k S(t) + \gamma_k B(t). \qquad (11)$$

We assume that β_k, γ_k are real-valued and constant for each $k = 0, \ldots, n$. Furthermore, $\{B(t)\}_{0 \leq t \leq T}$ reflects the bond process based on the constant interest rate r. By construction of the VolTarget portfolio, β_k and γ_k fulfill the condition:

$$\lim_{t \to t_k^-} V(t) = \lim_{t \to t_k^+} V(t), \; k = 1, \ldots, n. \qquad (12)$$

Based on this framework, we are going to run a numerical analysis to price American-type options linked to the pure risky asset S and we will compare these figures with derivatives of American type, which use a VolTarget portfolio V as underlying having S as a risky asset.

5. PRICING AMERICAN-TYPE OPTIONS BY THE LEAST SQUARES METHOD

For pricing American-type options in our numerical simulations, we have chosen the least squares method (LSM) introduced by Longstaff and Schwartz in 2001. For further development of the method, we also refer e.g. to Hilpisch (2015), Huynh et al. (2008).

In this section, we recall the main idea of the method and touch upon some implementation details.

A holder of an American option at any exercise time makes a decision whether to exercise the option or not, based on comparison of the payoff from immediate exercise with the option continuation value. The payoff from immediate exercise can be determined straightforwardly as soon as the value of the underlying asset becomes known.

The option continuation value is determined by the conditional expectation (given the underlying asset values) of the payoff from keeping the option and not exercising it at this point in time. Evaluating the option continuation value is the key issue in pricing American-type derivatives.

The least squares method (LSM) of Longstaff and Schwartz is based on the idea that the conditional expectation in question can be estimated by projecting (or, more precisely, regressing) the realized payoffs from keeping the option on basis functions of the underlying asset price values. In Longstaff and Schwartz (2001), a recursive procedure is developed where the continuation option value is estimated for each exercise date starting from the option expiration date and going backward in time on simulated underlying asset price paths.

This way for each simulated asset price path, an optimal exercise strategy is determined and a set of cash flows (or payoffs) generated along each path is estimated. Then, the American option price at time zero is determined as an average of all discounted payoffs from each underlying asset price path.

The choice of the set of basis functionc for the regression procedure described above is an important issue. Longstaff and Schwartz (2001) suggest a variety of choices, starting from simple power functions $1, S, S^2, \ldots$ and up to Laguerre, Hermite, Legendre, Chebyshev, Gegenbauer, and Jacobi polynomials. To implement the LSM, a finite number of the basic functions is used.

After simulating a certain number of the underlying asset price paths, only the paths where the option is in the money are used for building optimal exercise strategies.

6. NUMERICAL EXPERIMENTS

In this section, we provide the results of our numerical simulations. We use the LSM method of Longstaff and Schwartz (Longstaff & Schwartz, 2001) to price American put options (subsection 6.1), as well as American lookback options with floating strike price (subsection 6.2). As was indicated at the end of Section 4, we provide prices for options of two types: (a) with the underlying asset being a pure risky asset; (b) with the underlying asset being a VolTarget portfolio linked to the same risky asset as in case (a).

The risky asset under consideration follows the Heston model with stochastic volatility (section 4). The following parameter values are used within our simulation:

$$v = 1.25, \ \beta = 0.04, \ \sigma_\sigma = 0.2, \ \rho = -0.5,$$
$$r = 0.03.$$

Table 1 contains the initial values of the risky asset volatility $\sigma(0) > 0$, which are used within the numerical analysis.

Table 1. Risky asset annual volatility level used as basis for numerical studies on American options

Risky asset volatility $\sigma(0)$	0.10	0.20	0.30	0.40	0.50

In line with the description given in equation (9)-(10), we will simulate 1,000 paths for the risky asset S assuming a starting value $S(0) = 100$ and considering the different volatility levels from Table 1. We assume that any dividends are re-invested in the strategy and we do not consider any transaction costs.

