"Threshold Autoregressive Modeling of Bond Series - Japanese Case"

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THRESHOLD AUTOREGRESSIVE MODELING OF BOND SERIES – JAPANESE CASE[1]

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Abstract

In this paper, I explore the presence of non-linearity for the daily series of 10-year Japanese government bond (JGB) yields by using the Tsay (1989) test, I find the threshold non-linearity due to a significant change in Japanese debt management policy.

I test to find the sixth lag of the series as the threshold variable, then locate the threshold, and estimate a 2-regime self-exciting threshold autoregressive model (SETAR) for this time series.

Key words: non-linearity, time series, threshold variable, self-exciting threshold autoregressive model, and Japanese debt management policy.

JEL classification: G12, C22, C51, G18.

1. Introduction

The linear Gaussian models such as AR models, ARMA models and ARIMA models have been proved by many previous researches that they are not ideally suited for modeling some financial time series that exhibit asymmetry, limit cycles and jump phenomena, in other words, non-linearity. Econometricians and statisticians therefore resort to non-linear models to interpret financial time series. The threshold autoregressive model (TAR) introduced by Tong and Lim (1980), among the family of non-linear models, has been found to best capture asymmetries, limit cycles and jump phenomena in the dynamic structure of economic and financial time series.

Besides, TAR models are pretty popular in the non-linear time-series literature, and they are relatively simple to specify, estimate, and interpret compared with many other non-linear time-series models.

The non-linearity in stock prices and exchange rates has been often detected by various statistical tests (Hinich and Patterson, 1985; Scheinkmann and LeBaron, 1989; Hsieh, 1989, 1991; Crato and de Lima, 1994; Brooks, 1996). However, only few attempts have been made to subsequently model the non-linearity explicitly.

In this paper, I explore the threshold non-linearity for the daily series of 10-year JGB yields due to the extraordinary change in Japanese debt management policy and estimate a 2-regime SETAR model for the time series.

I find strong evidence for a TAR model using the sixth lag of the series as the threshold variable, and estimate the threshold. Finally, I conclude that the autoregressive structure of JGB yields changes once during the studied time period.

The remainder of the paper is organized as follows. The next section introduces TAR models. Section 3 describes the characteristics of the daily series of 10-year JGB yields. In section 4, I apply a SETAR model to the series. I first introduce both the theory and application of the Tsay (1989) test of non-linearity against linear autoregressive (AR) models hypothesis, then I estimate a 2-regime SETAR model. I also do model checking by calculating the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of standardized residuals of the model. The final section contains a brief conclusion.

2. The TAR Models

The TAR models use threshold space to improve linear approximation. The basic idea of threshold models is the introduction of regimes via thresholds, in other words, the local approximation over states. If I name this idea as the threshold principle, I may group a number of finite parametric non-linear time series models under the threshold principle. The principle allows the analysis of a complex stochastic system by decomposing it into simpler subsystems. A TAR model is a generalization of an AR model which permits for different regimes for the series depending on its past values.

TAR models have been successfully applied to model non-linearities in financial variables such as exchange rates, volatility of return and arbitrage trading. For example, a TAR model of exchange rates explains an inner regime of sluggish adjustment for small disequilibria ─ or small deviations from some long run equilibrium path or attractor and an outer regime of mean reversion comprising large deviations.

The important application of TAR models in volatility is to handle the asymmetric responses in volatility between positive and negative returns. TAR models can also be used to study arbitrage trading in index futures and cash prices.

Besides financial variables, TAR models have also been used successfully to explore asymmetries in macroeconomic variables such as unemployment, GNP, etc., over the course of the business cycle. In this respect there is a question of whether the apparent persistence in an economic time series such as GNP or unemployment provides evidence of asymmetries that standard Gaussian linear parameter models cannot accommodate (Neftci, 1984).

A TAR model has several characteristics: first, geometrically ergodic and stationary. Second, the series exhibits an asymmetric increasing and decreasing pattern (Tsay, 2002).

3. The Characteristics of the Daily Series of 10-year JGB Yields

Some characteristics of JGB yields time series include:

(a) They tend to move cyclically with Japanese business cycles.

As I can observe, when economy is growing rapidly, investors dump bonds and invest in stocks, so bonds prices go down, therefore bonds yields go up; when economy is in down turn, the demand for bonds is high, this pushes bonds prices up, so bonds yields go down.

(b) The rates rise slowly, but decay quickly.

In the JGB market, there is one bond issue that is designated as the liquid bond at any given time. The liquid bond is usually called the benchmark bond. The benchmark bond is chosen from 10-year government bonds. Moreover, the Ministry of Finance and the Bank of Japan have a strong enough negotiating position to impose terms on the underwriting syndicate of government bonds.

