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Stable Modeling of different European Power Markets

Christian Mugele, Svetlozar T. Rachev and Stefan Trück

Abstract

In this paper we address the issue of modeling spot prices of different European power markets. With the German, Nordic and Polish power markets we consider three markets at a very different stage of liberalization. After summarizing the stylized facts about spot electricity prices, we provide a comparison of the considered markets in terms of price behavior. We find that there are striking differences: while for the Nordic and German power exchange prices show heavy tails, spikes, high volatility and heteroscedasticity, returns of spot prices in the Polish market can be modeled adequately by the Gaussian distribution. We introduce the stable Paretian distribution to capture heavy tails, high kurtosis and asymmetries in electricity spot prices. We further provide ARMA/GARCH time series models with Gaussian and stable innovations for modeling the behavior of the different markets.

AMS Classification: 62P20; 62-07; 91B70; 62M10.
Key words: Stable distribution; Electricity prices; GARCH model; Time Series.

1. Introduction

The last decade has witnessed radical changes in the structure of the power markets in Europe. While the process of regulation and liberalization in some countries is still subject to current debate and legislation, market integration in the European Union requires harmonizing electricity markets. The first to start liberalizing their electricity markets in Europe, were England and Wales in 1990 (see Bundesministerium für Wirtschaft und Arbeit, 2003)). Thereby, power exchanges play an increasingly important role, since electricity has transformed from a primarily technical business, to one in which the product is treated in much the same way as any other commodity (Pilipovic, 1998). However, for the modeling of electricity prices and the valuation of electricity derivatives we cannot simply rely on models developed for financial or other commodity markets. Electricity is non-storable (at least not economically), which causes demand and supply to be balanced on a knife-edge. Relatively small changes in load or generation can cause large changes in price and all in a matter of hours, if not minutes. In this respect there is no other market like it. The special characteristics of electricity spot market prices are the motivation for this paper. More precisely, we describe and compare electricity spot markets' data of Nord Pool, the European Energy Exchange and Gielda Energii SA. Adequate models of price dynamics capturing the main characteristics of electricity prices are a key issue, since spot prices are one of the main factors not only for risk management but also for strategic planning and decision support systems of the market players.

The paper is set up as follows. Section 2 summarizes the stylized facts about spot electricity prices. In section 3 we compare the data by using descriptive statistics. We find that the data exhibit high kurtosis and heavy tails and remove deterministic effects and outliers. Section 4 reviews the most important facts on the stable distribution and illustrates the superior fit of the distribution to spot prices in comparison to the normal distribution. In section 5 we provide ARMA and GARCH models with focus on the different performance of Gaussian and stable Paretian processes for the innovations. We find evidence that the latter also performs better in time series modeling of energy price data. Section 6 concludes and makes some suggestions for future work.

2. Particularities of electricity spot prices

In contrast to other financial markets the spot electricity market is actually a day-ahead market. A classical spot market would not be possible, since the system operator needs advanced notice to verify that the schedule is feasible and lies within transmission constraints. The spot is nor-

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mally an hourly contract with physical delivery and is not traded on a continuous basis, but rather in the form of a conducted once per day auction. It is the underlying of most electricity derivatives.

Several countries have deregulated their power markets in the last decade. Among the considered countries the process started in the Nordic region in 1995. The last Nordic country to fully open its power market was Denmark in 2003. As a reply to EU Directive 96/92/EC Germany liberalised its market in 1998. Still, this shift in regulation and the idea of separating monopolistic and competitive activities are not completely implemented. Access to the network is still an obstacle to free trade, market concentration is high and state interventions persist for environmental reasons or as special economic help for former Eastern Germany. The latter results from the German reunification. This economic transition to capitalism is even more pronounced in Poland. The existence of long-term power purchase agreements that account for 54% of electricity purchases is the most prominent example. The liberalisation of the Polish power market is still in progress. It started in 2003 for consumers with a yearly purchase of over 10 GWh and will be finished in 2006 only. The differences in market matureness and liberalisation are also reflected in the establishment of the power exchanges. Nord Pool was established as soon as 1993 while GE SA and EEX followed in 2000 and 2002.

2.1. Seasonality

Due to the realtime balancing needs of electricity and the resulting strong dependency on cyclical demand electricity prices are very cyclical itself. This seasonal component in electricity prices is more pronounced than in any other commodity and several different seasonal patterns can be found in electricity prices during the course of a day, week and year. They mostly arise due to changing climate conditions, like temperature and the number of daylight hours. Also in some countries the supply side shows seasonal variations in output. Hydro units, for example, are heavily dependent on precipitation and snow melting, which varies from season to season. Thus, the seasonal fluctuations in demand and supply translate into the seasonal behavior of spot electricity prices.

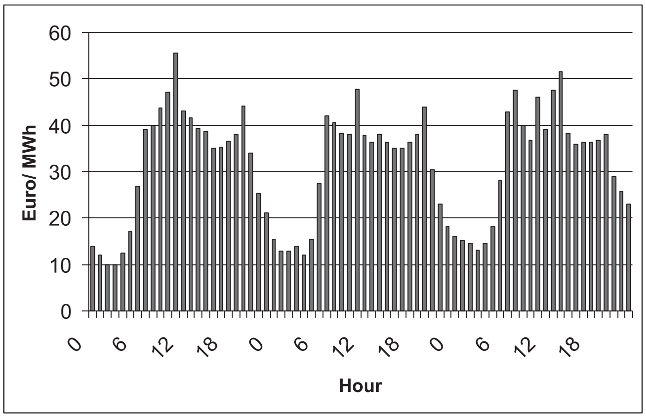

Fig. 1. EEX – Hourly spot prices from September 23 to September 25, 2002

An exemplary intraday pattern for the EEX can be seen in Figure 1 where the intraday evolution of hourly spot prices for a sample of three days from September 23 to September 25, 2002 at the EEX is plotted. Prices always start rising between 6 a.m. and 8 a.m. and peak during midday. Starting at about 6 p.m. to 8 p.m. prices start falling again. Similar effects can also be observed for a weekly as well as an annual seasonal pattern.

2.2. Volatility

Another stylized fact about electricity spot prices is the unusually high volatility of prices. The volatility seen in electricity prices is unprecedented in financial and other commodity markets. It is not unusual to observe annualized volatilities of more than 1000% on hourly spot. The high volatility can be traced back to the storage and transmission problems and the need for markets to be balanced in real time. Inventories cannot be used to smooth price fluctuations. Temporary demand and supply imbalances in the market are difficult to correct in the short-term. Therefore price movements in electricity markets are more extreme than in other commodity markets.