Correspondingly, we shall simulate the associated path of the VolTarget portfolio V with the initial value $V(0) = S(0) = 100$ for each path of the risky asset S. A volatility target level of $VT = 0.2$ and a maximum leverage $L = 2$ are assumed. The VolTarget portfolio is rebalanced monthly.

For all American options considered in this section, we assume a life time of 1 year, and within our numerical analysis, we assume that the options can be exercised at any trading day.

Following Huynh et al. (2008), for the basis functions in the LSM method (see section 4), we use three power functions: $1, S, S^2$.

6.1. Comparison of the American put option prices

In the context of American options that are embedded into investment-linked products (see section 2), we consider an American put option with the payoff function at time τ as follows:

$$B_\tau = \max\big(P \cdot S(0) - S(\tau),\ 0\big), \qquad (13)$$

where $\tau \in [0,1)$ with 1 standing for one year.

Table 2 gives the numerical prices for American put options on the pure risky asset described above for the initial volatility levels varying between 10% and 50% annually, and for protection levels P between 80% and 100%. In this case, the protection level of 80% $(P = 0.8)$ corresponds to the strike price of 80, and the protection level of 100% $(P = 1.0)$ corresponds to the strike price of 100. Let us recall that the lifetime of each option is 1 year, and the initial underlying asset price is $S(0) = 100$.

Table 2. American put option prices with the pure risky asset as an underlying

Protection level in %/$\sigma(0)$	0.10	0.20	0.30	0.40	0.50	RPR
80	0.03	0.96	3.29	5.75	9.16	1.00
85	0.13	1.66	4.44	7.75	11.06	0.99
90	0.43	2.91	6.05	9.48	12.99	0.97
95	1.24	4.67	7.81	12.21	16.12	0.92
100	2.94	6.81	10.95	14.84	18.57	0.84

As expected, one observes in each column an increase in the put option prices when the protection level (and therefore the strike price) increases. Also, in each row the put prices increase with the increasing initial volatility level of an underlying asset.

The last column in Table 2 contains the Relative Price Range (RPR) for each protection level. To obtain the RPR, we determine the difference between the put price corresponding to $\sigma(0) = 0.50$ and the put price assuming $\sigma(0) = 0.10$. This value is divided by the put price, which corresponds to $\sigma(0) = 0.50$. For example, the RPR of 0.99, which occurs in the second row, has been calculated as follows: (11.06–0.13)/11.06 = 0.99.

The data in Table 3 represent the corresponding numerical prices of the American put options where the underlying asset is a VolTarget portfolio linked to the same risky asset as in Table 2.

Table 3. American put option prices linked to the VolTarget strategy

Protection level in %/$\sigma(0)$	0.10	0.20	0.30	0.40	0.50	RPR
80	0.86	1.82	2.06	2.18	2.75	0.69
85	1.56	2.89	3.23	3.32	3.98	0.61
90	2.69	4.35	4.40	5.16	5.54	0.51
95	4.42	6.27	6.76	6.85	7.55	0.42
100	6.67	8.43	8.56	9.32	10.11	0.33

In Table 3, similar to the data in Table 2, one observes an increase in the put prices in each row, as well as in each column. However, the RPR values in Table 3 are significantly lower than those in Table 2. In particular, the RPR in Table 3 is on average 45.85% lower than that in Table 2.

Additionally, option prices in Table 3 that correspond to higher levels of initial underlying asset volatility are much lower than those in Table 2. For example, comparing the option prices that correspond to $\sigma(0) = 0.40$, one observes the following. When an underlying asset is a VolTarget portfolio (Table 3), option prices are on average 49.12% lower that those where an underlying asset is a pure risky asset (Table 2).

In order to visualize some of our numerical results, we have included the spider diagram that shows the American put option prices that correspond to the protection level of 100% in each of the tables (see Figure 1). The blue line corresponds to the case of a pure risky asset as an underlying, while the red line represents the case of an American put option linked to the VolTarget strategy. The graph illustrates that the VolTarget-linked American put option prices are significantly less dependent on changing market volatilities than the prices of American put options linked to a pure risky asset.

