"The membership fee increases over time with the network size"

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Abstract

We study the dynamic pricing decisions for monopolistic network service providers. We assume that the firm follows a two-part pricing scheme, which consists of an ongoing membership fee, and a usage fee for communications within the network. We study the firms' problems as an optimal control problem and establish the optimal pricing policy. It's demonstrated that in the two-part tariff pricing strategy, the membership fee always increases over time with the increase in the network size. These results are particularly useful for managers.

Introduction

Two-part tariffs are widely practiced in the Internet and telecommunications. With the growth of these industries the pricing of services that take into account the growth of subscribers as well as the demand for services by members of the service poses a challenge to managers. There are several key issues associated with this. The first has to do with the fact that services typically offer a two-part price that consists of a membership fee and a usage price. The membership fee is a fee just to join the network while the usage price is variable. This raises an immediate question of what should be the optimal two-part tariff for a firm serving a growing network of subscribers? Appealing to Optimal Control, Fruchter and Rao (2001) offer an answer to this question. The paper deals with a situation, which is similar to that of a durable product in the sense that a customer becomes a member of the network only once just as a customer buys only one unit of a durable product. Stated differently, the customer adopts the service. However, the situation described in the paper is also similar to a non-durable product in the sense that the customer pays an on-going fee and a usage price that could be thought of as repeat purchase price. Durable goods pricing has been analyzed by Dolan and Jeuland (1981) and Kalish (1983) for a monopolist and by Bass and Rao (1985) and Dockner and Jorgensen (1988) under competition. However, past work has not explicitly considered a two-part pricing policy that changes over time with the growth of a network. In their recent paper on the pricing of cellular phones Jain, Muller and Vilcassim (1999) have examined the question of how the pricing of a complementary product such as the handset influences the pricing of the metered service, phone calls. They conclude that under certain cost conditions and competition in a two-period world the price of the telephones decreases over time while that of the calls is non-decreasing. In developing the model they assume that the average demand, per customer, in minutes of phone calls decreases over time as the network size grows. The assumption is consistent with data and has also been observed by Manova et al. (1998). Fruchter and Rao (2001) make a similar assumption. However, instead of the focus on complementary products they focus on network membership and usage by network members. They found that for some specifications the membership fee constitutes a penetration strategy, being low at the beginning, and increasing with network size. The intuition behind this was explained by the assumption that early adopters are heavy users, and thus it is better to increase network size through a low membership fee, and charge a high usage price initially for the heavy users.

This study generalizes the two-part tariff pricing strategy of Fruchter and Rao (2001) by demonstrating that the membership fee always increases over time with the increase in the network size. Next we present the model.

1. Model formulation and notation

We use the same model and assumptions as in Fruchter and Rao (2001). Thus, we consider a monopolist who markets a service to which consumers can subscribe, and then use at a certain usage level. The firm's revenues are from the customers' usage of the service and from membership fees. The firm must decide how to price its service over time given the dynamics of the network service and usage demand. We model the dynamics consistent with broad empirical observations and then ask what the optimal pricing policy is.

Denote $N(t)$ as the number of subscribers to the service at time t. This can be considered the cumulative adoption of the service at time t. The demand of this population of consumers would be $D(t)$, and produce revenues resulting from their usage of services. An additional source of revenue could be a membership fee of $k(t)$ per customer. Denote $p(t)$ as the unit price of usage charged by the monopolist, c^p the firm's cost of providing the

services, and c^k the cost per subscriber. Note that there are two cost parameters. The first, c^p, is the cost of providing a metered service that would correspond to, for example, the minutes of telephone calls, the minutes of connection to the Internet or the number of quotes provided by a stockbroker. The second, c^k, is the cost of providing the service to a member that would correspond to, for example, the billing, computer server space and help provided by the firm. This cost is generally not metered and in that sense is fixed with respect to usage. The net revenue $R(t)$ is then given by

$$R(t) = N(t)[k(t) - c^k + D(t)(p(t) - c^p)].$$

The monopolist's problem is to choose the two-part pricing strategy $[k(t), p(t)]$ that will maximize the discounted profits over a time horizon $[0, T]$.

