"Risk Profiles of Life Insurance Participating Policies: Measurement and Application Perspectives"

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RISK PROFILES OF LIFE INSURANCE PARTICIPATING POLICIES: MEASUREMENT AND APPLICATION PERSPECTIVES

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Abstract

The paper deals with the calculation of suitable risk indicators for Life Insurance policies in a Fair Value context. In particular, aim of this work is to determine the quantile reserve for Life insurance Participating Policies. This goal poses both methodological and numerical problems: for this reason the paper discusses both the choice of the mathematical models and the calculation tecnique. Numerical application illustrates the results.

Key words: Participating policies, Fair Value, Quantile Reserve, Mathematical Reserve.
JEL classification: G22, G28, G13.

1. Introduction

At the end of March 2004, The International Accounting Standard Boards (IASB) issued the International Financial Reporting Standard 4 Insurance Contracts (IFRS 4) (e.g. [8]), providing, for the first time, guidance on accounting for insurance contracts, and marking the first step in the IASB's project to achieve the convergence of widely varying insurance accounting practices around the world. In particular, on the one hand, the IFRS 4 "permits an insurer to change its accounting policies for insurance contracts only if, as a result, its financial statements present information that is more relevant and no less reliable, or more reliable and no less relevant"; on the other hand "it permits the introduction of an accounting policy that involves remeasuring designated insurance liabilities consistently to reflect current interest rates and, if insurer so elects, other current estimates and assumptions". Thus IFRS 4 give rise to a potential reclassification of some or all financial assets "at fair value through profit and loss" when an insurer changes accounting policies for insurance liabilities.

The IASB defines the Fair Value "an estimate of the price an entity have realized if it had sold an asset or paid if it had been relieved a liability on the reporting date in an arm's length exchange motivated by normal business considerations". In particular, the IASB allows for using stochastic models in order to estimate future cash flows.

In the actuarial perspective, the introduction of an accounting policy and of a fair valuation system implies that the Fair Value of the mathematical reserve could be defined as the net present value of the debt towards the policyholders evaluated at current interest rates and, eventually, at current mortality rates.

In actuarial literature, many papers deal with models for the Fair valuation of insurance liabilities; in particular Milevsky and Promislow (e.g. [10]) propose a stochastic approach to model the future mortality hazard rate in insurance contract with option to annuitise, Bacinello (e.g. [1]) deals with the problem of pricing a guaranteed life insurance participating policy, Ballotta and Haberman (e.g. [2]) give a thoretical model for evaluating guaranteed annuity conversion option. In this field, our paper aims at giving a contribution to the question of calculation of suitable risk indicators for the mathematical reserve of a guaranteed life insurance participating policy in a fair value context.

The paper is organised as follows: section 2 gives a survey about the application of the quantile reserve to actuarial liabilities. In section 3 the mathematical formalization is introduced. Finally, in section 4, a numerical evidence is offered.

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2. The quantile reserve and the actuarial liabilities

The quantification of a liability fair value can be approached introducing the replicating portfolio, that is a portfolio of financial instruments giving origin to a cash flow matching that one underlying the liability itself. Of course, in the case of liability traded in an existing market, the fair value coincide with the market value itself. The fair valuation of insurance liabilities, since considering cash flows depending on the human life, and so not trading in an existing market, can be considered existing in the economic reality, as stated in Buhlmann (e.g. [6]), and can be measured by means of a financial instruments portfolio.

Within this scenario, it is possible to introduce quantitative tools, such as the quantile reserve. Indicating by $W(t)$ the financial position at time t, that is the stochastic mathematical reserve of a life insurance contract, or a portfolio of contracts, the quantile reserve at confidence level α, $0 < \alpha < 1$, is expressed by the value $W_{1-\alpha}{}^{*}(t)$ in the following equation

$$P\left\{W(t) > W_{1-\alpha}{}^{*}(t)\right\} = \alpha .$$

As one can see, the quantile reserve is a threshold value in the sense that in $(1 - \alpha)100\%$, $W(t)$ is smaller or equal to the quantile reserve. This representation gives rise to a market consistent value of the insurance liability.

