“Re-examine weekday effects and intraday returns: a probability distribution approach for Dow and Nasdaq ten-minute indices”

AUTHORS	Hai-Chin Yu https://orcid.org/0000-0003-3274-4656 https://publons.com/researcher/2441225/hai-chin-yu/ Chia-Yi Wu Der-Tzon Hsieh

ARTICLE INFO	Hai-Chin Yu, Chia-Yi Wu and Der-Tzon Hsieh (2011). Re-examine weekday effects and intraday returns: a probability distribution approach for Dow and Nasdaq ten-minute indices. *Banks and Bank Systems*, *6*(3)
RELEASED ON	Monday, 10 October 2011
JOURNAL	"Banks and Bank Systems"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

NUMBER OF REFERENCES	NUMBER OF FIGURES	NUMBER OF TABLES
0	0	0

© The author(s) 2024. This publication is an open access article.

Hai-Chin Yu (Taiwan), Chia-Yi Wu (Taiwan), Der-Tzon Hsieh (Taiwan)

Re-examine weekend effects and intraday returns: a probability distribution approach for Dow and Nasdaq ten-minute indices

Abstract

For DJIA and Nasdaq stock markets, the weekend effects does not disappear over the past decade. Beneath the surface however there remain systematic day-of-the-week-effects only visible when returns are partitioned by the different sessions within a day. A similar intraday pattern was found for both DJIA and Nasdaq markets and provide another explanation for the day-of-the-week effect.

Using probability distribution techniques, this paper re-examines the weekday effect by considering intraday returns across opening, lunch and closing sessions for the Dow and Nasdaq indices. This approach, which examines the peak, height and width rather than just looking at the mean and standard deviation, provides more complete information by concluding the weekday effects in a better way. Generally speaking, the traditional Monday effect still exists, but in the case of the Dow it has been postponed to a shorter period at Monday closing rather than the entire Monday. As for the Nasdaq, the traditional Friday effect still exists, but it has been shortened to a Friday closing period rather than covering the entire Friday. The authors find that the Nasdaq exhibits apparently much more positive feedback and aggressive behavior than the Dow. Furthermore, the repetitive trading behavior of large financial institutions and the more frequent trading resulting from electronic systems influence intraday and interday effects appearance in a shorter period. The anomaly may not be necessarily related to firm size.

Keywords: probability distribution, intraday return, intraday volatility, intraday effect, weekday effect.
JEL Classification: C23, G10, G14, G15, F3.

Introduction

The modern electronic trading and monitoring systems have increasingly influenced the investors' behaviors in stock markets. Market participants are now better equipped to monitor price movements within a day. These developments make the information transmission becomes more efficient among both institutional and individual investors, and in turn foster the trading activity in a higher frequency. These more frequently trading behaviors make intraday pattern a significant difference than before, and in turn, influence the trading behavior over the weekdays. As such, re-examine the weekday (weekend) effects by checking intraday pattern at the present time is meaningful.

An extensive literature documents how weekday returns vary with the days of the week across various types of assets and markets (see, for example, Pettengill, 2003)[1]. Over the past several decades, the traditional Monday effect has been found in stock returns, where the average returns on Monday are significantly negative and lower than those for the rest of the weekdays (Cross, 1973; French, 1980; and Gibbons and Hess, 1981). A well-regarded explanation for the negative returns on Mondays is that unfavorable news most often appears on weekends (Fishe, Gosnell and Lasser, 1993). It has been theorized that this bad weekend news causes investors to sell on Mondays. Arsad and Coutts (1996) present strong evidence for the existence of the weekend effect, but only in the presence of a bad news environment rather than of one of good news. To this day, the Monday effect remains an enigma. Why would otherwise savvy investors buy securities on Friday if they expected to have negative returns on the next trading day, Monday?

Recently, Brusa et al. (2000) find a 'reverse' weekend effect whereby returns for Monday are significantly positively larger than the prior Friday for major stock indexes – Dow, CRSP, S&P 500 and NYSE. Gu et al. (2004) also find that the weekend effect has been reversing in U.S. indices from late 1980s to late 1990s. Some researches even find strong evidence that the weekend effect have disappeared after the anomalies published (see, for example, Marquering et al., 2006). These different findings motivate us to explore whether this effect really disappears when returns are partitioned by the different sessions within a day.

Daily returns, especially closing to closing returns, have typically been used in the extant weekend effect studies, but investors are not limited to buying or selling products until markets close. Therefore, using daily closing returns to examine the weekend effects may lead to invalid results and conclusions.

© Hai-Chin Yu, Chia-Yi Wu, Der-Tzon Hsieh, 2011.
We would like to thank Thomas C. Chiang for providing the high frequency data for the Dow and Nasdaq. The previous version was presented at the Behavioral Finance & Emerging Markets Conference in Taipei and the 82nd Annual Meeting of the Western Economic Association in Seattle. We are grateful for the helpful suggestions from the participants.
[1] Pettengill (2003) has organized a complete literature review on this issue.

For example, information disclosed from after Friday closing until Monday closing actually includes the weekend (Friday overnight, Saturday and Sunday) and Monday (morning, lunch and afternoon) effects. Therefore, using only the Monday closing price to measure the Monday return is actually confounded by the weekend effect and the entire Monday effect. Using Monday closing without considering the intraday returns from Monday morning, lunch and near-closing also ignores the real information occurring on Monday. Moreover, the advent of new electronic trading systems and information technologies has lowered quote spreads and improved price efficiency (Jones et al., 2008). These changes in investor behavior mean that re-examining the prior analogy pattern within a day or a trading week becomes more motivated. Besides, the flow of information through an electronic trading system also enables market participants to better monitor intraday movements in security prices so that observing intraday influences is necessary.

Although some extant researchers use intraday (opening to closing) returns to examine the weekend effects, most of the measures they use are only the mean or variance at the end of the day without taking into consideration the entire return distribution (see, for example, Rogalski, 1984; Smirlock and Starks, 1986)[1]. Harris (1986) also finds that the Monday effect arises only for the first 45 minutes after the market opens, and then disappears after that. This implies that the Monday effect may not be able to be detected while using the daily closing price only. To avoid these biases, the entire trading day is separated into three sub-sessions: the opening, lunch and closing times, to explore the weekday and weekend effect while considering the intraday effect simultaneously. That being said, the existence of opening, lunch and closing returns enables us to separate out the overnight/weekend effects from those within the day. It thus facilitates a more powerful test of the day-of-the-week effect.

Recently, Heston et al. (2010) identified short-term intraday return reversals and long-term daily return continuations for NYSE stocks. Their results suggest an "intraday timing of institutional order flow" that can explain why weekday/weekend effects might exist. That is, large financial institutions might be buying the same set of stocks at the same time of day during each trading day because they are following an indexing strategy or some quantitative investment strategy which causes these firms to trade similar securities in the same direction. For example, an institution's traders will typically place multiple purchase orders for the same set of securities over potentially several days. This, in turn, could create excess demand or excess supply for specific stocks at certain points during the trading day.

Our paper differs from the extant literature in a number of respects. First, by using the probability distributions rather than just the mean or variance, we can better analyze the existence of a weekday/weekend effect, which has not yet been done in prior research. Second, we use 10-minute intraday returns data grouped into three intraday sub-sessions rather than just daily data to detect more precisely the presence of weekday/weekend effects. Third, we precisely fit and estimate the function of return and volatility which is lacking in the extant literature. For example, based on the symmetric and fat-tailed characteristics of the returns in our data, the Gaussian function is employed to fit the distribution and the parameters are estimated. This approach can provide an entire overview of the distribution. Moreover, the asymmetric and right-skewed characteristics of the volatility in our data have inspired us to fit it with a log-normal function and the parameters for the peak, width and height in the distribution are estimated. All these estimates help provide us with more information about the micro behavior of the weekday/weekend effect. Fourth, we look at both the Dow and Nasdaq indices, and not just a single index over the same period from August 1997 to December 2003. These two indexes represent not only the core of the U.S. economy but also enable us to verify whether there are size-related differences in any weekday/weekend effects that might be present in the data. Finally, we compare the similarities and differences in regard to the weekday behavior in both the Dow and Nasdaq markets.

Our analysis is presented in two stages. In the first stage, we quantify the equations employed to measure the return and volatility, and develop the 10-minute return patterns from the opening, lunch and closing times for both the Dow and Nasdaq. The second stage is the main contribution of the paper. In this section, the probability distribution approach is employed to fit the return and volatility distributions from Mondays to Fridays, and some interesting findings result.

The remaining sections of this paper are organized as follows. Section 1 summarizes the literature review. Section 2 presents the data and the methodology employed. Section 3 reports the estimated results of the probability distributions of return and

[1] Rogalski (1984) employs intraday data by decomposing daily returns into shorter periods. Smirlock and Starks (1986) indicate that the weekend effect may be swamped and lead to bias while investigating the weekend effect without looking at the intraday returns.

volatility over the opening, lunch and closing times from Monday to Friday, and the final section provides a discussion and conclusion.

1. Literature review

Many researchers have tried to explain why the asset return is negative on Mondays. Wingender and Groff (1989) conclude that the Monday effect is not due to outliers. Other researchers document that this phenomenon may be due to statistical errors (Gibbons and Hess, 1981; Chen et al., 2002). Abraham and Ikenberry (1994) indicate that market makers may face less liquidity and volume on Mondays, and that this may be the cause of lower returns. Sullivan et al. (2001) argue that the Monday effect may result from data mining. Still, other researchers attribute the effect to the capital market efficiency, micro market effects, or settlement procedures (Fama, 1991; Gibbons and Hess, 1981; Lakonishok and Levi, 1982; Dyl and Martin, 1985). It may also be the case that the behavior of positive feedback investors who buy when prices increase and sell when prices decrease contributes to the negative returns on Mondays (DeLong et al., 1989). To sum up, while the existence of the Monday effect in equity returns has been widely documented, there is no consensus yet as to what actually causes the Monday effect.

Some researchers attribute the Monday effect to information processing costs (e.g., Lakonishok and Maberly, 1990, etc.). Abraham and Ikenberry (1994) also indicate that the negative returns on Mondays are the consequence of information revealed on previous trading days, particularly on Fridays. They employ S&P 500 index intraday returns and find that the selling pressure occurred before 11:00 a.m. most of the time. In addition to the information processing costs explanation, some researchers attribute the decline in the transaction cost to another reason; for example, Kamara (1997) demonstrates that the Monday effect declined significantly after April 1982 when S&P futures began to be traded. The existence of the futures contracts may reduce the risk of weekend surprises, and consequently mitigate the Monday effect.

Still, other researchers attribute the weekday effect to the news arrivals. For instance, Steely (2001) finds that there is a strong weekly pattern on the announcement dates of major macroeconomic news in the UK. Of particular note, most of the market-wide events are clustered on Tuesdays, Wednesdays and Thursdays rather than on Mondays and Fridays. Therefore, news arrivals provide a rational explanation to support the day-of-the-week effect. In addition, the weekday effect has been shown to be pervasive: it appears not only in the U.S. but also in a number of other countries[1].

Recently, some researchers have documented that, for large cap firms, there is no Monday effect. However, small-firm securities continue to exhibit the same pattern of higher returns on Fridays and negative returns on Mondays. Kamara (1997) finds that the Monday seasonal effect is not significant in the S&P 500 over 1962-1993 for large firms, while it is still significant among small stocks. Mahdian and Perry (2001) divide the full sample period from 1964-1998 into two sub-sample periods and show that the Monday effect existed during 1964-1987. However, the Monday effect moved in the reverse direction after the 1987 stock market crash in the case of large firms. Sullivan and Liano (2003) indicate that the average return on Monday for the value-weighted index was higher than that on the rest of the weekdays, proving the dispersal of the Monday effect in large-firm securities. Yet, the ratio of declining issues on Monday was still higher than on the rest of the weekdays for small-firm securities, implying the existence of a Monday effect in small-firm securities. Brusa, Liu and Schulman (2000) re-examine the persistence of the weekend effect in stock returns using the Dow, CRSP, S&P 500 and the NYSE indexes, and find that Monday returns were monotonically increasing as firm size advanced. Monday returns were inclined to be negative in small firms but positive in large firms. Furthermore, the weekend effect existed in the portfolios of small firms, while the 'reverse' weekend effect existed in the portfolios of large firms.

Overall, the prior literature provides mixed evidence on the weekend and weekday effects, with more recent research suggesting that there might also be size-related differences in these effects.

