Yong-Chern Su (Taiwan), Ling-Chin Kao (Taiwan), Peiwen Chen (Taiwan)

NA-GARCH modeling value-at-risk of financial holdings

Abstract

The paper employs EGARCH, representing rotation asymmetry effect, and NA-GARCH, representing shift asymmetry effect, with variations in their mean equations: ARMA(1,1), AR(1), MA(1), and "in-mean" models as value-at-risk (VaR) forecast models. Forward tests of one-day ahead VaR performance in 99 % and 95 % confidence levels are evaluated with realized P&L for 216 observations in two simulated portfolios standing for financial holdings in Taiwan. Based on violation number, it also compares other performance indicators, such as mean VaR, aggregate, mean and max violation, to strike a balance between model effectiveness and capital charge efficiency. The main findings are as follows. First, all of the VaR forecast models, except ARMA(1,1) under 99%, in EGARCH and NA-GARCH achieve the targeted violation rate and are qualified to be internal models. Second, ARMA(1,1) models show synchronous volatile trend as real P&L time series, yet the one-day lag makes more violations. In addition, the excessive volatility is the implication of over fitting problem. Third, no particular VaR model can distinctively outperform others and serves as the best-fitting model, nor can the authors tell the shift or the rotation asymmetric effect dominates the portfolios during the observation period.

Keywords: market risk, value-at-risk, GARCH, NA-GARCH, financial holdings.
JEL Classification: G21, G28.

Introduction

Today's dynamic financial markets and rapid developments of financial products give financial institutions greater opportunities to access vigorous business activities, which bring about more risk at the same time. The importance of risk management in financial institutions has been valued not only for it strengthens the soundness and stability of the banking system, but for the capability of risk management becomes core competence of the financial institutions. With the implementation of Basle II in the end of 2006, more sophisticated approaches, namely internal models, are encouraged to use. These internal models generally require lower capital charge. Thus, financial institutions can enjoy higher profitability via efficient use of capital.

We believe the financial holdings in Taiwan will have the urgent need to develop their own internal models in the near future, starting from the market risk VaR models. However, few empirical studies have been conducted on VaR models actually in use. Berkowitz and O'Brien (2002) firstly provide VaR performance testing of six large commercial banks in U.S. and propose an outperforming single ARMA-GARCH model.

The asymmetric GARCH models perform better than GARCH models in VaR estimates have already been proved. What arouses our interests is the asymmetry can be captured in either a shift or a rotation of news curves; nevertheless based on Hentschel (1995) study, these two types can't be substituted for each other and the "shift" is the dominant source of asymmetry for small shocks. Hence, we employ non-linear asymmetric GARCH (NA-GARCH) group models, providing shift asymmetry effect, to compare with EGARCH group models, providing rotation asymmetry effect in our study. The purpose is to verify which asymmetric effect is more influential in VaR forecast models and hopefully, in both the model effectiveness and capital efficiency perspectives, to find the most appropriate VaR model for financial holding company in Taiwan through asymmetric GARCH models: EGARCH and NA-GARCH, combining various return structures.

We find that all the VaR forecast models in EGARCH and NA-GARCH can be viewed as qualified internal models for bank because all VaRs achieve the targeted violation rate (for 99% confidence level, target violation rate is 1%, outlier number must be smaller than 2,1% times 216 observations). In portfolio A, NA-GARCH and EGARCH models perform equally well in violation number under 99% confidence level while no consistent results under 95% confidence level. In this category ARMA (1,1)-EGARCHM (1,1) generates 10 violations, the largest number among all models in both portfolios. NA-GARCH models outperform EGRCH models in portfolio B under 99% and 95%.

As for pair comparison and VaR movements to capture the dynamic volatility, there is no big difference in both models in portfolio A, while after closely observing the line graphs, NA-GARCH performs slightly better than EGARCH since it rapidly follows the negative shocks, though the response is relatively small. In portfolio B, EGARCH responds actively to changing profit and loss. Notice that ARMA models have almost exactly the same volatile trend as real P&L time series, yet the one day lag makes more violations than other models – MA(1), AR(1), and "in mean" – which have more stable VaRs in all situations.

The remainder of the paper is organized as follows. Section 1 defines the data and presents the main methodology of our VaR forecast models. Section 2 evaluates model performance with realized P&L. The final section provides general conclusions and further discussions.

1. Data and methodology

Under the constraint on the high confidentiality of trading data in financial holding companies in Taiwan and the difficulty to access the information, simulated portfolios are construed for further study of VaR models. Two most active financial holding groups' trading books, Fun Bon and Cathy, were simulated in our two portfolios. All the positions in both portfolios were from November 28, 2001 through April 15, 2003.

We employ EGARCH and NA-GARCH models in our study. Nelson (1991) has developed E-GARCH model to improve the shortcomings of GARCH model, such as inability to express leverage effect, shock persistence, and the no negativity constraints on coefficients. The E-GARCH function can be expressed as follows:

$$R_t = \alpha + Volume\ R_{t-1} + \beta\ h_t + MA\ \varepsilon_{t-1} + \varepsilon_t ,$$

$$\ln h_t = A + B(1)\ \ln h_{t-1} + C(2)(\varepsilon_{t-1} / h_{t-1}^{\ 0.5}) + \\ + \delta\ (|\varepsilon_{t-1}| / h_{t-1}^{\ 0.5} - (2/\pi)^{0.5}),$$

where $\ln h_t$ is the logarithm of conditional variance at time t, R_t is the return on a portfolio at time t, π is the ratio of the circumference of a circle to its diameter, A, $B(1)$, $C(2), \delta, \alpha, \beta$ MA and $Volume$ are parameters.

Combining mean equation and variance equation above we have ARMA (1,1)-EGARCHM (1,1) model. The E-GARCH model is asymmetric because the level of $\varepsilon_{t-1} / h_{t-1}^{\ 0.5}$ is included with a coefficient $C(2)$. Since the coefficient is typically negative, positive return shocks generate less volatility than negative return shocks, all else being equal.

NA-GARCH model first appeared in the paper proposed by Engle and Ng (1993), they recommended the "news impact curve" as a measure of how news is incorporated into volatility estimates by traditional GARCH model and the other five parametric models which are capable of capturing the leverage and size effects, the NA-GARCH is one among them. Hentschel (1995) classified NA-GARCH model as a shifted news impact curve to attain asymmetry and had the conclusion that the shift is the dominant source of asymmetry for small shocks. The NA-GARCH function can be expressed as follows:

$$R_t = \alpha + Volume\ R_{t-1} + \beta\ h_t + MA\ \varepsilon_{t-1} + \varepsilon_t ,$$

$$\ln h_t = A + B(1)h_{t-1} + C(1)(\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}})^2 ,$$

where $\ln h_t$ is the logarithm of conditional variance at time t, R_t is the return on a portfolio at time t, A, $B(1)$, $C(1)$, $C(2)$, α, β MA and $Volume$ are parameters.

Combining mean equation and variance equation above we have ARMA (1,1)-NA-GARCHM (1,1) model. The asymmetry lies in the coefficient $C(2)$. If $C(2) > 0$, then the positive innovations will bring about higher volatility than negative innovations of the same magnitude will do and vice versa.

Under the ARMA (1,1)-EGARCHM (1,1) and ARMA (1,1)-NA-GARCHM (1,1) models, various sub-models could be derived from the mean equation:

$$R_t = \alpha + Volume\ R_{t-1} + \beta\ h_t + MA\ \varepsilon_{t-1} + \varepsilon_t ,$$

where α, β, MA and $Volume$ are parameters.

In order to build E-GARCH and NA-GARCH VaR forecast models, parameters need to be estimated for the initial period. Data period from November 28, 2001 to April 15, 2003, corresponding to 617 observations, which are divided into two groups: in-sample and out-of-sample. 401 in-sample return data are put into the mean and variance equations to obtain stable parameters, then the mean and variance for the remaining 216 days can be predicted. Rolling out-of-sample VaR forecast begins at day 402, and out-of-sample estimates are updated daily. Thus, one-day-ahead 99% and 95% VaR forecast at time t is given by $\hat{R}_{t+1} - 2.326 \times \hat{h}_{t+1}$ and $\hat{R}_{t+1} - 1.645 \times \hat{h}_{t+1}$, respectively.

The remaining 216 observations of real time series P&L can play as a natural benchmark for evaluating our VaR forecast models since the most basic test of a VaR is to see if the stated probability level is actually achieved. If the actual loss at time t is greater than the forecasted VaR, we call this a "hit", in other words, violations. In the forward testing, out-of-sample VaR forecast for 216 days (from 16/06/2002 to 15/04/2003) are tested in 95% and 99% confidence level. However, unilaterally based on violation numbers to judge the best-fit VaR model is not complete. In banks' points of view, the effectiveness of model and the efficiency of capital charge are both curial in market risk management. Therefore, other than violation number, we employ mean VaR, mean violation, aggregate violation and max violation as indicators to compare which VaR models in our study best fit the two simulated portfolios.

2. Empirical results

2.1. Time series pattern of daily P&L. We calculate the whole 617 sample points based on marking to market to get the daily profit and loss. Neither of the return distribution resembles normal distribution, and the phenomenon of leptokurtosis is observed with both return portfolios having clusters around their mean values. Furthermore, the skewness values suggest portfolio A is left-skewed, while portfolio B is right-skewed. This property is also revealed by the statistic results that portfolio B has higher 95th percentile and 99th percentile return than portfolio A. We find that portfolio A has slightly higher average return and smaller standard deviation; this also conforms to more diversified asset allocation in portfolio A. In theory, leptokurtosis and skewed distribution has very different characteristics from normal distributions. Hence, applying VaR with other methodology, conditional variance model is considered appropriate in our study.

2.2. Testing results of VaR models. We conduct EGARCH and NA-GARCH two groups of VaR forecast models, and four sub-models are in each category based on variation of the mean equation, namely ARMA (1,1), MA (1), AR (1) and simply EGARCHM (1,1) or NA-GARCHM (1,1) models. Estimated parameters and detailed statistics in out-of-sample forward testing of eight models are illustrated respectively from Table 1 to Table 8 (see Appendix). Notice that *C*(2) coefficient is negative in both EGARCH and NA-GARCH models, that comply with the asymmetric effect we assumed. Pair comparisons between EGARCH and NA-GARCH on the same mean structure are later implemented to identify which is the better-fit model for two simulated portfolios.

