"Capital Requirement, Portfolio Risk Insurance, and Dynamic Risk Budgeting"

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Capital Requirement, Portfolio Risk Insurance, and Dynamic Risk Budgeting[1]

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Abstract

Due to risk based capital requirements, financial institutions need to budget their risk-taking to assure their financial survival. This is necessary because the economic capital of the institutions which have to back risky positions is widely assumed to be a short resource. Therefore, financial institutes are advised to pursue a strategy which guarantees that a specified risk budget is never violated.

In this paper, we concentrate on the trading portfolios of financial institutions and develop a dynamic risk budgeting approach. We argue that the limitation of risk-taking should depend on actual profit & loss. Based on the standard modeling of financial market stochastics we provide a method of risk budget adjustment adopting the idea of synthetic portfolio insurance. By varying the strike price of an implicit synthetic put option we are able to keep within budgets accepting a certain default probability. Our approach comprises reducing capital requirements and the cost of regulatory capital.

Key words: Capital requirement, Conditional-value-at-risk, Portfolio insurance, Risk budgeting, Value-at-risk.
JEL classification: G21, G31, C10.

1. Introduction

The modern risk management of financial institutions increasingly applies methods of active risk controlling that base upon the measurement of market risk. Instruments of active risk controlling are, for example, hedging techniques and risk budgeting procedures. Motivations for active risk controlling were first driven by the increasing magnitude of market risk for the most part and as a result, the Value-at-risk concept has become the standard tool to specify risk. But although academics and practitioners undertook tremendous efforts to adequately measure shortfall risk, the questions of how to control and particularly how to budget this risk attracted surprisingly low interest.

In this paper, we develop a profit & loss-dependent, dynamic risk budgeting approach for financial institutions. Its aim is an optimal adjustment of risk budgets to reduce capital requirements and to reduce the costs of (regulatory) capital. Based on standard modeling of financial market stochastics, we provide a risk budget adjustment method adopting the idea of synthetic portfolio insurance. By varying the strike price of an implicit synthetic put option we are able to keep within budgets accepting a certain default probability.

The paper is organized as follows. Section 2 provides some important preliminary issues and motivates our aim. We analyze the capital requirement decision in Section 3 and characterize our modeling framework in Section 4. The dynamic risk budgeting approach is presented in Section 5 and applied within a simple simulation study in Section 6. Conclusions and practical implications are drawn in Section 7.

2. Preliminary remarks

In this section, we start on discussing some preliminary issues that seem to be important for further understanding. Why should financial institutions engage in risk management at all?

Why should they set risk budgets? And, why should the institutions adjust these budgets permanently?

While from the perspective of modern finance theory there does not exist any need for a company to hedge unsystematic risks of its cash flows because shareholders would diversify their investment risks on their own, there are, however, several economic reasons for doing so. Financial institutions may need risk management to reduce the costs of external capital. They may need it to lower the costs of financial distress and, by reducing earnings volatility, to avoid high tax liabilities[1]. In particular, banks and financial institutes face risk-based capital requirements so that hedging and budgeting risks may be preferred rather than raising additional capital.

The budgeting of risk-taking of a financial institution to sustainable levels is necessary due to its restricted risk-taking ability according to its endowment with capital. The capital reserve, hereinafter referred to as "risk capital" (see Section 3), is assumed to be a short resource and does not have transactional relations to other capital resources. Unlike the capital invested in risky assets, the risk capital is backing the investment[2]. From that point of view, risk capital is an increasing function of the risk of investment. The riskier the investment portfolio is the higher the risk capital needed to back it. The cap of acceptable risk is specified within the capital requirement decision as a risk budget.