2. Data and methodology

2.1. Data description. The data employed in the study consist of Dow Jones Industrial Average 30 (Dow) and Nasdaq Composite (Nasdaq) 10-minute intraday returns provided by Trade and Quotation (TAQ). The TAQ database consists of continuously recorded information on the trades and quotations for the securities. The 10-minute returns for both

[1] Jaffe and Westerfield (1985) find that the U.S., the UK, Japan, Canada and Australia have weekend effects, where Fridays have the highest returns. However, their results show that the lowest and negative returns appear on Tuesdays instead of on Mondays in the Japanese and Australian stock markets. Subsequently, Kato (1990) also points out that the lowest returns appear on Tuesdays, while the highest returns occur on Wednesdays in Japan. The lowest returns on Tuesday seem to relate to the low returns on Mondays. Singapore, Turkey and France have also been shown to have the lowest negative returns on Tuesdays rather than on Mondays.

the Dow and the Nasdaq cover the period from August 1, 1997 to December 31, 2003, and include 1,614 trading days with 62,946 observations starting at 9:30 and extending to 15:50 EST (Eastern Time Zone)[1].

The reason for choosing these two indices is that the former represents the most well-established and financially-sound companies, whereas the latter consists of smaller, high-tech and higher growth companies. These two indexes thus represent not only the core of the U.S. economy but also enable us to verify whether size effects that might be present in the data. In addition, the 10-minute horizon is short enough that the realized returns and volatility can be captured, and yet it is also long enough that the confounding influences from market microstructure behavior such as the "bid-ask bounce" first noted in Blume and Stambaugh (1983) can be largely avoided.

2.2. Methodology. To explicitly display the distribution of intraday returns and volatility, this section builds the probability distributions of the intraday returns and volatility over the whole trading time, respectively.

2.2.1. Quantifying the intraday returns and probability distribution. Our intraday returns are calculated by taking the first difference of the natural log of the index. The notation for intraday returns is expressed in equation (1).

$$R_t = \ln Y_t - \ln Y_{t-1}, \tag{1}$$

where R_t is defined as the return at time *t*. Y is the stock price index at time *t*. To construct the probability distributions, we first employ the histogram method to separate the total sample into 100 equal intervals with each interval having an average return as below in equation (2):

$$\Delta R = \frac{R^{\max} - R^{\min}}{100}, \tag{2}$$

where ΔR is the average return range in each interval. $R^{\max}$ and $R^{\min}$ are the maximum and minimum intraday returns, respectively. Then, we count the number of each interval $N(R_n)$ ranging between $R_{n-1} = R^{\min} + (n-1)\cdot(\Delta R)$ and $R_n = R^{\min} + n\cdot(\Delta R)$.

Here, n is an integer ranging from 1 to 100. Therefore, the probability of the intraday returns ${}_{n-1}R_n$ between R_{n-1} and R_n can be expressed as equation (3):

$$P_r({}_{n-1}R_n) = \frac{N({}_{n-1}R_n)}{\sum_{n=1}^{100} N({}_{n-1}R_n)}. \tag{3}$$

We further define the probability distribution P_r as a normalized distribution of the intraday return R_t which satisfies:

$$\sum_{n=1}^{100} P_r({}_{n-1}R_n) = 1. \tag{4}$$

Figures 1(a) and 1(b) in the Appendix display the shapes we fit into the probability distribution of returns for the Dow and Nasdaq, respectively. Table 1 (see Appendix) reports the summary statistics of 10-minute returns. It shows that both distributions are slightly right-skewed with a skewness of 0.13 for the Dow and 0.27 for the Nasdaq, meaning that there is a greater likelihood of winning positive returns than negative ones. The mean and standard deviation for the Nasdaq are higher than those for the Dow, demonstrating that the Nasdaq has better returns associated with higher risk than the Dow. Moreover, the kurtosis is as high as 19.95 for the Dow and 23.95 for the Nasdaq, demonstrating that both of them have leptokurtic or fat-tailed characteristics. The kurtosis of the Nasdaq is larger than that of the Dow, implying more extreme observations in the Nasdaq. Finally, the J-B values show that neither the Dow nor the Nasdaq satisfy the normal distribution.

2.2.2. Quantifying the intraday volatility and probability distribution. There are thirty-nine 10-minute intraday intervals from 9:30 a.m. to 3:50 p.m. in each trading day. We then set a time window equal to 39 for estimating the distribution of volatility. The volatility is defined as the average absolute value over a time window $T = 39\delta t = 39$, where $\delta t = 1$. Following Liu et al. (1999), the intraday volatility is expressed as equation (5):

$$V_t = \frac{1}{39}\sum_{n=0}^{38}\left|R_{t-n\delta t}\right|, \qquad \delta t = 1, \quad t = 39,...,62946\cdot \tag{5}$$

[1] Following the analysis in Andersen and Bollerslev (1997), we constructed 10-minute returns spanning from 9:30 to 15:50 (EST) with 39 observations for each trading day.

Next, we use the probability distribution technique to fit this volatility. We divide the total sample observations into 100 equal intervals and calculate the frequency in each interval to estimate the shape of the probability distribution. The volatility in each interval is shown as in equation (6):

$$\Delta V = \frac{V^{\max} - V^{\min}}{100}, \tag{6}$$

where the $V^{\max}$ and $V^{\min}$ represent the global maximum and minimum intraday volatilities, respectively. We then count the number of $N(V_n)$ in each interval ranging from $V_{n-1} = V^{\min} + (n-1)\cdot(\Delta V)$ to $V_n = V^{\min} + n\cdot(\Delta V)$, where n is an integer ranging from 1 to 100. The estimated probability of the volatility in each interval of ${}_{n-1}V_n$ (between V_{n-1} and V_n) can be expressed as equation (8):

$$P_r({}_{n-1}V_n) = \frac{N({}_{n-1}V_n)}{\sum_{n=1}^{100} N({}_{n-1}V_n)}, \tag{8}$$

The normalization of equation (8) can be formulated to become equation (9),

$$\sum_{n=1}^{100} P_r({}_{n-1}V_n) = 1. \tag{9}$$

The fitted volatility distributions of the Dow and Nasdaq are exhibited in Figures 2(a) and 2(b) in the Appendix, respectively.

These graphs show some similarities and distinctions between the Dow and Nasdaq. First, both of the volatility distributions are asymmetric and rightly-skewed, meaning that the probability is decreasing with the increasing volatility. This finding is not surprising because the higher volatility always occurs when there is a lower likelihood. Second, both of them exhibit long right-tailed characteristics such that a log-normal distribution may be fitted. We then employ the log-normal function as shown in equation (10) to fit them.

$$P(V) = \frac{1}{Vw\sqrt{2\pi}} \exp\left[-\frac{1}{2w^2} \left(\ln \frac{V}{V_c} \right)^2 \right], \tag{10}$$

where the parameters V_c and w represent the peak and width of the distribution, and μ and σ represent the average and standard deviation of the volatility, respectively,where

$$\mu = \exp\left[\ln V_c + \frac{w^2}{2} \right] \text{ and } \sigma = \sqrt{\exp\left(2 \ln V_c + w^2\right) \left(\exp\left(w^2\right) - 1\right)}. \tag{11}$$

After setting $P'(V) = 0$, we can derive the peak V function as shown in equation (12):

$$V = e^{-w^2} V_c. \tag{12}$$

The results of the estimated measures of V, w, μ and σ are reported in Table 2 (see Appendix).

In Table 2, both the μ and σ of the Nasdaq are larger than those of the Dow, implying that the Nasdaq exhibits a higher return and a larger risk than the Dow. This result is consistent with our previous description that Nasdaq is composed of diversified high-tech companies with higher growth rates while the Dow is composed of well-established companies with stable growth rates. Furthermore, both of the V_c (peak) and w (width) for the Nasdaq are larger than those for the Dow, demonstrating that the Nasdaq has a higher volatility peak and area ($w \pm V_c$) compared to the results for the Dow.

3. Empirical results of probability distribution

3.1. The intraday volatility over the entire day. To examine if different trading times in each day provide different information regarding investors' behaviors, we then plot the thirty-nine 10-minute returns from the opening to the closing interval in each day as in Figure 3(a) in the Appendix. Figure 3(a) exhibits the average absolute returns with the opening interval (9:30-9:40) for the Dow and it shows that the returns of the Dow emerge with a striking peak at the opening time, then decline quickly after 9:40 a.m., and approach an almost flat-curve at lunch time. It then experiences a rising trend while approaching the closing time (14:20-15:40), before finally declining slightly at the closing time of 15:50. This evidence implies that investors in the Dow may prefer to settle down their trading before closing, so that a small 'u-shape' volatility pattern is displayed.

Since the opening returns include much overnight information and noise, in order to control for these possible biases, we re-draw the graph without the opening time in Figure 3(b) in the Appendix. Nonetheless, a small 'U-shaped' pattern still appears, demonstrating that the overnight information is digested very quickly and that no significant difference is found.

As for the Nasdaq, the intraday return pattern with and without the opening time are displayed in Figures 4(a) and 4(b), respectively. In a noteworthy finding, the Nasdaq exhibits a continuously ascend-

ing upward trend at closing time and a large 'U-shaped' pattern is revealed. This continuous upward trend at closing time demonstrates that investors in the Nasdaq are prepared to take more risk and may prefer to buy-in and take the overnight risk hoping for excess profit. In addition, the curvature of the shape in the Nasdaq is larger than that for the Dow, implying that more diversified investors' beliefs exist in the Nasdaq compared to the Dow.

3.2. The intraday returns over opening, lunch, and closing times from Mondays to Fridays. To examine the graphical evidence in more detail, we report the descriptive statistics for both the Dow and Nasdaq stocks for the full sample, as well as for each weekday for the three periods in Tables 3 and 4 (see Appendix).

3.2.1 Summary statistics of weekday and intraday returns of the Dow. We then separate the entire trading day into three sub-periods, namely, the opening, lunch, and closing sections, to explore the weekday, weekend, and intraday effects simultaneously. Table 3 presents the summary statistics from Monday to Friday with the associated graph being shown in Figure 5(a).

The results in Table 3 show that the full-day average return is nearly zero (0.05 bps), but is fairly volatile (20 bps) and somewhat non-normal (due to skewness and kurtosis measures that deviate from the normal distribution). The intraday returns for each of the five trading days exhibit a similar pattern with near-zero returns, high volatility, and non-normal distributions. Surprisingly, we find Mondays having the highest positive returns (0.000027), while Fridays show the lowest negative returns (-0.00001) across all the weekdays. We suspect this result is what others have referred to as the 'reversal' weekend effect[1]. Two decades ago, Smirlock and Starks (1986) argued that the sign of the morning return might be swamped by the afternoon return. To clarify this issue, the intraday returns of opening, lunch, and closing times across the weekdays are calculated and plotted in Figure 5(b) in the Appendix.

Some meaningful results are found: first, Friday closing shows a significantly positive return compared with the other trading times over the weekdays. However, this positive return lasts only until the Monday opening, then declines quickly to Monday lunch and becomes negative by Monday closing. This implies that the traditional selling time on Monday morning may have been postponed to the Monday closing. Hence, if one only considers the average returns without exploring the intraday returns (opening, lunch and closing), one may erroneously conclude that weekday effects no longer exist like the extant research suggests. However, after considering the intraday effect, we find that the traditional Monday effect basically still exists, as it is just postponed until Monday closing rather than until Monday morning. If a positive Friday closing return is a signal for the subsequent ascending price on Monday, then the magnitude of the Friday closing return might also signal the magnitude of the Monday opening return. Hence, Monday morning continues to exhibit the positive return from the Friday closing until Monday noon. Second, Tuesday then exhibits a positive return which is obviously positively-skewed, meaning that Tuesday effects may exist for the Dow.

These results imply that in explaining the weekend or Monday effect, one must not ignore the microstructure return within a day, including the opening, lunch and closing returns. The extant literature fails to capture this intraday information and thus the conclusion that the 'Monday effects disappear' may be misleading. In actual fact, this effect still exists, but it shifts to a shorter period of time (i.e., to Monday closing).

The possible interpretation may be attributed to transaction or information costs because the Dow is composed of well-established large firms with extremely high liquidity. Of course, large firm combinations of the Dow may be another reason for investors to postpone their selling time since they may be less volatile.

