




# “Portfolio selection using the multiple attribute decision making model”

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# PORTFOLIO SELECTION USING THE MULTIPLE ATTRIBUTE DECISION MAKING MODEL

## Abstract

This paper uses a Multiple Attribute Decision Making (MADM) model to improve the out-of-sample performance of a naïve asset allocation model. Under certain conditions, the naïve model has out-performed other portfolio optimization models, but it also has been shown to increase the tail risk. The MADM model uses a set of attributes to rank the assets and is flexible with the attributes that can be used in the ranking process. The MADM model assigns weights to each attribute and uses these weights to rank assets in terms of their desirability for inclusion in a portfolio. Using the MADM model, assets are ranked based on the attributes, and unlike the naïve model, only the top 50 percent of assets are included in the portfolio at any point in time. This model is tested using both developed and emerging market stock indices. In the case of developed markets, the MADM model had 24.04 percent higher return and 53.66 percent less kurtosis than the naïve model. In the case of emerging markets, the MADM model return is 90.16 percent higher than the naïve model, but with almost similar kurtosis.

## Keywords

portfolio selection, MADM models, naïve  
diversification, tail risk

## JEL Classification

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## INTRODUCTION

Fund managers and individual investors spend considerable time looking for asset allocation strategies that will maximize their realized returns. There are several portfolio selection models that use various optimization techniques to find optimal portfolios. Despite their theoretical appeal, these models often fail to outperform the naïve strategy of equal investment in all the assets in the portfolio. Yet the naïve investment strategy can be unwieldy when there are many assets in a portfolio. This paper uses multiple attribute decision making (MADM) to develop a model that refines the naïve investment strategy by reducing the number of assets in a portfolio without compromising its performance.

MADM models consider various attributes of individual assets and assign weights to each attribute. Using these weights, the model ranks each asset in terms of its desirability for inclusion in a portfolio. Using the MADM model, assets are ranked based on the attributes, and unlike the naïve model, only the top 50 percent of assets are included in the portfolio at any point in time. Similar to the naïve model, the total investment is divided equally among these assets. The advantage of this asset allocation method is that it eliminates the least preferable assets from a portfolio, and still maintains the simplicity of equal allocation of the naïve model.

The assets used in this paper are 16 developed and 16 emerging market stock indices. A separate motive of this paper is also to study how the

benefits of international diversification have changed during the past 10 to 15 years. The data covers a period from July 1999 to December 2015 for developed markets, and June 2002 to December 2015 for emerging markets. The results indicate that the ex-post returns of portfolios created using the MADM model outperformed that of the naïve strategy and had higher Sharpe ratios. In the case of developed market portfolios, the MADM model returns had lower skewness and kurtosis than the naïve model. In the case of emerging markets, the MADM model has higher average returns and Sharpe ratios, but had slightly higher skewness and kurtosis than the naïve model.

## 1. LITERATURE REVIEW

For an investor, return and risk are the two most important factors to consider before choosing an investment. In Markowitz's (1952) classical mean-variance framework, risk is measured by the standard deviation of expected returns, and the optimal allocation of assets in a portfolio is obtained by maximizing the expected return and minimizing the variance of the expected returns of the portfolio. Empirical studies have shown that the unconditional asset returns cannot be adequately characterized by their mean and variance alone. Standard deviation measures both upside and downside deviations from the expectations, but investors are more concerned about the downside deviations from the expected returns. One way of measuring downside risk is to use the semi-deviation, which Markowitz (1959) advocates as a better measure of risk. For investors who are more concerned about the downside risk, those assets with high downside risk will have higher risk premiums. Using a downside beta estimate, Ang et al. (2006) show that the downside risk premium for U.S. stocks can be approximately 6 percent.

Several empirical studies have found that stock returns have negative skewness and excess kurtosis<sup>1</sup>, and therefore higher moments are relevant for estimating portfolio risk. Negative skewness increases the downside risk of an investment. Negative skewness is a risk associated with bear markets, and inclusion of an asset that increases the skewness of a portfolio is less desirable for the investor and should have a higher risk-premium (Harvey & Siddique, 2000). Galagedera and Brooks (2007) argue that investors with non-increasing risk aversion will dislike assets with negative co-skewness with market returns and will expect higher returns for such assets.