3. The mathematical model

Let us consider an endowment policy issued at time 0 and maturing at time ξ, with initial sum insured C_0. Moreover, let us define $\left\{r_t; t = 1,...,\xi\right\}$ and $\left\{\mu_{x+t}; t = 1,...,\xi\right\}$ the random spot rate process and the mortality process respectively, both of them measurable with respect to the filtrations F^r and F^μ. The above mentioned processes are defined on an unique probability space $\left(\Omega, F^{r,\mu}, P\right)$ such that $F^{r,\mu} = F^r \cup F^\mu$ (e.g. [10] and [2]). For the participating policy, we assume that, in case of single premium, at the end of the t-th year, if the contract is still in force, the mathematical reserve is adjusted at a rate ρ_t defined as follows (e.g. [1])

$$\rho_t = \max\left\{\frac{\eta S_t - i}{1 + i}, 0\right\} \qquad t = 1,...,\xi . \tag{1}$$

The parameter η, $0 \le \eta \le 1$, denotes the constant participating level, and S_t indicates the annual return of the reference portfolio. The relation (1) explains the fact that the total interest rate credited to the mathematical reserve during the t-th year, is the maximum between ηS_t and i, where i is the minimum rate guaranteed to the policyholder. Since we are dealing with a single premium contract, the bonus credited to the mathematical reserve implies a proportional adjustment at the rate ρ_t also of the sum insured. According to Bacinello (e.g. [1]), it is assumed that if the insured dies within the term of the contract, the benefit increase of an additional last adjustment at the end of the year of death.

Denoting by C_t, $t = 1,...,\xi$, the benefit paid at time t if the insured dies between ages x+t-1, x+t or, in case of survival, for $t = \xi$, the following recursive relation holds for benefits of successive years

$$C_t = C_{t-1}(1 + \rho_t) \qquad t = 1,...,\xi .$$

The iterative expression for them is instead

$$C_t = C_0 \prod_{j=1}^{t}(1 + \rho_t) \qquad t = 1,...,\xi ,$$

where we have indicated by ϕ_t the readjustment factor

$$\phi_t = \prod_{j=1}^{t} \left(1 + \rho_j\right) \qquad\qquad t = 1,...,\xi.$$

In this context, as the elimination of the policyholder can happen in case of death in the year $t \in [0, \xi[$ or in case of survival $t = \xi$, the liability borne out by the insurance company can be expressed in this manner

$$W_0^L = \sum_{t=0}^{\xi} C_{t\ t-1/1}Y_x + C_\xi \, _\xi J_x,$$ (2)

where

$$_{t-1/1}Y_x = \begin{cases} e^{-\Delta(t)} & \text{if } t-1 < \text{Tx} \le t \\ 0 & otherwise \end{cases}, \qquad _\xi J_x = \begin{cases} 0 & if\ 0 < Tx \le \xi \\ e^{-\Delta(\xi)} & T_x \ge \xi \end{cases}.$$

In the previous expression T_x is a random variable which represents the remaining lifetime of an insured aged x, $\Delta(t) = \int_0^t r_u du$ is the accumulation function of the spot rate.

3.1. Financial and mortality scenario

The valuation of the financial instruments involving the policy will be made by assuming a two factor diffusion process obtained by joining Cox-Ingersoll-Ross (CIR) model for the interest rate risk and a Black-Scholes (BS) model for the stock market risk; the two sources of uncertainty are correlated.

The interest rate dynamics $\{r_t; t = 1,2,...\}$ is described by means of the diffusion process

$$dr_t = f^r\left(r_t, t\right)dt + l^r\left(r_t, t\right)dZ_t^r,$$ (3)

where $f^r\left(r_t, t\right)$ is the drift of the process, $l^r\left(r_t, t\right)$ is the diffusion coefficient Z_t^r is a Standard Brownian Motion; in particular, in the CIR model, the drift function and the diffusion coefficient are defined respectively as (e.g.[7])

$$f^r\left(r_t, t\right) = k\left(\theta - r_t\right), \qquad\qquad l^r\left(r_t, t\right) = \sigma_r \sqrt{r_t},$$

where k is the mean reverting coefficient, θ is the long term period "normal" rate, σ_r is the spot rate volatility. It must be pointed out that for pricing interest rate derivatives, the Vasicek model is widely used. Nevertheless, this model assigns positive probability to negative values of the spot rate; for long maturities this can have a relevant effect and therefore the Vasicek (e.g. [12]) model appears to be inadequate to value life insurance policies.