When the economy is over-heated, the benchmark yield is raised gradually by the action that Japanese debt management authority undertake in order to give the economy a soft landing. While when economy needs to be heated up, the benchmark yield is pressed down relatively quickly serving as a stimulating tool.

The latter characteristic suggests that the dynamic structure of the 10-year bond yields series exhibits an asymmetric increasing and decreasing pattern, in other words, nonlinearity.

Besides, by checking the time plot of the bond yields series (Figure 1), I find that the conditional mean of the bond yields changes over time. This is an evidence of regime change.

Fig. 1. The Time Plot of the Daily Series of 10-year JGB Yields (November 17, 2002-July 18, 2004)

Furthermore, the histogram of the series (Figure 2) confirms the asymmetry of the distribution of the series.
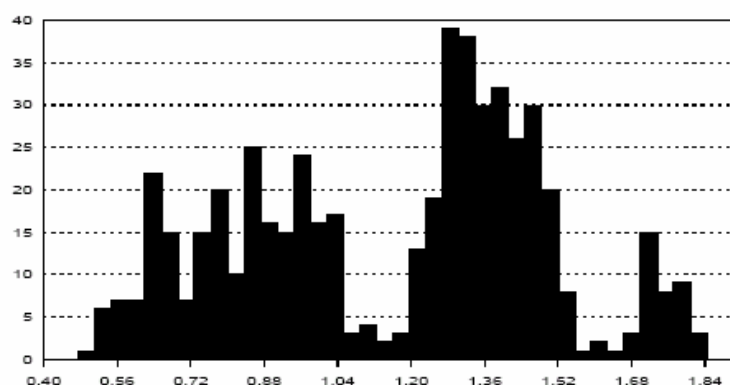

Fig. 2. The Histogram of the 10-year JGB Yields Series (November 17, 2002-July 18, 2004)

Table 1

	The Distribution of the JGB Yields Series	The Standard Normal Distribution
Standard Deviation	0.366	1.000
Skewness	-0.132	0.000
Kurtosis	-0.978	3.000

Note: The skewness is a measure of the asymmetry of the distribution of a series. The negative skewness implies that the left tail of the distribution is fatter than the right tail, or that large negative yields tend to occur more often than large positive ones. The kurtosis measures the fatness of tails of the distribution of a series.

Last, by comparing the standard deviation, skewness and kurtosis of the JGB yields series with those of the standard normal distribution (Table 1), I conclude that standard Gaussian linear parameter models cannot fully explain the behavior of the JGB yields series.

4. SETAR Modeling of the 10-year JGB Yields Series

A time series Y_t is a SETAR process if it follows the model

$$Y_t = \beta_0^{(j)} + \sum_{i=1}^{p} \beta_i^{(j)} Y_{t-i} + \varepsilon_t^{(j)} \,, \tag{1}$$

$$\text{if } \gamma_{j-1} \leq Y_{t-d} < \gamma_j$$

where $j = 1, \dots, k$
and d are positive integers. The thresholds are

$$-\infty = \gamma_0 < \gamma_1 < \dots < \gamma_k = \infty \,.$$

The superscript j is used to signify the regime, $\varepsilon_t^{(j)}$ are i.i.d. sequences with mean 0 and variance σ_j^2 and are mutually independent for different j. The parameter d is referred to as the delay parameter or the threshold lag. The SETAR model is non-linear provided that $k > 1$.

Such a process partitions the one-dimensional Euclidean space into K regimes and follows a linear AR model in each regime. When there are at least two regimes with different linear models, the overall process Y_t is non-linear.

4.1. A Test for Threshold Non-linearity

In order to apply a threshold model to the bond yields time series, it is necessary to assess the need for a threshold model for the data.

4.1.1. Tsay's (1989) test
In this section, I test threshold non-linearity of the JGB yields time series.

In order to test linearity against non-linearity of switching type, I can apply many techniques developed for testing parameter constancy against structural change. For example, a linear model whose parameters change once at a given point of time is piecewise linear in the same way as a switching regression model with two regimes. The available observation vectors (assuming i.i.d. errors or a martingale difference error process) may be rearranged in the ascending or descending order according to the threshold variable or the switching variable. If this is done, linearity tests may be obtained using ideas previously applied to detecting structural change (Granger and Tersvirta, 1993).

The proposed test is a combined version of the non-linearity tests of Keenan (1985), Tsay (1986), and Petruccelli and Davies (1986). It is simple and widely applicable. Its asymptotic distribution under the linear model assumption is nothing but the usual F distribution (Tsay, 1989).