2.3. Mean reversion

Besides seasonality, electricity spot prices are in general regarded to be mean reverting (Schwartz, 1997). Mean reversion is a critical difference between the electricity and most financial markets. While interest rate markets exhibit mean reversion in a weak form, the actual rate of reversion appears to be related to economic cycles and is therefore slow. In electricity markets, however, the rate of reversion is very strong. The mean reverting nature of electricity spot prices can be explained by the markets fundamentals. When there is an increase in demand generation assets with higher marginal costs will enter the market on the supply side, pushing prices higher. When demand returns to normal levels, these generation assets with relatively high marginal costs will be turned off and prices will fall. It is this rational operating policies for generation assets that support the assumption of mean reversion of electricity spot prices. Thus, in the short-run, mean reversion results from the cyclical mean reverting nature of demand as the determinants of demand, the weather and climate are cyclical as well.

2.4. Jumps and Spikes

In addition to mean reversion and strong seasonality on the annual, weekly and daily level, spot electricity prices exhibit infrequent, but large jumps. The spot price can increase tenfold during a single hour. These spikes are the result of occasional outages or capacity limits of generation or transmission assets or a sudden, unexpected and substantial change in demand. Then demand reaches the limit of available capacity and the electricity prices exhibit positive price spikes. When the relevant asset is returned to service or demand recedes, prices rapidly revert to their previous levels. In periods where demand is reduced, electricity prices fall. Due to the operating cost or constraints of generators, who cannot adjust to the new demand level, negative price spikes can also occur. Spikes are normally quite short-lived, and as soon as the weather phenomenon or outage is over, prices go usually back to a normal level.

3. Descriptive statistics of the Data

In this section we will investigate three different markets in terms of spot price behavior. Nord Pool[1] is one of the best functioning power exchanges we could identify. A high market share compared to other non-mandatory exchanges, a high number of participants and a high variety of traded products can be observed. Moreover, it interconnects four different countries. The European Energy Exchange (EEX) in Leipzig is less developed but growing. Germany is the biggest producer of electricity in the European Union and liberalised its market completely in 1998. The least mature power exchange is Gielda Energii SA (GE SA). Market activity is still low and market opening is below fifty percent. However, electricity consumption in Poland is expected to grow by 2% a year during the next ten years (see Finish Energy Industries Federation, 2003). As Poland joined the European Union on May 1, 2004 an early investigation of the market also seems to be of high interest.

3.1. The data

A first comparison of the three data sets already indicates that there are big differences between the three markets. Spot prices at EEX and Nord Pool with their excess kurtosis are heavy tailed and skewed to the right. The limited number of observations clearly reduces the significance of results for GE SA but prices are obviously less 'extreme' with low kurtosis and a pretty symmetric shape.

Basic descriptive statistics of the observed data can be found in Table 1, while histograms for spot prices of Nord Pool and Gielda Energii Power Exchange are plotted in Figure 2.

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[1]Nord Pool operates in the Nordic region that consists of Denmark, Norway, Finland and Sweden.

Table 1

Spot Markets Data

	EEX	Nord Pool	GE SA
Unit	Euro/MWh	NOK/MWh	PLN/MWh
Mean	22,3338	132,7327	116,9176
Maximum	240,26	633,3642	145,6596
Minimum	3,47	21,2708	82,8708
Std. Dev.	11,5923	44,5758	15,81
Skewness	9,8952	1,271352	-0,2196
Kurtosis	167,3329	11,9688	2,0349
Observations	926	2101	152
Sample range	19/06/2000 – 31/12/2002	30/12/1996 – 30/09/2002	01/07/2002 – 30/11/2002

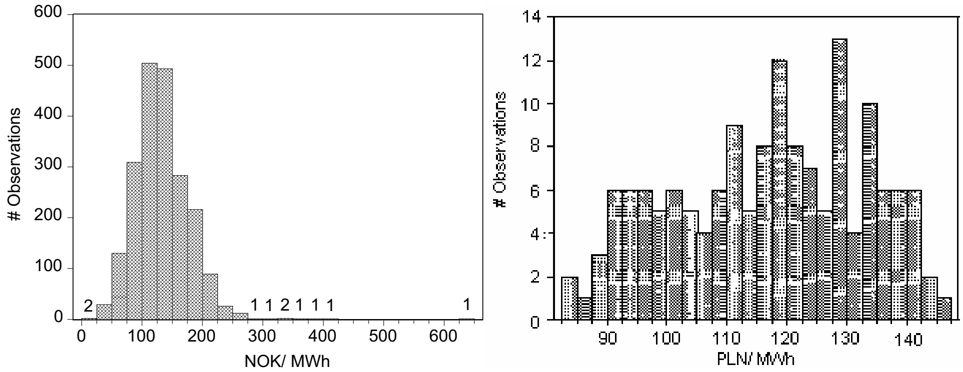

Fig. 2. Histograms of the raw data for the Nord Pool and Gielda Energii Power Exchange

3.2. Preprocessing

3.2.1. Outlier detection

Some of the used methods and measurements to describe and further analyze the data are very sensitive against outlying values, e.g. the autocorrelation coefficient. One might also consider such outlying values as market anomalies that cannot be captured in our analysis. When identifying seasonal and weekly patterns outliers would also disturb these results.

To identify an outlying value we create one time series for each weekday to take into account cyclic effects within one week. Every observation that deviates more than three times the standard deviation from the mean is considered to be an outlier. These observations are then replaced by the mean of the respective series. After one round we stop. This way we try to balance the disturbing influence that outliers have on the analysis and the disturbing influence induced by replacing extreme values.

Nord Pool

During the sample range nine outliers are identified. They all occur in winter, basically in January and February and break the upper constraint. The highest spot market price was reached on February 5, 2001. It was 633.36 NOK/ MWh while the average price in February is 152.56 NOK/ MWh. Outliers do not occur on a special weekday. Three blocks with two or more outliers within one week are observed. This gives evidence to heteroscedasticity.

European Energy Exchange

At the EEX outliers are also often observed sequently. Two blocks are striking, one in mid-January 2001 and another in January 2002. The high loads during the winter seasons are one technical explanation. However, high daily average prices were also paid in June, July and August

2002. This might be linked to the fact that in Germany short term regulating power is not widely available as in the hydro dominated Nordic system.