Figure 5(c) exhibits the skewness of the opening, lunch and closing times in each day. Interestingly, the skewness for Tuesday lunch times exhibits a relatively higher coefficient of 3 than at the other times across all the weekdays, implying that the Tuesday lunch may have a better likelihood of winning the positive returns than others. Besides, the skewness at the opening time is similar to that for the entire day, implying that the investors' beliefs at the opening time can be viewed as a proxy for the whole day. In particular, Tuesdays exhibit a more positive-skewed result among the three trading times, with the highest at Tuesday lunch, followed by Tuesday morning, and then Tuesday closing. This interesting finding implies that the Monday effect may have been postponed until Monday closing, and the Friday effect may have been replaced by the Tuesday effect in the Dow.

3.2.2. Results of weekday and intraday returns for the Nasdaq. The Nasdaq's summary statistics in Table 4 (see Appendix) reports a similar pattern as for the Dow except that volatility is somewhat higher (e.g., a full-day standard deviation of 30 bps ver-

[1] As section 1 pointed out, the reverse weekend effect means that the return is negative on Friday and positive on Monday.

sus the Dow's 20 bps). However, we also observe some differences. Most notably, the Dow reports a statistically significant Monday opening return of +1.33 bps (at the 5% level) while the Nasdaq index contains significant Tuesday opening and closing returns (+2.37 bps and -1.64 bps). These results, along with the graphs in Figures 5(a)-5(b) and 6(a)-6(b), suggest that there might be statistically significant differences in the risk/return characteristics during the week for both Dow and Nasdaq stocks. However, these results do not support a weekend effect because Friday returns are not significantly positive and Monday returns are not significantly negative.

The corresponding Nasdaq graph in Figure 6(a) in the Appendix shows that although Monday is found to have negative returns that are also the lowest, Friday does not exhibit a positive or the highest return. Instead, the Thursday effect may replace the Friday effect to reveal a pre-weekend effect.

However, after considering the intraday returns across the opening, lunch and closing times, we find that the traditional Friday effect does not disappear, for it appears at the Friday closing time rather than for the whole of Friday. The corresponding 3-dimensional graph is shown in Figure 6(b). In addition, the skewness of the Nasdaq among the opening, lunch and closing times over the weekdays in Figure 6(c) shows that the Tuesday lunch has the largest skewness as for the Dow. The similar shape of the skewness between the opening time and entire day implies that the investors' beliefs at the opening may be a proxy for the day as a whole. Besides, Tuesday shows the highest right-skewed peak, with the highest scenes at Tuesday lunch, followed by Tuesday morning. It seems that a negative feedback exists between Monday and Tuesday for the Nasdaq.

To sum up, the average returns on a daily and intraday basis are reported for the Dow and Nasdaq stocks in Figures 5(a)-5(b) and 6(a)-6(b), respectively. These graphs support prior research that shows that there might be weekday effects, particularly on Mondays, for Dow stocks, and possibly on Tuesdays and Thursdays for Nasdaq stocks.

3.3. Volatility distribution of intraday returns over opening, lunch, and closing times from Monday to Friday. *3.3.1. Results of volatility distribution of the Dow from Mondays to Fridays.* We next analyze the volatility distribution. Figure 7(a) displays the average absolute volatility from Mondays to Fridays. Although higher volatilities appear on Thursdays and Fridays, they are not significant. To better understand the details, we estimate the parameters in equations (10), (11) and (12) and report the results in Table 5. In particular, Tuesdays and Mondays exhibit the least volatility compared to the rest of the weekdays with the lowest peaks of V_c= 0.00114 and 0.00116, respectively. The corresponding width on Tuesdays is the narrowest (with a coefficient of 0.37973) as well. Moreover, although Tuesdays and Mondays exhibit the least volatility, their tails seem to be longer than the other weekdays, implying that the unexpected shocks normally appear on Tuesdays or Mondays instead of on weekends.

Although Thursday and Friday seem to be the most volatile days when compared with other weekdays (both have the same volatility of 0.00133), the coefficients of the probability distribution show that the width on Fridays (0.406) is broader than that on Thursdays (0.380). Given that the range $V_c \pm w$ represents the volatility area, Fridays should be more volatile than Thursdays. Finally, Figures 8(a)-8(f) exhibit the volatility distribution from Mondays to Fridays. The volatility distributions shift rightward from Thursdays and Fridays to Mondays and Tuesdays.

We further check the intraday volatility across the opening, lunch and closing times from Mondays to Fridays and find that the Tuesday close exhibits the largest volatility of 0.001267, although Mondays and Tuesdays seem to maintain the lowest daily volatility (see Table 5, Figures 7(b) and 9 in Appendix). This indicates again that different kinds of frequency data may lead to different conclusions. That is, Tuesday is the most stable day in terms of daily returns, but at the same time Tuesday's close is the most volatile time based on the intraday returns, and Friday is the most volatile day based on the daily data.

3.3.2. Volatility distribution of Nasdaq returns from Mondays to Fridays. The estimates of Nasdaq are reported in Table 6 (see Appendix). Monday exhibits the smallest volatility (σ) of 0.0023, the lowest peak (V_c) of 0.00209, and the narrowest width (w) of 0.461; while Thursday has the largest σ (0.0026), V_c (0.00231) and volatility area (between -0.46896 and 0.47358). The volatility distributions from Mondays to Fridays are displayed in Figures 11(a)-11(f). Similar to the Dow, Tuesday has the longest right tail, while Thursday shows the shortest tail and the lowest height, meaning that the unexpected shocks are most likely to occur on Tuesday.

Figures 10(a)-10(b) display the weekday returns in two and three dimensions over the weekdays, respectively. As shown, Monday and Tuesday exhibit the lowest volatility, while Thursday shows the highest one. However, the intraday effect among the weekdays is not clear across the opening, lunch and

closing times. To further explore this, we check the distributions over the three sessions shown in Figures 12(a)-12(f). Interestingly, Tuesday exhibits a significantly different shape compared to the other weekdays across the opening, lunch and closing times: the lowest volatility and peak, and the shortest tail. Besides, we find that the distribution of the Tuesday closing shifts rightward against the Tuesday lunch, and lunch time shifts rightward against the opening. The Tuesday closing effect continues until Wednesday morning exhibiting the longest right tail.

Conclusions and discussion

This work provides additional evidence of high frequency stock returns that is based mainly on five facts. First, we observe that the Dow declines slightly at closing time experiencing a small 'U-shape', while the Nasdaq ascends upward continuously at closing time, showing a large 'U-shaped' pattern. This evidence implies that investors in both markets have different trading strategies. The Nasdaq apparently provides much more positive feedback and is more aggressive than the Dow. Our findings are in line with the study of Goodhart and O'Hara (1997), Andersen and Bollerslev (1997) and Andersen et al. (2000) who use the absolute intraday return to measure the intraday volatility.

Fact two pertains to the Dow, and examining the intraday returns helps us avoid arriving at a misleading and incorrect conclusion. We find that the traditional Monday effect still exists, but it has been postponed to the Monday closing, a shorter period of one day. Not surprisingly, after a negative return at the Monday closing, Tuesday by contrast reveals a positive return associated with a significantly positive skewness, implying that a new positive Tuesday effect exists for the Dow. The evidence of our findings is different from that of Jaffe and Westerfield (1985) and Kato (1990) who found that Tuesdays reveal the lowest returns.

Fact three is that, in regard to the Nasdaq, although the Monday effect still exists based on checking the average return, the intraday patterns are not obvious. Similarly, examining the intraday returns helps us observe the traditional weekend effects over a short period. We find that a significantly positive return appears at the Friday closing time rather than in the morning or at lunch. Therefore, the weekend effect appears only at the Friday closing in the case of the Nasdaq. Without doing this, we might be misled into reporting that the weekend effect appears earlier on Thursday instead of on Friday and conclude that the investors' trading behavior varies in a more efficient or dynamic way.

Fact four is that, for both the Dow and Nasdaq stocks, no significant weekend pattern can be revealed, but there appear to be statistically significant differences within the week (thus supporting the idea of weekday effects).

Finally, fact five is that the parameters of intraday volatilities from the log-normal distribution show that Dow and Nasdaq have similar patterns: the lowest average volatility and peak appear on Monday and Tuesday, while the highest average volatility and peak appear on Thursday. However, the differences between the distributions of intraday volatility are not significant.

References

1. Arsad, Z. and A.J. Coutts (1996). The weekend effect, good news, bad news and the Financial Times Industrial Ordinary Shares Index, *Applied Economics Letters*, 3, pp. 797-801.
2. Abraham, A. and D.L. Ikenberry (1994). The individual investor and the weekend effect, *Journal of Financial and Quantitative Analysis*, 29, pp. 263-277.
3. Andersen, T.G. and T. Bollerslev (1997). Intraday periodicity and volatility persistence in financial markets, *Journal of Empirical Finance*, 4, pp. 115-158.
4. Andersen, T.G., T. Bollerslev and J. Cai (2000). Intraday and interday volatility in the Japanese stock market, *Journal of International Financial Markets, Institutions and Money*, 10, pp. 107-30.
5. Branch, B.B., J. Jung, and T. Yang (2001). The Monday merger effect, *International Review of Financial Analysis*, 10, pp. 1-18.
6. Brusa, J., P. Liu and C. Schulman (2000). The weekend effect, 'reverse' weekend effect, and firm size, *Journal of Business Finance and Accounting*, 27, pp. 555-575.
7. Chen, C., C. Lee and A. Wang (2002). A note on stock market seasonality: the impact of stock price volatility on the application of dummy variable regression model, *The Quarterly Review of Economics and Finance*, 42 (1), pp. 155-162.
8. Coutts, J. Andrew (1999). Friday the Thirteenth and the Financial Times Industrial Ordinary Shares Index 1935-94, *Applied Economics Letters*, 6, pp. 35-37.
9. Cross, F. (1973). The behavior of stock prices on Fridays and Mondays, *Financial Analysts Journal*, 29 (6), pp. 67-69.
10. Delong, J., A. Schleifer, L.H. Summers and R.J. Waldmann, (1989). The size and incidence of the losses from noise trading, *Journal of Finance*, 14, pp. 681-696.
11. Dyl, E. and S. Martin, Jr., (1985). Weekend effects on stock returns: a comment, *Journal of Finance*, 40 (1), pp. 347-349.
12. Fama, E. (1991). Efficient capital markets: II, *Journal of Finance*, 46 (5), pp. 1575-1617.