There is a general agreement on the nonexistence of a global optimal test, since the number and locations of the thresholds are unknown before the estimation. However, according to Tsay (1989), the test that he proposed is more powerful and simpler than other tests available in the literature such as the portmanteau test proposed by Petruccelli and Davies (1986), etc. The Tsay (1989) test also avoids the problem of nuisance parameters encountered by the likelihood ratio test. Tsay (1989) makes use of arranged autoregression and recursive estimation to derive a test for threshold non-linearity. The null hypothesis is that the model is a linear AR process. While the alternative hypothesis is that the model exhibits threshold non-linearity. The arranged autoregression seeks to transfer the SETAR model into a model change problem with the thresholds serving as the change points.

4.1.2. The Arranged Autoregression
This section discusses the arranged autoregression concept introduced by Tsay (1989) which facilitates efficient estimation of TAR models.

Under the null hypothesis of linearity, residuals of a properly specified linear model should be independent. Any violation of independence in the residuals indicates inadequacy of the entertained model, including the linearity assumption. The idea behind the Tsay (1989) test is that under the null hypothesis there is no model change in the arranged autoregression so that the stan-

dardized predictive residuals should be close to i.i.d. with mean zero and variance 1. In this case, they should also have no correlation with the regressors (Tsay, 2002).

Let me show the arranged autoregression by matrices. I denote the threshold variable as $\upsilon_{t-d.}$

$$\left(Y|\nu\right) = \left(Y_{t-1}, Y_{t-2},..., Y_{t-p}|\nu_{t-d}\right) =$$

$$
\begin{array}{ccccc|c}
Y_{11} & Y_{12} & \dots & Y_{1p} & & \nu_1 \\
Y_{21} & Y_{22} & \dots & Y_{2p} & & \nu_2 \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
Y_{n1} & Y_{n2} & \dots & Y_{np} & & \nu_n
\end{array}.
$$

Rearrange the rows of Y, let $(Y \mid \upsilon)$ follow the order of the last column of $(Y \mid \upsilon)$. This yields:

$$\left(Y^{\nu}|\nu\right) = \left(Y_{t-1}^{\nu}, Y_{t-2}^{\nu},..., Y_{t-p}^{\nu}|\nu_{(i)}\right) =$$

$$
\begin{array}{cccc}
Y_{11}^{\nu} & Y_{12}^{\nu} & \dots & Y_{1p}^{\nu} \\
Y_{21}^{\nu} & Y_{22}^{\nu} & \dots & Y_{2p}^{\nu} \\
\vdots & \vdots & \vdots & \vdots \\
Y_{n1}^{\nu} & Y_{n2}^{\nu} & \dots & Y_{np}^{\nu}
\end{array}
$$

where υ_i denotes the i th smallest observation of υ_{t-d} and the superscript υ denotes ordering according to $\upsilon_{t-d.}$ A crucial property of this arranged form is that by reordering rows or cases of the initial matrix-form setup, it preserves the dynamics of Y_t. Furthermore, the ordinary regression residuals of the arranged autoregression are not correlated over time.

4.1.3. The Test

There are two steps to do the test. The first step is to run an arranged autoregression. The second is to use the predictive residuals to calculate the associated F statistic. If the F statistic is larger than the critical F value, I reject the null of linearity.

For the first step, an important practical matter is the question of how to appropriately choose the order of autoregressive approximation, p. A rough rule is to take p in the range of 4 to 8, although even bigger p values seem to do as well (Keenan, 1985).

(1) Running an Arranged Autoregression

Write an AR (p) regression with n observations as

$$Y_t = \left(1, Y_{t-1},..., Y_{t-p}\right)\alpha + \varepsilon_t \qquad (2)$$

for

$$t = p + 1,..., n,$$

where α is the (p+1)-dimensional vector of coefficients and ε_t is the noise. While $(Y_t, 1, Y_{t-1}, ..., Y_{t-p})$ is a case of data for the AR(p) model. Then, an arranged autoregression is an autoregression with case rearranged, based on the values of a particular regressor (in my test, I arranged cases based on the values of $Y_{t-1}, ..., Y_{t-p}$, one regressor one time). For example, if I sort the series by Y_{t-1}, I place the smallest Y_{t-1} first and the largest Y_{t-1} last. This gives an arranged autoregression,

$$Y_t^* = \left(1, Y_{t-1}^*,..., Y_{t-p}^*\right)\beta + \gamma_t. \qquad (3)$$

If there are non-linearities of the TAR type, then the β-vector associated with the small and large values of Y_{t-1}^* should be different from that associated with medium sized Y_{t-1}^*. This hypothesis can be checked by testing the arranged autoregression for structural breaks.

For a self-exciting model, arranged autoregression becomes useful if it is arranged according to the threshold variable. But the problem is that I do not know the threshold variable be-

fore I do the test. So, I assume that Y_{t-1}, ..., Y_{t-p} are all candidates of threshold variable, then run arranged autoregression based on each threshold variable candidate, finally pick the candidate that exhibits the most significant non-linearity as the threshold variable.