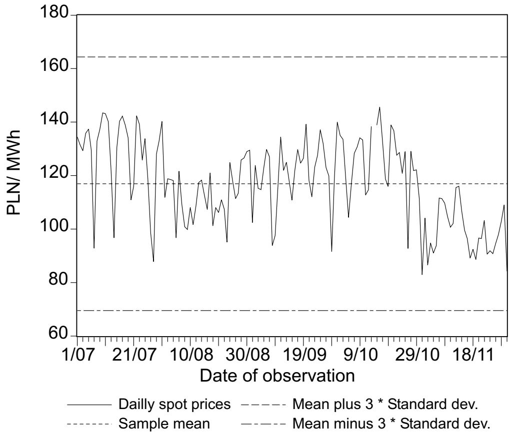

Fig. 3. Spot market prices, GE SA 01.07.2002-30.11.2002

Gielda Energii SA

Unalike, none of the daily spot prices as traded at GE SA is an outlier. To illustrate this, Figure 3 shows the daily spot prices at GE SA as well as the overall mean ± three times the sample standard deviation for all weekdays. However, the visual inspection of the graph gives evidence that there may be a structural break in the data considering the last two months. The short observation time makes such interpretations difficult.

3.2.2. Seasonal fluctuations

Electricity prices highly depend on cyclic effects such as seasons, weekdays and hours. Obviously, temperature, daylight, or rainfall vary and thereby show a cyclic behavior. We remove weekly and yearly effects assuming constant patterns of the form

$$x_t = m_t \cdot s_t \cdot \varepsilon_t ,$$

where x_t is the observation in time, m_t is the current mean, s_t is the seasonal effect and ε_t is a random error. This allows us to ignore those fluctuations when estimating the time series later on. The short sample range for the Polish power exchange allows to identify weekly effects only.

Two examples should illustrate the need to take into account those effects. Figure 4 shows the monthly average prices for Nord Pool as well as the daily average prices for GE SA.

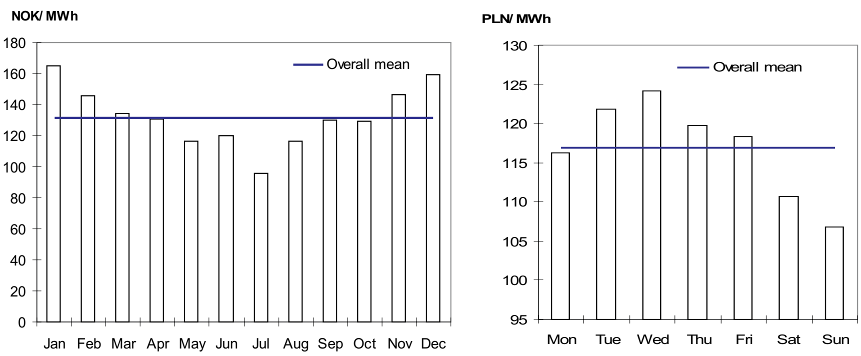

Fig. 4. Seasonal Patterns of Spot Prices (Nord Pool – Monthly average prices, GE SA – Daily average prices)

Shortcomings of this seasonal adjustment are that the fluctuations within one period should not be too big and that the number of observations for each period (especially for each month) should not differ. It is furthermore assumed that the errors ε_t are eliminated by calculating the arithmetic mean.

Weekly pattern

The weekly patterns are calculated for all three spot markets. The weekly patterns of all three markets show strong weekend effects. Prices are lower on Saturdays and Sundays what might be little surprising. Tuesday is the day with the highest prices at EEX and Nord Pool.

Table 2

$\hat{S}_{weekday}$

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Nord Pool	0.965	0.958	0.963	0.967	0.991	1.063	1.094
EEX	0.895	0.845	0.856	0.850	0.921	1.166	1.467
GE SA	1.002	0.957	0.938	0.973	0.985	1.053	1.091

Yearly pattern

The yearly pattern for Nord Pool is most reliable by virtue of the wider sample range. The Nord Pool data comprise almost six years whereas for EEX prices of two and a half years are considered.

Table 3

$\hat{S}_{month}$

	Jan	Feb	Mar	Apr	May	Jun
Nord Pool	0.7866	0.8903	0.9666	0.9902	11.136	10.813
EEX	0.8243	10.220	10.610	0.9531	11.299	10.345
	Jul	Aug	Sep	Oct	Nov	Dec
Nord Pool	13.566	11.145	0.9996	10.004	0.8866	0.8136
EEX	11.616	10.962	0.9223	10.334	0.9162	0.8455

Yearly patterns highly depend on the geographic location of the supply area. In California, for example, air-conditioning has a significant effect on electricity demand and as a result in prices. In such a region prices are high in summer and low in winter.

In the Nordic region heating and light lead to high monthly prices in January and December while prices decline during summer. Geography matters not only for the demand side. Using natural resources such as water also the electricity generation is affected.

At the EEX the resulting pattern is different. In April and September the annual effect is positive. A wider range might smooth the shape of this pattern as we can see no obvious reason for these two little peaks.

3.2.3. Preprocessed data

The preprocessing consisted of outlier replacement and seasonal adjustment. The resulting time series are the starting point for further modeling and estimations. They are plotted in Figures 5, 6 and 7 along with their first differences. The first differences are also referred to as returns in the course of this study[1].

[1]Some authors, however, use a different notation and define the returns r_t as the first differences of the natural logarithm of the prices ($\ln P_t - \ln P_{t-1}$) multiplied with 100, where P_t is the price at time *t*.

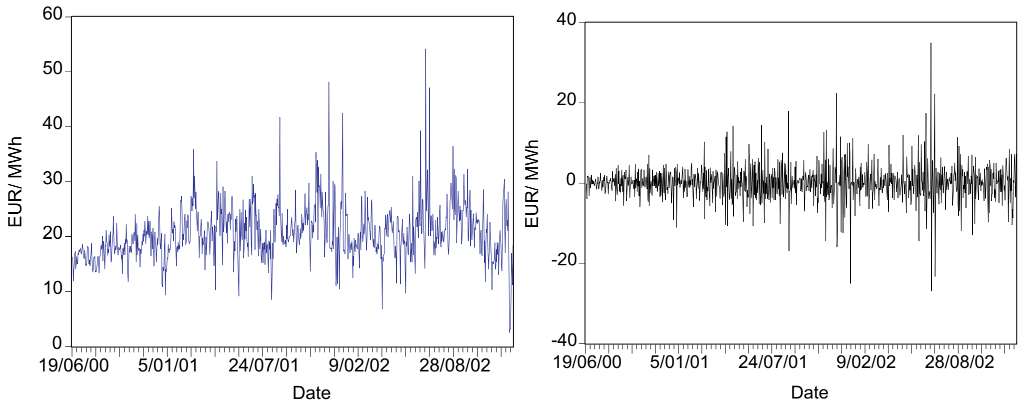

Fig. 5. EEX – Pre-processed data (Levels, Returns)

Volatility at the EEX seems to increase over time. This might be the result of an increasing market activity. The returns of the Nord Pool data as well as the one of the EEX data exhibit heteroscedasticity. That means that there are times of higher volatility. The visual inspection of the returns underlines the different behavior of the GE SA data in comparison to EEX and Nord Pool. The properties of the preprocessed data are summarised in Table 4.