13. Fishe, R.P.H., T.F. Gosnell and D.J. Lasser, (1993). Good news, bad news, volume and the Monday effect, *Journal of Business Finance and Accounting*, 22 (6), pp. 881-893.
14. French, K. (1980). Stock returns and the weekend effect, *Journal of Financial Economics*, 8, pp. 55-70.
15. Gibbons, M., and P. Hess, (1981). Day of the week effects and asset returns, *Journal of Business*, 54 (4), pp. 579-596.
16. Goodhart, C.A.E. and M. O'Hara (1997). High frequency data in financial markets: issues and applications, *Journal of Empirical Finance*, 4, pp. 73-114.
17. Gu, A.Y. (2004). The reversing weekend effect: evidence from the U.S. equity markets, *Review of Quantitative Finance and Accounting*, 22 (1), pp. 5-14.
18. Harris, L. (1986). A transaction data study of weekly and intraday patterns in stock returns, *Journal of Financial Economics*, 16, pp. 99-117.
19. Heston, S.L., Korajczyk, R.A., and R. Sadka (2010). Intraday patterns in the cross-section of stock returns, *Journal of Finance*, 65 (4), pp. 1369-1407.
20. Ito, T., R. K. Lyons, and M. T. Melvin (1998). Is there private information in the FX market? The Tokyo experiment, *Journal of Finance*, 53, pp. 1111-1130.
21. Jaffe, J., and R. Westerfield (1985). The weekend effect in common stock returns: the international evidence, *Journal of Finance*, 40, pp. 433-455.
22. Juha-Pekka, T. Martikaninen (1997). Trading patterns of small and large traders: implications for the weekend effect in Finland, *Applied Economics Letters*, 4, pp. 595-597.
23. Kamara, A. (1997). New evidence on the Monday seasonal in stock returns, *Journal of Business*, 70, pp. 63-84.
24. Kato, K. (1990). Weekly patterns in Japanese stock returns, *Management Science*, 36 (9), pp. 1031-1043.
25. Keim, D.B., and A. Madhavan (1995). Execution costs and investment performance: An empirical analysis of institutional equity trades, Working Paper 9-95, University of Pennsylvania, Wharton School.
26. Kelly, F. (1930). Why you win or lose: The Psychology of Speculation, Boston: Houghton Mifflin.
27. Lakonishok, J. and M. Levi (1982). Weekend effects on stock returns: a note, *Journal of Finance*, 37 (3), pp. 883-889.
28. Lakonishok, J. and E. Maberly (1990). The weekend effect: trading patterns of individual and institutional investors, *Journal of Finance*, 45, pp. 231-243.
29. Lauterbach, B. and M. Ungar (1992). Calendar anomalies: some perspectives from the behavior of the Israeli stock market, *Applied Financial Economics*, 2, pp. 57-60.
30. Liu, Y., P. Gopikrishnan, P. Cizeau, M. Meyer, C.K. Peng, and H. E. Stanley (1999). Statistical properties of the volatility of price fluctuations, *Physical Review* E, 60, pp. 1390-400.
31. Maberly, E. (1995). Eureka! Eureka! Discovery of the Monday effect belongs to the Ancient Scribes, *Financial Analysts Journal*, pp. 10-11.
32. Mehdian S.M.J. Perry (2001). The reversal of the Monday effect: New evidence from U.S. equity markets, *Journal of Business Finance and Accounting*, 28 (7), pp. 1043-1065.
33. Marquering, W., Nisser, J. & Valla, T. (2006). Disappearing anomalies: A dynamic analysis of the persistence of anomalies, *Applied Financial Economics*, 16 (4), pp. 291-302.
34. Pettengill G.N. (2003). A survey of the Monday effect literature, *Quarterly Journal of Business and Economics*, 42, pp. 3-27.
35. Rogalski, R. (1984). New findings regarding day-of-the-week returns over trading and non-trading periods: a note, *Journal of Finance*, 39, pp. 1603-1614.
36. Smirlock, M. and L. Starks (1986). Day-of-the week and intraday effects in stock returns, *Journal of Financial Economics*, 17, pp. 197-210.
37. Steely, J.M. (2001). A note of information seasonality and the disappearance of the weekend effect in the UK stock market, *Journal of Banking and Finance*, 25, pp. 1941-1956.
38. Sullivan, J.H. and K. Liano (2003). Market breadth and the Monday seasonal in stock returns, *Quarterly Journal of Business and Economics*, pp. 65-72.
39. Sullivan, R., A. Timmermann and H. White (2001). Dangers of data mining: the case of calendar effects in stock returns, *Journal of Econometrics*, pp. 249-286.
40. Wingender J, J.E. Groff (1999). On stochastic dominance analysis of day-of-the-week return patterns, *Journal of Financial Research*, 12 (1), pp. 51-55.
41. Yu, H.-C. and M.-C. Huang (2004). Statistical properties of volatility in fractal dimensions and probability distribution among six stock markets, *Applied Financial Economics*, 14, pp. 1087-1095.
42. Yu, H.-C., I. Chiou I and J. Jordan-Wagner (2008). Does the weekday effect of the yen/dollar spot rates exist in Tokyo, London, and New York? An analysis of panel probability distributions, *Applied Economic*, 40 (20), pp. 2631-2643.
43. Yu, H.-C. Shih T.L. (2011). Gold, crude oil and the weekend effect: A probability distribution approach, *Investment Management and Financial Innovation*, 8 (2), pp. 36-47.

Appendix

Table 1. Summary statistics of 10-minute intraday returns for the Dow and Nasdaq (1997-2003)

Index	Obs.	Mean	Std. dev.	Minimum	Maximum	Skewness	Kurtosis	JB
Dow	60,527	0.000005	0.002	-0.032	0.034	0.127	19.95	1004360.88
		(0.4995)				(0.0000)**	(0.0000)**	(0.0000)**
Nasdaq	62,945	0.000012	0.003	-0.064	0.057	0.271	23.59	1461387.78
		(0.4383)				(0.0000)**	(0.0000)**	(0.0000)**

Notes: This table presents the summary statistics of 10-minute returns for the Dow and Nasdaq. The sample extends from August 1, 1997 through December 31, 2003 for a total of 60,527 observations. The Jarque-Bera (JB) statistic is $JB = \frac{T}{6}(S^2 + \frac{(k-3)^2}{4})$, where S is the skewness and k is the kurtosis. *, ** represent significance at the 10% and 5% levels, respectively.

Table 2. Estimated coefficients in the lognormal distribution of volatility for the Dow and Nasdaq (1997-2003)

	Obs.	Peak (V_c)	Width(w)	Mean	Std. dev.	Minimum	Maximum
Dow	60451	0.00119	0.38555	0.001292	0.000580	0.000207	0.005665
Nasdaq	62907	0.00221	0.47100	0.002472	0.001275	0.000376	0.014587

Notes: This table presents the estimated results of parameters in a lognormal distribution using 10-minute intraday returns. The sample extends from August 1, 1997 through December 31, 2003 for a total of 60,451 observations. Following equation (10), i.e., $P(V) = \frac{1}{Vw\sqrt{2\pi}} \exp\left[-\frac{1}{2w^2}\left(\ln\frac{V}{V_c} \right)^2 \right]$, the peak ($V_c$) represents the peak of the probability location while the width (w) indicates the width of the peak at the half height of the distribution.

Table 3. Summary statistics of 10-minute returns over the opening, lunch and closing times from Monday to Friday for the Dow (1997-2003)

Period	Obs.	Mean	Std. dev.	Minimum	Maximum	Skewness	Kurtosis	JB
Monday returns								
Entire	11544	0.000027	0.0019	-0.025	0.028	-0.077	22.48	243095.90
		(0.1336)				(0.0007)**	(0.0000)**	(0.0000)**
Opening	2960	0.000133	0.0028	-0.025	0.028	0.008	15.65	30225.94
		(0.0105)**				(0.8501)	(0.0000)**	(0.0000)**
Lunch	2960	0.000018	0.0012	-0.009	0.014	0.442	9.70	11709.20
		(0.4398)				(0.0000)**	(0.0000)**	(0.0000)**
Closing	2960	-0.000039	0.0017	-0.017	0.010	-0.922	9.61	11825.55
		(0.2226)				(0.0000)**	(0.0000)**	(0.0000)**
Tuesday returns								
Entire	12402	0.000017	0.0019	-0.023	0.032	1.159	26.15	356354.03
		(0.3317)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3180	0.000075	0.0027	-0.023	0.032	1.220	18.80	47632.06
		(0.1253)				(0.0000)**	(0.0000)**	(0.0000)**
Lunch	3180	0.000026	0.0014	-0.015	0.028	2.736	65.67	575439.58
		(0.3005)				(0.0000)**	(0.0000)**	(0.0000)**
Closing	3180	-0.000011	0.0018	-0.009	0.011	0.056	3.07	1255.82
		(0.7357)				(0.1964)	(0.0000)**	(0.0000)**
Wednesday returns								
Entire	12402	-0.000007	0.0019	-0.032	0.018	-0.397	16.69	144320.90
		(0.6787)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3180	-0.000066	0.0027	-0.032	0.018	-0.730	14.16	26850.32
		(0.1764)				(0.0000)**	(0.0000)**	(0.0000)**
Lunch	3180	0.000013	0.0013	-0.007	0.012	0.347	6.14	5069.07
		(0.5905)				(0.0000)**	(0.0000)**	(0.0000)**
Closing	3180	0.000003	0.0018	-0.009	0.015	0.331	4.21	2414.04
		(0.9222)				(0.0000)**	(0.0000)**	(0.0000)**
Thursday returns								
Entire	12168	0.000001	0.0020	-0.023	0.034	0.119	17.76	167397.34
		(0.9523)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3120	-0.000034	0.0028	-0.023	0.034	0.058	14.13	25958.57
		(0.5023)				(0.1795)	(0.0000)**	(0.0000)**
Lunch	3120	0.000014	0.0013	-0.010	0.007	-0.166	3.67	1771.11
		(0.5547)				(0.0001)**	(0.0000)**	(0.0000)**
Closing	3120	0.000024	0.0019	-0.014	0.014	0.439	5.51102	4048.80
		(0.4911)				(0.0000)**	(0.0000)**	(0.0000)**
Friday returns								
Entire	12011	-0.000010	0.0019	-0.032	0.022	-0.191	17.12	160085.06
		(0.7054)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3139	-0.000019	0.0028	-0.032	0.022	-0.264	12.49	20470.48
		(0.7139)				(0.0000)**	(0.0000)**	(0.0000)**
Lunch	3140	0.000001	0.0013	-0.006	0.007	-0.013	3.30	1426.72
		(0.9674)				(0.7654)	(0.0000)**	(0.0000)**
Closing	3140	0.000059	0.0017	-0.008	0.012	0.188	3.78	1894.12
		(0.0613)*				(0.0000)**	(0.0000)**	(0.0000)**

Notes: This table presents the summary statistics of 10-minute returns over the entire day (9:30-15:50), opening time (9:30-11:00), lunch hour (12:00-13:30) and closing time (14:20-15:50). The sample extends from August 1, 1997 through December 31, 2003 for a total of 60,527 observations. *, ** denote significance at the 10% and 5% levels, respectively.

Table 4. Summary statistics of 10-minute returns over the opening, lunch and closing times from Monday to Friday for the Nasdaq (1997-2003)

Period	Obs.	Mean	Std. dev.	Minimum	Maximum	Skewness	Kurtosis (excess)	JB
Monday returns								
Entire	12012	-0.000009	0.003	-0.064	0.041	-0.616	24.92	311689.79
		(0.7285)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3080	0.000011	0.005	-0.064	0.041	-0.759	16.47	35136.85
		(0.9100)				(0.0000)**	(0.0000)**	(0.0000)**
Lunch	3080	0.000007	0.002	-0.016	0.014	-0.133	4.51	2619.85
		(0.8572)				(0.0025)**	(0.0000)**	(0.0000)**
Closing	3080	0.000025	0.003	-0.021	0.022	0.174	5.71	4203.55
		(0.6732)				(0.0001)**	(0.0000)**	(0.0000)**
Tuesday returns								
Entire	12909	0.000006	0.004	-0.050	0.051	1.514	25.61	357735.34
		(0.8613)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3310	0.000237	0.006	-0.050	0.051	1.327	14.90	31593.90
		(0.0169)**				(0.0000)**	(0.0000)**	(0.0000)**
Lunch	3310	0.000013	0.003	-0.021	0.050	4.64635	79.89	892356.66
		(0.8013)				(0.0000)**	(0.0000)**	(0.0000)**
Closing	3310	-0.000164	0.004	-0.023	0.027	-0.1695	5.48	4157.71
		(0.0082)**				(0.0001)**	(0.0000)**	(0.0000)**
Wednesday returns								
Entire	12909	0.000021	0.004	-0.060	0.057	0.042	23.61	299928.98
		(0.5692)				(0.0505)*	(0.0000)**	(0.0000)**
Opening	3310	0.000072	0.006	-0.060	0.057	-0.040	16.60	38045.40
		(0.4979)				(0.3470)	(0.0000)**	(0.0000)**
Lunch	3310	0.000010	0.003	-0.025	0.024	0.231	8.38347	9722.70
		(0.8328)				(0.0000)**	(0.0000)**	(0.0000)**
Closing	3310	-0.000009	0.004	-0.017	0.02174	0.171	2.80	1098.91
		(0.8813)				(0.0001)**	(0.0000)**	(0.0000)**
Thursday returns								
Entire	12675	0.000046	0.004	-0.043	0.057	0.290	18.19	182681.28
		(0.1968)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3250	0.000159	0.006	-0.043	0.057	0.203	11.63	18359.92
		(0.1368)				(0.0000)**	(0.0000)**	(0.0000)**
Lunch	3250	0.000020	0.002	-0.013	0.012	-0.060	2.51	856.17
		(0.6400)				(0.1595)	(0.0000)**	(0.0000)**
Closing	3250	0.000023	0.004	-0.014	0.023	0.375	3.15257	1422.37
		(0.7136)				(0.0000)**	(0.0000)**	(0.0000)**
Friday returns								
Entire	12908	-0.000001	0.004	-0.061	0.053	-0.101	26.53	378701.52
		(0.9689)				(0.0000)**	(0.0000)**	(0.0000)**
Opening	3249	-0.000052	0.006	-0.061	0.053	-0.114	15.80	33803.09
		(0.6201)				(0.0077)*	(0.0000)**	(0.0000)**
Lunch	3250	0.000029	0.002	-0.015	0.028	0.533	10.13	14052.93
		(0.4917)				(0.0000)*	(0.0000)**	(0.0000)**
Closing	3250	0.000076	0.003	-0.020	0.021	-0.007	4.74	3053.66
		(0.1849)				(0.8583)	(0.0000)**	(0.0000)**

Notes: This table presents the summary statistics of 10-minute returns over the entire day (9:30-15:50), opening time (9:30-11:00), lunch hour (12:00-13:30) and closing time (14:20-15:50). The sample extends from August 1, 1997 through December 31, 2003 for a total of 60,527 observations. *, ** denote significance at the 10% and 5% levels, respectively.