According to modern portfolio theory, individuals aim at maximizing the expected risk premium for financial assets per unit of risk. As there is evidence that investors weigh negative returns or losses stronger than positive returns or gains (loss aversion, disposition effect), measures of risk that focus on the downside tail of the return distribution were incorporated into the portfolio selection theory[3]. From that advancement we know that risk itself becomes a function of the investor's risk aversion. For Value-at-risk as a measure of downside risk, Campbell et al. (2001) show that investors choose the allocation of risky assets first and then the amount of additional borrowing or lending to achieve some desired levels of downside risk. Hence, the two-fund separation holds like in the mean-variance framework. Furthermore, one can argue that the aimed level of downside risk together with the confidence level associated with it reflect the individual (absolute) aversion to risk. We interpret this target downside risk as the risk budget determined by the investor.

In that context, risk budget adjustments – given a constant confidence level of downside risk – could be explained by adjustments in risk aversion[4]. If one assumed the expected-utility-maximizing bank management to have a concave utility function[5] of risk capital and to be risk averse with constant relative risk aversion (CRRA)[6], the optimal share of risk capital provided to back risky investments would be independent of risk capital level. Then absolute risk aversion depends on the amount of risk capital available. The higher the amount of risk capital to dispose the lower the absolute risk aversion and the higher the risk taken[7]. Cumulative losses accompany decreasing risk capital (see Section 3). If relative risk aversion is constant, then absolute risk aversion must increase together with cumulative losses. This means lower acceptance of risk-taking and therefore lower risk budgets.

Another reason for which financial institutions should dynamically adjust risk budgets may be seen in the need for monitoring portfolio managers. Managers of trading portfolios act as agents to whom the bank management has delegated the portfolio selection decision. Compared with the bank management, portfolio managers usually have got better information about investment opportunities,

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[1] See, for example, Smith and Stulz (1985), Froot et al. (1993), DeMarzo and Duffie (1995), Stulz (1996), and recently Danielsson et al. (2002) for discussions of risk management incentives.

[2] For distinction between cash capital and risk capital see Merton and Perold (1993); for the link of risk capital and Value-at-risk see Kupiec (1999).

[3] Portfolio selection under shortfall constraints has its origin in Roy (1952). For portfolio selection under the Value-at-risk constraint see Campbell et al. (2001). Earlier see also Leibowitz and Kogelman (1991) who maximize expected portfolio return subject to the constraint that a minimum return is gained for a given confidence level over a given time horizon.

[4] Lo (1999) and Ait-Sahalia and Lo (2000), who incorporate risk aversion into the Value-at-risk framework, argue similarly.

[5] For justifying the concavity of the management's utility function in our context see Stulz (1984), Smith and Stulz (1985) and Froot et al. (1993).

[6] There is much evidence supporting that hypothesis. See, for example, the early works of Blume and Friend (1975), Friend and Blume (1975) and Kraus and Litzenberger (1976) as well as Szpiro (1986) or more recently Gollier and Zeckhauser (2002).

[7] Froot et al. (1993) and Froot and Stein (1998) find the absolute risk aversion of a financial institution to be a convex decreasing function of its equity capital endowment.

about the market, and so on. The portfolio managers themselves are assumed to maximize their individually expected utility of income, and since in most cases their income directly depends on their trading profits, they may maximize trading profits. So there may be the danger that portfolio managers take more risk than the institute is able to bear. The bank management, on the other side, may seek to maximize the bank's shareholder value under the constraint of capital adequacy requirements. It has to meet solvency criteria not only for regulatory but also for economic reasons such as the long-term survival of the institution. Therefore, and because profit & loss is a part of the short termed, first order risk capital of the bank (see Section 3), the management has to re-adjust its risk budgets set according to the current profit & loss situation of the institution.

3. Capital requirement

To determine the amount of capital needed to insure the solvency of the financial institute, i.e. to cover potential losses, risk measures for capital requirement decisions are developed. The capital requirement can be internal, motivated by risk management needs as discussed above, or regulatory. Measures of risk for capital requirement are also used to conduct internal capital allocation and to set risk budgets in the trading book. As the capital required to cover potential losses is determined by risk measures, it is also referred to as risk capital or economic capital[1]. The understanding of the term "risk capital" crucially depends on the properties of the accepted risk measure.