6.2. Comparison of the American lookback option prices

In this subsection, we consider American floating-strike lookback options, having in mind
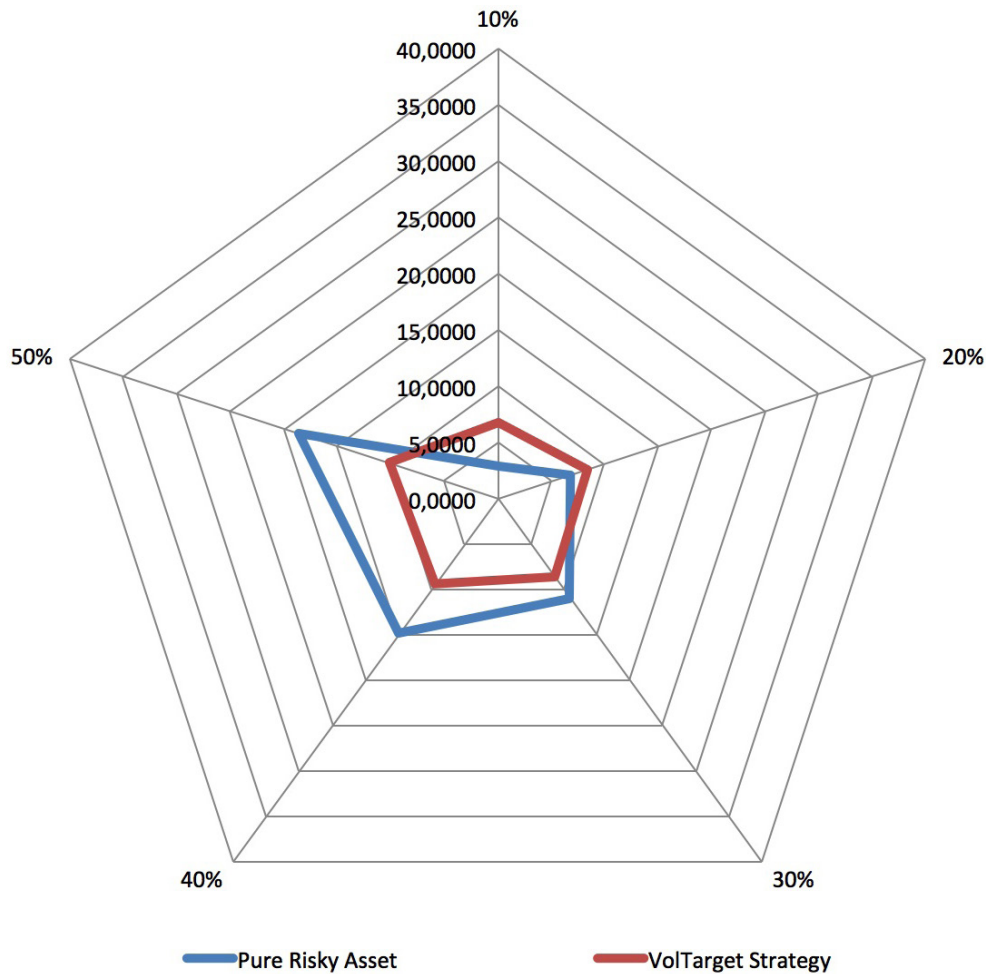
Figure 1. American put option prices with a maturity
of one year and a protection level of a 100%

their application as embedded options in invest-
ment-linked products with high-watermark lock-
in (see section 2).

Our main interest will be in floating-strike look-
back options of American type that have the fol-
lowing payoff at time $\tau \in [0,1)$:

$$B_\tau^{LB} = \grave{u}\grave{u}\grave{u} \left(P \cdot \underset{0 \le s \le \tau}{\quad} S(s) - S(\tau) \quad \right) \quad (14)$$

Table 4 gives the numerical prices for floating-strike
lookback options of American type linked to a pure
risky asset for the initial volatility levels varying be-
tween 10% and 50% annually, and for protection
levels between 80% and 100%. Again, the lifetime
of each option is 1 year, and the initial underlying
asset price is $S(0) = 100$.