Apart from skewness, stock returns also exhibit kurtosis (Mandelbrot & Taylor, 1967). As in the case of co-skewness, assets that have higher co-kurtosis with market portfolio returns should be less desirable for investors, and hence should require a higher risk premium. Doan et al. (2010) tested the effect of co-skewness and co-kurtosis on U.S. and Australian stocks and found that both had significant roles in explaining stock returns in both markets.

The greatest advantage of a naïve diversification model is its simplicity – there are no input parameters to the model other than the set of assets. There are several recent papers that try to understand the reasons for the superior performance of the naïve models over the optimization models. Platen and Rendek (2012) use Naïve Diversification Theorem to suggest that naïve diversification optimizes log utility. According to Murtazashvili and Vozlyublennaia (2013), the naïve strategy is superior to the optimization models, when the data for estimating the parameters for optimization models are limited. Hwang et al. (2018) find that in large portfolios the naïve strategy outperforms the mean-variance optimization but has higher tail risk. This suggests that the outperformance of the naïve strategy represents investor compensation for increased tail risk. Comparing a naïve model with other diversification strategies, Banerjee and Hung (2013) did not find any statistically significant difference in the economic profits between momentum strategies and the naïve strategy.

Reliance on 60- and 120-month trailing returns for estimating the input parameters of optimization models may be the reason for the unfavorable comparison of the optimization models

<sup>1</sup> Ibbotson (1975), Hwang and Satchell (1999), Harvey and Siddique (1999), and Prakash et al. (2003).

against the naïve model (Kritzman et al., 2010). A very long history of returns is required to accurately estimate the expected returns for optimization models (Merton, 1980). According to Platanakis et al. (2019) mean-variance models are superior to equal weighting for asset allocation, but less so when it comes to stock selection.

There are some studies that disagree that the naïve model outperforms other optimal models. Disatnik and Katz (2012) studied the global minimum variance portfolio that is constructed using a block structure to create a covariance matrix of asset returns and found that it outperforms the naïve model. Kirby and Ostdiek (2012) suggest that the research design by DeMiguel et al. (2009) focusing on mean-variance efficient portfolios created a false conclusion that naïve strategies outperform. The focus on the Markowitz model as a comparison was subject to high estimation risk and high turnover.

Based on the above discussions, the following hypotheses are tested:

$H_1$ : *Portfolios selected using an MADM model will yield higher returns than the naïve model for a given set of investments.*

$H_2$ : *Portfolios selected using an MADM model will have lower tail risk than the portfolios created using a naïve model.*

## 2. EMPIRICAL MODEL

In MADM models, there is a DM who considers available information and creates a ranking or weighting system to guide the decision making. There are three different approaches to selecting the weights of attributes: subjective, objective and integrated. This paper uses a correlation coefficient and a standard deviation (CCSD) integrated approach by Wang and Luo (2010). The basic CCSD model used in this paper assumes that there are  $n$  individual decision alternatives designated as:  $A_1, \dots, A_n$ , and these alternatives are evaluated using  $m$  attributes designated as:  $O_1, \dots, O_m$ . Since the attributes are of different scale, they need to be normalized to eliminate dimensional units as follows:

For attributes that are considered benefits:

$$z_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \quad i = 1, \dots, n. \quad (1)$$

For attributes that are considered costs:

$$z_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}}, \quad i = 1, \dots, n, \quad (2)$$

where  $x_j^{\min}$  and  $x_j^{\max}$  are the minimum and maximum values of each set of attributes. The decision matrix can be written as follows:

$$Z = \begin{bmatrix} z_{11} & \dots & z_{1m} \\ \dots & \dots & \dots \\ z_{n1} & \dots & z_{nm} \end{bmatrix}. \quad (3)$$