From the economic perspective, risk is meant to be the negative deviation from a planned or desired reference target, i.e. the shortfall below a certain benchmark return. In the following, we consider measures of risk which are special cases within the class of Stone's generalized risk measures. They model risk by the two-parameter function[2]

$$F(l,n) = \mathrm{E}\big[\max\{l - x,0\}^n\big] = \int_{-\infty}^{l}(l - x)^n f(x)dx, \qquad (1)$$

if F is the (single) return distribution of a portfolio with uncertain return X. Parameter l is the reference level from which deviations are measured, and parameter $n \in \mathrm{N}_0$ specifies the relative impact of large and small deviations. This type of measure of risk is referred to as shortfall risk measure. It defines what is known as the family of lower partial moments of order n.

In the need for risk measures for capital requirement decisions, instead of X we consider the portfolio's uncertain profit & loss $L(T)$ at a given time horizon T. The reduction of the degrees of freedom in (1) by setting $n = 0$ and $l = -\nu_p$ derives the well known zero lower partial moment

$$\mathrm{LPM}_{0,-\nu_p} = \int_{-\infty}^{-\nu_p} f_L(x)dx = F_L(-\nu_p) = p. \qquad (2)$$

If it clearly exists, for a given profit & loss distribution ν_p represents the p-fractile and as such a loss limit which is exceeded with respect to a (small) probability p. This measure is referred to as Value-at-risk[3] which is formally defined by

$$\nu_p := -\inf\{x \mid \mathrm{prob}(L(T) \leq x) \geq p\}. \qquad (3)$$

If Value-at-risk is accepted as a measure of risk for capital requirement, risk capital will be that amount of capital which is sufficient to cover potential losses at time T with at least probability $1 - p$.

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[1] See earliest Merton and Perold (1993).
[2] See Stone (1973) and Fishburn (1977).
[3] For the Value-at-risk see among others e.g. Duffie and Pan (1997), Linsmeier and Pearson (2000) and Jorion (2000).

Further, setting $n = 1$ and holding $l = -v_p$ lead to the well known first lower partial moment

$$\text{LPM}_{1,-v_p} = \int_{-\infty}^{-v_p}(-v_p - x)f_L(x)dx = -v_p\int_{-\infty}^{-v_p}f_L(x)dx - \int_{-\infty}^{-v_p}xf_L(x)dx = (-v_p + c_p)F_L(-v_p). \quad (4)$$

For a given profit & loss distribution, c_p is the conditional expectation below the specified level v_p. It defines the expected loss under the condition that loss falls short of the limit specified by Value-at-risk. This corresponds to the risk measure referred to as Conditional-value-at-risk[1] which is formally given with

$$c_p := -\text{E}\big[L(T) \mid L(T) \le -v_p\big]. \quad (5)$$

If Conditional-value-at-risk is accepted as a measure of risk for capital requirement, risk capital will be that amount of capital which is sufficient to cover potential losses at time T with probability $1 - p$ and additionally to cover potential losses at time T falling below Value-at-risk with probability p.

Value-at-risk seems to be a fair approximation of risk, and it has become a widely used industry standard. In the case of financial institutions it can be motivated through regulatory capital requirements. However, there is serious critique[2] of Value-at-risk as it does not have the properties which a meaningful (coherent) risk measure for capital requirement should possess. It violates the coherence axioms, in particular the subadditivity axiom, proposed by Artzner et al. (1999). Conditional-value-at-risk is proofed to be a coherent risk measure[3] and is therefore considered as a more appropriate means of regulatory control. A further and perhaps much more important point is the fact that in contrast to Value-at-risk Conditional-value-at-risk measures risk in the Rothschild-Stiglitz-manner, i.e. it is compatible with the criterion of second order stochastic dominance (SSD)[4]. Therefore, it seems to be a more appropriate measure for internal capital requirement decisions, too.