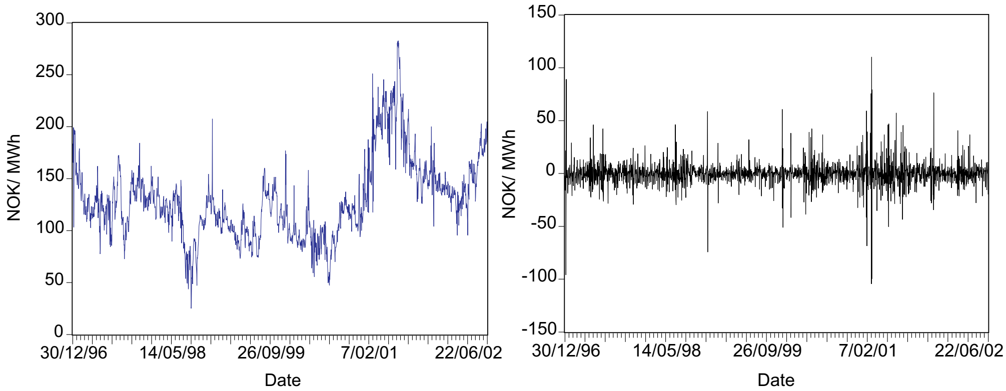

Fig. 6. Nord Pool – Pre-processed data (Levels, Returns)

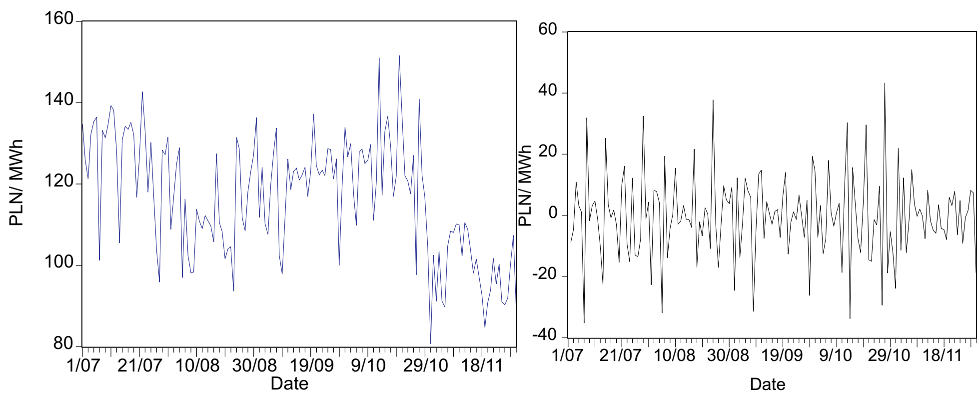

Fig. 7. Gielda Energii SA – Pre-processed data (Levels, Returns)

Table 4

Pre-processed data

	EEX	Nord Pool	GE SA
Unit	Euro/MWh	NOK/MWh	PLN/MWh
Mean	205.388	1.292.929	1.165.638
Maximum	541.636	2.827.303	1.516.295
Minimum	24.794	251.947	806.492
Std. Dev.	48.698	381.894	147.094
Skewness	10.789	0.9116	-0.1778
Kurtosis	83.773	43.111	22.907
Observations	926	2101	152
Sample range	19/06/2000 - 31/12/2002	30/12/1996 - 30/09/2002	01/07/2002 - 30/11/2002

4. Fit of the Stable and Normal Distributions

Before applying time series modeling we compare the goodness of fit on the spot market data of the normal and the stable distribution. The stable Paretian or α -stable distribution is therefore introduced first. In a variety of applications it was proofed that in general the stable distribution provides a superior fit to financial data compared to the Gaussian distribution, Rachev and Mittnik (2000). For risk management in energy markets the stable distribution was already tested by Khindanova et al. (2001).

4.1. Stable Paretian Distribution

The definition and basic properties of the stable Partian distribution or α -stable distribution are given in this appendix. A definition of the stable distribution can be found for example in Rachev and Mittnik (2000).

4.1.1. Definition

Let X be a random variable with stable distribution. The following theorem fully characterizes a random variable with stable distribution. Let X be a random variable. The following conditions are equivalent:

Let $a, b \in R^+$ and X_1, X_2 be independent copies of the random variable X. There exist $c > 0$ and $d \in R$ such that

$$aX_1 + bX_2 \overset{d}{=} cX + d, \tag{1}$$

where $\overset{d}{=}$ denotes equality in distribution.

Let n be a positive integer, $n \geq 2$, and $X_1, X_2, \ldots, X_n$ be independent copies of X. There exist $c_n \in R^+$ and $d_n \in R$ such that

$$X_1 + X_2 + \cdots + X_n \overset{d}{=} c_n X + d_n. \tag{2}$$

X has a domain of attraction, i.e. there is a sequence of i.i.d. random variables Y_i $with\ i \in N$, a real positive sequence d_i $with\ i \in N$ and a real sequence a_i $with\ i \in N$ such that

$$\frac{1}{d_n}\sum_{i=1}^{n} Y_i + a_n \xrightarrow{d} X ,$$

where $\xrightarrow{d}$ denotes convergence in distribution.

The characteristic function of X admits the following form:

$$E\left(e^{iXt}\right) = \begin{cases} \exp\left(-\sigma^{\alpha}|t|^{\alpha}\left[1 - i\beta sign(t)\tan\frac{\pi\alpha}{2}\right] + i\mu t\right), \text{if } \alpha \neq 1, \\ \exp\left(-\sigma|t|\left[1 + i\beta \text{sign}(t)\ln|t|\right] + i\mu t\right), \text{if } \alpha = 1, \end{cases} \quad (3)$$

The most common ways to define a stable Paretian distribution are definitions one and four.

4.1.2. Basic Properties

The parameters of a stable Paretian distribution describe the stability, skewness, scale and drift. Referring to Definition 4 these characteristics are represented by α , β , σ and μ . These parameters satisfy the following constraints:

α *is the index of stability* $(0 < \alpha \leq 2)$.

For values of α lower than 2 the distribution is becoming more leptocurtic in comparison to the Normal distribution. That means that the peak of the density becomes higher and the tails heavier.

When $\alpha > 1$, the location parameter μ is the mean of the distribution.

β *is the skewness parameter* $(-1 \leq \beta \leq 1)$.

A stable distribution with $\beta = \mu = 0$ is called symmetric α -stable ($S\alpha S$). If $\beta > 0$, the distribution is skewed to the right. If $\beta < 0$, the distribution is skewed to the left.

σ *is the scale parameter* $(\sigma \geq 0)$.