Table 5. Estimated coefficients in lognormal distribution of 10-minute volatility over the opening, lunch and closing times from Monday to Friday for the Dow

Period	Obs.	Mean (μ)	Std. dev. (σ)	Volatility area ($V_c \pm w$)	Peak (V_c)	Widt (w)
Monday volatility						
Entire	11544	0.001258	0.000576	-0.39604 ~ 0.39836	0.00116	0.39720
Opening	2960	0.001285	0.000576	-0.40188 ~ 0.40424	0.00118	0.40306
Lunch	2960	0.001250	0.000559	-0.39910 ~ 0.40146	0.00118	0.40028
Closing	2960	0.001242	0.000596	-0.39930 ~ 0.40164	0.00117	0.40047
Tuesday volatility						
Entire	12402	0.001242	0.000603	-0.37859 ~ 0.38087	0.00114	0.37973
Opening	3180	0.001228	0.000602	-0.39849 ~ 0.40083	0.00117	0.39966
Lunch	3180	0.001235	0.000604	-0.39791 ~ 0.40025	0.00117	0.39908
Closing	3180	0.001267	0.000602	-0.39741 ~ 0.39977	0.00118	0.39859
Wednesday volatility						
Entire	12402	0.001292	0.000581	-0.38054 ~ 0.38292	0.00119	0.38173
Opening	3180	0.001274	0.000584	-0.39847 ~ 0.40083	0.00118	0.39965
Lunch	3180	0.001296	0.000588	-0.39938 ~ 0.40176	0.00119	0.40057
Closing	3180	0.001303	0.000568	-0.39668 ~ 0.39906	0.00119	0.39787
Thursday volatility						
Entire	12168	0.001333	0.000559	-0.37900 ~ 0.38144	0.00122	0.38022
Opening	3120	0.001334	0.000561	-0.39869 ~ 0.40107	0.00119	0.39988
Lunch	3120	0.001338	0.000552	-0.39768 ~ 0.40006	0.00119	0.39887
Closing	3120	0.001328	0.000569	-0.40117 ~ 0.40355	0.00119	0.40236
Friday volatility						
Entire	11935	0.001333	0.000570	-0.40523 ~ 0.40763	0.00120	0.40643
Opening	3120	0.001329	0.000575	-0.40459 ~ 0.40697	0.00119	0.40578
Lunch	3121	0.001327	0.000571	-0.40596 ~ 0.40834	0.00119	0.40715
Closing	3121	0.001303	0.000573	-0.40926 ~ 0.41162	0.00118	0.41044

Notes: This table presents the estimated coefficients of 10-minute returns over the entire day (9:30-15:50), opening time (9:30-11:00), lunch hour (12:00-13:30) and closing time (14:20-15:50) from Monday to Friday. The sample extends from August 1, 1997 through December 31, 2003 for a total of 60,527 observations. *, ** denote significance at the 10% and 5% levels, respectively. The lognormal function is shown as,

$$P(V) = \frac{1}{Vw\sqrt{2\pi}} \exp\left[-\frac{1}{2w^2} \left(\ln \frac{V}{V_c} \right)^2 \right],$$

where $\mu = \exp\left[\ln V_c + \frac{w^2}{2} \right]$ and $\sigma = \sqrt{\exp\left(2 \ \ln \ V_c + w^2\right) \ \left(\exp\left(w^2\right) - 1\right)}$.

The parameters of V_c and w represent the peak and width of the distribution, respectively; μ and σ represent the average intraday volatility and standard deviation of the intraday volatility, respectively.

Table 6. Estimated coefficients in lognormal distribution of 10-minute volatility over the opening, lunch and closing times from Monday to Friday for the Nasdaq

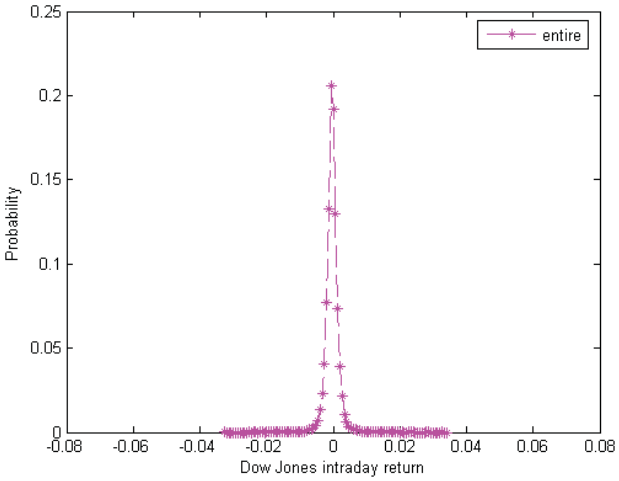
Period	Obs.	Mean (μ)	Std. dev. (σ)	Volatility area ($V_c \pm w$)	Peak (V_c)	Width (w)
Monday volatility						
Entire	12012	0.002327	0.001181	-0.45935 ~ 0.46353	0.00209	0.46144
Opening	3080	0.002365	0.001201	-0.46525 ~ 0.46949	0.00212	0.46737
Lunch	3080	0.002306	0.001163	-0.45599 ~ 0.46013	0.00207	0.45806
Closing	3080	0.002315	0.001185	-0.45937 ~ 0.46351	0.00207	0.46144
Tuesday volatility						
Entire	12909	0.002365	0.001270	-0.47283 ~ 0.47703	0.00210	0.47493
Opening	3310	0.002328	0.001221	-0.47841 ~ 0.48255	0.00207	0.48048
Lunch	3310	0.002353	0.001239	-0.47037 ~ 0.47457	0.00210	0.47247
Closing	3310	0.002421	0.001356	-0.46957 ~ 0.47387	0.00215	0.47172
Wednesday volatility						
Entire	12909	0.002545	0.001375	-0.47126 ~ 0.47578	0.00226	0.47352
Opening	3310	0.002475	0.001356	-0.46104 ~ 0.46546	0.00221	0.46325
Lunch	3310	0.002565	0.001397	-0.47424 ~ 0.47880	0.00228	0.47652
Closing	3310	0.002589	0.001357	-0.47721 ~ 0.48181	0.00230	0.47951
Thursday volatility						
Entire	12675	0.002610	0.001278	-0.46896 ~ 0.47358	0.00231	0.47127
Opening	3250	0.002641	0.001346	-0.48002 ~ 0.48466	0.00232	0.48234
Lunch	3250	0.002612	0.001260	-0.46453 ~ 0.46917	0.00232	0.46685
Closing	3250	0.002578	0.001238	-0.46154 ~ 0.46612	0.00229	0.46383
Friday volatility						
Entire	12402	0.002505	0.001232	-0.46463 ~ 0.46905	0.00221	0.46684
Opening	3240	0.002527	0.001235	-0.46718 ~ 0.47166	0.00224	0.46942
Lunch	3240	0.002493	0.001223	-0.47371 ~ 0.47813	0.00221	0.47592
Closing	3240	0.002422	0.001238	-0.48591 ~ 0.49019	0.00214	0.48805

Notes: This table presents the estimated coefficients of 10-minute returns over the entire day (9:30-15:50), opening time (9:30-11:00), lunch hour (12:00-13:30) and closing time (14:20-15:50) from Monday to Friday. The sample extends from August 1, 1997 through December 31, 2003 for a total of 60,527 observations. *, ** denote significance at the 10% and 5% levels, respectively. The lognormal function is shown as,

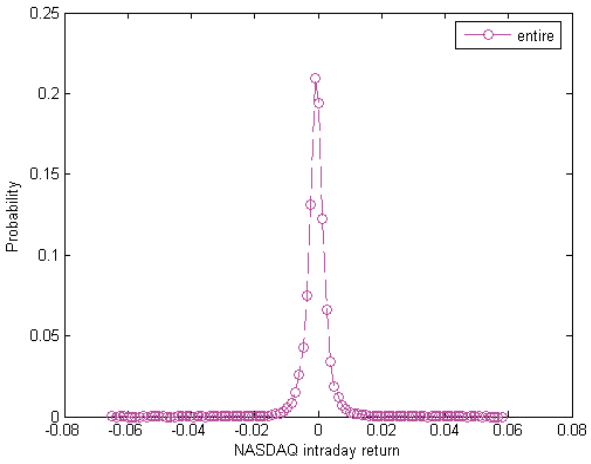
$$P(V) = \frac{1}{Vw\sqrt{2\pi}} \exp\left[-\frac{1}{2w^2} \left(\ln \frac{V}{V_c} \right)^2 \right]$$

where $\mu = \exp\left[\ln V_c + \frac{w^2}{2} \right]$ and $\sigma = \sqrt{\exp\left(2 \ln V_c + w^2\right) \left(\exp\left(w^2\right) - 1\right)}$

The parameters of V_c and w represent the peak and width of the distribution, respectively; μ and σ represent the average intraday volatility and standard deviation of the intraday volatility, respectively.

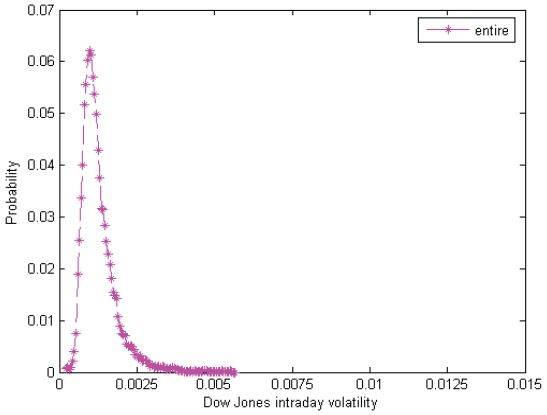


(a) Dow

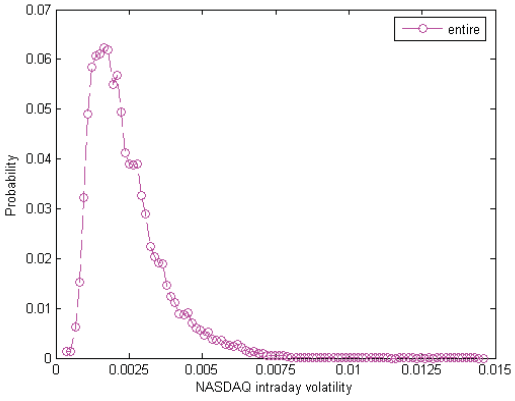


(b) Nasdaq

Fig 1. Probability distribution of the 10-min intraday return for the Dow and Nasdaq over the whole period