We assume that the growth of the number of subscribers will depend on both $k(t)$ and $p(t)$. The higher the fee and the price are, the slower is the growth. We capture these effects by assuming

$$\dot{N} = f(N, k, p), \quad \partial f(N, k, p)/\partial k < 0 \text{ and } \partial f(N, k, p)/\partial p < 0,$$

where $\dot{N} = \dot{N}(t) = dN(t)/dt$. Furthermore, we assume that the membership fee $k(t)$ is non-negative. Optimal control theory (cf. Kamien and Schwartz, 1991) provides a good method of formulating and analyzing the monopolist's problem of determining the two-part pricing policy, (k, p), because the policy is changing over time. Since we are concerned with the flow of profits over time they would have to be discounted. Let r denote the discount rate. Assuming that the firm has a long-lived sum of discounted profits, over the infinite horizon, Π are:

$$\Pi = \int_0^{\infty} N(t)[(k(t) - c^k) + (p(t) - c^p)D(N, p)]e^{-rt}dt.$$

Note that the firm receives revenues from membership fees based on k and usage based on p and D. The monopolist's goal is to maximize Π, while recognizing that he can influence the growth of customer base $\dot{N}(t)$ by a judicious choice of $k(t)$ and $p(t)$. Denote the optimal policy by the pair $(k^*(t), p^*(t))$. This optimal policy must then be a solution to the following mathematical problem:

$$\begin{cases} \max_{k, p} \Pi \\ s.t. \ \dot{N}(t) = f(N(t), k(t), p(t)), \quad N(0) = 0 \end{cases}$$

2. Optimal policy

Fruchter and Rao (2001) examined the optimal pricing decision solution of this problem. They found that the solution that maximizes Π reduces to one, which satisfies the necessary conditions that k and p maximize the Hamiltonian, because it recognizes the value for increasing the customer

base is given by λ, the co-state associated with the state variable N. These conditions translate into the following mathematical conditions (see Fruchter and Rao (2001) for details):

$$\dot{\lambda} = r\lambda - (p - c^p)(D + ND_N) - (k - c^k) - \lambda f_N,$$

$$\lim_{t \to \infty} \lambda(t)e^{-rt} = 0 \qquad (1)$$

$$\lambda f_k + N = 0 \qquad (2)$$

$$ND + (p - c^p)ND_p + \lambda f_p = 0. \qquad (3)$$

Equations (2) and (3) are the necessary first order optimality conditions for k and p, respectively. Condition (1) is a differential equation for λ with a boundary condition at ∞. The boundary condition says that at the end of the planning horizon there is no value to increasing the network size. The differential equation tells us what the value of increasing the network size is at each point in time. In addition to these conditions, we must verify second order conditions to ensure that we are indeed maximizing profits. Such a sufficient condition for local optimum, see Fruchter and Rao (2001), is that the Hessian matrix of the Hamiltonian, denoted by $\hat{H}$, is negative definite, i.e.,

$$\hat{H} = \begin{bmatrix} \lambda f_{kk} & \lambda f_{kp} \\ \lambda f_{pk} & 2ND_p + (p - c^p)ND_{pp} + \lambda f_{pp} \end{bmatrix} < 0 \quad (4)$$

for all (k, p) near (k^*, p^*). In (4) $f_{kp}, f_{pk}, f_{kk}, f_{pp}, D_{pp}$ denote the corresponding second order partial derivatives.

The authors found that for some specifications the membership fee constitutes a penetration strategy, being low at the beginning, and increasing with network size. The intuition behind this was explained by the assumption that early adopters are heavy users, and thus it is better to increase network size through a low membership fee, and charge a high usage price initially for the heavy users.

Now we can show the following general result.

Proposition. A positive membership fee always follows a penetration strategy.

Proof. We will prove this proposition by a contradiction. Assume that k decreases when N increases. Consider the necessary condition (2); since $N > 0$, we have $\lambda f_k < 0$. From the sufficient condition (4), we obtain that the first principal minor, λf_{kk}, should be negative. Thus, if k decreases when N increases, the value of λf_k increases closer to zero, with its absolute value diminishing. However, as N continues to increase, the necessary condition $\lambda f_k + N = 0$ can no longer hold, and thus we arrive at a contradiction. Thus, k should increase with N. This

ends our proof that the membership fee always follows a penetration strategy.

Conclusion

The dynamic two-part tariff pricing policy presented here extends the existing dynamic pricing research (e.g., Kalish, 1983; Dockner and Jorgensen, 1988; and Fruchter and Rao, 2001. The study demonstrates that a monopolistic firm should always charge a low membership fee in order to build the network, and gradually increase the membership fee as the network grows during the diffusion process.

The optimal policy being consistent with the results of casual observation of Internet Service Provider pricing patterns has managerial importance by giving guidelines how to proceed. At the beginning there is a period when customers are acquired, during which the membership fee is low. Later, when the increase in the number of customers has been accomplished, the membership fee increases.

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