To find a ranking system for the decision making, a set of weights should be given for each of the attributes. Let  $W = (w_1, \dots, w_m)$  be the set of non-negative weights, and the sum of these weights adds up to 1. Multiplying each of the normalized attribute value with this weight will give the overall assessment value of each decision alternative as follows:

$$d_i = \sum_{j=1}^m z_{ij} w_j, \quad i = 1, \dots, n. \quad (4)$$

The greater the value of  $d_i$ , the greater the importance of that attribute in the overall assessment. Since the value of the weights are not known beforehand, it is necessary to see what happens as a specific attribute is removed from the overall decision matrix. An attribute  $O_j$  will be removed from the decision matrix, and the overall assessment value of the remaining decision alternative is calculated as follows:

$$d_{ij} = \sum_{k=1, k \neq j}^m z_{ik} w_k, \quad i = 1, \dots, n. \quad (5)$$

To assess the impact of removing an attribute from the decision matrix, its correlation with the overall assessment values can be calculated as follows:

$$R_j = \frac{\sum_{i=1}^n (z_{ij} - \bar{z}_j)(d_{ij} - \bar{d}_j)}{\sqrt{\sum_{i=1}^n (z_{ij} - \bar{z}_j)^2 \cdot \sum_{i=1}^n (d_{ij} - \bar{d}_j)^2}} \quad j = 1, \dots, m, \quad (6)$$

where

$$\bar{z}_j = \frac{1}{n} \sum_{i=1}^n z_{ij}, j = 1, \dots, m. \quad (7)$$

$$\bar{d}_j = \frac{1}{n} \sum_{i=1}^n d_{ij} = \sum_{k=1, k \neq j}^m \bar{z}_k w_k, j = 1, \dots, m. \quad (8)$$

If the correlation  $R_j$  is high and close to one, then eliminating that decision alternative  $O_j$  will have a very little impact on the overall assessment and can be given a small weight. The other factor that should be considered in the weighting scheme is the variation of a specific attribute among the decision alternatives. Here again, if the standard deviation of the attribute is low, it should be given a lower weight than one that has a high standard deviation. Based on this, the weights can be defined as:

$$w_j = \frac{\sigma_j \sqrt{1 - R_j}}{\sum_{k=1}^m \sigma_k \sqrt{1 - R_k}} j = 1, \dots, m, \quad (9)$$

where  $\sigma_j$  is the standard deviation of the values of  $O_j$  determined by:

$$\sigma_j = \sqrt{\frac{1}{n} \sum_{i=1}^n (z_{ij} - \bar{z}_j)^2} j = 1, \dots, m. \quad (10)$$

The model is solved as follows:

$$\text{Minimize } J = \sum_{j=1}^m \left( w_j - \frac{\sigma_j \sqrt{1 - R_j}}{\sum_{k=1}^m \sigma_k \sqrt{1 - R_k}} \right)^2. \quad (11)$$

Subject to

$$\sum_{j=1}^m w_j = 1, w_j \geq 0, j = 1, \dots, m. \quad (12)$$

The above model can be solved using any optimization model.

To use the above model in a portfolio context, one must pick the attributes for the model. In the classic mean-variance model, there are two attributes, expected return as the benefit attribute and the covariance as the cost attribute. Since the MADM model has no restriction on the number of attributes that can be included in the model, there is

flexibility in choosing the attributes that are included. This paper uses the following attributes in the MADM model.

- a) *Mean-returns for the past one year:* Momentum strategies that involve investing in recent winners and shorting recent losers have been shown to be profitable in many studies (Jegadeesh & Titman, 1993, 2001). This study uses average monthly returns for the past one year to capture the momentum effect as follows:

$$r_i = \frac{1}{12} \sum_{t=1}^{12} r_{it}, \quad (13)$$

where  $r_{it}$  is the return for the index  $i$  in month  $t$ .

- b) *Target semi-standard deviation:* Risk-averse investors generally prefer assets that have low volatility for a given level of risk. The most commonly used measure of volatility is the standard deviation of asset returns. A drawback using standard deviation is that it takes into consideration both positive and negative deviation from the expected return. This model uses the target semi-standard deviation to measure the downside risk using the following equation:

$$SV_i = \sqrt{\frac{1}{n} \sum_{r_i < Target} (Target - r_{it})^2}, \quad (14)$$

where the monthly target return is 0.75 percent and the number of months  $n$  is sixty.