Table 4. American floating-strike lookback option
prices linked to the pure risky asset

Protection level in %/σ(0)	0.10	0.20	0.30	0.40	0.50	RPR
80	0.03	1.84	5.62	10.76	16.97	1.00
85	0.21	3.36	8.47	14.27	21.94	0.99
90	0.79	5.42	11.45	18.78	27.61	0.97
95	2.54	8.71	16.21	24.62	32.38	0.92
100	6.12	13.45	21.26	29.33	39.64	0.85

We see that, similar to the case of American put
options, the floating-strike American lookback
option prices increase with increasing protection
level. Also, the lookback option prices increase
with the increasing initial volatility level of an un-
derlying asset. In addition, comparing prices in
Tables 2 and 4, we see that the lookback feature
makes these options significantly more expansive
than American put options.
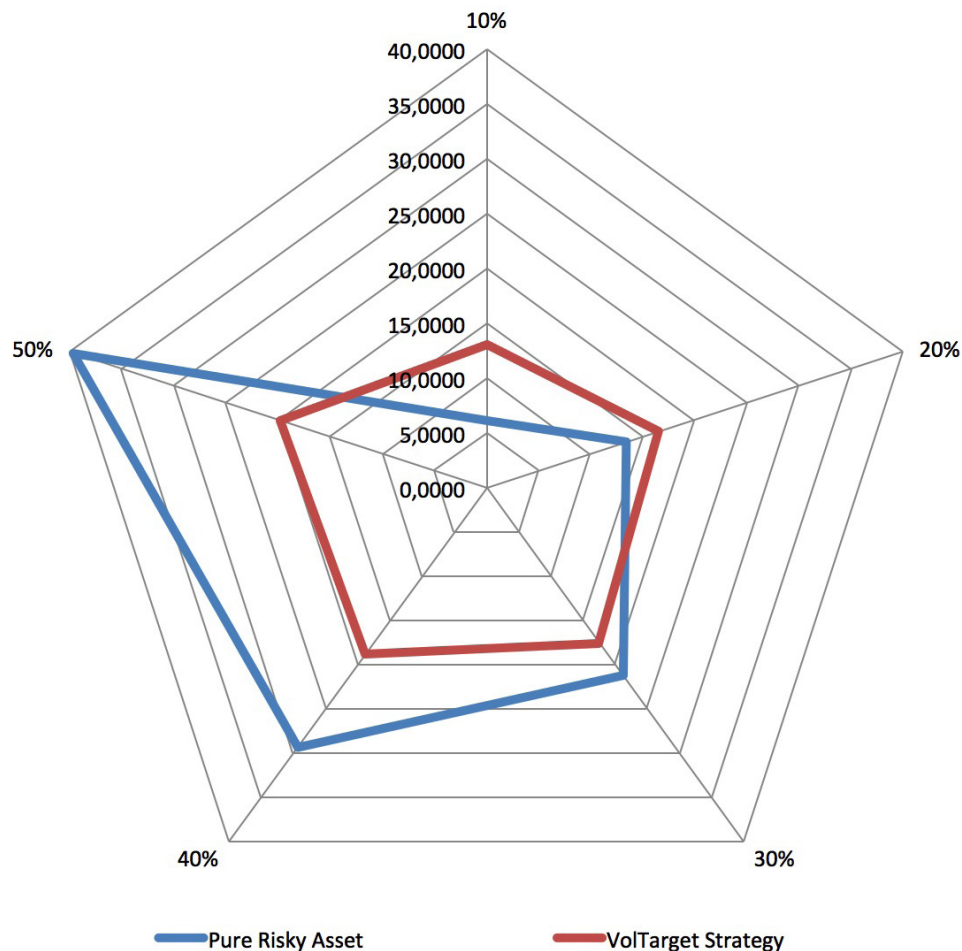
Figure 2. American lookback option prices with a maturity
of one year and a protection level of 100%

The last column in Table 4 contains the Relative Price Range (RPR) for each protection level, similar to the case of the American put option prices presented in subsection 6.1.