2.2.1. Various VaR models. In Table 9 and 10 (see Appendix), we demonstrate all the 8 models under 99% and 95% confidence level of portfolio A in terms of violation number and other performance indicators: mean VaR, aggregate, mean and max violation. In portfolio A, except ARMA(1,1) models generate three violations, rest of the models generate only one outlier under 99% confidence levels. However, under the 95% confidence level, violations dramatically increase up to 10 in ARMA(1.1)-EGARCHM(1,1) model, with the biggest max violation and aggregate violation amount. Portfolio B has milder change between 99 % and 95 % confidence levels, yet ARMA(1.1)-EGARCHM(1, 1) remains the worst model, with 5 violations, 3.6773% aggregate violation in 95% range. The time series movement of realized P&L and corresponding one-day ahead VaR forecast models depicted in Figures 1-8 (see Appendix) can give us further understanding how models dynamically react to volatility changes. ARMA(1,1) models volatile follow the realized P&L closely with one day lag while other MA(1), AR(1), "in mean" models relatively have smoother line. In portfolio A, lines plotted by VaR forecast models including MA(1), AR(1), "in mean" models, both in EGARCH models and NA-GARCH models are almost tangling. Portfolio B has more distinctive difference among the three MA(1), AR(1), "in mean" models in EGARCH models.

2.2.2. EGARCH vs. NA-GARCH model comparisons. Pair comparison results of EGARCH and NA-GARCH VaR forecast models are illustrated in Tables 11 and 12 (see Appendix). Under 99% confidence level, EHARCH and NA-GARCH generate same number of violations in portfolio A. In portfolio B, NA-GARCH groups outperform based on none violation number (except for violation number of ARMA(1,1) = 3) in 216 observation days, yet mean VaR, implication of capital charge, is more than 4% (only ARMA(1,1) = 2.76%), much bigger than EGARCH models. MA (1)-EGARCHM(1,1), AR(1)-EGARCHM(1,1) and AR(1)-NAGARCHM(1,1) have comparable better performance in portfolio A under 95% confidence level. In portfolio B, NAGARCH groups remain zero violation rate (except ARMA (1,1), violation number = 5), and this time with smaller mean VaR and mean VaR differences.

Again, the movement of VaR forecast models can clearly be observed in line graphs (see Figures 11-18 displaying pair comparisons in Appendix) to evaluate the ability to capture time varying volatility among various models. In portfolio A, there is no big difference between EGARCH and NA-GARCH model in the behavior of responding volatility, the reacts are slow and mild, and the lines are relatively stable, ARMA models excluded. However, in portfolio B, EGARCH group models quickly respond the shocks of daily profit and loss, and moreover, with smaller VaR amount than NA-GARCH models throughout the forward forecasting period.

Conclusions

This paper has employed EGARCH and NA-GARCH models with variations in their mean equations: ARMA(1,1), AR(1), MA(1) and simply "in mean" models as VaR forecast models. Then, one-day ahead VaR performance under 99% and 95% confidence levels is evaluated with realized P&L in two simulated portfolios representing two most active financial holding companies in Taiwan. We conduct the forward testing of VaRs for 216 observations based on violation number. However, other indicators such as mean VaR, aggregate, mean and max violation are considered as well to strike a balance between effectiveness and efficiency.

We find that all the VaR forecast models in EGARCH and NA-GARCH can be viewed as qualified internal models for bank because all VaRs achieve the targeted violation rate (for 99% confidence level, target violation rate is 1%, outlier number must be smaller than 2,1% times, 216 observations). In portfolio A, NA-GARCH and EGARCH models perform equally well in violation number under 99 % confidence level while no consistent results under 95% confidence level. In this category ARMA(1,1)-EGARCHM(1,1) generates 10 violations, the largest number among all models in both portfolios. NA-GARCH models outperform EGRCH models in portfolio B under 99% and 95%. As for pair comparison and VaR movements to capture the dynamic volatility, there is no big difference in both models in portfolio A, while after closely observing the line graphs, NA-GARCH performs slightly better than EGARCH since it rapidly follows the negative shocks, though the response is relatively small. In portfolio B, EGARCH responds actively to change profit and loss. Notice that ARMA models have almost exactly the same volatile trend as real P&L time series, yet the one day lag makes more violations than other models – MA(1), AR(1), and "in mean" – which have more stable VaRs in all situations.

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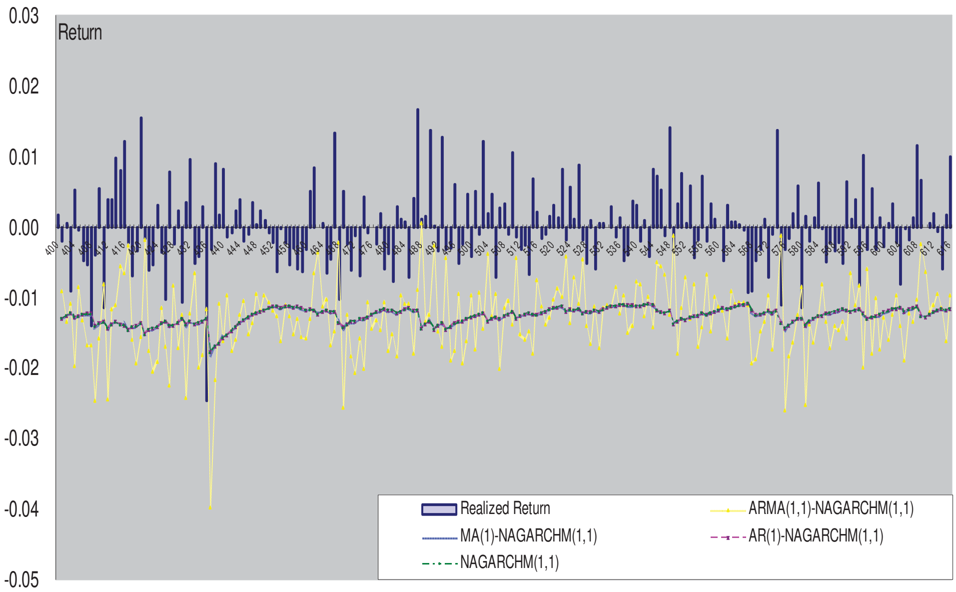
Appendix



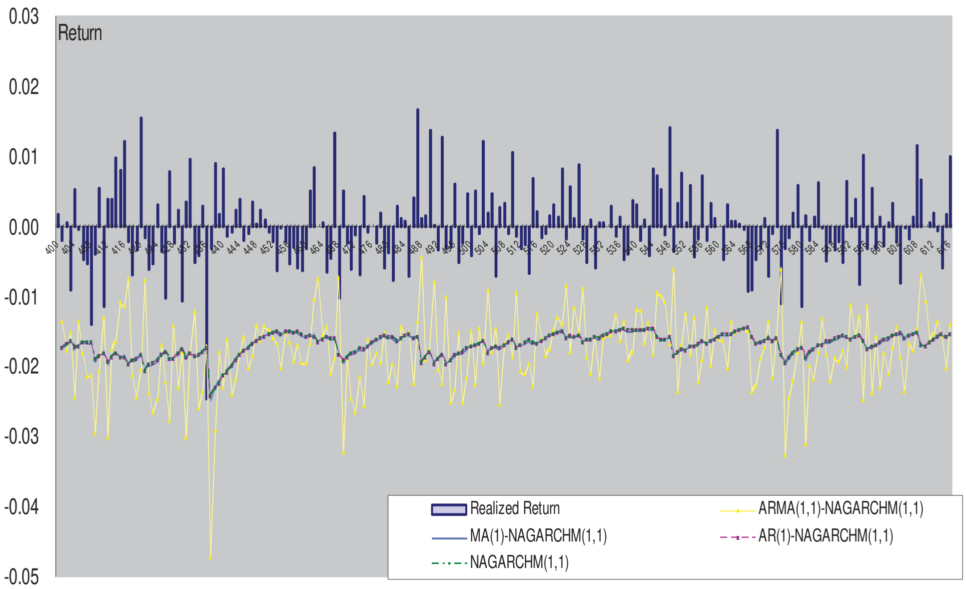
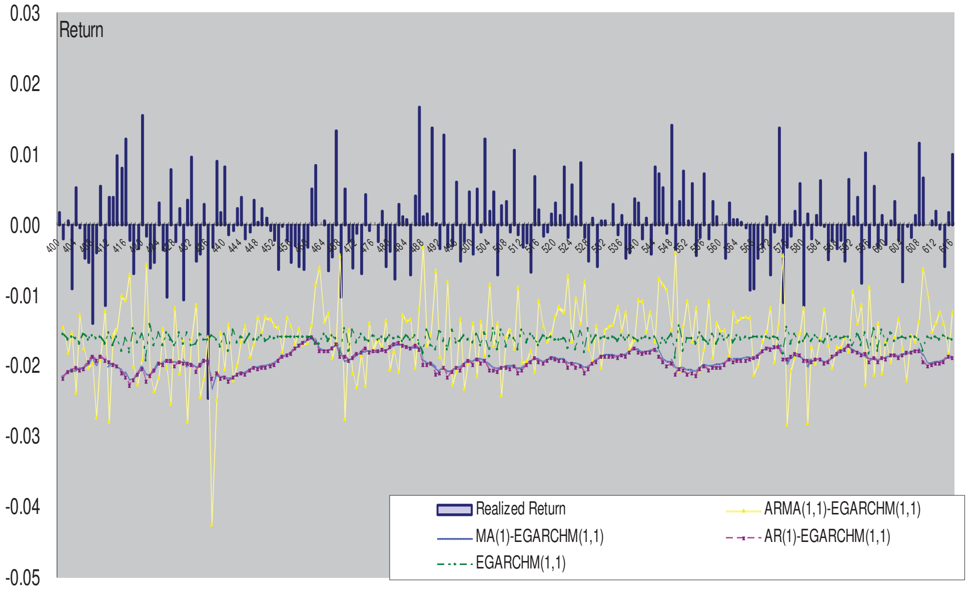