After I find the threshold variable, I consider a simple case, k = 2, i.e., the TAR model has two regimes, separated by a threshold γ_1, and for each regime, follows an AR model of different p. Note that the separation does not require knowing the precise value of γ_1, only the number of observations in each group depends on γ_1. If the thresholds γ_1 were known, then consistent estimates of the parameters could easily be obtained. But note that the actual value of γ_1 is not required in order to perform Tsay (1989) test; all that is needed is the existence of a nontrivial threshold. While according to Tsay (1989) test, if I get a test result of rejecting the null of linearity, I proof the existence of a nontrivial threshold.

(2) Performing the F test

The F test does not require knowing the thresholds, it simply tests that the predictive residuals have no correlations with regressors if the null of linearity holds.

Therefore I first pick out the recursive predictive residuals of arranged autoregression, $\hat{\gamma}_t$, then perform the following regression

$$\gamma_t = \left(1, Y_{t-1}^*, ..., Y_{t-p}^*\right)\omega + \varepsilon_t \tag{4}$$

and test whether $\omega \neq 0$ with a F test. And denote the F statistic F_1. To increase the power of the test, sort the cases in reversed order, i.e., the largest regressor first and the small last, then repeat the entire procedure with the reversed series and denote the corresponding F statistic F_2. Pick the larger F statistic of the two and use it as the test statistic. If it is larger than the critical value, I reject the null of linearity.

Because I do not know which lag of Y_t is the threshold variable, I assume that Y_{t-1}, ..., Y_{t-d} (where d = 1, 2, 3, ..., 8 for the JGB yields series) are all candidates of threshold variable, then do the F test based on each threshold variable candidate, finally pick the candidate that exhibits the most significant non-linearity as the threshold variable.

4.2. The Estimation method

Several statistical assumptions are the foundation of ordinary regression analysis. One key assumption is that the ordinary regression residuals are independent of each other. However, the regression errors of time series usually are correlated over time. Since the assumptions of which the classical linear regression model are based will usually be violated, it is not proper to use ordinary regression analysis for time series data.

However, the ordinary regression residuals of the arranged autoregression do not violate the key assumption.

Therefore for the arranged regression, least squares estimates and maximum likelihood estimates are the same. Since the least squares estimation is much easier to apply than the maximum likelihood estimation, Tsay (1989) used least squares estimates. I also choose to use least square estimates.

4.3. The data sets

The data sets are the daily close yield series of 10-year JGB, from November 17, 2002 to July 18, 2004, in total 610 observations (source: Reuters). The data are in percentages.

The reason why I choose 10-year maturity is because original issue with 10-year maturities account for the majority of new issues of JGBs.

Furthermore, according to the research by Boudoukh and Whitelaw (1991), trading in 10-year issues constitutes over 99% of the total trading in Japanese government issues that also include other issues such as 20-year issues, short maturity bonds and municipal bonds.

Another reason why I choose to study 10-year maturity bond is that the 10-year JGB is usually designated as the liquid bond called benchmark bond. Therefore, the result on the benchmark bond might have significant application on studying other bonds.

4.4. The Test Results

The test results are summarized in Table 2. Based on the test results and comparing with the critical F value, I observe that Y_{t-6} series shows the most significant non-linearity. So I conclude that Y_{t-6} is the threshold variable.

Table 2

	Threshold Lags, d							
F Statistics	1	2	3	4	5	6	7	8
AR(4)								
$F_{5,333}$	2.41	2.42	2.51	3.06				
p value	0.036	0.036	0.030	0.010				
AR(5)								
$F_{6,315}$	2,30	2,44	2,97	2,74	2,81			
p value	0.034	0.026	0.021	0.013	0.011			
AR(6)								
$F_{7,301}$	2.70	2.97	3.02	3.04	3.29	3.80		
p value	0.010	0.005	0.004	0.002	0.001	0.001		
AR(7)								
$F_{8,287}$	2.37	2.78	2.71	2.53	2.92	3.11	2.93	
p value	0.018	0.006	0.007	0.011	0.004	0.002	0.004	
$F_{9,273}$	2.29	2.70	2.63	2.38	2.92	3.01	2.77	2.62
p value	0.017	0.005	0.006	0.013	0.003	0.002	0.004	0.006

Note: $F_{a,b}$ denotes the proposed F statistic with a and b degrees of freedom. The AR orders used are those commonly employed in the literature.

I run five arranged AR models such as AR(4), AR(5), AR(6), AR(7) and AR(8). For each AR process, I perform Tsay (1989) test for the variable Y_{t-d}, where d is the threshold lag. In this case, $d = 1, 2, 3, ..., 8$.

From the test result, I observe that, for AR(6) process, Y_{t-6} has the largest F value, i.e., 3.80, with the most significant p value of 0.1%, comparing with other lags of Y_t such as $Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4}$ and Y_{t-5}.