Clearly, on the fair pricing of our policy, the specification of the reference portfolio dynamics is very important. The diffusion process for this dynamics is given by the stochastic differential equation

$$dS_t = f^S\left(S_t, t\right)dt + g^S\left(S_t, t\right)dZ_t^S,$$ (4)

where S_t denotes the price at time t of the reference portfolio, Z_t^S is a Standard Brownian Motion with the property

$$Cov\left(dZ_t^r, dZ_t^S\right) = \varphi dt \qquad\qquad \varphi \in R.$$

Since we assume a BS type model (e.g.[3]), we have

$$f^S\left(S_t, t\right) = \mu_S S_t, \qquad\qquad g^S\left(S_t, t\right) = \sigma_S S_t,$$

where μ_S is the continuously compounded market rate, assumed to be deterministic and constant and σ_S is the constant volatility parameter.

For the dynamics of the process $\{\mu_{x+t:t}; t=1,2,...\}$, we propose to choose a model based on the Lee Carter methodology.

A widely used actuarial model for projecting mortality rates is the reduction factor model for which

$$\mu_{y:t} = \mu_{y:0} RF(y,t)$$

subject to $RF(y,0) = 1 \quad \forall y$, where $\mu_{y:0}$ is the mortality intensity of a person aged y in the base year 0, $\mu_{y:t}$ is the mortality intensity for a person attaining age y in the future year t, and the reduction factor is the ratio of the mortality intensity. It is possible to target RF, in a Lee Carter approach, $\mu_{y:0}$ being completely specified (e.g.[11]). Thus, $\mu_{y:0}$ is estimated as

$$\hat{\mu}_{y:0} = \sum_t d_{y:t} \bigg/ \sum_t e_{y:t} \, ,$$

where $d_{y:t}$ denotes the number of deaths at age y and time t, and $e_{y:t}$ indicates the matching person years of exposure to the risk of death. Taking the logarithm of equation (3) we have

$$\log \mu_{y:t} = \log \mu_{y:0} + \log RF(y,t)$$

s.c. $\log RF(y,0) = 0$. Defining

$$\alpha_y = \log(\mu_{y:0}) \qquad\qquad \log\{RF(y,t)\} = \beta_y k_t$$

the Lee Carter structure is reproduced (e.g.[9]).

In fact the Lee Carter model for death rates is given by

$$\ln(m_{yt}) = \alpha_y + \beta_y k_t + \varepsilon_{yt},$$

where m_{yt} denotes the central mortality rates for age y at time t, α_y describes the shape of the age profile averaged over time, k_t is an index of the general level of mortality while β_y describes the tendency of mortality at age y to change when the general level of mortality k_t changes. ε_{yt} denotes the error. In this framework, for our purposes, with y=x+t, one can use the following model for the time evolution of the hazard rate

$$\mu_{x+t:t} = \mu_{x+t:0} e^{\beta_{x+t} k_t} \, .$$

4. Numerical proxies for the quantile reserve via simulation procedures

4.1. The problem background

In this section we present a simulation procedure to calculate the quantile reserve, providing a practical application of the mathematical and accounting tools presented previously.

In particular our objective is to quantify the two critical values of the quantile reserve $W_{0,95}*(t)$ and $W_{0,99}*(t)$.

The computation of the quantile reserve values requires the knowledge of the distribution of $W(t)$.

To this aim we use a Monte Carlo simulation procedure which, as well known, is tipically employed to model random processes that are too complex to be solved by analytical methods. Moreover the use of simulation techniques allows to test in an easier way the effects of changes in the input variables or in the output function.

As a first step, as usually done in simulation procedures, we develop the statement of the problem giving the mathematical relation between the input and output variables. The mathematical model should be realistic and practically solvable. On the basis of the model presented in section 3, the output is given by the financial position of the insurer at time t, $W(t)$, and the input variables are given by the time of valuation t, the survival probabilities and the term structure of inter-

est rates, while the reference portfolio dynamics, as previously stated, is considered deterministic and constant. In this order of ideas, we assume that the best prediction for the time evolution of the surviving phenomenon is represented by a fixed set of survival probabilities, opportunely estimated taking into account the improving trend of mortality rates. As a consequence, in our application the first two inputs are deterministic while the random input is represented by the model describing interest rates distribution. In our case, the estimation of the risk-adjusted mean reverting parameter is not needed, since its value has no effect on the reserve determination.