The distinction between risk capital and other concepts of capital is crucial to the risk budgeting approach presented afterwards. Independently of the concrete risk measure for capital requirement, risk capital itself has to be distinguished into different capital components according to costs and liquidity. The bank management may be interested in avoiding loss disclosures on the balance sheet and costs in terms of increasing refinancing rates, for example[5]. To compensate high probability-"normal" losses therefore preferably risk capital components are drawn which do not apparently affect equity. These high liquid, first order risk capital components may be thought of cumulative profits and valuation reserves in liquid assets. Management has not to fall back to second or third order risk capital components like open reserves and equity before low probability-extreme losses have to be compensated. As profit & loss is a part of the short termed, first grade risk capital of the institution, together with cumulating losses, risk capital and the ability of risk-taking are decreasing.

4. Modeling framework

In the following, we choose a comparatively simple modeling framework which allows for an unobstructed view on the structure of the problem and the derived results. We consider a complete and arbitrage-free capital market where the institute trades a portfolio, e.g. we think of a stock market index, in continuous time. The market value $S(t)$ of that risky asset at time t is modeled by a geometric Brownian motion

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[1] For the Conditional-value-at-risk see (with different definitions) Artzner et al. (1999), who call it tail conditional expectation or Tail-value-at-risk, Tasche (2002), who calls it expected shortfall, and Rockafellar and Uryasev (2002).
[2] For a comprising critique on Value-at-risk see, for example, Szegö (2002).
[3] See Acerbi and Tasche (2002).
[4] See Rothschild and Stiglitz (1970).
[5] See Stulz (1996) and Ahn et al. (1999).

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \qquad (6)$$

with drift μ, standard deviation σ, and $dW(t)$ being the increment of a (standard) Wiener process. Further, a risk-free investment exists. The value $B(t)$ of which follows the dynamic

$$dB(t) = rB(t)dt, \qquad (7)$$

where r denotes the risk-free return.

The market risk which the financial institution faces can be identified as potential losses in portfolio value caused by price changes in the risky asset. We assume the institution's risk management and capital requirement criterion is a measure of downside risk.

First, Value-at-risk is taken into consideration for internal capital requirement decisions. Thereby, we define the future portfolio loss relative to the future portfolio value that results from investing its time t value in the risk free asset. That future value specifies the natural benchmark because a risk free portfolio yields a Value-at-risk of zero. As in our modeling framework the conditional distribution of asset prices is log-normal and the conditional distribution of asset log-returns is normal, Value-at-risk can be easily estimated.

Value-at-risk at time t for a given probability p and a given progress in time T calculates along the price process (6) to

$$v_p(t) = S(t)\exp(rT) - S(t)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)T + z(p)\sigma\sqrt{T}\right), \qquad (8)$$

where $z(p)$ is the p-fractile of the standard normal distribution.

If, and only if, the assumptions made above hold, Value-at-risk will fulfill the subadditivity axiom[1]. Therefore, Value-at-risk applies for our purposes to serve as an internal capital requirement and risk budgeting criterion only in this special case.

Second, we take Conditional-value-at-risk into consideration for internal capital requirement decisions. Without any restrictive assumptions it generally exhibits a coherent measure of risk. In our modeling framework and with the same definition of relative future portfolio loss as applied to Value-at-risk above, it can be easily estimated, too.

Conditional-value-at-risk at time t for a given probability p and a given progress in time T calculates along the price process (6) to

$$c_p(t) = S(t)\exp(rT) - S(t)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)T + \frac{1}{p}\phi(z(p))\sigma\sqrt{T}\right), \qquad (9)$$

where $z(p)$ is the p-fractile of the standard normal distribution and ϕ represents the probability density function of the standard normal distribution.