For AR(7) process, again, Y_{t-6} has the largest F value of 3.11 with the most significant p value that is 0.2%, among other lags: $Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4}, Y_{t-5}, Y_{t-7}$.

Last, for AR(8) process, Y_{t-6} also has the largest F value that is 3.01 with the most significant p value of 0.2%, comparing with $Y_{t-1}, Y_{t-2}, Y_{t-3}, Y_{t-4}, Y_{t-5}, Y_{t-7}, Y_{t-8}$.

4.5. Finding the Best Lag Length

In order to find the best lag length, I can calculate the Akaike Information Criterion (AIC) or the Schwartz Bayesian Criterion (SBC) from each equation and then examine the output of the model with the smallest AIC and/or SBC.

It is also common to determine a lag length based on the outcome of t-tests. This method-ology picks the lag length such that the t-statistic for the last lag is significant at some pre-specified level.

I estimate twelve AR models such as AR(1), AR(2), AR(3),..., AR(12), then I calculate the AICs, SBCs and t-tests for these 12 models. The results are shown in Table 3.

Table 3

Lags	AIC	SBC	t
1	-170.660	-162.763	178.936
2	-177.658	-165.950	-1.961
3	-172.206	-156.774	0.061
4	-168.054	-148.983	0.048
5	-182.525	-159.915	-0.895
6	-187.063	-160.930	0.066
7	-185.243	-155.666	-1.555
8	-184.491	-151.556	1.368
9	-177.569	-141.365	-0.141
10	-173.026	-133.649	0.330
11	-175.991	-133.543	-2.571
12	-170.837	-125.427	-0.650

Note: AICs, SBCs and t-tests for AR(1), AR(2),..., AR(12). AIC is the Akaike Information Criterion and SBC is the Schwartz Bayesian Criterion.

The AIC selects the model with 6 lags while the SBC selects the model with 2 lags. In this case, the choice is unclear since t-statistic on lag 6 has a prob-value that is even greater than 10%.

I decide to select the model with 6 lags, because generally speaking, the longer the lag is, the better the fitting of the AR model will be.

4.6. Locating the Thresholds and Estimating the SETAR Model

Sometimes economic theory is helpful in choosing a particular model, but more often it is not. Choosing models involves subjective judgment. While for non-linear time series modeling, there are still some general guidelines to follow besides subjective judgment. Non-linear time series modeling starts with building an adequate linear model on which non-linearity tests are based. If non-linearity is statistically significant, then one chooses a class of non-linear models to enter-tain. The selection here may depend on the experience of the analyst and the substantive matter of the problem under study. For TAR models, one may use the procedures given in Chan (1993), Tong (1983, 1990) and Tsay (1989, 1998) to build an adequate model.

Tong (1983) developed a two-regime version of the SETAR model as follows:

$$Y_t = I_t \left[\alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} \right] + (1 - I_t) \left[\beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i} \right] + \varepsilon_t, \qquad (5)$$

where: Y_t is the series of interest, α_i and β_i are coefficients to be estimated, τ is the value of the threshold, p is the order of the SETAR model and I_t is the Heaviside indicator function:

$$I_t = \begin{cases} 1 \text{ if } Y_{t-1} \geq \tau \\ 0 \text{ if } Y_{t-1} < \tau \end{cases}, \qquad (6)$$

where Y_{t-1} is the threshold variable, τ is the threshold.

Of course, for different time series, Y_{t-1} is not necessarily always the threshold variable. The threshold variable could be any lag of Y_t such as Y_{t-1}, Y_{t-2}, Y_{t-3}, etc.

The nature of the system is that there are two states of the world. In one state of the world, Y_{t-1} exceeds the value of the threshold τ so that $I_t = 1$ and $(1 - I_t) = 0$. As such, Y_t follows the auto-regressive process: $\alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}$. Similarly, in the other state, Y_t falls short of the threshold τ, so that $I_t = 0$, $(1 - I_t) = 1$ and Y_t follows the autoregressive process: $\beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i}$. It seems that there are two attractors or potential equilibrium values. In the 'high' state, the system is drawn toward $\alpha_0 / (1 - \Sigma \alpha_i)$; in the 'low' state, the system is drawn toward $\beta_0 / (1 - \Sigma \beta_i)$. Moreover, the degree of autoregressive decay will differ across the two states if for any value of i, $\alpha_i \neq \beta_i$. The key feature of the SETAR model is that a sufficiently large shock denoted by ε_t can cause the system to switch between states.

When threshold τ is unknown, how to locate it? Chan (1993) shows how to obtain a super-consistent estimate of the threshold parameter.