The scale parameter σ allows to write any stable random variable X as $X = \sigma X_0$ where X_0 has a unit scale parameter and α and β are the same for X and X_0.

μ *is the drift* $(\mu \in R)$.

To indicate the dependence of a stable random variable X from its parameters, we write:

$$X \sim S_{\alpha}(\beta, \sigma, \mu).$$

The stable Paretian distribution is reduced to the Normal distribution if $\alpha = 2$ and $\beta = 0$. Obviously the stable distribution offers more parameters to fit it to the actual data than e.g. the normal distribution.

The word *stable* is used because the shape is preserved (apart from scale and shift) under addition such as in Equation 1. The stable characteristic is also given under additional schemes (Maximum, minimum, etc.).

4.2. Estimated parameters and performance measures

Having introduced the stable Paretian distribution we compare its goodness of fit on the spot market data to the normal distribution. The parameters of the α -stable distribution are calculated using a numerical maximum likelihood method that was implemented by Stoyan Stoyanov from BRAVO Group. For further reading on maximum likelihood estimation we refer to Rachev et al. (2000)[1].

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[1]p. 91 ff.

We then compare the Kolmogorov distance (KD) and the Anderson-Darling (ANS) statistic of both estimations. The latter measurement is more sensitive against the goodness of fit in the tails while the Kolmogorov distance compares the maximum deviation of the empirical sample distribution from the estimated distribution function.

4.2.1. Original data

First, we refer to the original daily spot market prices that still incorporate the high third and fourth moments. Table 5 shows the estimates for both distritions on all three data sets.

Table 5

Estimates of unconditional distributions – Original data

		Parameters			
		α	μ	σ	β
Nord Pool		2	150.4494	88.8184	0
	Stable	1.5344	151.8318	31.0115	1.000
EEX	Normal	2	22.3067	11.5841	0
	Stable	1.7630	22.1589	4.4283	1.000
GE SA	Normal	2	116.9176	15.8121	0
	Stable	2.0000	116.9166	11.1462	-0.2694

The skewness of the EEX and Nord Pool data is very high. The parameter β which ranges from -1 to 1 reaches its maximum value in both cases. Again, the GE SA data differ. Very little and even negative skewness is indicated by the parameter β which is relatively close to zero. A behavior that might not be expected beforehand regarding the marginal production costs of electricity depending on the energy source which usually results in price spikes during peak times.

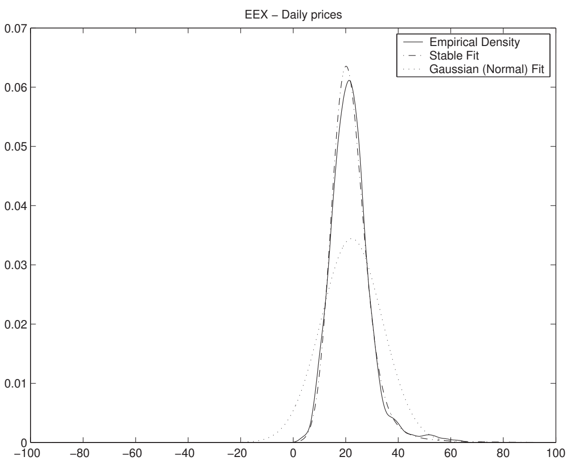

Fig. 8. EEX – Empirical and theoretical density functions

Table 6 presents the Kolmogorov distance and the Anderson-Darling statistic for the normal and the α -stable case.

Table 6

Goodness of fit of unconditional distributions – Original data

	Kd		Ans	
	Normal	Stable	Normal	Stable
Nord Pool	0.1772	0.2532	4872.4	0.7264
EEX	0.1618	0.1041	0.3466	0.2419
GE SA	0.0662	0.068	0.1901	0.1896

The results show that the α -stable distribution describes the original price data better according to these two measurements. Only in the -untypical- case of GE SA with its low market activity the normal distribution does not perform worse.

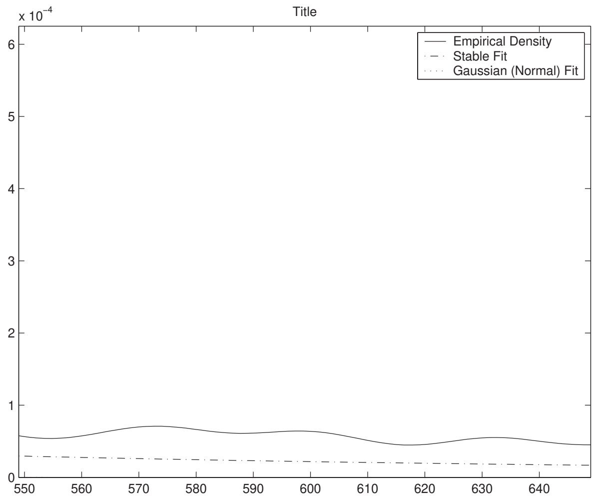

Fig. 9. Nord Pool – Empirical and theoretical density functions

4.2.2. Returns Preprocessed data

Second, the two distributions are fitted on the returns of the pre-processed data. The preprocessed data do no longer contain extremely high values and periodic weekly and yearly fluctuations are removed. Again, the estimated parameters, the Kolmogorov distances and the Anderson-Darling statistics are calculated. The parameters can be found in Table 7. The resulting goodness of fit is given in Table 8.

Table 7

Estimates of unconditional distributions – Preprocessed data

		Parameters			
		α	μ	σ	β
Nord Pool	Normal	2	0.0006	11.4745	0
	Stable	1.3664	-0.5626	4.1468	-0.1110
EEX	Normal	2	0.0023	4.5582	0
	Stable	1.5966	0.0203	2.2675	0.0407
GE SA	Normal	2	-0.3039	13.4711	0
	Stable	1.7110	-0.3827	8.2110	0.0571

The parameter α is the smallest in the case of Nord Pool. This reflects a high kurtosis. The skewness, indicated by parameter β, is not very distinct in all three cases. The reduced skewness mainly results from the fact that now the returns are examined. The parameter β when fitting the stable distribution on the preprocessed data is 1.000, 0.6417 and -0.2573 for Nord Pool, EEX and GE SA, respectively.

Comparing the goodness of fit, the α-stable distribution leads to better results than the Normal distribution in all cases.