- c) *Correlation with S&P 500 index:* High correlation in the returns of assets in a portfolio reduces the benefits of diversification. In the mean-variance model, a full correlation matrix is used. Even though using a full correlation matrix is theoretically more appealing, initial testing using the MADM model showed that many of the correlations end up with very low weights and hence may not contribute much to the model. It can also be argued that for a domestic investor, the correlation between the domestic and foreign markets is more important, and it should be one of the attributes that may be considered in the model. Correlation of monthly returns of  $i^{\text{th}}$  country index and the S&P500 index is calculated as:

$$\rho_{i,SP500} = \frac{Cov(r_i, r_{SP500})}{\sigma_i \sigma_{SP500}} \quad (15)$$

d) *Co-skewness with S&P 500 index:* There are several empirical and theoretical studies that show higher moments of the return distribution are important in selecting portfolios (Prakash et al., 2003). An investor with non-quadratic utility function and non-increasing absolute risk aversion may prefer assets with returns that have positive skewness and low excess kurtosis. Investors may consider assets with returns that have high negative co-skewness and large co-kurtosis with market returns to be less attractive. (Doan et al., 2010). This model uses co-skewness between the monthly returns of  $i^{th}$  index and the S&P500 index, which is calculated as:

$$\gamma_{i,SP500} = \frac{Cov(r_i, r_{SP500}^2)}{\sigma_i \sigma_{SP500}^2} \quad (16)$$

e) *Co-kurtosis with S&P 500 index:* Co-kurtosis between the monthly returns of  $i^{th}$  index and the S&P500 index is calculated as:

$$\theta_{i,SP500} = \frac{Cov(r_i^2, r_{SP500}^2)}{\sigma_i^2 \sigma_{SP500}^2} \quad (17)$$

The above attributes, except the mean returns, are calculated using a rolling window of 60 months. For developed markets, the initial estimates of the attributes are made using the monthly returns from July 1999 to June 2004 and for emerging markets from June 2002 to May 2007. These estimates are used in the MADM model to rank the indices for that month, and only the top eight indices are chosen to be included in the MADM portfolio<sup>2</sup> with equal investment in each of the eight indices. For example, if the initial investment is \$1, then investment in each index will be \$1/8. The value of each investment  $i$  for the month  $t$  will be:

$$w_{i,t} = w_{i,t-1} (1 + r_{i,t}), \quad (18)$$

where  $r_{i,t}$  is the realized return of  $i^{th}$  index for the month  $t$ . The total value of the MADM portfolio of 8 indices for the month  $t$  will be:

$$w_{M,t} = \frac{w_{M,t-1}}{8} \sum_{i=1}^8 (1 + r_{i,t}). \quad (19)$$

For the first month, the value of the portfolio will be 1. The return of the MADM portfolio for the month  $t$  will be:

$$r_{M,t} = \frac{w_{M,t} - w_{M,t-1}}{w_{M,t-1}} \quad (20)$$

For naïve portfolios equal investment is made in all 16 indices and the return for the naïve portfolio for the month  $t$  will be:

$$r_{N,t} = \frac{w_{N,t} - w_{N,t-1}}{w_{N,t-1}} \quad (21)$$

At the end of each month, the new values of the attributes for the MADM model are recalculated by dropping the first month and adding the current month. Using the new set of attributes, the indices are re-ranked using the MADM model. If an index that is currently in the top eight drops out of it, it is replaced with the one that comes into the top eight. The investment in those eight indices are made equal by selling the ones that gained in value and buying the ones that went down in value. The same process is used for the naïve model, except that all 16 indices are kept.