Fig. 1. NA-GARCH VaRs in portfolio A under 99% and 95% confidence levels



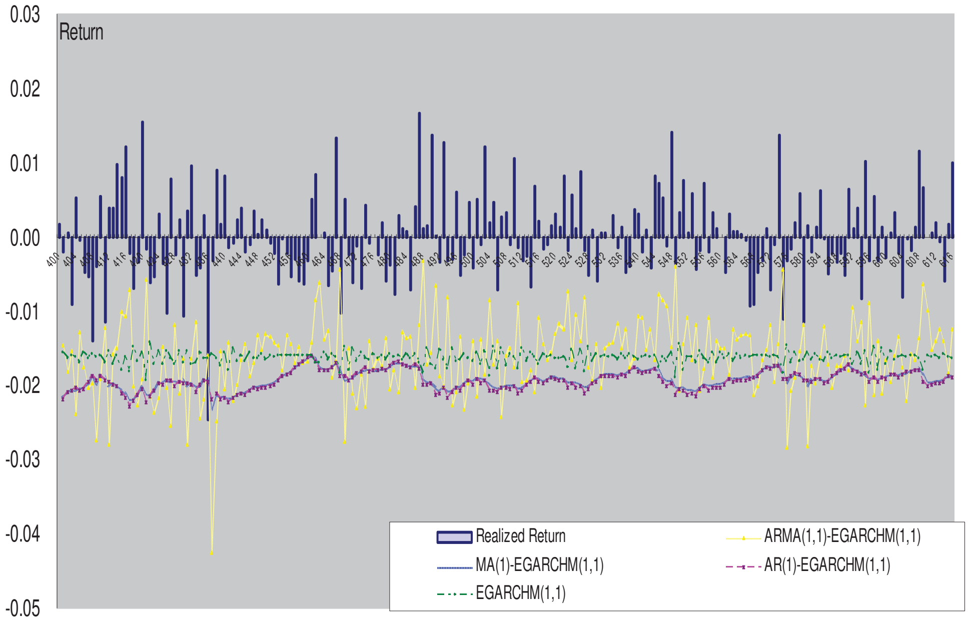
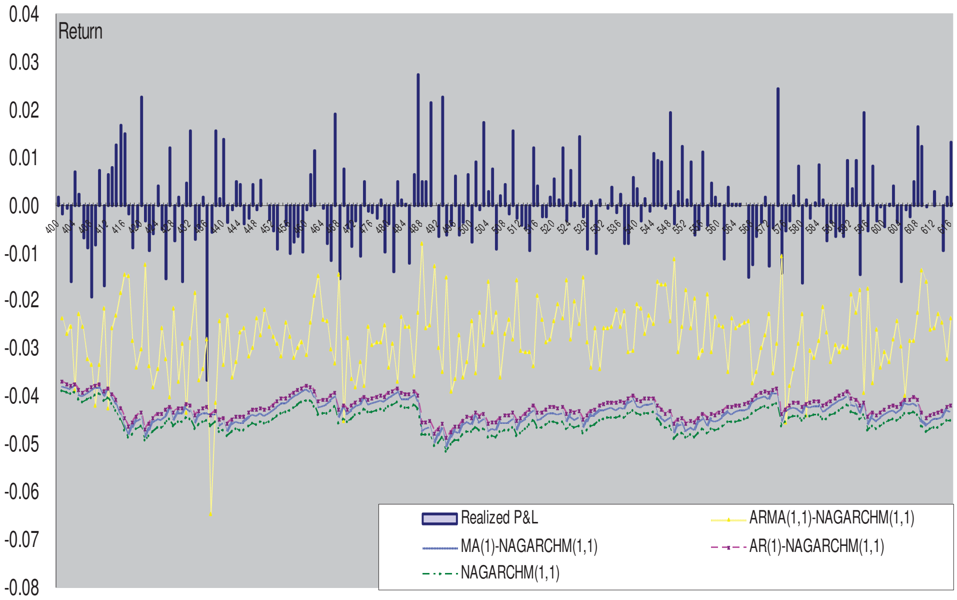


Fig. 2. EGARCH VaRs in portfolio A under 99% and 95% confidence levels



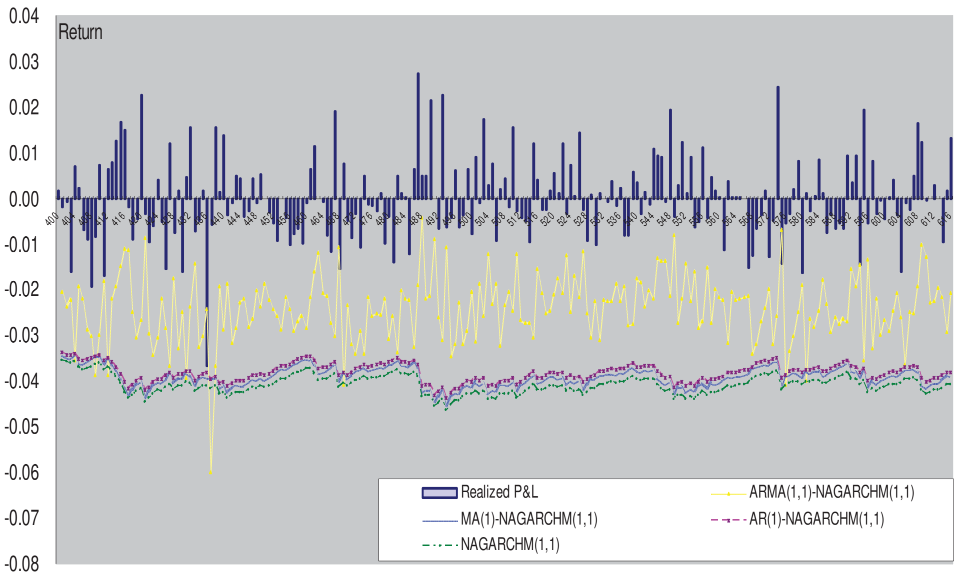


Fig. 3. NA-GARCH VaRs in portfolio B under 99% and 95% confidences level

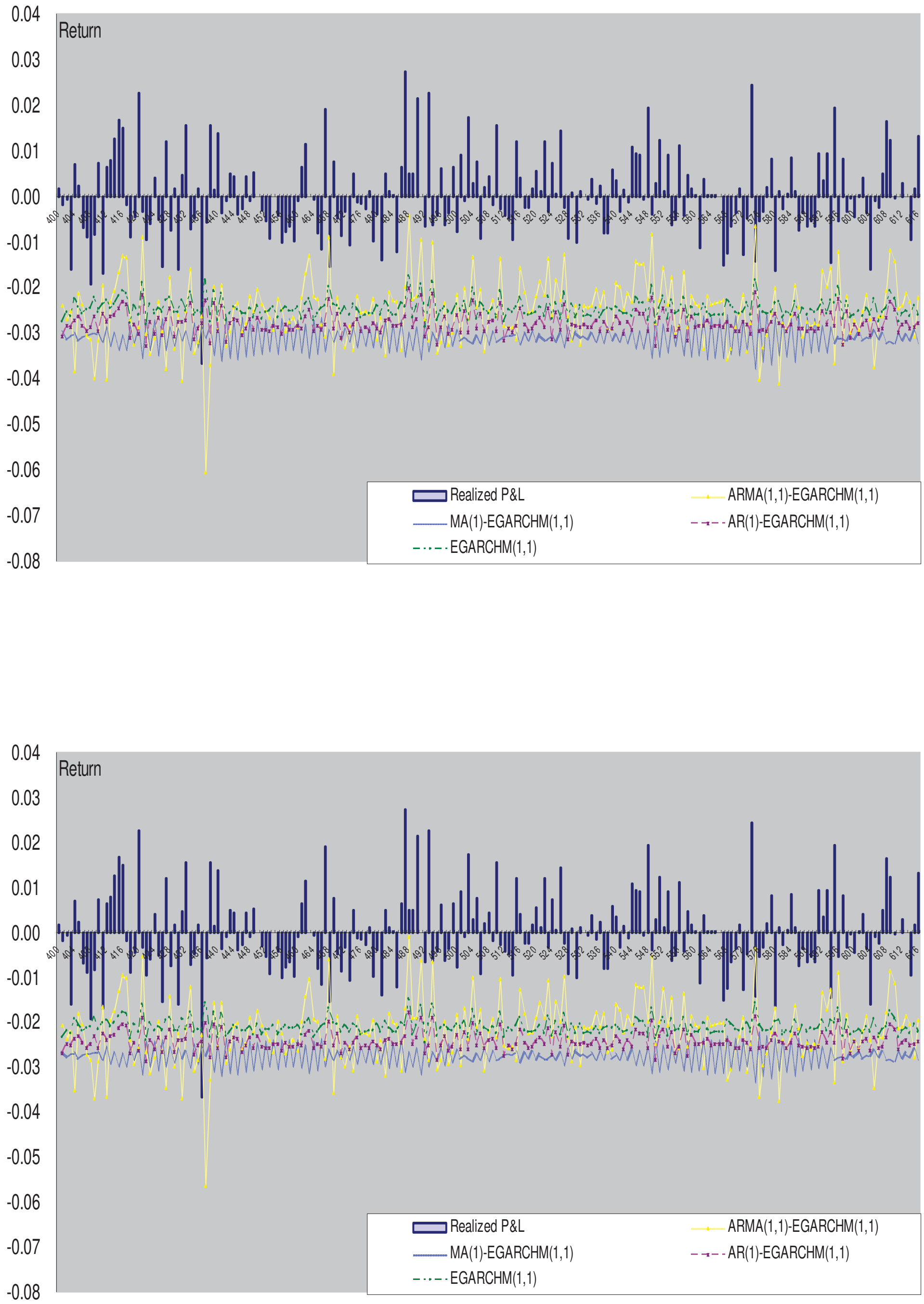
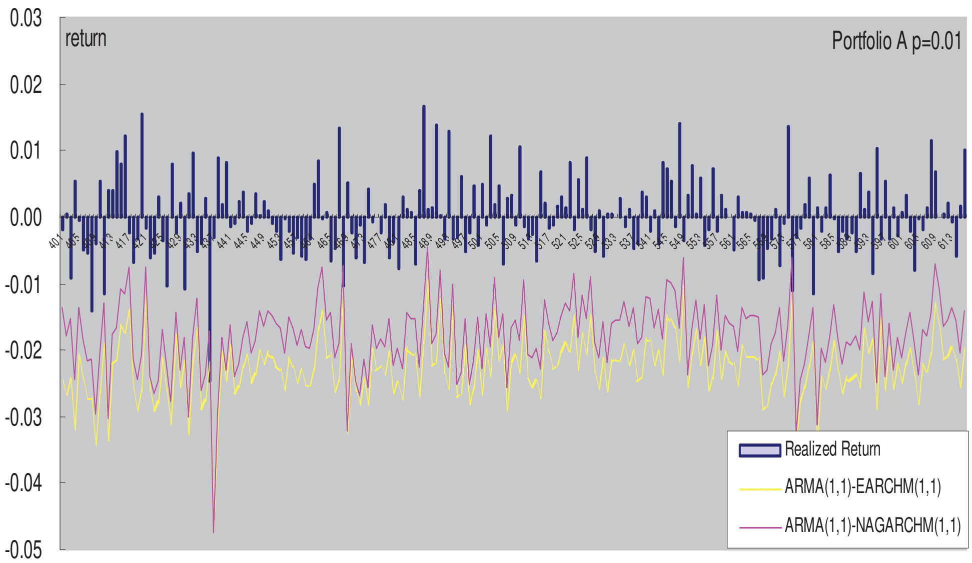