5. Dynamic risk budgeting

In this section we develop a dynamic risk budgeting approach. Therefore we adopt the idea of synthetic portfolio insurance. Since the works of Leland (1980) and Rubinstein and Leland (1981) we have known that options can exactly be replicated. They showed that a put option can be duplicated at every time by trading the underlying asset and the risk free investment, e.g. a (near) risk free government bond. That is, because in case of a geometric Brownian motion for the asset process both the option and the underlying asset linearly depend on a single source of market risk[2]. The duplication portfolio consists of a short position in the underlying asset and a long position in the risk free bond. Option delta is thereby calculated within the well known model of Black and Scholes (1973). So, in that "classical" option based portfolio insurance, portfolios are not

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[1] For the proof see Read (1998).
[2] The assumption of other stochastic processes than geometric Brownian motion for the underlying asset leads to the failure of the duplication strategy because then usually more than one source of market risk exists. See, e.g., for application of jump-diffusion processes Merton (1976) or for stochastic volatility Hull and White (1987).

hedged by put options but the hedging effect is achieved by dynamic reallocation of the invested capital between the risky asset and the risk free bond.

For simplicity, we assume options to be priced according to the Black-Scholes model. The market price $P(t) = P(S(t), X, r, T, \sigma)$ of a put option at time t equals

$$P(t) = X \exp(-rT)\Phi(d_1) - S(t)\Phi(d_2), \tag{10}$$

$$d_1 = \frac{\ln\left(\frac{X}{S(t)}\right) - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = \frac{\ln\left(\frac{X}{S(t)}\right) - \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

where X denotes the strike price and $\Phi(.)$ represents the cumulative standard normal distribution. We continuously calculate the delta factor of this put option as

$$\Delta(t) = \frac{\partial P(S(t), X, r, T, \sigma)}{\partial S(t)} < 0. \tag{11}$$

The put delta expresses the sensitivity of the market price of the put option with respect to changes in the market price of the underlying asset. The reciprocal of the delta factor ceteris paribus indicates the number of put options needed to completely neutralize the price change per asset over the next infinitesimal time step.

As it is known, the put option can be duplicated by a portfolio consisting of a short position in the underlying asset amounting to

$$-\Delta(t)S(t) = S(t)\Phi(d_2), \tag{12}$$

and a long position in the risk free bond amounting to

$$B(t) = X \exp(-rT)\Phi(d_1). \tag{13}$$

Hence, for the synthetic put option follows

$$P(t) = B(t) + \Delta(t)S(t). \tag{14}$$

As the synthetic put option itself partly consists of a short position in the asset, hedging with synthetic put options means reducing the risky position in assets in favor of a risk free position in bonds. Then, the hedged portfolio has a quota in the asset amounting to

$$a(t) = \frac{(1 + \Delta(t))S(t)}{(1 + \Delta(t))S(t) + B(t)} \tag{15}$$

and a quota in the risk free bonds amounting to $1 - a(t)$.

For the risk budgeting problem recall that we assumed the financial institute to calculate its capital requirement with $v_p(t)$ or $c_p(t)$. Independently of the measure of risk used, we call the initially required risk capital the risk budget $\delta_p(t)$. Now, the described portfolio insurance procedure is not implemented really but implicitly. Further, we just adopt the results obtained, leaving the risky position in the asset at the portfolio manager's charge, and adjust the risk budget $\overline{\delta}_p(t = 0)$ initially set by the management at time $t = 0$ in continuous time.

The dynamic risk budget adjustment of a trading portfolio proceeds as

$$d\delta_p(t) = \begin{cases} (1 - a(t))\delta_p(t)dt, & \sum L \leq 0 \\ \\ (1 - a(t))\delta_p(t)dt + dL(t), & \sum L > 0 \end{cases}, \quad t > 0 \tag{16}$$

and describes an implicit portfolio risk insurance.

Risk budgeting is made in such a way that the risk budget decreases with cumulating losses, and vice versa. The strike price of the implicit synthetic put option is set at the lowest value bound of the portfolio given by the initial risk budget. At time $t = 0$, the risky position maximally

possible in the asset is much greater than the strike price. The option is well "out of the money" and its delta factor is near zero. At the same time, d_1 becomes very small, and hence the value of the cumulative standard normal distribution becomes nearly zero. From this follows that $B(t = 0)$ becomes zero, and $a(t = 0)$ becomes one. Thus this means, the risk budget is completely available at the beginning. If the risky asset rises in market value the financial institution will make profits, and the risk budget will expand at these profits because the risk capital will increase at this amount. $B(t)$ remains unchanged at zero, and $a(t)$ remains unchanged at one. Whereas, if the risky asset falls in market value and the financial institution is making losses, the synthetic put option will move more and more towards "at the money". Thereby, both the delta factor and the quota in the asset of the hedged portfolio are declining. The risk budget is reduced then due to the lower $a(t)$ factor.