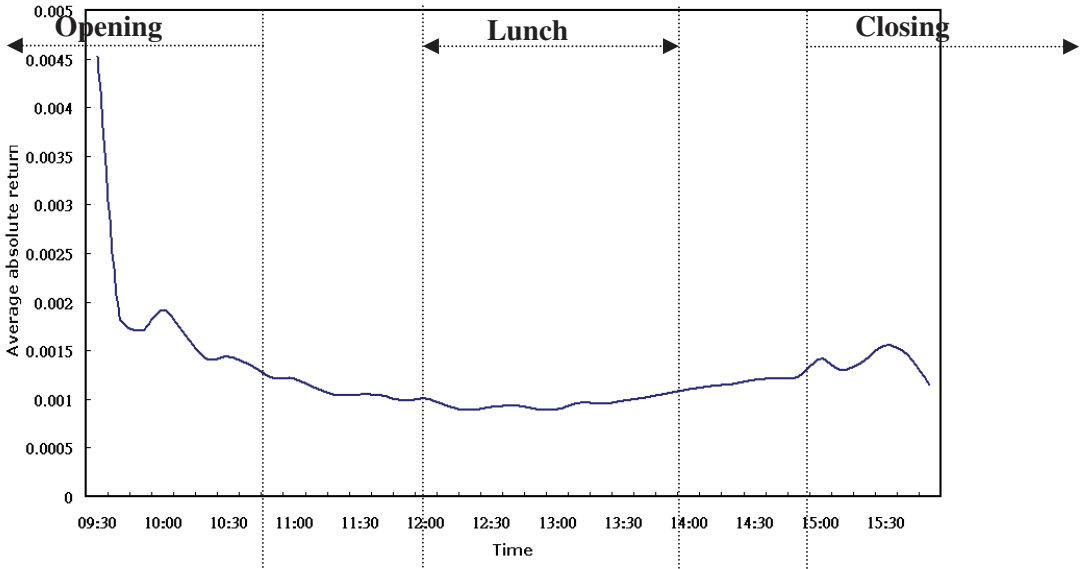


(a) Dow

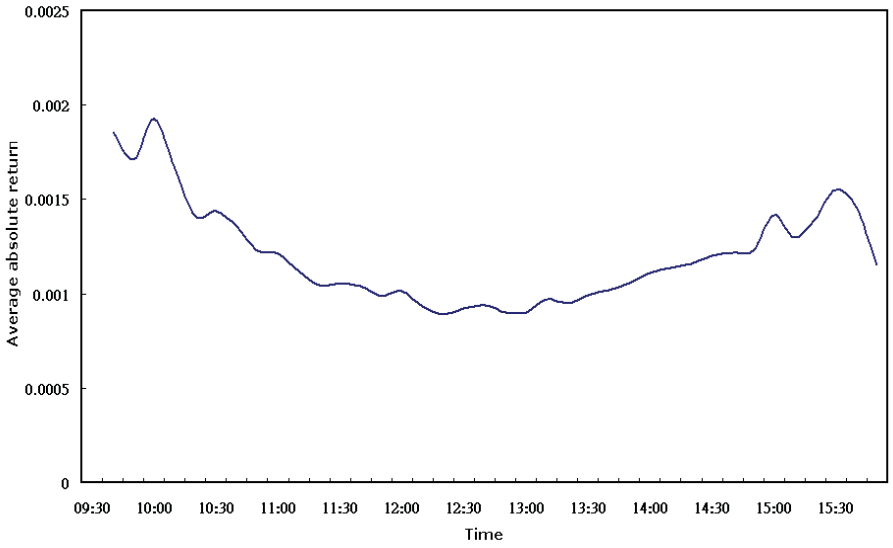


(b) Nasdaq

Fig 2. Log-normal distribution of the volatility of the 10-min return over the whole period

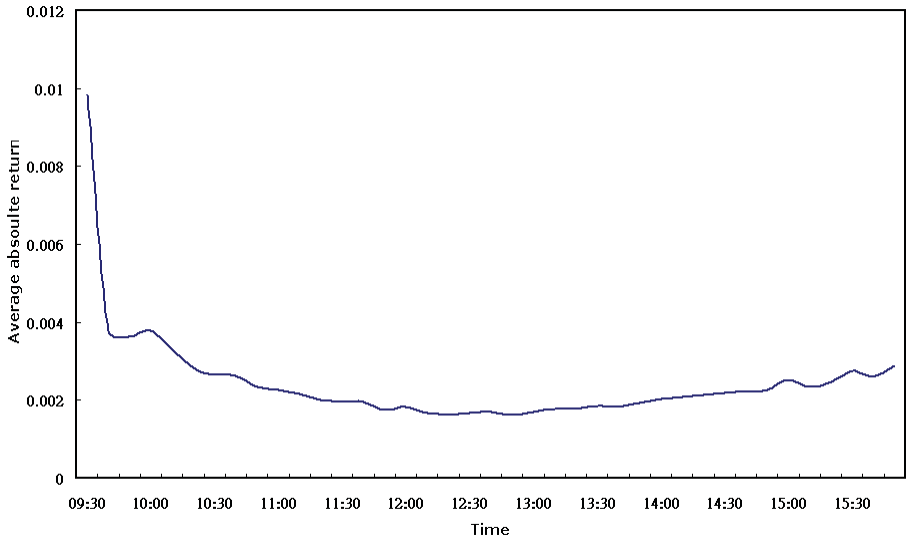


(a) With opening returns

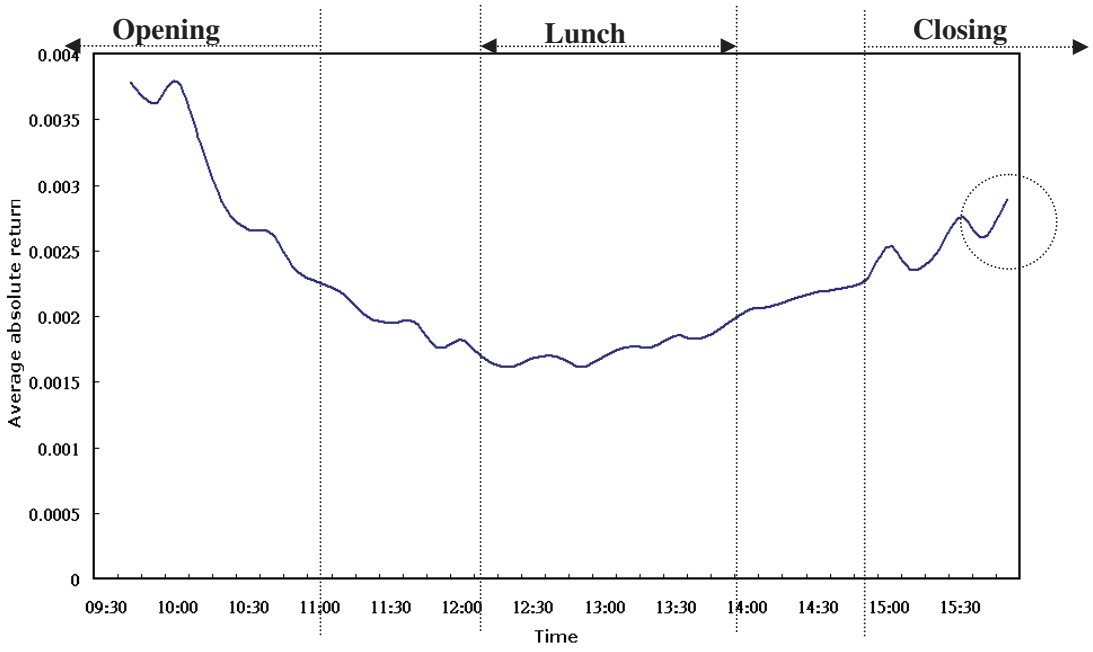


(b) Without opening returns

Notes: This figure presents the volatility of Dow 10-minute returns for three intraday periods: opening, lunch, and closing times (9:30-11:00, 12:00-1:30, and 14:20-15:50, respectively). The returns are calculated as average absolute returns.



(a) With opening returns



(b) Without opening returns

Fig 4. The intraday volatility U-shape over the day for the Nasdaq

Notes: This figure presents the volatility of Nasdaq 10-minute returns for three intraday periods: opening, lunch, and closing times (9:30-11:00, 12:00-1:30, and 14:20-15:50, respectively). The returns are calculated as average absolute returns.

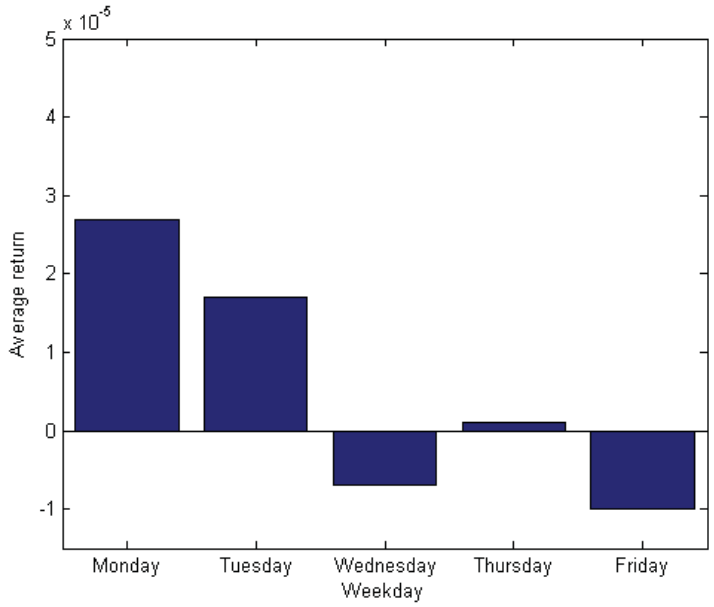


Fig. 5(a). Average intraday returns for the weekdays (Monday to Friday) for the Dow

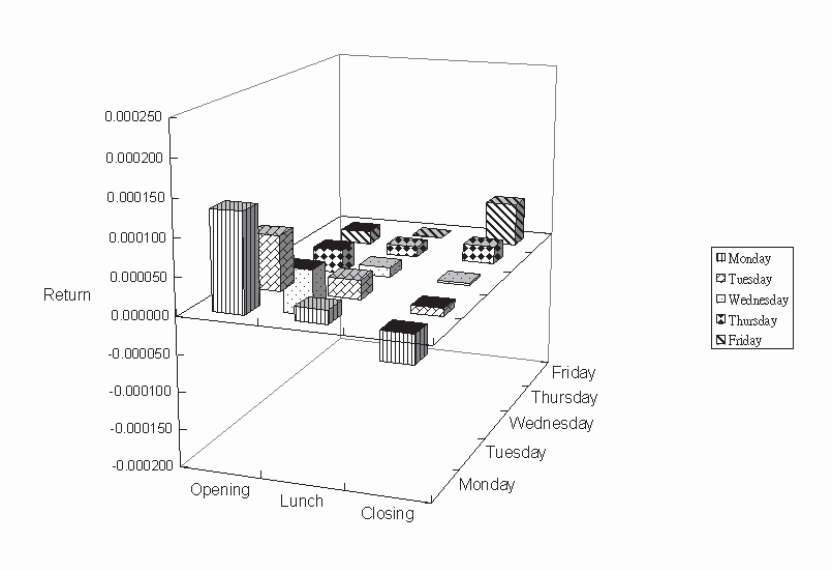


Fig 5(b). Three-dimensional intraday returns for the weekdays (Monday to Friday) across opening, lunch and closing times for the Dow

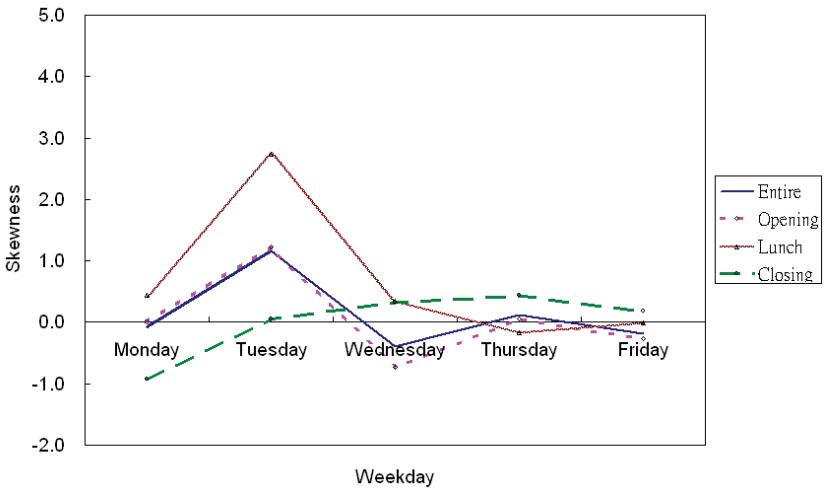


Fig. 5(c). Skewness of the weekdays (Monday to Friday) returns across the entire, opening, lunch, and closing times for the Dow

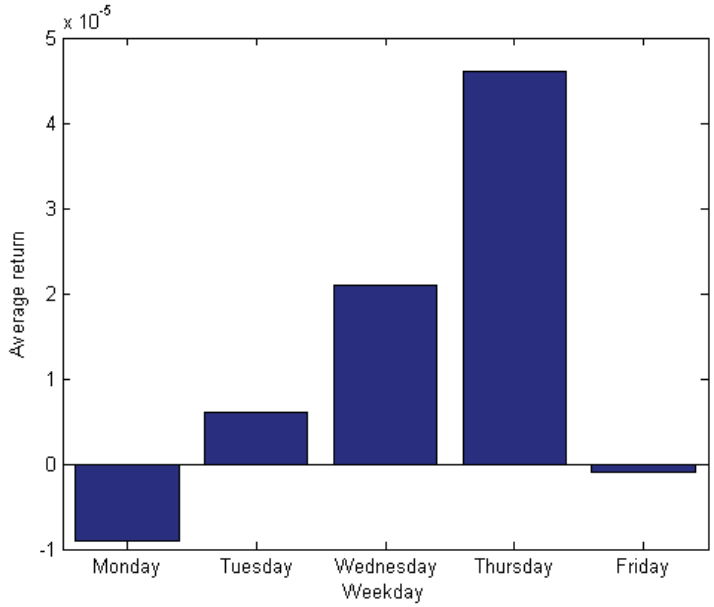


Fig 6(a). Average intraday returns of the weekdays (Monday to Friday) for the Nasdaq