The data in Table 5 represent the corresponding numerical prices of the American lookback options where the underlying asset is a VolTarget portfolio linked to the same risky asset as in Table 4.

Table 5. American floating-strike lookback option prices with the VolTarget portfolio as an underlying

Protection level in %/$\sigma(0)$	0.10	0.20	0.30	0.40	0.50	RPR
80	1.37	3.22	3.72	3.96	4.57	0.70
85	2.99	5.30	5.81	6.51	6.89	0.57
90	5.17	8.54	8.62	9.24	10.56	0.51
95	8.12	12.21	12.25	13.37	14.33	0.43
100	13.05	16.54	17.55	18.84	19.82	0.34

In Table 5, similar to the data in Table 4, one observes an increase in option prices in each row, as well as in each column. However, the RPR values in Table 5 are significantly lower than those in Table 4. On average, the difference is about 46.02%.

Similar to the case of American put options considered in the previous subsection, the prices of American lookback options linked to the VolTarget portfolios are overall lower than those that correspond to the pure risky asset as an underlying, for higher levels of initial risky asset volatility. For example, let us compare the option prices that correspond to $\sigma(0) = 0.40$ in Table 5 with corresponding prices in Table 4. When an underlying asset is a VolTarget portfolio, option prices are on average 49.98% lower that those when an underlying asset is a pure risky asset.

For visualization of our numerical results, please see the spider diagram in Figure 2 that shows the prices of the American floating-strike lookback options that correspond to the protection level of 100% in each of two tables. Again, one can easily see that the VolTarget-linked derivatives are less dependent on changing market volatilities than their standard counterparts.

6.3. Summary of the numerical experiments

Our numerical results presented in subsections 6.1 and 6.2 demonstrate that replacing a pure risky underlying asset with its VolTarget counterpart in American-type derivatives (namely, in American put options and in American floating-strike lookback options) results in the following:

- for initial volatility levels that are higher than the chosen target volatility level, American option prices in the case of a VolTarget underlying asset are overall lower than the corresponding option prices, which are linked to a pure risky asset;

- American option prices are significantly less dependent on the fluctuations of the market volatility, when an underlying asset is a VolTarget portfolio compared to the case of a pure risky asset as an underlying;

- our numerical results for American derivatives with a VolTarget strategy as an underlying demonstrate similar trends as in our previous research on VolTarget-linked options of European type (see Albeverio et al., 2013; Albeverio et al., 2018).

CONCLUDING REMARKS

On the current market of structured products and especially investment-linked products, one observes a wide use of VolTarget strategies in financial products with and without guarantees. Initially, VolTarget strategies have been used as underlying assets for European options that were embedded in investment-linked products. Currently, investment-linked products with embedded American-type options linked to VolTarget strategies started to emerge on the market. Practitioners use VolTarget strategies in order to make the key product parameters less dependent on market environments. This article is one the first attempts to consider the new version of VolTarget-linked options of American type from an academic perspective.

In this paper, we focused on American put options, as well as on American lookback options, linked to VolTarget strategies. These options occur as embedded derivatives in investment-linked products with guarantees. For such products, an embedded option price corresponds to a guarantee fee paid by an investor who purchases an investment-linked product.

For our numerical simulations, we have used the Heston stochastic volatility market model. For numerical pricing of American-type derivatives, we have utilized the least squares method of Longstaff and Schwartz.

Our numerical experiments demonstrate that in case of an American-type derivative with a VolTarget strategy as an underlying asset, the corresponding guarantee fee of the investment-linked products may be reduced and made significantly less dependent on changing market volatilities. These results are in line with previous research on VolTarget strategies, which are used as underlying of European-type derivatives.

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