Fig. 4. EGARCH VaRs in portfolio B under 99% and 95% confidence levels



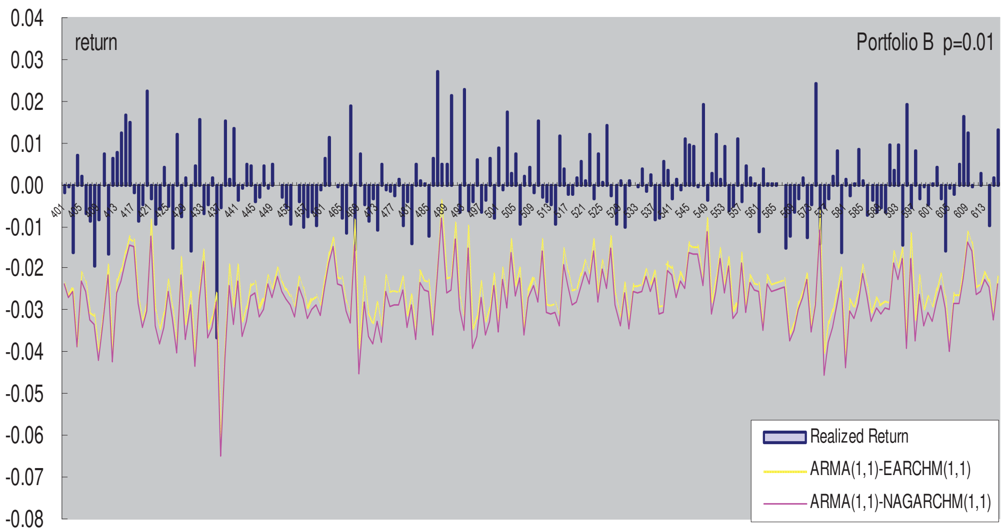
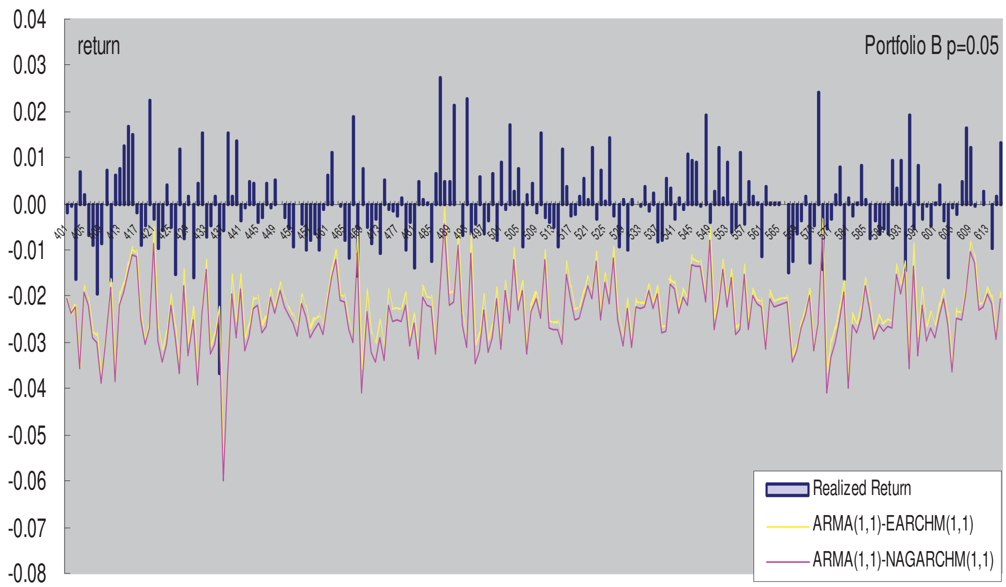


Fig. 5. ARMA(1,1)-EGARCHM(1,1) vs. ARMA(1,1)-NA-GARCHM(1,1) pair comparison under 99 % confidence level



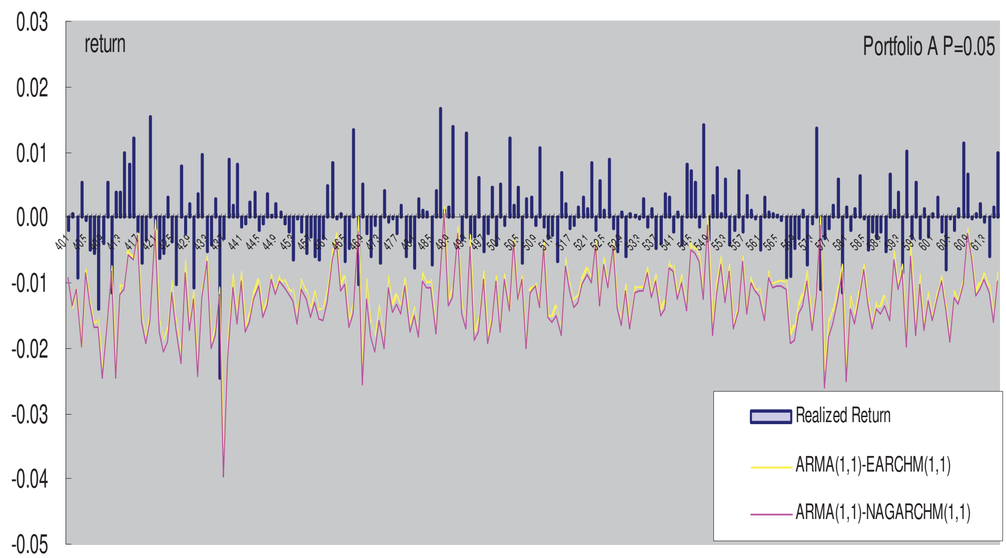
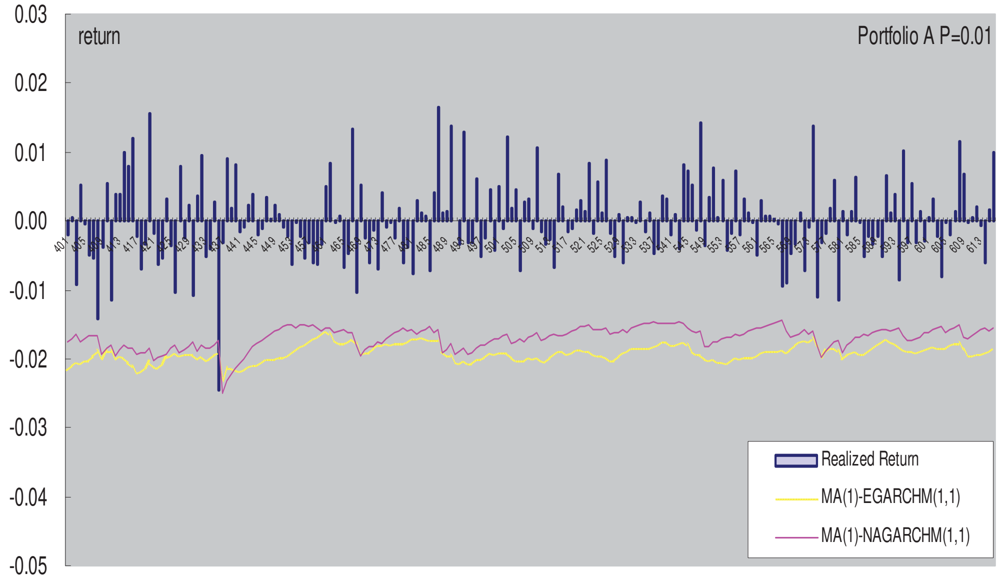


Fig. 6. ARMA(1,1)-EGARCHM(1,1) vs. ARMA(1,1)-NA-GARCHM(1,1) pair comparison under 95% confidence level