Since the portfolios are rebalanced at the end of each month, it is necessary to include the transaction costs before evaluating the performance of each portfolio. In the naïve strategy, the turnover is based on the changes in value of the asset due to its return during the period. In the MADM strategy, only the top 8 assets are chosen to be included in the portfolio in a month. It is possible that in any given month an asset may be dropped completely from the portfolio and replaced by another in its place. In this respect it is possible that the turnover in the MADM model may be higher than that of the naïve model. To capture the economic cost of turnover, the following model is used. The weight of an asset  $j$  at the beginning of the period  $t$  is  $w_{j,t}$  and at the end of the month  $w_{j,t+}$ . Due to difference in returns of each asset during the month, weights should be rebalanced to be equal for all constituents of the portfolio at the begin-

2 Also, portfolios with only top 4 and top 12 indices were tried. In the former case, this led to excessive turnover, and in the latter, the difference with the naïve portfolio became insignificant.

ning of the month  $t+1$ . Let the new weight of each asset at the beginning of month  $t+1$  be  $w_{j,t+1}$ . The trade of each asset at the end of time  $t$  will then be  $|w_{j,t+1} - w_{k,j,t}|$ . Assuming that  $C$  is the transaction cost of each trade, the overall transaction cost for the time period  $t$  will be:

$$C \cdot \sum_{j=1}^N |w_{j,t+1} - w_{j,t}|. \quad (22)$$

Following DeMiguel et al. (2009),  $C$  is given a value of 50 basis points. The overall return of the portfolio is then reduced by the transaction cost for the month to get the wealth of the strategy as:

$$W_{t+1} = W_t (1 + R_t) (1 - C \cdot \sum_{j=1}^N |w_{j,t+1} - w_{j,t}|). \quad (23)$$

The performance of each portfolio is calculated using the Sharpe ratio as follows:

$$SR_k = \frac{\mu_k}{\sigma_k}, \quad (24)$$

where  $\mu_k$  is the return of the portfolio and  $\sigma_k$  is its standard deviation. The higher moments of the portfolios are compared to see the difference between various investment strategies.

### 3. DATA

The above MADM model is testing two sets of portfolios – one comprising developed market country index returns and the other comprising emerging market country index returns. Based on geographic diversity and data availability, sets of sixteen country indices are used as investment assets in the portfolios. The monthly dividend adjusted return for each country index is obtained from Wharton Research Data Services (WRDS). WRDS uses Compustat Global data for security prices to construct a market capitalization weighted index for each country. To be included in a country index, a stock has to trade in one of the exchanges listed in that country and should be in the top 50 percent of the firms by market capitalization in that country.

Since the data availability varied across countries, the data from July 1999 to December 2015 for the developed markets and June 2002 to December

2015 for the emerging markets is used. To calculate the attributes that are inputs for the model, a rolling window of 60 monthly returns is used. The out-of-sample period for developed markets is from July 2004 to December 2015, and for emerging markets it is from June 2007 to December 2015. Since the time period in this study covers both the global financial crisis and the European debt crisis, it is of interest to study how the models behave in periods of high volatility.

Center for Research in Security Prices (CRSP) data is used to calculate the monthly returns of the S&P 500 index. For the risk-free rate, the 3-month T-bill interest rate is used, which is obtained from the Federal Reserve Economic Data (FRED). Monthly excess returns are calculated by subtracting the risk-free rate from the monthly returns of the portfolios.

## 4. RESULTS

Normality tests indicate that none of the returns in this study are normally distributed and therefore higher moments need to be incorporated. Averages of the decision criteria used in the MADM model are given in Table 1.

Panel A has average values of decision criteria for developed markets. The average mean monthly return for the past one year is the highest for Denmark at 1.3 percent and the lowest for Japan at 0.6 percent. Average semi-standard deviation for Australia is the lowest at 4.52 percent and the highest for Finland at 7.6 percent. Generally, all developed market returns had high correlation with the S&P 500 index returns. The correlation ranged from 0.63 for Hong Kong to 0.86 for United Kingdom. Finland had the lowest negative co-skewness of 0.30 with the S&P 500 index, and Singapore has the highest at 0.64. Except for Belgium, Germany, Netherlands, Singapore, Sweden and the United Kingdom, all other countries has negative co-kurtosis with the S&P 500 index. The overall results indicate that mean monthly returns for the past one year and co-kurtosis have the largest variation among the developed markets.