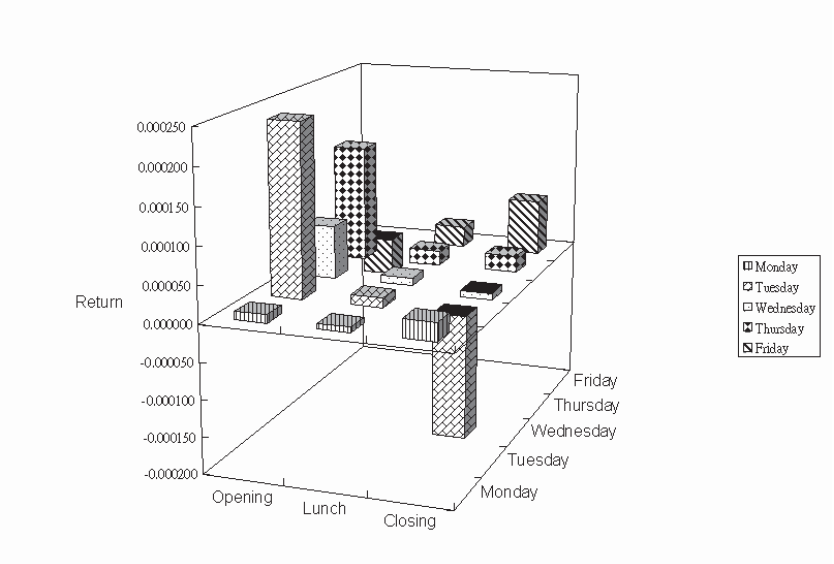


Fig. 6(b). Three-dimension intraday returns for the weekdays (Monday to Friday) across opening, lunch and closing times for the Nasdaq

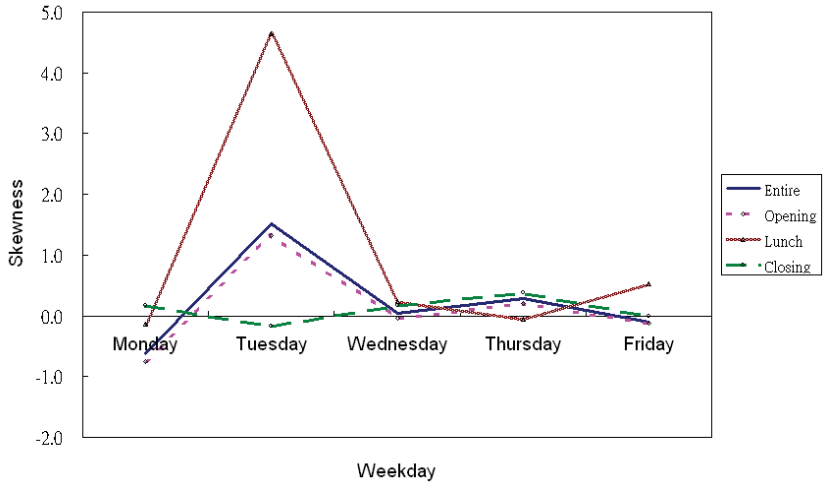


Fig. 6(c). Skewness of the weekdays (Monday to Friday) returns across the entire, opening, lunch, and closing times for the nasdaq

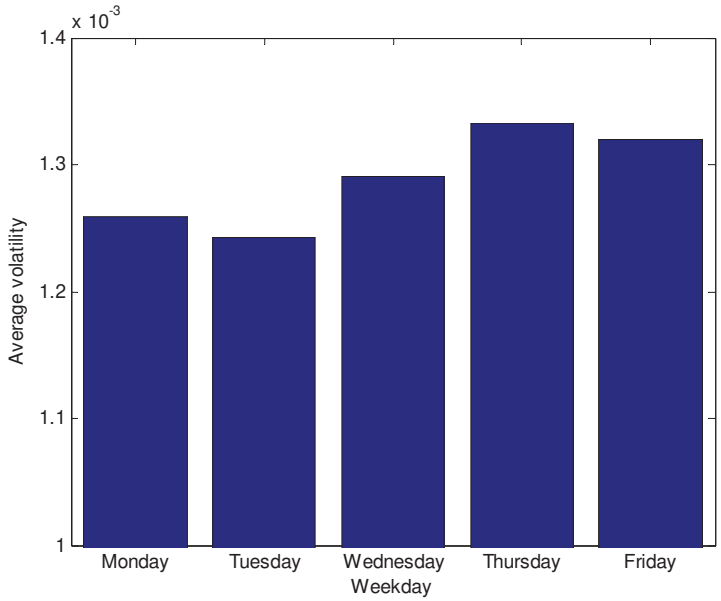


Fig. 7(a). The average volatility from Monday to Friday for the Dow

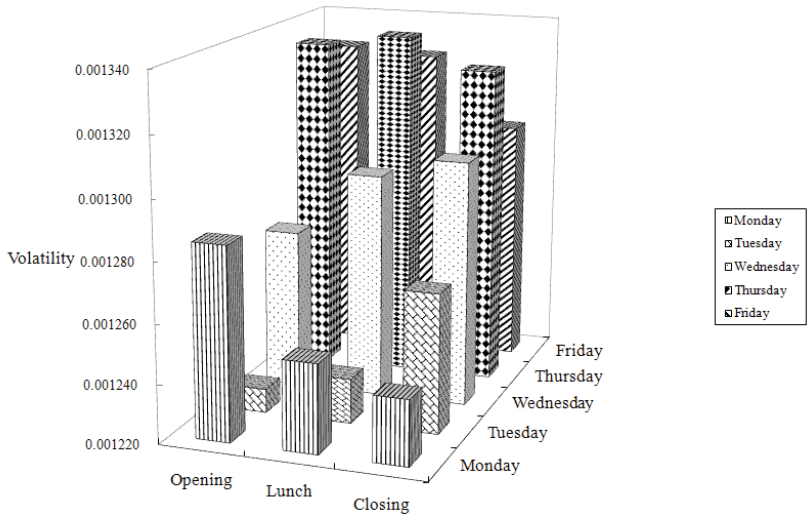
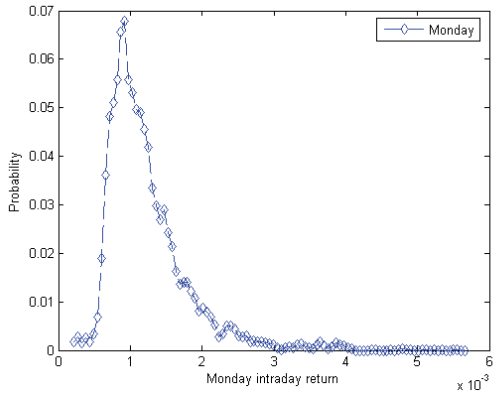
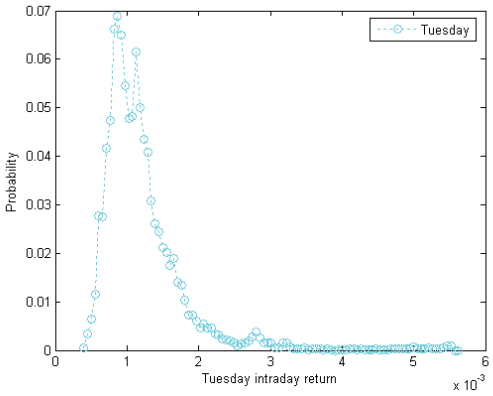


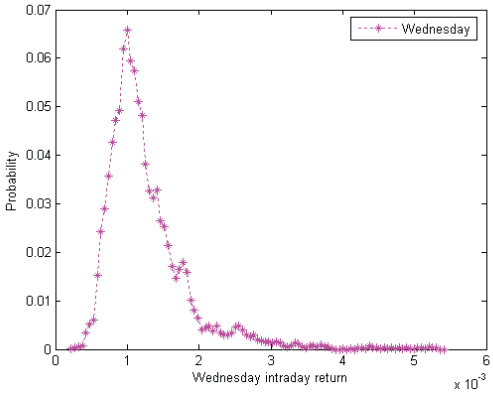
Fig. 7(b). Three-dimensional weekday volatilities from Monday to Friday across opening, lunch and closing times for the Dow



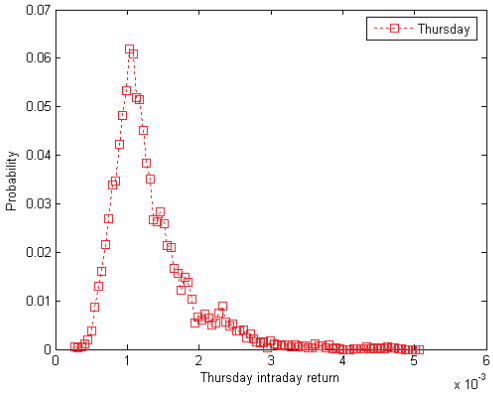
(a) Monday



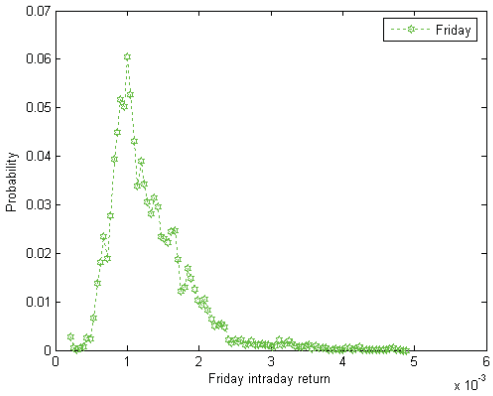
(b) Tuesday



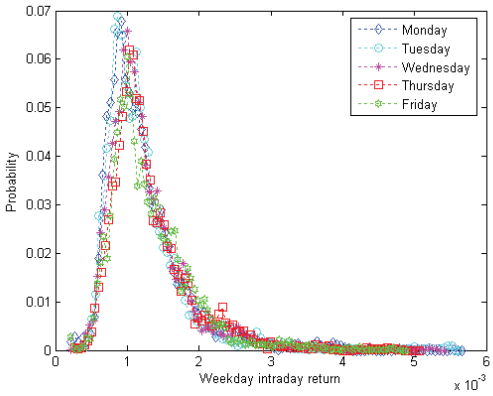
(c) Wednesday



(d) Thursday

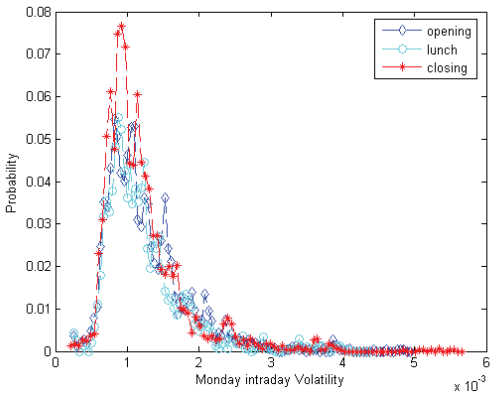


(e) Friday

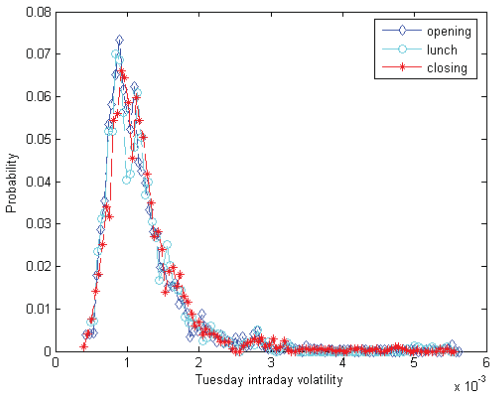


(f) Monday to Friday

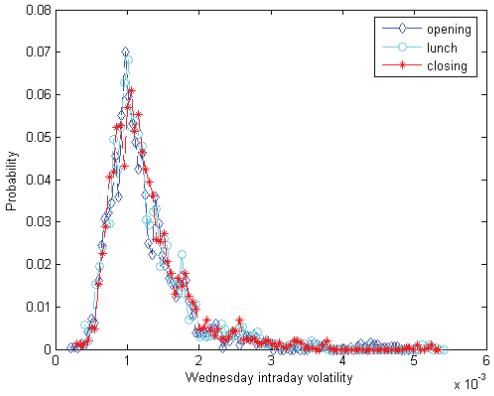
Fig. 8. Probability distributions of intraday volatility for the weekdays (Monday to Friday) for the Dow