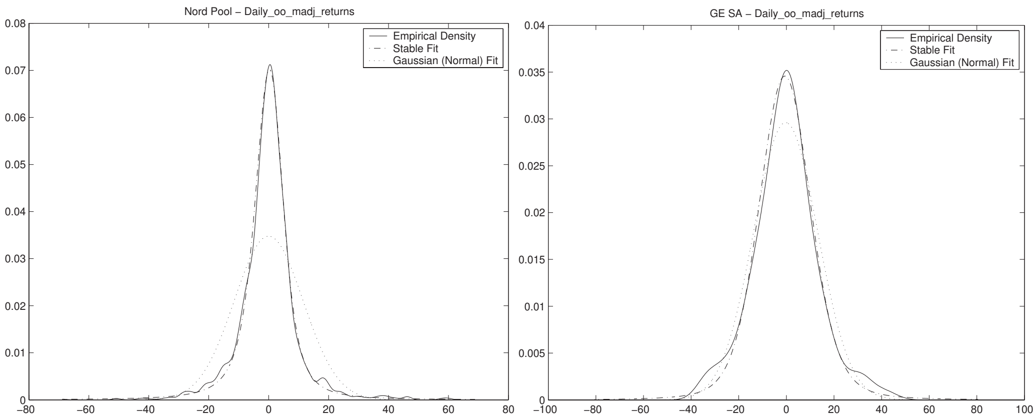

Fig. 10. Empirical and theoretical density functions (Nord Pool, GE SA)

Table 8

Goodness of fit of unconditional distributions – Preprocessed data

	Kd		Ans	
	Normal	Stable	Normal	Stable
Nord Pool	0.1439	0.0676	2362800.0	0.1344
EEX	0.0811	0.0186	26.574	0.0538
GE SA	0.0739	0.0499	0.1715	0.1069

4.3. Summary

This first quantitative description of the data reveals a high kurtosis for the EEX and Nord Pool data. Additionally, the data are skewed to the right and typical seasonal fluctuations can be observed. The GE SA data are very limited and consequently results are not very significant. Especially the low market activity[1] explains this contrasting behavior. Finally, the stable Paretian distribution was introduced and fitted on the original data as well as on the first differences of the preprocessed data. The comparison of this fit with the Normal case gives empirical evidence that the data are more adequately described by the α-stable distribution.

In the next section some models are presented that allow modeling the conditional mean and in a next step the conditional variance of a process.

5. Time Series Modeling

We now turn to time series modeling of the spot market data. *Autoregressive Moving Average* (ARMA) and *Generalized Autoregressive Conditional Heteroskedasticity* (GARCH) models are fitted on the returns. We look again at whether the residuals are more likely to be normally or α-stable distributed.

[1]The traded volume of about 1 TWh in 2002 corresponds to less than 1% of the electricity consumption in Poland. Moreover, considerable market activity can be observed on the Polish balancing market what reduces the interest in the spot market.

5.1. ARMA modeling

Before we start modeling an Augmented Dickey-Fuller (ADF) test as introduced by Dickey and Fuller (1979) is performed to test whether the data is stationary[1].

According to the ADF test[2] the hypothesis of a unit root in the returns is rejected at the 1% level for the raw and the adjusted returns (see Table 9).

Table 9

ADF test results

	Nord Pool	EEX	GE SA
Daily prices	-30.42 ***	-22.85 ***	-10.30 ***
Preprocessed data	-28.04 ***	-20.86 ***	-8.94 ***

ARMA models allow modeling a conditional mean. The mean of the next period $t+1$ depends on the information that is available up to the current period t. For the AR and MA terms a lag structure of up to four was considered. For a fixed AR order the best MA order was determined according to the Akaike criterion.

We finally choose these three models:

Nord Pool: ARMA(4,3) with AR(3) omitted

EEX: ARMA(3,3) with MA(2) omitted

GE SA: ARMA(3,4)

The estimation was done by using EViews 3.1. The results are provided in Table 10. The standard deviation of each parameter is given in parenthesis below each value. Parameters fixed to zero are indicated by 0.

Table 10

ARMA estimation results

	Nord Pool	EEX	GE SA
AR(1)	0.3264 (0.0421)	0.8221 (0.0520)	0.4604 (0.0672)
AR(2)	-0.5594 (0.0432)	0.1872 (0.0469)	-0.2583 (0.0723)
AR(3)	0 --	-0.0793 (0.0430)	-0.6500 (0.0657)
AR(4)	-0.1374 (0.0238)	-- --	-- --
MA(1)	-0.6144 (0.0420)	-1.3538 (0.0415)	-1.1443 (0.0126)
MA(2)	0.5609 (0.0343)	0 --	0.4940 (0.0210)
MA(3)	-0.1343 (0.0232)	0.3566 (0.0387)	0.5580 (0.0097)
MA(4)	-- --	-- --	-0.4578 (0.0334)

[1]Stationary is again used in the sense of weakly stationary.
[2]ADF test with lag 4, no trend and no intercept assumed.

The ARMA estimation assumes the residuals to be normally distributed. To test whether this assumption holds in the case of our data we fit the Normal and the stable distribution on the residuals. The estimated parameters are shown in Table 11.

Table 11

Estimated parameters of the ARMA residuals

		Parameters			
		α	μ	σ	β
Nord Pool	Normal	2	0.0403	10.6195	0
	Stable	1.5369	-0.0830	4.8379	0.0009
EEX	Normal	2	0.001	3.9259	0
	Stable	1.6158	-0.055	2.0156	0.0722
GE SA	Normal	2	-0.7282	10.4568	0
	Stable	1.7575	-1.0312	6.4485	-0.6301

If the residuals were normally distributed μ should be close to zero. To see whether the Normal or the α -stable distribution perform better their goodness of fit is compared. As before we calculate the Kolmogrov distance and the Anderson-darling statistic (see Table 12).

Table 12

Goodness of fit of ARMA residuals

	Kd		Ans	
	Normal	Stable	Normal	Stable
Nord Pool	0.0952	0.0124	8920000	0.0531
EEX	0.0812	0.0249	560.956	0.0769
GE SA	0.0736	0.0977	0.5319	0.2047

Again, the normal assumption does not verify. The goodness of fit is much better for the stable case. A visual inspection of the empirical and theoretical distribution functions as in Figure 11 underlines these results.

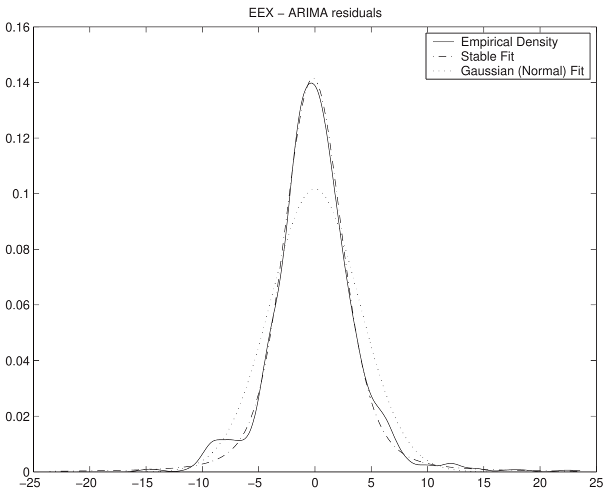

Fig. 11. EEX – ARMA residuals

As the autocorrelations (ACs) of the squared residuals are significant for the Nord Pool and EEX data[1] we model conditional variances using a GARCH model.