The averages of decision criteria used in the MADM model for the emerging markets are given in Panel B. The mean of monthly returns for the

**Table 1.** Averages of decision criteria for the out-of-sample period**Panel A:** Developed markets (July 2004 – December 2015)

Asset	Mean returns for the past 1 year	Semi std. dev.	Correlation with SP500 index	Co-skewness with SP500 index	Co-kurtosis with SP500 index
AUSTRALIA	0.0082	0.0452	0.7460	-0.5587	-0.5033
BELGIUM	0.0105	0.0585	0.7244	-0.6042	0.2761
DENMARK	0.0130	0.0567	0.7084	-0.6212	-0.0255
FINLAND	0.0071	0.0760	0.7238	-0.2998	-0.6691
FRANCE	0.0072	0.0561	0.8560	-0.5147	-0.0244
GERMANY	0.0082	0.0620	0.8481	-0.6112	0.8967
HONG KONG	0.0119	0.0721	0.6353	-0.4890	-0.7075
IRELAND	0.0062	0.0675	0.7019	-0.4703	-0.6397
ITALY	0.0043	0.0608	0.7763	-0.4749	-0.2766
JAPAN	0.0060	0.0545	0.5489	-0.6192	-0.8123
NETHERLANDS	0.0084	0.0587	0.8155	-0.5758	0.2348
NORWAY	0.0114	0.0673	0.7459	-0.5402	-0.4058
SINGAPORE	0.0086	0.0609	0.6875	-0.6431	0.1373
SWEDEN	0.0111	0.0584	0.7802	-0.4521	0.1654
SWITZERLAND	0.0065	0.0483	0.7998	-0.5319	-0.3547
UNITED KINGDOM	0.0072	0.0482	0.8653	-0.5344	0.1309

**Panel B:** Emerging markets (June 2007 – December 2015)

Asset	Mean returns for the past 1 year	Semi std. dev.	Correlation with SP500 index	Co-skewness with SP500 index	Co-kurtosis with SP500 index
BRAZIL	0.0058	0.0609	0.6631	-0.4294	0.0166
CHILE	0.0063	0.0387	0.5107	-0.2886	0.0164
CHINA	0.0134	0.0975	0.3677	-0.3533	0.0138
EGYPT	0.0022	0.0748	0.4478	-0.5032	0.0222
GREECE	-0.0146	0.0925	0.6594	-0.6371	-0.0016
HUNGARY	0.0010	0.0794	0.6948	-0.5961	0.0111
INDIA	0.0119	0.0812	0.5938	-0.4716	0.0244
INDONESIA	0.0148	0.0785	0.6040	-0.5082	0.0228
MALAYSIA	0.0088	0.0438	0.5804	-0.4388	0.0105
MEXICO	0.0086	0.0488	0.7170	-0.5596	0.0191
PHILIPPINES	0.0126	0.0612	0.5649	-0.4619	0.0168
POLAND	0.0025	0.0680	0.7192	-0.5772	0.0110
SOUTH AFRICA	0.0108	0.0444	0.6634	-0.5338	0.0154
SOUTH KOREA	0.0065	0.0652	0.6633	-0.4767	0.0129
TAIWAN	0.0063	0.0652	0.6381	-0.3744	0.0085
THAILAND	0.0103	0.0695	0.5582	-0.5913	0.0137

past one year has the lowest value of -1.46 percent for Greece, and China has the highest at 1.34 percent. Chile has the lowest semi-standard deviation of 3.87 percent, while China has the highest at 9.75 percent. All emerging market returns have positive correlation with the S&P 500 index, but are lower than that of developed markets. China has the lowest correlation of 0.37 and at 0.72, Poland has the highest correlation with the S&P 500 index. As in the case of developed markets, emerging markets also has negative co-skewness with S&P 500 index. China has the lowest co-skewness at

0.29, and Greece has the highest at 0.64. Except for Greece, all other emerging markets have positive co-kurtosis with the S&P 500 index. Overall, attribute variability is higher for emerging markets.

Out-of-sample period excess returns are given in Table 2.