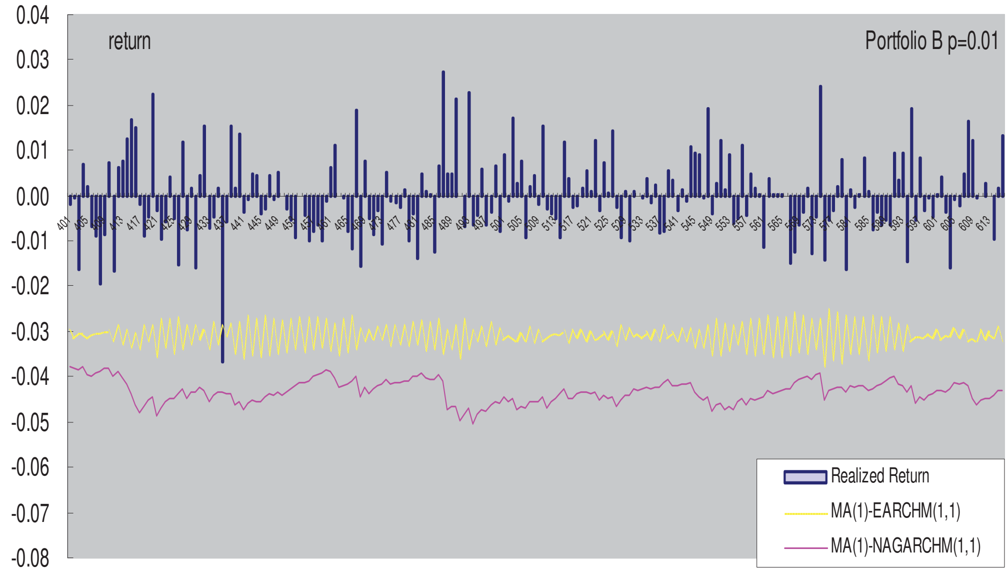
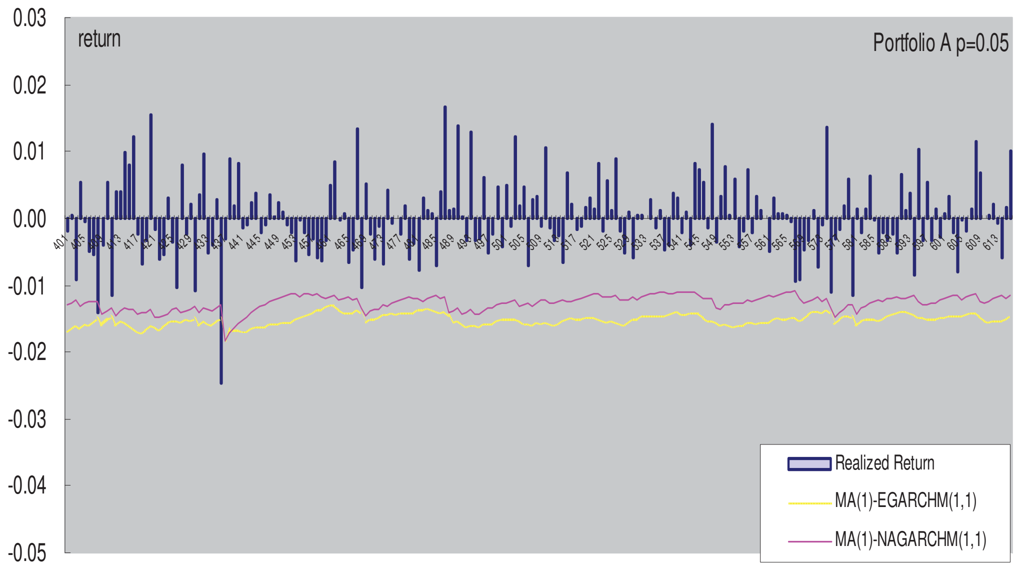


Fig. 7. MA(1)-EGARCHM(1,1) vs. MA (1)-NA-GARCHM(1,1) pair comparison under 99% confidence level



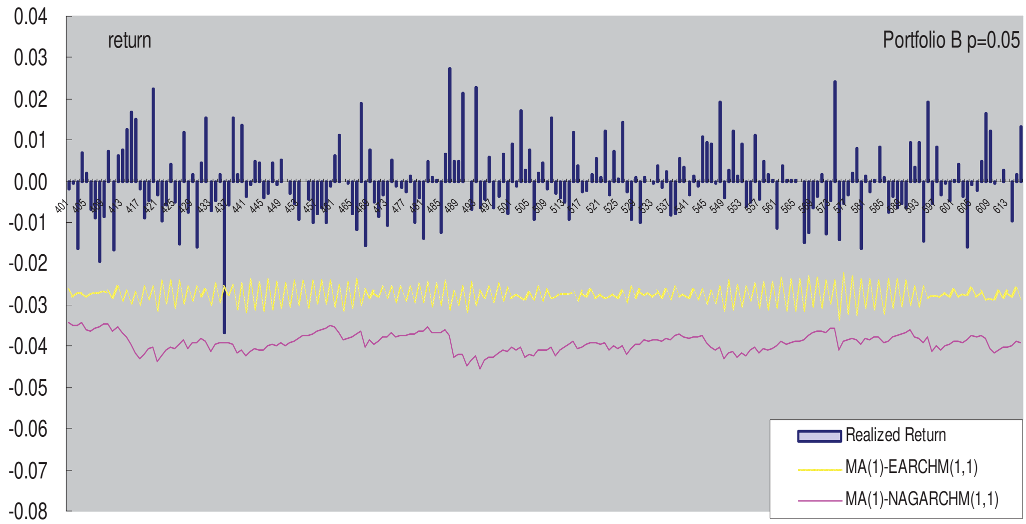


Fig. 8. MA(1)-EGARCHM (1,1) vs. MA(1)-NA-GARCHM(1,1) pair comparison under 95 % confidence level

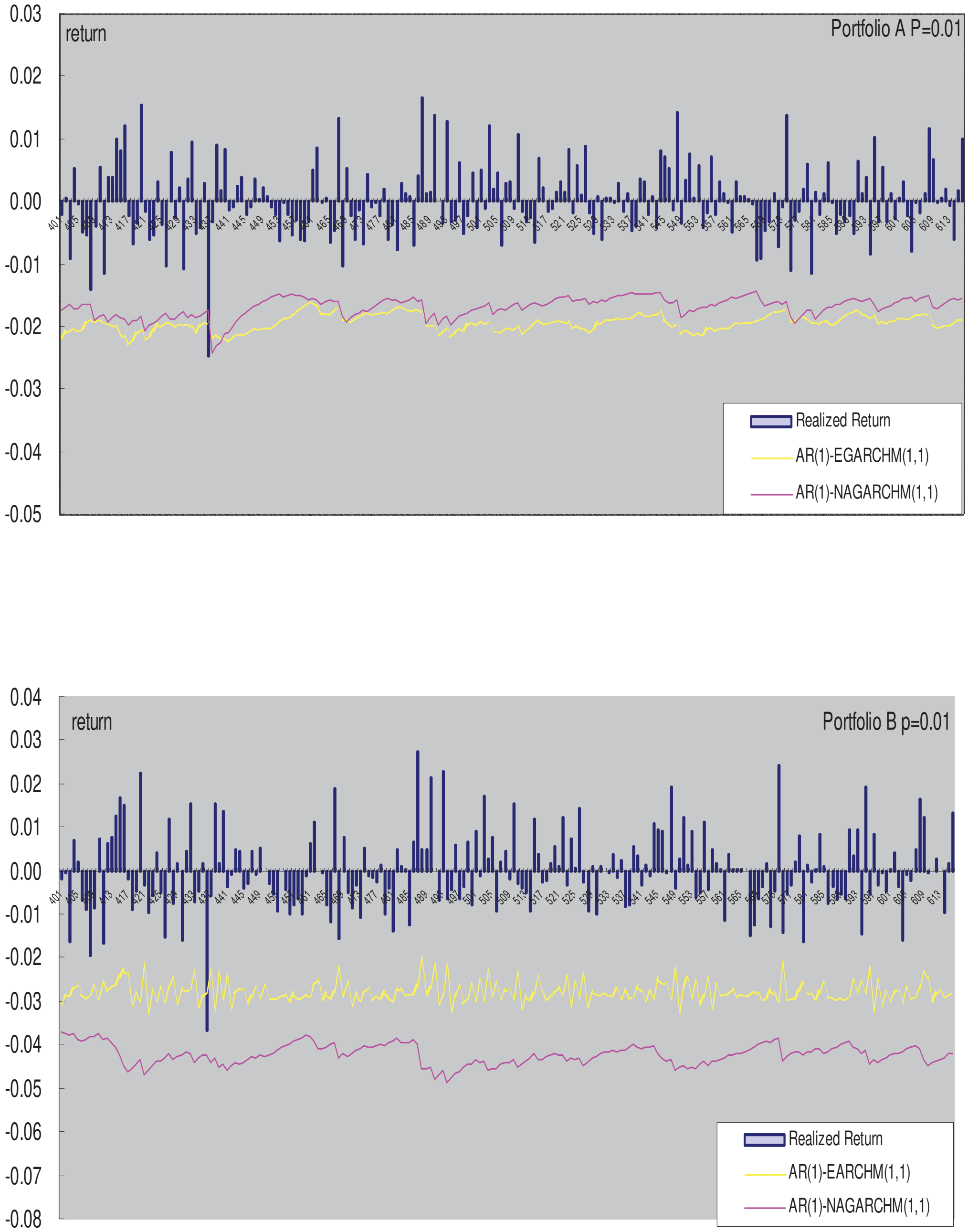
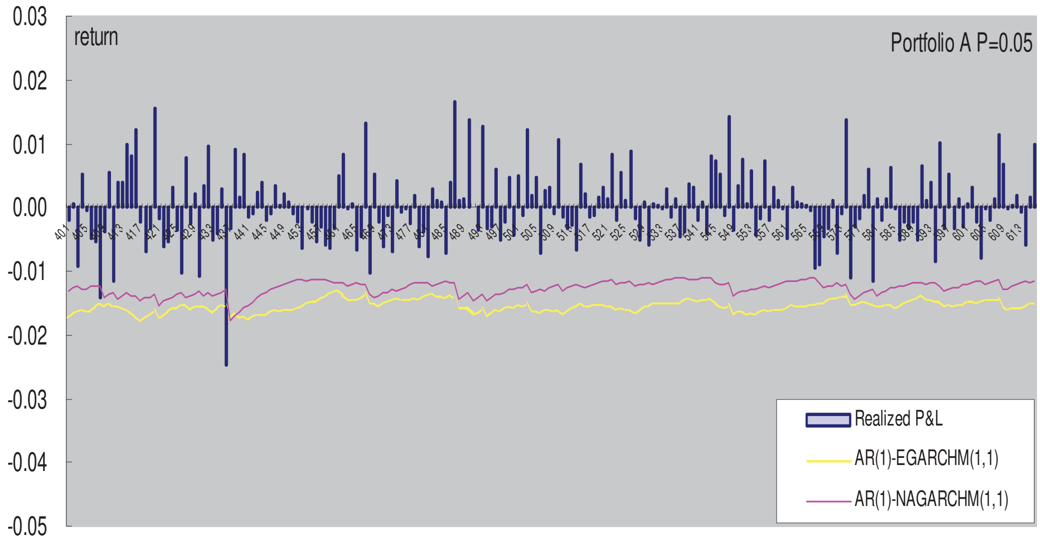