We define the strike price of the implicit put option as

$$X \in \left(0, \frac{\overline{v}_p(t=0)}{\exp(rT) - \exp\left((\mu - (1/2)\sigma^2)T + z(p)\sigma\sqrt{T}\right)} - \overline{v}_p(t=0) \right] \quad (17)$$

or

$$X \in \left(0, \frac{\overline{c}_p(t=0)}{\exp(rT) - \exp\left((\mu - (1/2)\sigma^2)T + (1/p)n(z(p))\sigma\sqrt{T}\right)} - \overline{c}_p(t=0) \right]$$

whereby the upper bound of the interval marks the lowest accepted bound in portfolio value given by the initial risk budget. If the strike price equals the upper bound of the interval defined in (17), the portfolio will be hedged at its lowest accepted value bound and the risk budget will keep that level. Therefore, we call it portfolio risk insurance.

That portfolio risk insurance implies that the lowest accepted value bound would never be violated. But by calculating the risk capital requirement at a confidence level of p we initially accepted a default probability of the risk budget of $1 - p$. Therefore, we desire to achieve that level of default. Ahn et al. (1999) provide a model of optimally hedging a given risk exposure under a Value-at-risk constraint using options. In the same setting of a complete and arbitrage-free capital market they show that hedging costs are independent of the strike price of the put option used. The optimal strike price rather depends on the riskiness of the asset, the time horizon of the hedge, and the confidence level desired by the management. Thus, by varying the strike price of the implicit synthetic put option it is possible to determine the level of portfolio protection. Reducing the strike price continually yields to accept the fall of the implicit hedge below the lowest value bound of the portfolio with increasing probability. Now, we can reduce the strike price as long as the default probability associated with the risk budget set is achieved. Then, the risk budget will be violated by this probability.

6. Simulation analysis

In this section, we test the proposed dynamic risk budgeting approach within our analysis framework. For the price process of the single risky asset, e.g. a stock market index, we assume a mean per time unit of $\mu = .0005$ and a standard deviation per time unit of $\sigma = .015$. The start price of the asset is set at $S_0 = 100$ and the risk free rate at $r = .03$. In 7,500 test runs we calculate price processes with 256 time steps each. In parallel, we apply dynamic risk budgeting by using an implicit synthetic put option with a strike price at the lowest accepted bound in portfolio value. The risk budget is calculated at a five percent confidence level applying both Value-at-risk and Conditional-value-at-risk as measures of capital requirement. Clearly, the requirement of risk capital will always be higher if the latter is accepted, as Figure 1 shows. We observe an average capital requirement of 2.243 in case of Value-at-risk and of 2.977 in case of Conditional-value-at-risk.
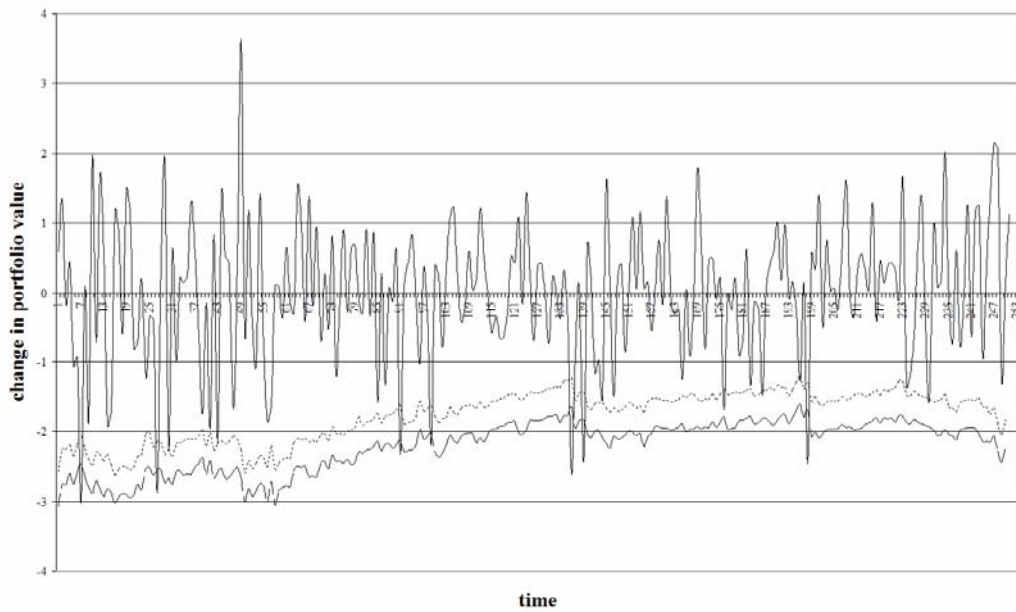
Fig. 1. Change in portfolio value and capital requirement applying Value-at-risk (---)
and Conditional-value-at-risk (**---**)