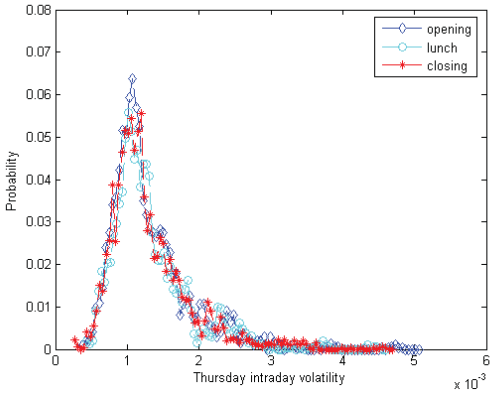
(a) Monday



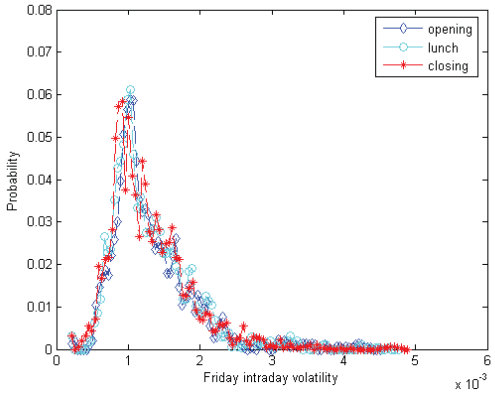
(b) Tuesday



(c) Wednesday



(d) Thursday



(e) Friday

Fig. 9. Probability distributions of intraday volatility from Monday to Friday across opening, lunch and closing times for the Dow

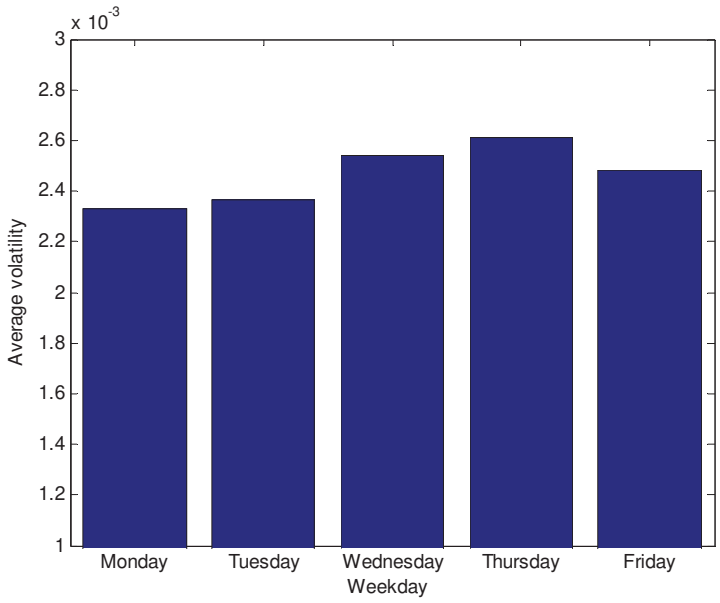


Fig. 10(a). Bar chart of the average volatility from Monday to Friday for the Nasdaq

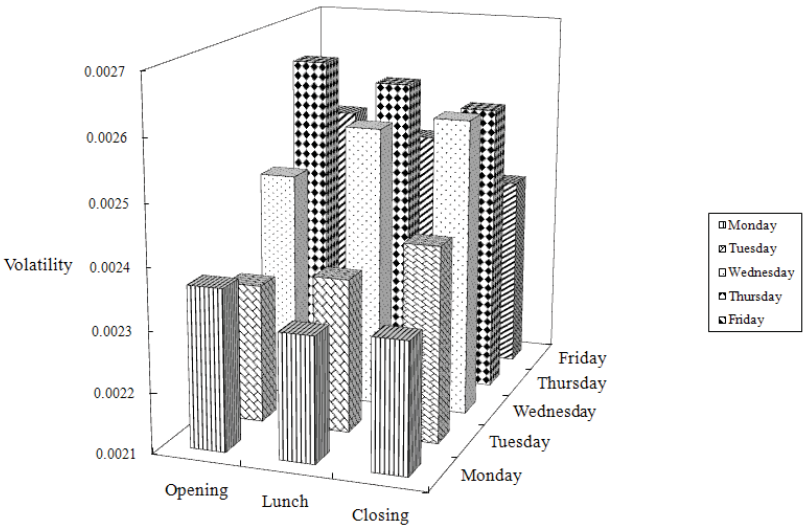
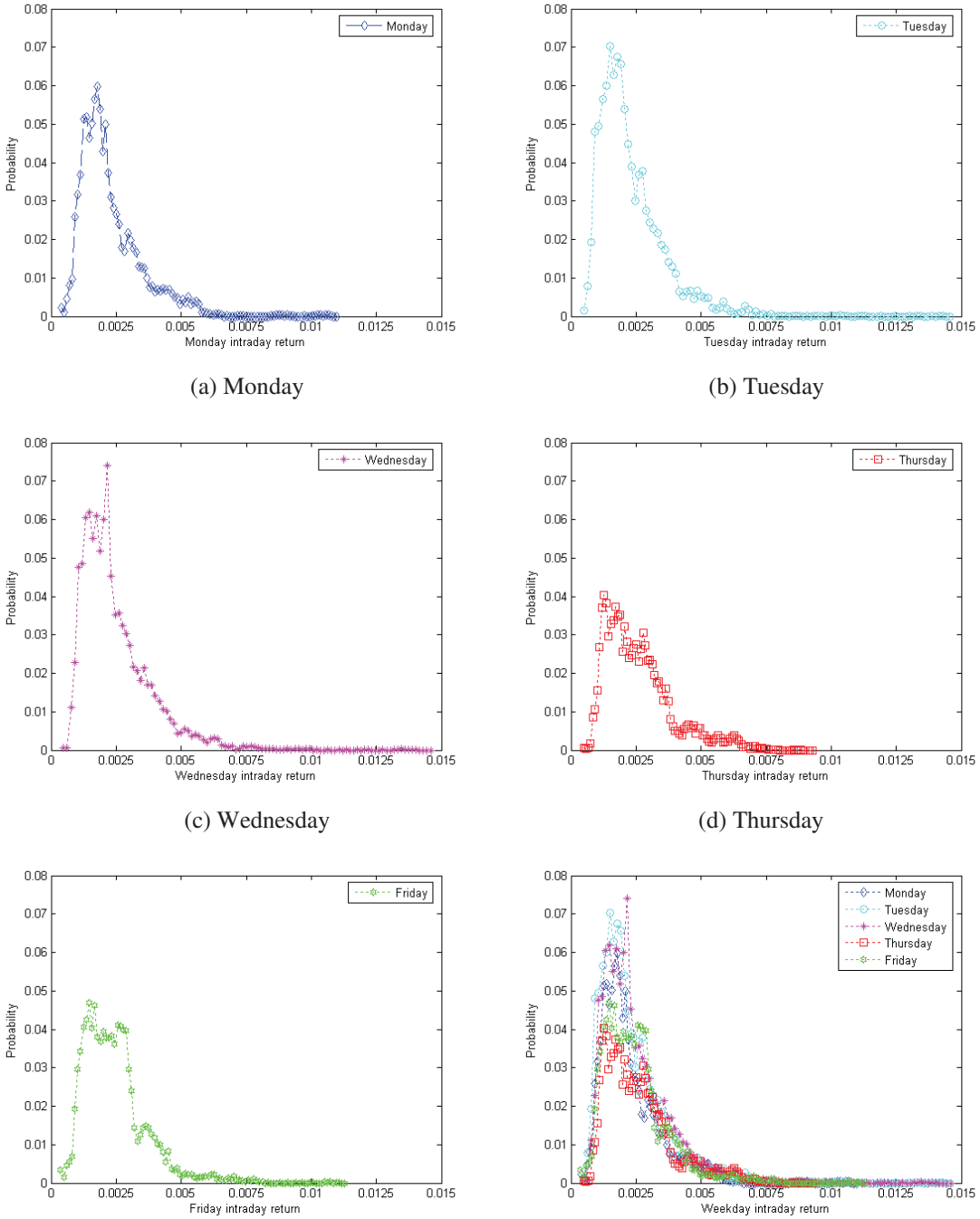


Fig. 10(b). Three-dimensional weekday volatilities from Monday to Friday across opening, lunch and closing times for the Nasdaq

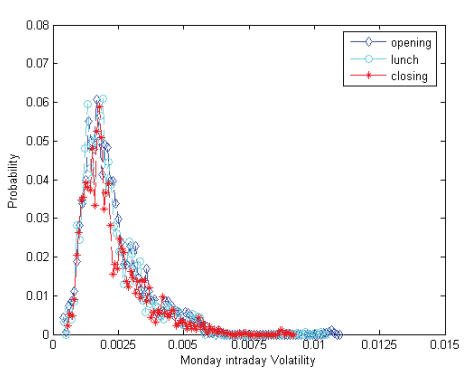


(a) Monday (b) Tuesday

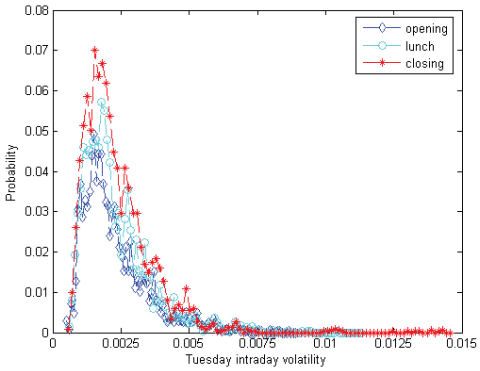
(c) Wednesday (d) Thursday

(e) Friday (f) Monday to Friday

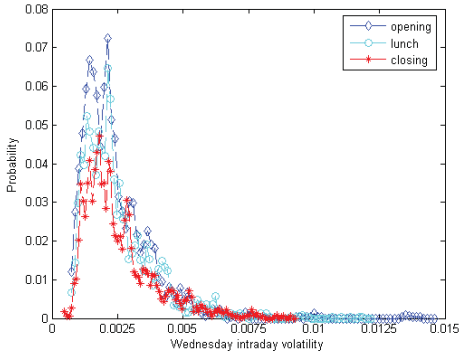
Fig. 11. Probability distributions of intraday volatility for the weekdays (Monday to Friday) for the Nasdaq



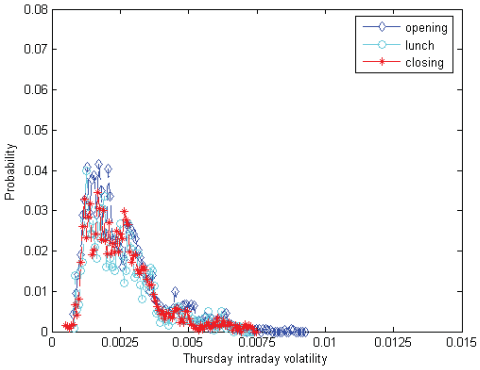
(a) Monday



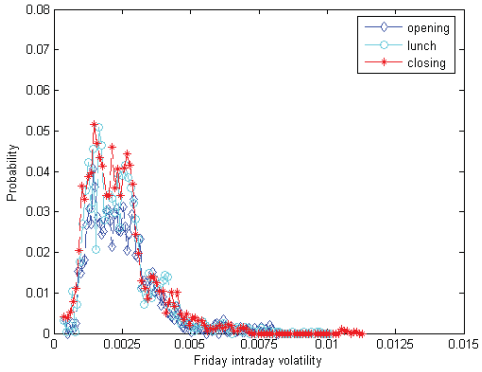
(b) Tuesday



(c) Wednesday



(d) Thursday



(e) Friday

Fig. 12. Probability distributions of intraday volatility from Monday to Friday across opening, lunch and closing times for the Nasdaq