Fig. 9. AR (1)-EGARCHM (1,1) vs. AR(1)-NA-GARCHM(1,1) pair comparison under 99% confidence level



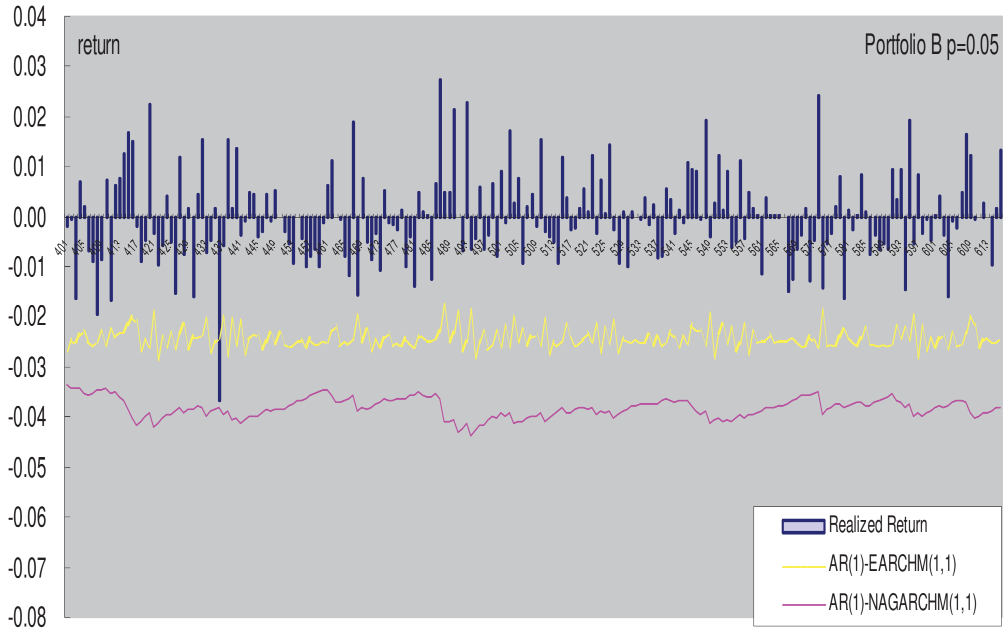
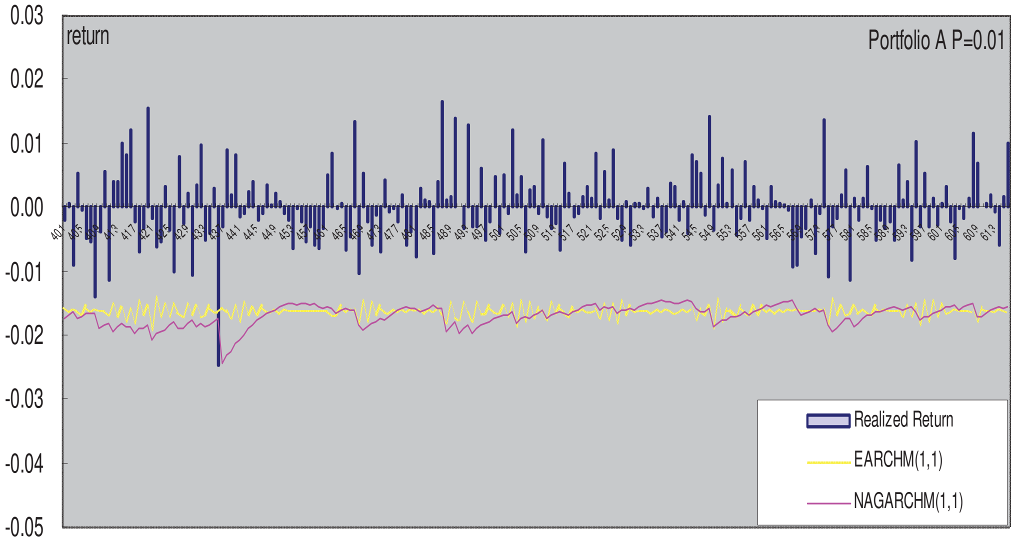


Fig. 10. AR(1)-EGARCHM(1,1) vs. AR(1)-NA-GARCHM(1,1) pair comparison under 95% confidence level



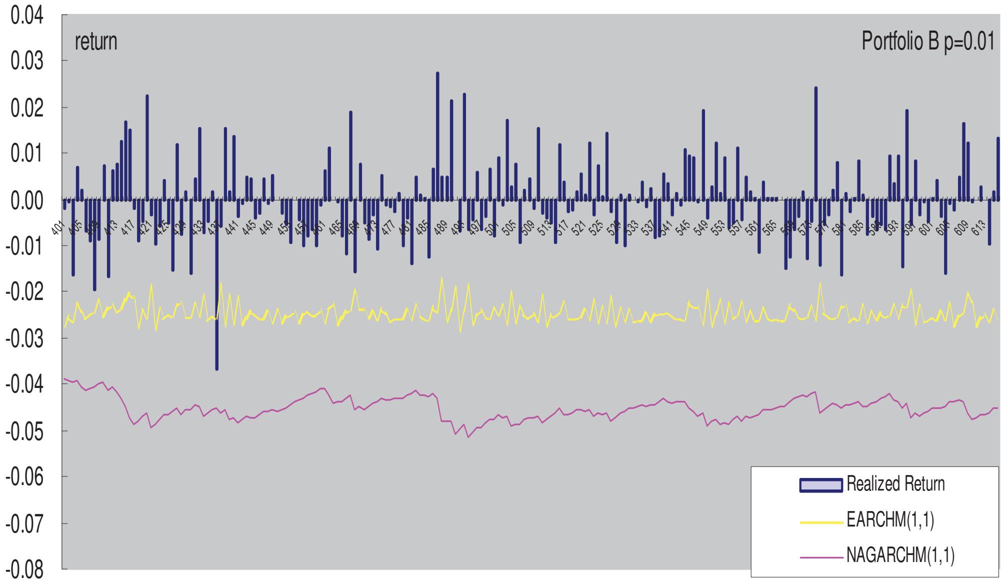


Fig. 11. EGARCHM (1,1) vs. NA-GARCHM(1,1) pair comparison under 99% confidence level

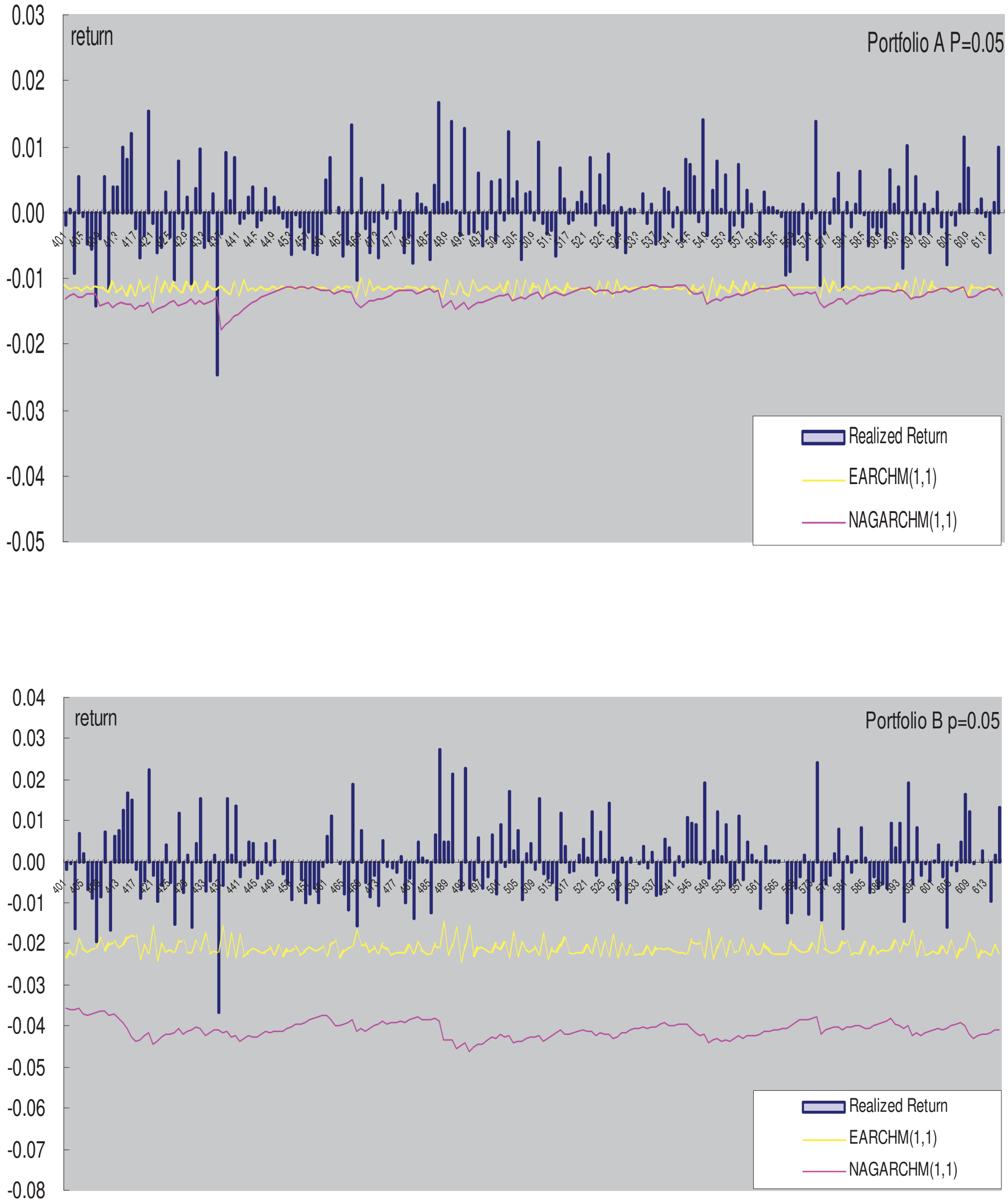


Fig. 12. EGARCHM (1,1) vs. NA-GARCHM(1,1) pair comparison under 95 % confidence level