Now, we hold the first measured capital requirement (2.538 or 3.178, respectively) as the initial risk capital and apply the proposed dynamic risk budgeting which depends on the profit & loss development of the portfolio. Comparing the dynamic behavior of the portfolio values, the risk budgeting approach shows the expected properties. In Figure 2 we draw an example of an asset price process. At high losses the proposed dynamic risk budgeting strategy reduces the risk budgets and we achieve an assurance of the portfolio value on the level of the strike price of the implicit option. At the same time we participate in increasing asset values. The portfolio value resulting from the risk budgeting strategy lies always above the value which we had without such a strategy.
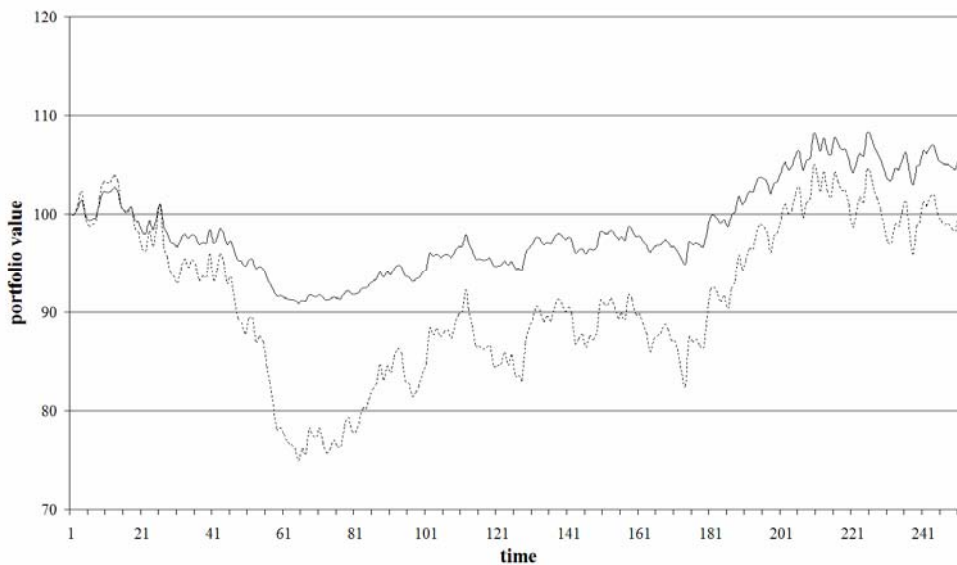