The example of application we propose is referred to a life insurance participating contract. In particular we quantify at the beginning of the contract the two critical values $W_{0,95}*(t)$ and $W_{0,99}*(t)$ of the reserve distribution.

The output of the simulation procedure is a sample which gives N values for $W(t)$, being N the number of simulations.

In order to perform the simulation procedure it is necessary to get the discrete time equation for the chosen SDE describing the evolution in time of the interest rates (3). We choose the first order Euler's approximation scheme, obtaining the following sample path simulation equation:

$$r_{k\Delta t} = r_{(k-1)\Delta t} + \alpha(\mu - r_{(k-1)\Delta t})\Delta t + \sigma\sqrt{r_{(k-1)\Delta t}\Delta t} \cdot \varepsilon_k \qquad k=1,2,..,T , \qquad (5)$$

where $\{\varepsilon_k\} \approx N(0,1)$.

This approximation scheme is characterized by an easy implementation and a simple interpretation of the results.

The discretized process we consider can be represented by the sequence $\{r_{\Delta t}, r_{2\Delta t}, ..., r_{k\Delta t}\}$, where k is the number of time steps, Δt is a constant and T is the time horizon.

The following simulation procedure is carried out in order to gain a sample of N values of $W(t)$:

a)   generation of T pseudo-random values $\{\varepsilon_k\} \approx N(0,1)$;

b)   computation of one simulated path for the stochastic interest rate $\{r_{k\Delta t}\}$ using the T values obtained in step (a);

c)   computation of one value of the reserve on the basis of the previous results. The simulation procedure will be repeated N times to gain N values for $W(t)$.

At this point our purpose is to quantify the two critical values of the reserve distribution $W_{0,95}*(t)$ and $W_{0,99}*(t)$. Since the reserve is a liability, we are interested in the right hand tail of the distribution.

In the following, we propose a numerical application considering two different values of N. Being the discretized CIR model composed by a deterministic part and by a stochastic one $\{\varepsilon_k\} \approx N(0,1)$, according to the Glivenko-Cantelli theorem, we expect that the empirical distribution of $W(t)$ asymptotically tends to a normal one.

4.2. Numerical results

The numerical example we propose refers to a participating contract issued on a person aged 40 with time to maturity 20 years.

We assume for the CIR process α= 0.0452, σ= 0.0053 and the initial value r_0=0.0279, estimated on the 3-month T-Bill January 1996-January 2006, $\mu_S = 0.03$, $\sigma_S = 0.20$ for the time evolution of the reference fund. For the correlation coefficient φ we adopt a slightly negative value $(\varphi = -0.06)$ coherently with the literature for the Italian Stock market. For the survival probabilities we use the mortality Italian data for the period of 1947-1999 to evaluate the projection of the mortality factor in a Lee Carter context.

We report the results obtained by means of the procedure proposed in section 4.1 considering N=1000, 10000.

We show that, increasing the number of simulations N, we obtain a more significant sample of $W(t)$ values, getting more exact information about its distribution.

In Table1 the characteristic values of the simulated distribution of the reserve are reported, corresponding to the number of simulation paths indicated in the column.

Table 1

Reserve distribution results obtained for N=1000 and N=10000 simulation paths

N	1000	10000
Mean	1018.107	1018.120
Median	1017.070	1017.378
Maximum	1202.904	1299.42319.64853
Minimum	832.0687	742.9516
Kurtosis	2.644382	2.988519
Skewness	-0.030861	0.013978

Table's 1 contents confirm the asymptotic behaviour of the empirical distribution of the random variable $W(t)$. Now, as already recalled, the Glivenko-Cantelli theorem is verified, in the sense that, as we can easily observe, as N increases $W(t)$ approximates a normal distribution. In particular, in the case of N=10000, kurtosis takes the value 2.988519 and skewness takes the value 0.013978. It is well known that a normal variable has a kurtosis of 3 and a skewness equals zero, therefore the obtained values in the case of N=10000 can be considered acceptable. Moreover, we get the following results:

Table 2

Jarque-Bera test for N=10000

J-B test	0.380561
Probability	0.826727

As well known, the J-B (e.g. [3]) is a statistic for testing whether the series is normally distributed. The JB test is known to have very good properties in testing for normality; it is easy to compute and it is commonly used in the regression context in econometrics (e.g. [5]). The test statistic measures the difference of the skewness and kurtosis of the series with those from the normal distribution. Under the null hypothesis of a normal distribution, the J-B statistic is distributed as a chi-square with two degrees of freedom ($\chi^2(2)$). The reported probability is the probability that the J-B statistic exceeds (in absolute value) the value under the null hypothesis. A small probability value leads to the rejection of the null hypothesis of a normal distribution. In our case, being the probability equal to 0.826727, we can accept the hypothesis of normal distribution of W(t).