Table 1. Parameters estimated in ARMA(1,1)-EGARCHM(1,1)

$$R_t = \alpha + Volume R_{t-1} + \beta h_t + MA\varepsilon_{t-1},$$
$$\ln h_1 = A + B(1)\ln h_{t-1} + C(2)\left(\varepsilon_{t-1} / h_{t-1}^{0.5}\right) + \delta\left(\left|\varepsilon_{t-1}\right| / h_{t-1}^{0.5} - (2/\pi)^{0.5}\right),$$

where ln h_t is the logarithm of conditional variance at time *t*, R_t is the return on a portfolio at time *t*, π is the ratio of the circumference of a circle to its diameter, *A*, *B*(1), *C*(2), δ, α, β, *Volume* and *MA* are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	-0.0016	0.0014	-1.1540		α	-0.0044	0.0043	-1.0210	
Volume	0.7975	0.1837	4.3420	***	*Volume*	0.8145	0.1850	4.4030	***
β	0.2318	0.2014	1.1510	*	β	0.4561	0.4480	1.0180	
MA	-0.8355	0.1178	-4.7000	***	*MA*	-0.7913	0.2075	-3.8140	***
A	-0.6749	0.3914	-1.7240	*	*A*	-0.8642	0.8303	-1.0410	
B(1)	0.9316	0.0395	23.6020	***	*B*(1)	0.9066	0.0898	10.1010	***
C(2)	-0.0052	0.0283	-0.1830		*C*(2)	-0.0320	0.0326	-0.9810	
δ	0.1143	0.0427	3.3750	***	δ	0.1027	0.0718	1.4310	

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 2. Parameters estimated in MA(1,1)-EGARCHM(1,1)

$$R_t = \alpha + \beta h_1 + MA\varepsilon_{t-1} + \varepsilon_t,$$
$$\ln h_t = A + B(1)\ln h_{t-1} + C(2)\left(\varepsilon_{t-1} / h_{t-1}^{0.5}\right) + \delta\left(\left|\varepsilon_{t-1}\right| / h_{t-1}^{0.5} - (2/\pi)^{0.5}\right),$$

where ln h_t is the logarithm of conditional variance at time *t*, R_t is the return on a portfolio at time *t*, π is the ratio of the circumference of a circle to its diameter, *A*, *B*(1), *C*(2), δ, α, β and *MA* are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	-0.0057	0.0029	-2.0000	**	α	-0.0076	0.0081	-0.9380	
β	0.8354	0.4105	2.0350	**	β	0.7749	0.8249	0.9390	
MA	-0.0548	0.0519	-1.0570		*MA*	0.0403	0.0560	0.7200	
A	-0.3909	0.2612	-1.4960		*A*	-18.1861	0.2014	-90.2980	***
B(1)	0.9602	0.0264	36.3100	***	*B*(1)	0.9692	0.0168	-57.6460	***
C(2)	-0.0168	0.0218	-0.7680		*C*(2)	0.0265	0.0183	1.4460	
δ	0.1223	0.0466	2.6270	***	δ	0.0317	0.0325	0.9750	

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 3. Parameters estimated in AR(1,1)-EGARCHM(1,1)

$$R_t = \alpha + Volume R_{t-1} + \beta h_t + \varepsilon_t,$$
$$\ln h_t = A + B(1)\ln h_{t-1} + C(2)\left(\varepsilon_{t-1} / h_{t-1}^{0.5}\right) + \delta\left(\left|\varepsilon_{t-1}\right| / h_{t-1}^{0.5} - (2/\pi)^{0.5}\right),$$

where ln h_t is the logarithm of conditional variance at time *t*, R_t is the return on a portfolio at time *t*, π is the ratio of the circumference of a circle to its diameter, *A*, *B*(1), *C*(2), δ, α, β and *Volume* are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	-0.0059	0.0031	-1.9310	*	α	-0.0041	0.0052	-0.7840	
Volume	-0.0513	0.0518	-0.9910		*Volume*	0.0389	0.0443	0.8780	
β	0.8586	0.4379	1.9610	***	β	0.4138	0.5342	0.7750	
A	-0.3866	0.2611	-1.4810		*A*	-12.5391	2.9327	-4.2760	***
B(1)	0.9606	0.0264	36.3420	***	*B*(1)	-0.3557	0.3170	-1.1220	
C(2)	-0.0173	0.0218	-0.7910		*C*(2)	-0.0524	0.0671	-0.7810	
δ	0.1215	0.0465	2.6610	***	δ	-0.2638	0.1060	-2.4880	**

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 4. Parameters estimated in EGARCHM(1,1)

$$R_t = \alpha + \beta h_t + \varepsilon_t ,$$

$$\ln h_t = A + B(1)\ln h_{t-1} + C(2)\left(\varepsilon_{t-1} / h_{t-1}^{\ 0.5}\right) + \delta\left(\left|\varepsilon_{t-1}\right| / h_{t-1}^{\ 0.5} - (2/\pi)^{0.5}\right),$$

where ln h_t is the logarithm of conditional variance at time t, R_t is the return on a portfolio at time t, π is the ratio of the circumference of a circle to its diameter, A, $B(1)$, $C(2)$, δ, α, β are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	0.0002	0.0065	0.0250		α	-0.0006	0.0046	-0.1320	
β	-0.0068	0.9193	-0.0070		β	0.0609	0.4706	0.1290	
A	-16.1627	2.9305	-5.5150	***	A	-12.3246	3.0527	-4.0370	***
$B(1)$	-0.6352	0.2949	-2.1540	**	$B(1)$	-0.3326	0.3302	-1.0070	
$C(2)$	0.0758	0.0445	1.7040	*	$C(2)$	-0.0527	0.0682	-0.7730	
δ	0.0669	0.0665	1.0050		δ	-0.2720	0.1044	-2.6070	**

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 5. Parameters estimated in ARMA(1,1)-NAGARCHM(1,1)

$$R_t = \alpha + VolumeR_{t-1} + \beta h_t + MA\varepsilon_{t-1} + \varepsilon_t ,$$

$$h_t = A + B(1)h_{t-1} + C(1)\left(\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}}\right)^2 ,$$

where h_t is the conditional variance at time t, R_t is the return on a portfolio at time t, A, $B(1)$,$C(1)$, $C(2)$, α, β, *Volume* and *MA* are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	-0.0011	0.0011	-0.9570		α	-0.0040	0.0040	-0.9870	
Volume	0.8222	0.1827	4.5010	***	*Volume*	0.8104	0.1973	4.1080	***
β	0.1550	0.1625	0.9540		β	0.4068	0.4133	0.9840	
MA	-0.8343	0.1895	-4.4020	***	*MA*	-0.7855	0.2223	-3.5330	***
A	0.0000	0.0000	2.2210	**	A	0.0000	0.0000	1.1440	
$B(1)$	0.8166	0.0508	16.0710	***	$B(1)$	0.8313	0.1230	6.7560	***
$C(1)$	0.0808	0.0246	3.2810	***	$C(1)$	0.0562	0.0396	1.4200	
$C(2)$	-0.3275	0.2265	-1.4450		$C(2)$	-0.4111	0.2917	-1.4090	

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 6. Parameters estimated in MA(1,1)-NAGARCHM(1,1)

$$R_t = \alpha + \beta h_t + MA\varepsilon_{t-1} + \varepsilon_t ,$$

$$h_t = A + B(1)h_{t-1} + C(1)\left(\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}}\right)^2 ,$$

where h_t is the conditional variance at time t, R_t is the return on a portfolio at time t, A, $B(1)$,$C(1)$, $C(2)$, α, β, and *MA* are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	-0.0024	0.0025	-0.9680		α	-0.0154	0.0094	-1.6380	
β	0.3464	0.3612	0.9590		β	1.5746	0.9670	1.6280	
MA	-0.0229	0.0634	-0.3610		*MA*	0.0299	0.0597	0.5010	
A	0.0000	0.0000	2.0780	**	A	0.0000	0.0000	1.8260	*
$B(1)$	0.8328	0.0492	16.9140	***	$B(1)$	0.8931	0.0419	21.3070	***
$C(1)$	0.0814	0.0280	2.9100	***	$C(1)$	0.0407	0.0214	1.9000	*
$C(2)$	-0.3890	0.1953	-1.9920	**	$C(2)$	-0.2852	0.2381	-1.1980	

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 7. Parameters estimated in AR(1,1)-NAGARCHM(1,1)

$$R_t = \alpha + Volume R_{t-1} + \beta h_t + \varepsilon_t ,$$
$$h_t = A + B(1)h_{t-1} + C(1)\left(\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}}\right)^2 ,$$

where h_t is the conditional variance at time t, R_t is the return on a portfolio at time t, A, $B(1)$,$C(1)$, $C(2)$, α, β and *Volume* are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	-0.0024	0.0025	-0.9500		α	-0.0148	0.0090	-1.6450	*
Volume	-0.0205	0.0630	-0.3260		*Volume*	0.0359	0.0592	0.6060	
β	0.3469	0.3686	0.9410		β	1.5126	0.9250	1.6350	
A	0.0000	0.0000	2.0730	**	*A*	0.0000	0.0000	1.8200	**
B(1)	0.8328	0.0491	16.9550	***	*B*(1)	0.8912	0.0428	20.8080	***
C(1)	0.0814	0.0278	2.9250	***	*C*(1)	0.0414	0.0217	1.9060	*
C(2)	-0.3914	0.1958	-1.9990	**	*C*(2)	-0.2957	0.2389	-1.2370	

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 8. Parameters estimated in NAGARCHM(1,1)

$$R_t = \alpha + \beta h_t + \varepsilon_t ,$$
$$h_t = A + B(1)h_{t-1} + C(1)\left(\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}}\right)^2 ,$$

where h_t is the conditional variance at time t, R_t: The return on a portfolio at time t, A, $B(1)$,$C(1)$, $C(2)$, α, β are parameters.