Fig. 2. Portfolio value with (—) and without (---) dynamic risk budgeting

In Figure 3 we draw the resulting probability density of the portfolio profit & loss in the case of dynamic risk budgeting against the case of a constant risk budget over time.
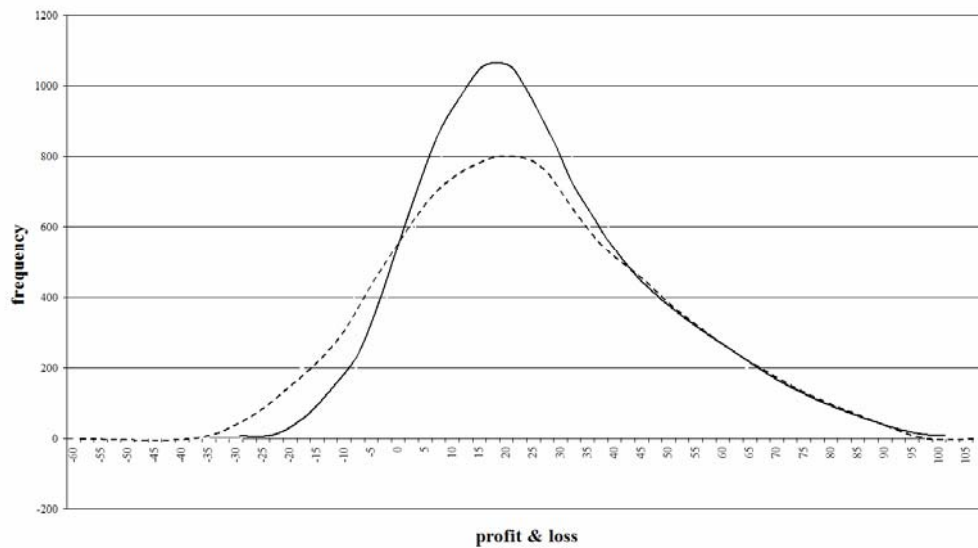

Fig. 3. Profit & loss density with (—) and without (---) dynamic risk budgeting

By applying the dynamic risk budgeting strategy the distribution of portfolio profit & loss becomes more asymmetric. As Figure 3 suggests, Table 1 shows that the skewness and kurtosis of the distribution are increasing. This is due to the portfolio insurance property. There, probability mass is moved from the left tail of the distribution to its center.

Table 1

Higher moments of the profit & loss distribution with and without dynamic risk budgeting

	Without budgeting strategy	With budgeting strategy
Skewness	0.1786	0.2903
Kurtosis	3.7304	5.2167

By this finding we can conclude that the objective of risk management to avoid lower-tail losses could be achieved. The results from applying the dynamic risk budgeting procedure suggest that there is a reduction in capital requirement and subsequently a reduction of cost of regulatory capital for the financial institute.

In order to stay close to the desired confidence level of risk measurement we now accept a default probability of the risk budgeting strategy at the confidence level of the initial risk budget, i.e. five percent. We reduce the strike price of the implicit synthetic put option and find that a strike price of half the lowest asset value bound results in a default probability that equals the confidence level of the applied risk measures for capital requirement. Table 2 makes the relations clear.

Table 2

Strike price of implicit synthetic put option and default probability of risk budgeting

Strike as percent of lowest value bound	100	95	90	85	75	50
Default probability	0.0054	0.0185	0.0294	0.0349	0.0446	0.0498

7. Conclusion

Depending on actual profit & loss of a risky portfolio we develop a dynamic risk budgeting approach in this paper. We provide a method of risk budget adjustment adopting the idea of synthetic portfolio insurance with an implicit put option. From shortfall risk measures for capital requirement we define the risk capital of a financial institute. Risk capital itself is enlarged by profits and shortened by losses resulting from risky investments. Based on a standard modeling framework we show the portfolio insurance properties of our approach. By accepting a default probability of the risk budget according to the confidence level of the risk measure we are able to lower the strike price of the implicit put option and hence to lower the portfolio insurance level.

For financial institutions facing risk based capital requirements the situation has improved. Our approach announces both reducing requirements of risk capital and reducing costs of capital.

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