5.2. GARCH modeling

In the next step, the estimated ARMA models are extended. The Nord Pool and EEX time series are reestimated including a GARCH(1,1) term. GARCH models were first introduced by Engle (1982) and generalized by Bollerslev (1986).

Consequently, the estimated models are:

Nord Pool: GARCH(1,1) extended by ARMA(4,3) with AR(3) omitted

EEX : GARCH(1,1) extended by ARMA(3,3) with MA(2) omitted

The estimation results can be found in Table 13[2].

As an example we explicitly describe the resulting process for the EEX case:

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \alpha_3 X_{t-3} + \beta_1 \varepsilon_{t-1} + \beta_3 \varepsilon_{t-3} + \varepsilon_t$$

where ε_t follows a GARCH(1,1) process.

That means in our case that $\varepsilon_t = \sigma_t \cdot \upsilon_t$, with $\sigma_t^2 = \omega + \delta_1 \varepsilon_{t-1}^2 + \phi_1 \sigma_{t-1}^2$, where υ_t is i.i.d. N(0,1).

The GARCH terms are all significant. The assumption in the case of our GARCH model is that the innovations υ_t are normally distributed. To test whether this assumption holds the innovations have to be extracted out of the residuals.

Table 13

GARCH estimation results

	Nord Pool	EEX
AR(1)	0.9449	0.8357
	(0.0263)	(0.0427)
AR(2)	-0.0647	0.2010
	(0.0375)	(0.0532)
AR(3)	0	-0.1016
	--	(0.0417)
AR(4)	0.0680	--
	(0.0227)	--
MA(1)	-1.0938	-1.3450
	(0.0000)	(0.0299)
MA(2)	0.0912	0
	(0.0366)	--
MA(3)	0.0115	0.3492
	(0.0362)	(0.0262)
ω	21.1794	0.0736
	(1.1827)	(0.0444)
ARCH(1)	0.3660	0.0811
	(0.0265)	(0.0109)
GARCH(1)	0.4677	0.9261
	(0.02196)	(0.0091)

[1]They exceed the approximate two standard error bounds computed as $\pm\frac{2}{\sqrt{T}}$. If the AC is within these bounds, it is not significantly different from zero at (approximately) the 5% significance level.

[2]Standard deviation in parentheses.

To do so, we need to set a starting value for σ_1^2 for which we choose the variance of the residuals.

Equivalently to the ARMA residuals, we want to check whether the assumption that the innovations are normally distributed is realistic. Accordingly, we fit the Normal distribution on the innovations as well as the α -stable distribution. The results can be found in Table 14.

Table 14

Estimated parameters of the GARCH innovations

		Parameters			
		α	μ	σ	β
Nord Pool	Normal	2	0.0079	1.0118	0
	Stable	1.6143	-0.0163	0.5236	-0.0777
EEX	Normal	2	0.0199	0.9900	0
	Stable	1.7024	0.0153	0.5629	0.1565

The Kolmogorov distance and the Anderson-Darling statistic are then calculated to compare the goodness of fit. These values are presented in Table 15.

Table 15

Goodness of fit of GARCH innovations

	Kd		Ans	
	Normal	Stable	Normal	Stable
Nord Pool	0.0758	0.0355	26500	0.0726
EEX	0.0665	0.034	22.4231	0.0719

The resulting innovations υ_t show evidence that they are not normally distributed but that they follow an α -stable distribution. The goodness of fit that is provided is significantly enhanced when fitting the stable distribution in both cases. This is also underlined by the plot of the theoretical and empirical distribution functions that can be found in Figure 12. For this reason, the idea of a GARCH model assuming the innovations to follow a stable Paretian distribution should be further considered.

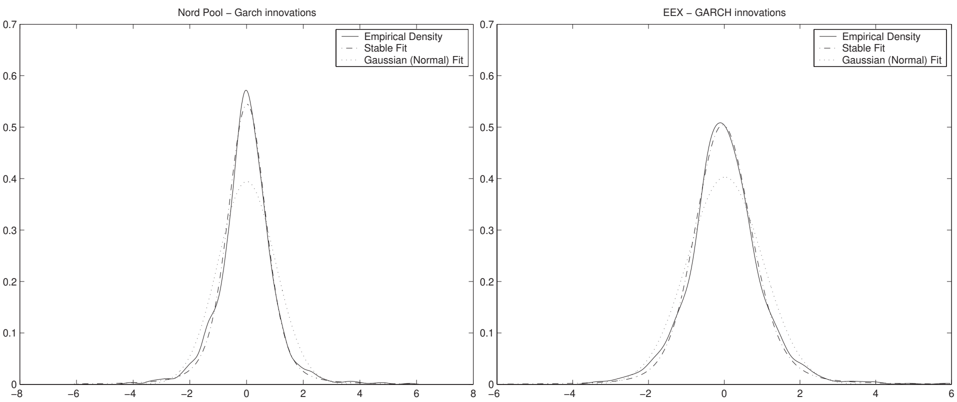

Fig. 12. GARCH residuals (Nord Pool, EEX)

5.3. Stable GARCH-M

As argued above the next step would be to estimate a GARCH model with stable innovations. However, Eviews 3.1 is not able to estimate such a model. For this reason we switch to Matlab 6.5 at this point to continue with a more advanced model. We use an extended program partly based on the UCSD GARCH toolbox for Matlab. For the normal as well as for the stable case we will estimate a GARCH-in-mean (GARCH-M) model and compare their performances in describing the data. The estimation procedures used in Matlab and Eviews are different. For example the applied Matlab toolbox includes linear constraints which are not included in Eviews. The default estimation method in Eviews for GARCH models is the Marquardt algorithm. It is a first derivative method, derivatives are computed numerically. In the applied Matlab routine they are computed analytically.

5.3.1. Definition

The GARCH-M model is an extension of the GARCH model. The general idea is that increases in the conditional variance are associated with increases in the conditional mean. Accordingly, the mean equation is extended.

The mean equation of a GARCH-M process is described by

$$X_t = c + \gamma f(\sigma_t) + \varepsilon_t \ . \tag{4}$$

The function $f(\sigma_t)$ defines in which way the equation depends on the conditional variance. Usually the standard deviation σ_t, the variance σ_t^2 or the logarithm of the the variance $\ln(\sigma_t^2)$ are used.