For a SETAR model, the procedure is to order the observations from smallest to largest such that:

$$Y^1 < Y^2 < Y^3 ... < Y^T. \tag{7}$$

For each value of Y^j, let $\tau = Y^j$, set the Heaviside indicator according to this potential threshold and estimate a SETAR model. Among the group of different SETAR models based on each value of Y^j, the regression equation with the smallest residual sum of squares contains the consistent estimate of the threshold. In practice, the highest and lowest 15% of the $\{ Y^j \}$ values are excluded from the grid search so as to ensure an adequate number of observations on each side of the threshold.

I do the grid search by running a loop program, and I take the associated value of threshold test and use it as a potential threshold. Then I estimate a SETAR model, and calculate the residual sum of squares associated with each regression. Finally, I plot a scatter diagram of residual sums of squares against potential thresholds, that is shown in Figure 3.
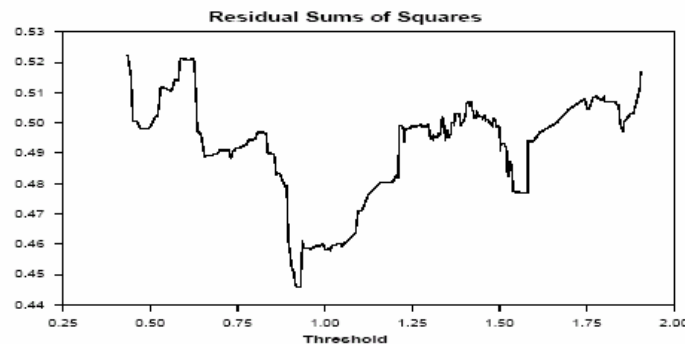

Fig. 3. The Plot of Residual Sums of Squares of SETAR Models of the JGB Yields Series against Potential Thresholds (November 17, 2002-July 18, 2004)

By checking the plot of residual sums of squares, I find that the threshold 0.920 results in the lowest residual sum of squares. And the plot clearly shows that there is only one threshold. Therefore, it implies the existence of two states for the 10-year JGB yields series.

With the threshold $\tau = 0.920$, I estimate a SETAR model for the daily series of 10-year JGB yields from November 17, 2002 to July 18, 2004.

The coefficients and their statistics are reported in Table 4.

Table 4

Independent Variable	Coefficient
α_0	-0.003 (0.016)
Y_{t-1}	1.025** (0.068)
Y_{t-2}	-0.089 (0.098)
Y_{t-3}	-0.132 (0.096)
Y_{t-4}	0.209* (0.097)
Y_{t-5}	0.042 (0.097)
Y_{t-6}	-0.053 (0.063)
β_0	0.001 (0.020)
Y_{t-1}	1.122** (0.099)
Y_{t-2}	0.139 (0.149)
Y_{t-3}	0.233 (0.147)
Y_{t-4}	-0.466** (0.149)
Y_{t-5}	-0.437** (0.161)
Y_{t-6}	0.411** (0.119)

Note: The numbers in the parentheses are standard errors, * indicates 5% level of significance, ** indicates 1% level of significance. The Centered R^2 of the SETAR model is 0.990395, the $\overline{R}^2$ of the model is 0.989980 and the Uncentered R^2 is 0.999061.

Because there are 2 regimes for the AR structure of the 10-year JGB yields series during the studied period, I estimate 2 different AR models for these two regimes. The AR orders are 6 for regime 1, 6 for regime 2; the number of observations are 226 in regime 1, 203 in regime 2. Details of the SETAR model are:

regime 1 :

$Y_t = - 0.003 + 1.025\ Y_{t-1} - 0.089\ Y_{t-2} - 0.132\ Y_{t-3} + 0.209\ Y_{t-4} + 0.042\ Y_{t-5} - 0.053\ Y_{t-6}$

When $Y_{t-6} \geq 0.920$

regime 2:

$Y_t = 0.001 + 1.122\ Y_{t-1} + 0.139\ Y_{t-2} + 0.233\ Y_{t-3} - 0.466\ Y_{t-4} - 0.437\ Y_{t-5} + 0.411\ Y_{t-6}$

When $Y_{t-6} < 0.920$

4.7. Checking the Model

In Table 5, I show the 2 regimes and the ACF and PACF of the standardized residuals of the 2-regime SETAR model.

In model checking, the ACF and PACF of the standardized residuals of the model all fail to suggest any model inadequacy, because the ACF and PACF plots (Figure 4) show little evidence that the regression residuals of the 2-regime SETAR model are correlated over time. Moreover, based on my calculating the mean and variance of the residuals, I conclude that the residuals of the model are nearly i.i.d. and normally distributed.