Portfolio A					Portfolio B				
Variable	Value	Std err	T-stat		Variable	Value	Std err	T-stat	
α	-0.0024	0.0025	-0.9580		α	-0.0166	0.0101	-1.6560	*
β	0.3513	0.3706	0.9480		β	1.7048	1.0348	1.6470	*
A	0.0000	0.0000	2.1440	**	*A*	0.0000	0.0000	1.8810	
B(1)	0.8297	0.0445	18.6620	***	*B*(1)	0.8992	0.0385	23.3530	***
C(1)	0.0820	0.0239	3.4270	***	*C*(1)	0.0380	0.0204	1.8590	*
C(2)	-0.4091	0.1959	-2.0880	**	*C*(2)	-0.2334	0.2298	-1.0160	

Note: *, ** and *** denote significant at the 10%, 5%, and 1% level, respectively.

Table 9. All VaR models in portfolio A

Model performance is measured by violation numbers of the forward testing VaR forecasts. The aggregate violations characterize the total amount of additional capital charge if violations really occur. Aggregate violation percentage defines as $\sum_{i=1}^{n}(\frac{R_i - VaR_i}{VaR_i})$, where n is the number of violations. The mean violation range describes the average amount of additional capital charge if violation really occurs. The mean violation percentage can express the degree of average amount of additional capital charge relative to VaR forecast. Mean violation percentage defines as $\frac{1}{n}\sum_{i=1}^{n}(\frac{R_i - VaR_i}{VaR_i})$. The max violation represents the max amount of additional capital charge if violation occurs. Max violation percentage defines as $-(\frac{R_i - VaR_i}{VaR_i})$ for $i = 1$ to n.

Portfolio A						
99% confidence level						
	Obs	Violation #	Mean VaR	Aggregate violation	Mean violation	Max violation
ARMA(1,1)-NAGARCHM(1,1)	216	3	-1.7843%	-1.5433%	-0.5144%	-0.7561%
MA(1)-NAGARCHM(1,1)	216	1	-1.6927%	-0.7291%	-0.7291%	-0.7291%
AR(1)-NAGARCHM(1,1)	216	1	-1.6918%	-0.7256%	-0.7256%	-0.7256%
NAGARM(1,1)	216	1	-1.6967%	-0.7227%	-0.7227%	-0.7227%
ARMA(1,1)-EGARCHM(1,1)	216	3	-1.6108%	-2.1243%	-0.7081%	-0.8660%
MA(1)-EGARCH(1,1)	216	1	-1.9144%	-0.5465%	-0.5465%	-0.5465%
AR(1)-EGARCHM(1,1)	216	1	-1.9405%	-0.5129%	-0.5129%	-0.5129%
EGARCHM(1,1)	216	1	-1.6314%	-0.8330%	-0.8330%	-0.8330%

Table 9 (cont.). All VaR models in portfolio A

Portfolio A						
95% confidence level						
	Obs	Violation #	Mean VaR	Aggregate violation	Mean violation	Max violation
ARMA(1,1)-NAGARCHM(1,1)	216	7	-1.2890%	-3.9491%	-0.5642%	-1.2977%
MA(1)-NAGARCHM(1,1)	216	2	-1.2631%	-1.3339%	-0.6670%	-1.1732%
AR(1)-NAGARCHM(1,1)	216	2	-1.2628%	-1.3431%	-0.6715%	-1.1684%
NAGARM(1,1)	216	2	-1.2686%	-1.3317%	-0.6659%	-1.1640%
ARMA(1,1)-EGARCHM(1,1)	216	10	-1.1804%	-4.8496%	-0.4850%	-1.3640%
MA(1)-EGARCH(1,1)	216	1	-1.5106%	-0.9561%	-0.9561%	-0.9561%
AR(1)-EGARCHM(1,1)	216	1	-1.5341%	-0.9225%	-0.9225%	-0.9225%
EGARCHM(1,1)	216	2	-1.1470%	-1.6134%	-0.8067%	-1.3173%

Table 10. All VaR models in portfolio B

Portfolio B						
99% confidence level						
	Obs.	Violation #	Mean VaR	Aggregate violation	Mean violation	Max violation
ARMA(1,1)-NA-GARCHM(1,1)	216	3	-2.7650%	-1.3022%	-0.4341%	-0.8612%
MA(1)-NA-GARCHM(1,1)	216	0	-4.3280%	0.0000%	0.0000%	0.0000%
AR(1)-NA-GARCHM(1,1)	216	0	-4.2261%	0.0000%	0.0000%	0.0000%
NA-GARM(1,1)	216	0	-4.5207%	0.0000%	0.0000%	0.0000%
ARMA(1,1)-EGARCHM(1,1)	216	3	-2.5130%	-2.4927%	-0.8309%	-1.0547%
MA(1)-EGARCH(1,1)	216	1	-3.1078%	-0.7699%	-0.7699%	-0.7699%
AR(1)-EGARCHM(1,1)	216	1	-2.7889%	-0.8818%	-0.8818%	-0.8818%
EGARCHM(1,1)	216	1	-2.4676%	-1.0905%	-1.0905%	-1.0905%
95% confidence level						
	Obs.	Violation #	Mean VaR	Aggregate violation	Mean violation	Max violation
ARMA(1,1)-NA-GARCHM(1,1)	216	4	-2.4109%	-2.4477%	-0.6119%	-1.2390%
MA(1)-NA-GARCHM(1,1)	216	0	-3.9124%	0.0000%	0.0000%	0.0000%
AR(1)-NA-GARCHM(1,1)	216	0	-3.8180%	0.0000%	0.0000%	0.0000%
NA-GARM(1,1)	216	0	-4.0878%	0.0000%	0.0000%	0.0000%
ARMA(1,1)-EGARCHM(1,1)	216	5	-2.2025%	-3.6773%	-0.7355%	-1.3981%
MA(1)-EGARCH(1,1)	216	1	-2.7586%	-1.0864%	-1.0864%	-1.0864%
AR(1)-EGARCHM(1,1)	216	1	-2.4328%	-1.2373%	-1.2373%	-1.2373%
EGARCHM(1,1)	216	1	-2.1059%	-1.4686%	-1.4686%	-1.4686%

Table 11. EGARCH and NA-GARCH model pair comparison under 99% confidence level

99% confidence level						
Portfolio A	Obs	Violation #	Violation rate	Mean VaR	Aggregate violation	Max violation
ARMA(1,1)-NA-GARCHM(1,1)	216	3	1.39%	-1.7843%	-1.5433%	-0.7561%
ARMA(1,1)-EGARCHM(1,1)	216	3	1.39%	-1.6108%	-2.1243%	-0.8660%
MA(1)-NA-GARCHM(1,1)	216	1	0.46%	-1.6927%	-0.7291%	-0.7291%
MA(1)-EGARCH(1,1)	216	1	0.46%	-1.9144%	-0.5465%	-0.5465%
AR(1)-NA-GARCHM(1,1)	216	1	0.46%	-1.6918%	-0.7256%	-0.7256%
AR(1)-EGARCHM(1,1)	216	1	0.46%	-1.9405%	-0.5129%	-0.5129%
NA-GARM(1,1)	216	1	0.46%	-1.6967%	-0.7227%	-0.7227%
EGARCHM(1,1)	216	1	0.46%	-1.6314%	-0.8330%	-0.8330%
Portfolio B	Obs	Violation #	Violation rate	Mean VaR	Aggregate violation	Max violation
ARMA(1,1)-NA-GARCHM(1,1)	216	3	1.39%	-2.7650%	-1.3022%	-0.8612%
ARMA(1,1)-EGARCHM(1,1)	216	3	1.39%	-2.5130%	-2.4927%	-1.0547%
MA(1)-NA-GARCHM(1,1)	216	0	0.00%	-4.3280%	0.0000%	0.0000%
MA(1)-EGARCH(1,1)	216	1	0.46%	-3.1078%	-0.7699%	-0.7699%
AR(1)-NA-GARCHM(1,1)	216	0	0.00%	-4.2261%	0.0000%	0.0000%
AR(1)-EGARCHM(1,1)	216	1	0.46%	-2.7889%	-0.8818%	-0.8818%
NA-GARM(1,1)	216	0	0.00%	-4.5207%	0.0000%	0.0000%
EGARCHM(1,1)	216	1	0.46%	-2.4676%	-1.0905%	-1.0905%

Table 12. EGARCH and NA-GARCH model pair comparison under 95% confidence level

95% confidence level						
Portfolio A	Obs.	Violation #	Violation rate	Mean VaR	Aggregate violation	Max violation
ARMA(1,1)-NA-GARCHM(1,1)	216	7	3.24%	-1.2890%	-3.9491%	-1.2977%
ARMA(1,1)-EGARCHM(1,1)	216	10	4.63%	-1.1804%	-4.8496%	-1.3640%
MA(1)-NA-GARCHM(1,1)	216	2	0.93%	-1.2631%	-1.3339%	-1.1732%
MA(1)-EGARCH(1,1)	216	1	0.46%	-1.5106%	-0.9561%	-0.9561%
AR(1)-NA-GARCHM(1,1)	216	1	0.46%	-1.6918%	-0.7256%	-0.7256%
AR(1)-EGARCHM(1,1)	216	1	0.46%	-1.9405%	-0.5129%	-0.5129%
NAGARM(1,1)	216	2	0.93%	-1.2686%	-1.3317%	-1.1640%
EGARCHM(1,1)	216	2	0.93%	-1.1470%	-1.6134%	-1.3173%
Portfolio B	Obs	Violation #	Violation rate	Mean VaR	Aggregate violation	Max violation
ARMA(1,1)-NA-GARCHM(1,1)	216	4	1.85%	-2.4109%	-2.4477%	-1.2390%
ARMA(1,1)-EGARCHM(1,1)	216	5	2.31%	-2.2025%	-3.6773%	-1.3981%
MA(1)-NA-GARCHM(1,1)	216	0	0.00%	-3.9124%	0.0000%	0.0000%
MA(1)-EGARCH(1,1)	216	1	0.46%	-2.7586%	-1.0864%	-1.0864%
AR(1)-NA-GARCHM(1,1)	216	0	0.00%	-3.8180%	0.0000%	0.0000%
AR(1)-EGARCHM(1,1)	216	1	0.46%	-2.4328%	-1.2373%	-1.2373%
NAGARM(1,1)	216	0	0.00%	-4.0878%	0.0000%	0.0000%
EGARCHM(1,1)	216	1	0.46%	-2.1059%	-1.4686%	-1.4686%