Average monthly returns, Sharpe ratios, and second and higher moments of the naïve and MADM portfolio with and without transaction costs are compared in Table 2. To understand the interna-



**Table 2.** Properties of the out-of-sample period excess returns for various portfolio strategies**Panel A:** Developed markets (July 2004 – December 2015)

Strategy	Mean	Std. dev.	Sharpe ratio	Skewness	Kurtosis	Investment value <sup>3</sup>
S&P 500 index	0.6588%	4.2985%	0.1533	-0.5438	0.5790	\$257.21
MADM without transaction cost	0.8419%	4.3628%	0.1930	-0.9630	1.4971	\$328.41
Naïve (1/N) without transaction cost	0.6672%	4.2898%	0.1555	-1.0256	3.2701	\$260.08
MADM with transaction cost	0.8041%	4.3677%	0.1841	-0.9773	1.5329	\$311.84
Naïve (1/N) with transaction cost	0.6507%	4.2925%	0.1516	-1.0438	3.3079	\$254.27
S&P500 and MADM with transaction cost <sup>1</sup>	0.7314%	2.9522%	0.2478	-0.5164	0.8487	\$303.70
S&P500 and Naïve with transaction cost <sup>2</sup>	0.6547%	2.9035%	0.2255	-0.5231	1.9889	\$274.22

Note: <sup>1</sup> Portfolio with 50 percent in S&P 500 index and 50 percent in the MADM portfolio. <sup>2</sup> Portfolio with 50 percent in the S&P 500 index and 50 percent in the naïve portfolio. <sup>3</sup> Value of \$100 investment in January 2005, which is reinvested each month at the out-of-sample return for each portfolio.

**Panel B:** Emerging markets (June 2007 – December 2015)

Strategy	Mean	Std. dev.	Sharpe ratio	Skewness	Kurtosis	Investment value <sup>3</sup>
S&P 500 index	0.3546%	4.6736%	0.0759	-0.6873	1.3061	\$135.96
MADM without transaction cost	0.5622%	4.2716%	0.1316	-1.2887	5.3903	\$170.73
Naïve (1/N) without transaction cost	0.2899%	4.6654%	0.0621	-1.0657	5.2574	\$127.08
MADM with transaction cost	0.5201%	4.2753%	0.1217	-1.3082	5.4705	\$163.54
Naïve (1/N) with transaction cost	0.2735%	4.6683%	0.0586	-1.0887	5.3320	\$124.96
S&P500 and MADM with transaction cost <sup>1</sup>	0.4373%	4.2251%	0.1035	-1.0374	3.7327	\$150.83
S&P500 and Naïve with transaction cost <sup>2</sup>	0.3141%	4.4469%	0.0706	-0.9726	3.5737	\$131.72

Note: <sup>1</sup> Portfolio with 50 percent in the S&P 500 index and 50 percent in the MADM portfolio. <sup>2</sup> Portfolio with 50 percent in the S&P 500 index and 50 percent in the naïve portfolio. <sup>3</sup> Value of \$100 investment in August 2004, which is reinvested each month at the out-of-sample return for each portfolio.

tional diversification benefits for a U.S. investor, the above returns are compared with the S&P 500 index returns and portfolios created with a 50 percent investment in the S&P 500 index and 50 percent in either the naïve portfolio with transaction costs (NTA) or the MADM portfolio with transaction costs (MADMTA).

The results for developed markets are given in Panel A (Table 2) for the out-of-sample period from January 2005 to December 2015. The monthly average returns for MADMTA is 15.34 basis points higher than that of the naïve portfolio with transaction costs and 14.53 basis points higher than that of the S&P 500 index. The standard deviation of the monthly returns for the MADMTA is 7.52 basis points higher than that of the naïve portfolios with transaction costs and 6.92 basis points higher than the S&P 500 index.