Table 5

Lags								
Regimes	0	1	2	3	4	5	6	7
AR Coefficients								
1	-0.003	1.025	-0.089	-0.132	0.209	0.042	-0.053	
2	0.001	1.122	0.139	0.233	-0.466	-0.437	0.411	
ACF of Standardized Residuals								
		-0.015	0.010	0.035	-0.062	0.001	0.035	-0.079
PACF of Standardized Residuals								
		-0.015	0.010	0.036	-0.061	-0.001	0.035	-0.074

Note: A 2-Regime SETAR Model for the JGB Yields Series (November 17, 2002-July 18, 2004) and ACF, PACF of the standardized residuals of the model. ACF is the autocorrelation function and PACF is the partial autocorrelation function.
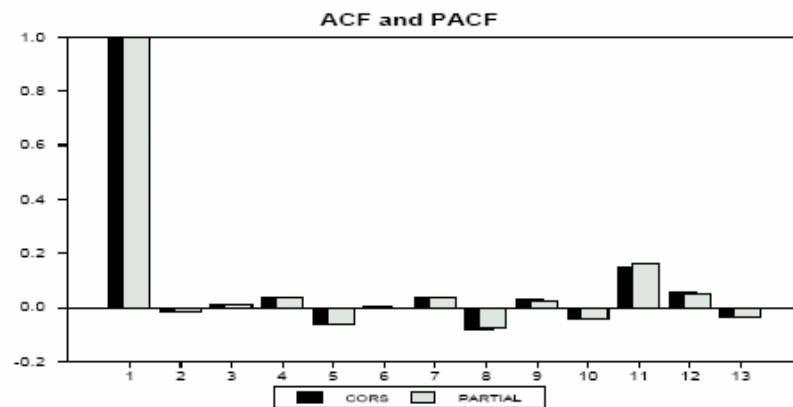

Fig. 4. The Autocorrelation Function (ACF) and the Partial Auto-correlation Function (PACF) Plots of Standardized Residuals of the 2-Regime SETAR Model for the JGB Yields Series (November 17, 2002-July 18, 2004)

Based on the model, I can observe that there are two states of the world for the 10-year JGB yields series. In one state of the world, Y_{t-6} exceeds the value of the threshold 0.920. As such the 10-year JGB yield follows the autoregressive process:

$$Y_t = - 0.003 + 1.025\ Y_{t-1} - 0.089\ Y_{t-2} - 0.132\ Y_{t-3} + 0.209\ Y_{t-4} + 0.042\ Y_{t-5} - 0.053\ Y_{t-6}$$

After calculating $\alpha_0 / (1 - \Sigma\ \alpha_i)$, I find that the yields are drawn toward 1.5. Similarly, in the other state of the world, Y_{t-6} falls short of the threshold 0.920. As such the 10-year JGB yield follows the autoregressive process:

$$Y_t = 0.001 + 1.122\ Y_{t-1} + 0.139\ Y_{t-2} + 0.233\ Y_{t-3} - 0.466\ Y_{t-4} - 0.437\ Y_{t-5} + 0.411\ Y_{t-6}$$

and the system is drawn toward -0.5 based on the calculation of $\beta_0 / (1 - \Sigma\ \beta_i)$.

Moreover, the degree of the autoregressive decay differs across the two states, since the coefficients of the AR process in one state are different with those of the AR process in the other state. A sufficiently large shock can cause the system to switch between the two states.

In this case, the large shock is the significant change in Japanese debt management policy during the end of 2002 to the beginning of 2003. I will come back to this issue in detail in the next section.

5. Conclusions

It is well known that in Japan, a bond issue is authorized by the Ministry of Finance but implemented by the Bank of Japan. Both the Ministry of Finance and Bank of Japan constitute Japanese debt management authority. They have a strong enough negotiating position to impose terms on the underwriting syndicate of government bonds (Boudoukh and Whitelaw, 1991).

The 2-regime SETAR model of the daily series of 10-year JGB yields indicates that the AR structure of the series changes once during the studied period.

One major reason why the AR structure changes is because the shock of interventions by Japanese debt management authority around the end of 2002 to the beginning of 2003.

During December 2002 to March 2003, there was an extraordinary move in Japanese debt management policy due to an expected redemption rush of mainly 10-year bonds in fiscal year 2008: a new government bond issuance plan was enacted; there was also a launch of various JGB-related measures stipulated in the settlement system reform laws; a buyback program was introduced, too. All in all, in order to accommodate the funds needed for the redemption rush, the bond yield was raised.

As I discuss in the previous section, the rates rise slowly, but decay quickly. The threshold variable Y_{t-6} indicates that the yield raising is done slowly, taking about a week, in other words, the intervention of Japanese debt management authority on the bond yields is reflected in reactions from the agents about a week later. The reactions include the market digesting the ripple of intervention and adjusting investment portfolio allocations.

I also can conclude that Japanese bond market is not perfectly efficient. The market does not respond to news instantly. Adjustment costs do exist.

Further research could be done on refining upon SETAR models and integrating two key properties of financial time series — non-linear and long-memory properties.