The variance equation does not change compared to a GARCH process and is given by

$$\varepsilon_t = \sigma_t \upsilon_t \ , \tag{5}$$

where $\upsilon_t \sim N(0,1)$ and σ_t^2 evolves according to

$$\sigma_t^2 = \omega + \sum \delta_i \varepsilon_{t-i}^2 + \sum \phi_j \sigma_{t-j}^2 \ . \tag{6}$$

In the stable case the normal assumption for the innovations υ_t is changed to the assumption of α -stable distributed innovations.

5.3.2. Modeling

First, we fit a normal GARCH-M model. We chose $f(\sigma_t)$ to be $f(\cdot) = \lambda \sigma_t$. This reflects the mean reverting property of electricity prices.

The resulting model is described by

$$X_t = \sum \alpha_i X_{t-i} + \sum \beta_j \varepsilon_{t-j} + \lambda \sigma_t + \varepsilon_t ,$$

where ε_t follows a normal GARCH(1,1) process as in Equations (7) and (8).

The resulting parameters can be found in Table 16. Standard errors of the parameters are in parentheses.

The results are very different compared to the previous estimations. However, the extension to a GARCH-M model should not be the reason. The reasons should be rather found in varying constraints and different estimation methods. The parameter λ is not significant in the EEX case. The extension to a GARCH-M model does not enhance the explanatory value of the model in this case. In the Nord Pool case all parameters are significant on a 1% level. However, λ is still rather close to zero.

Table 16

Normal GARCH-M

	EEX	Nord Pool
AR(1)	-0.79734	0.5291
	(0.0243)	(0.0509)
AR(2)	-0.29063	-0.70173
	(0.1231)	(0.0179)
AR(3)	0.25602	0.67242
	(0.0690)	(0.0058)
MA(1)	0.29802	-0.68536
	(0.0534)	(0.0350)
MA(2)	-0.31266	0.69499
	(0.1732)	(0.0222)
MA(3)	-0.59516	-0.77414
	(0.0799)	(0.0310)
λ	-0.00039	-0.011662
	(0.0124)	(0.0042)
σ	0.2701	15.792
	(0.1129)	(2.0956)
ARCH(1)	0.12502	0.35792
	(0.0166)	(0.0461)
GARCH(1)	0.87498	0.54518
	(0.0047)	(0.0361)

The resulting innovations are assumed to be normally distributed. We fit a stable distribution to these innovations. Table 17 shows the estimated values for the α -stable parameters. We find that the parameters are very different from the normal case and therefore we estimated a stable GARCH-M model.

Table 17

Estimated parameters

	α	β	σ	μ
EEX	1.7336	0.17248	0.57571	0.02153
Nord Pool	1.6189	-0.08482	0.52233	0.02311

Table 18

Log likelihood and standard error of regression

	EEX		Nord Pool	
	Log likel.	Std. error	Log likel.	Std. error
normal M-GARCH	-2499	39.848	-7600	10.955
stable M-GARCH	-2436	39.802	-7368	11.187

Table 19 shows the estimated parameters for the stable GARCH-M model. Standard errors in parentheses should be handled with care. As due to the infinite variance of the stable distributions the variance matrix of the standard errors cannot be interpreted exactly the same way as in the case of normally distributed innovations.

Using this approach the resulting model describes the data more adequately. The Log-likelihood value is enhanced for the EEX and Nord Pool data what can be seen in Table 18. Also the standard error of regression is better in the EEX case while it increases slightly for the Nord Pool data.

Table 19

Stable GARCH-M

	EEX	Nord Pool
AR(1)	-0.78359	-0.01374
	(0.0377)	(0.1818)
AR(2)	-0.16465	-0.16732
	(0.1407)	(0.1862)
AR(3)	0.29673	0.70763
	(0.0602)	(0.0498)
MA(1)	0.30244	-0.041222
	(0.0278)	(0.1178)
MA(2)	-0.43095	0.073051
	(0.1679)	(0.1441)
MA(3)	-0.61794	-0.78557
	(0.0884)	(0.0384)
λ	-0.00777	-0.013525
	(0.0072)	(0.0154)
σ	0.95771	20.646
	(0.2275)	(3.2819)
ARCH(1)	0.16175	0.43075
	(0.0296)	(0.0508)
GARCH(1)	0.80091	0.43127
	(0.0135)	(0.0469)

This way we demonstrate that in the case of the investigated spot prices the assumption of normally distributed innovations should be changed. Furthermore, we present a model with α-stable distributed error terms that leads to a more adequate description of our data and therefore is to be preferred to the normal model. The research on estimation methods for time series processes assuming the errors to be stable distributed should certainly be continued. Also the performance of the α -stable assumption against other distributions should be further examined.

5.4. Summary

The processes in this section allow to model conditional means and variances. Again, GE SA behaves very differently. The data are not very characteristic for electricity markets and higher market activity as well as a larger sample size would be necessary to reasonably model these data.

Against the Nord Pool and EEX data exhibit heteroscedasticity and the GARCH parameters are both significant. We had a special interest in the assumption that error terms are normally distributed. This empirical analysis clearly gives evidence that the α -stable distribution describes the resulting errors more adequately than the Normal distribution. Consequently, we fitted a model assuming the innovations to be α -stable distributed. Thereby we demonstrated that this model performs better in comparison to the normal case.

6. Conclusion and outlook

In this paper we addressed the issue of modeling spot prices of different European power markets. We found that there are quite striking differences between the least mature power exchange Gielda Energii SA in Poland and the German EEX or Nordic power exchange. While spot prices of GE SA could be modeled adequately by a Gaussian distribution, the more mature markets Nord Pool as well as the German EEX showed jumps and spikes as well as high volatilities and heteroscedasticity in spot price data.

Introducing the stable Paretian distribution, we found that for these markets it provided a superior fit to the returns of the spot prices. The reason is that the alpha-stable distribution is able to capture phenomena like heavy tails, high kurtosis and asymmetries in electricity spot prices. We further fitted a combined ARMA/ GARCH model to describe the time series behaviors of the three markets. Investigating the returns and error terms, we found that the assumption of normally distributed error terms does not hold. Again, the stable Paretian distribution gives a better fit also to the error terms, since they exhibit skewness and heavy tails.

In a last step a GARCH-M model assuming α -stable innovations was estimated. The comparison with the normal case clearly shows better log likelihood values what strengthens the argument to relax the normal assumption. The results recommend the use of heavy-tailed distributions for modeling electricity spot prices.

In future work the results should be compared to other approaches provided by the literature like jump diffusion or regime switching models (see e.g. Bierbrauer et al., 2004). Furthermore advanced stable models like a so-called ARMAX-GARCH-process with heavy-tailed innovations (Menn and Rachev, 2005) could be implemented for further spot price modeling and also derivative pricing.

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