Sharpe ratios show that the MADMTA is superior to both the NTA portfolio and the S&P 500 index. Comparing the higher moments of the portfolio

returns show that the MADMTA portfolio has lower skewness and excess kurtosis than the NTA portfolio. The skewness and excess kurtosis of S&P 500 index returns are lower than that of both MADMTA and NTA portfolios. What is interesting is that the excess kurtosis of the NTA portfolio is almost double that of the MADMTA, which validates the hypothesis of Brown et al. (2013) that the possible higher returns of naïve portfolios also increase the tail risk of these portfolios.

A simple diversification strategy of equal investment in the S&P 500 index and the MADMTA or NTA results in higher Sharpe ratios than investing only in one of these portfolios. This is an indication that there are diversification benefits to U.S. investors for investing in other developed stock markets.

The value of investing \$100 in January 2005 in any one of these portfolios and continuing to reinvest at the monthly rate of return shows the advantage of MADM portfolios over the naïve and S&P 500

index. By December 2015, the investment value of the MADMTA portfolio increased to \$311.84, while the S&P 500 index and NTA portfolios increased to \$257.21 and \$254.27, respectively. The equally weighted portfolio of the S&P 500 index and the MADMTA portfolio increased to \$303.70. Overall results indicate that a combination of the S&P 500 index and the MADMTA portfolio of developed market indices gives the best risk return combination for U.S. investors during the time period studied.

Comparing the effect of transaction costs of the MADM and naïve portfolio shows that transaction costs reduced the monthly returns of the MADM portfolio by an average of 3.78 basis points and by 1.65 basis points for the naïve portfolio. The MADM portfolio had higher transaction cost, but it was compensated by higher overall returns.

The results of investing in emerging market indices using various investment strategies are given in Panel B (Table 2). Average monthly returns of MADMTA portfolios clearly outperform that of NTA and the S&P 500 index by 25 and 17 basis points during the out-of-sample period from July 2007 through December 2015. Average monthly returns of the equally weighted portfolio of the S&P 500 index and MADMTA outperformed the S&P 500 index returns by an average of 8 basis points.

Risk as measured by the standard deviation of monthly returns for MADMTA was lower than both the S&P 500 index and NTA portfolios by 40 and 39 basis points, respectively. Investing in an equally weighted portfolio of the S&P 500 index and MADMTA reduced the standard deviation by a further 4 basis points than the MADMTA portfolio alone. As in the case of developed markets, the Sharpe ratios indicate the clear superiority of MADMTA portfolios over both S&P 500 index and NTA portfolios. The Sharpe ratio of MADMTA was almost double that of NTA portfolios.

Unlike the developed markets, MADMTA portfolios had higher skewness and kurtosis than that of the S&P 500 index. The difference in skewness and kurtosis between MADMTA and NTA is statistically insignificant. The value of \$100 invested in July 2007 increased to \$163.54 for the MADMTA portfolios, \$135.96 for S&P 500 index and \$124.96 for NTA portfolios. The conclusion that can be drawn from this analysis is that in the case of emerging markets, the out-of-sample performance of MADMTA portfolios is significantly higher than that of NTA, albeit with slightly higher risk. From a diversification standpoint, an equally weighted portfolio of the S&P 500 index and MADMTA produced significantly higher returns than the S&P 500 index alone, but with significantly higher tail risk. These results support both hypotheses laid out in this paper.

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## CONCLUSION

The quest for the best portfolio optimization model is unending, and there are several models that stake claim for the top spot. The naïve model that allocates the investment equally in a given number of assets is favored as an asset allocation method for its simplicity, and under certain conditions it gives higher out-of-sample returns than the other more complex allocation models. This paper uses the MADM model to rank assets for inclusion in the portfolio and find that this method can improve the out-of-sample performance of the naïve models.

A side-by-side comparison of MADM and naïve models using developed and emerging market indices clearly shows the better out-of-sample performance of the MADM model, especially in reducing the tail risk. The results also indicate that combining the international market indices with the domestic index can produce higher returns for U.S. investors.

MADM models are flexible in terms of the attributes that can be used in ranking the assets. This paper used only the moments of the return distribution and the correlation with the domestic market index as the attributes. Further research can be done with other attributes that can help refine the asset allocation for more specific purposes.

## AUTHOR CONTRIBUTIONS

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