References

1. Boudoukh, Jacob and Whitelaw, F. Robert (1993), "Liquidity as a choice variable: A Lesson from the Japanese Government Bond Market", The review of Financial Studies 1993, 6(2), 265-293.
2. Francis, X. Diebold and Atsushi, Inoue (2001), "Long memory and regime switching", Journal of Econometrics 105, 131-159.
3. Granger, Clive W.J. and Tersvirta, Timo (1993), "Modelling Nonlinear Economic Relationships", Oxford University Press.
4. Hansen, Bruce E. (1997), "Inference in TAR Models", Studies in Nonlinear Dynamic and Econometircs, April 1997, 2(1), 1-14.
5. Keenan, Daniel MacRae (1985), "A Tukey Nonadditivity-Type Test for Time Series Nonlinearity", Biometrika, Vol. 72, No. 1, April 1985, 39-44.
6. Quarterly Newsletters of the Ministry of Finance of Japan, July 2002-October 2004.
7. Tong, Howell (1983), "Threshold Models in Non-linear Time Series Analysis", Springer-Verlag.
8. Tong, Howell (1990), "Non-linear Time Series { A Dynamical System Approach", Clarendon Press Oxford.
9. Tsay, Ruey S. (1986), "Nonlinearity Tests for Time Series", Biometrika, Vol. 73, No. 2, Aug 1986, 461-466.
10. Tsay, Ruey S. (1989), "Testing and Modeling Threshold Autoregressive Processes", Journal of the American Statistical Association, March 1989, Vol. 84, No. 405, Theory and Methods, 231-240.
11. Tsay, Ruey S. (1991), "Detecting and Modeling Nonlinearity in Univariate Time Series Analysis", Statistica Sinica 1(1991), 431-451.
12. Tsay, Ruey S. (2002), "Analysis of Financial Time Series", John Wiley, & Sons, INC.

Appendix

1. A stationary linear time series always has a unique and stable equilibrium which is equal to its mean. A non-linear time series can have a single (stable or unstable) equilibrium, multiple equilibria or no equilibrium at all. Furthermore, even if the equilibrium is unique and stable, it is not necessarily equal to the mean of the time series (Franses and Dijk, 2000).

2. Limit cycle: A k-period limit cycle is defined as a set of k points. If the time series started in one of the points and no shocks occurred, the series would cycle among the k points. For high-order SETAR models, it can be quite difficult to establish the existence of equilibria, attractors and/or limit cycles analytically. A pragmatic way to investigate the properties of the skeleton of a high-order SETAR model is to use what might be called deterministic simulation.

3. Tong (1990, p. 379) defines an alternative AIC for a 2-regime SETAR model as the sum of the AICs for the AR models in the two regimes, that is,

$$p_1, p_2 = n_1 \ln \hat{\sigma}_1^2 + n_2 \ln \hat{\sigma}_2^2 + 2(p_1 + 1) + 2(p_2 + 1), \qquad (8)$$

here $n_j, j = 1; 2$, is the number of observations in the regime, and $\hat{\sigma}_j^2, j = 1; 2$, is the variance of the residuals in the regime. The SBC for a 2-regime SETAR model can be defined analogously as

$$p_1, p_2 = n_1 \ln \hat{\sigma}_1^2 + n_2 \ln \hat{\sigma}_2^2 + (p_1 + 1)\ln n_1 + (p_2 + 1)\ln n_2, \qquad (9)$$

The selected lag orders in the two regimes are those for which the information criterion is minimized.

4. The new government bond issuance plan: in designing the new government bond issuance plan, Japanese debt management authority decided to increase super-long-term issues in response to market trends and needs, while maintaining an appropriate balance among different maturity zones — short-term, medium-term, long-term and super-long-term. Consequently, the average maturity of JGBs to be issued in the market in this plan will be extended to 5 years and 8 months — 2 months longer than 5 years and 6 months which is the initial budget base for fiscal year 2002.

5. The launch of various JGB-related measures stipulated in the settlement system reform laws: in January 2003, Japan started to implement various JGB-related measures stipulated in the settlement system reform Laws, which was enacted in June 2002. A new settlement system for government bonds was put into operation on January 27 2003, prior to the corporate bonds, as the first step to make settlement of government and corporate bonds paperless — a scheme provided for by the Laws. As a result, all government bonds issued thereafter, will be fully paperless and managed only by the records kept in the transfer accounts at financial institutions. This should increase transaction speed and efficiency for JGBs, thus improving the government bond market even further.

6. The buy-back program: to implement the proactive government debt management policies not only at the issuance stage, but also for outstanding JGBs, a buy-back program was introduced, too. While the program can be used for various purposes, for the time being, Japan debt management authority use it to level the amount of redemption of mainly 10-year bonds, which are expected to reach a redemption rush in fiscal year 2008.