The asymptotic behaviour of the empirical distribution is shown graphically too, by means of the histograms and the Quantile-Quantile plots shown below for each value of N. As we can observe looking at Figure 2. R(t) well approximates a normal distribution when N=10000.
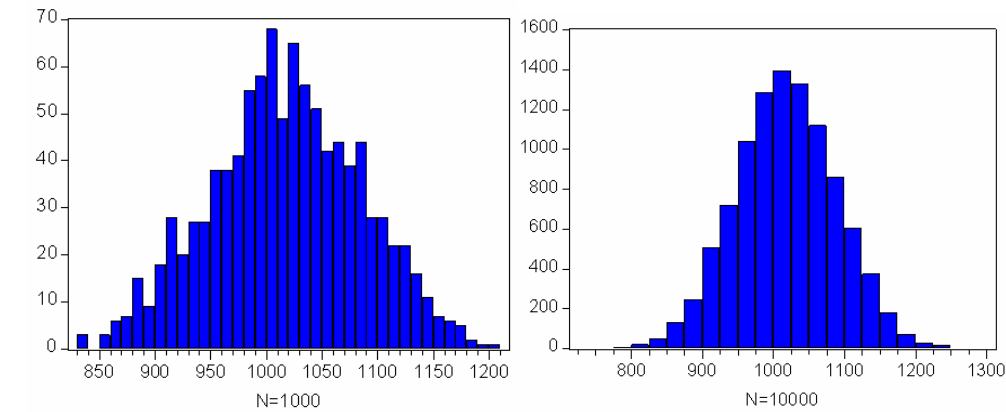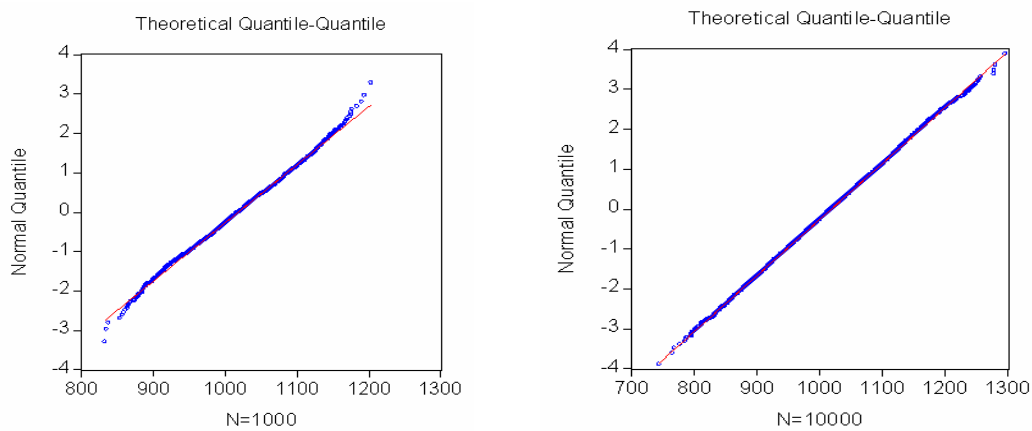
Fig.1. Histograms   N=1000, N=10000

Fig.2. Quantile Quantile plots N=1000, N=10000

Table 3

Quantile reserve for N=1000 and N=10000

N	1000	10000
W*(99%)	1167.501	1181.696
W*(95%)	1129.681	1133.742

Finally Table 3 shows the two critical values of the quantile reserve calculated for N=1000 and 10000 taking into account that the mathematical provision is a liability and that the critical values lie in the right-hand tail of the distribution. The difference between the W* values and the M[W], the mean value of the mathematical reserve, can be interpreted as an absolute index of the riskiness borne out by the insurer due to the uncertainty about interest and mortality rates. Obviously, the critical values of the quantile reserve obtained by means of 10000 simulation paths are more reliable.

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