“Regimes in Australian pension fund returns: a hidden semi-Markov approach”

AUTHORS	Robert J. Bianchi Michael E. Drew Adam N. Walk
ARTICLE INFO	Robert J. Bianchi, Michael E. Drew and Adam N. Walk (2012). Regimes in Australian pension fund returns: a hidden semi-Markov approach. *Investment Management and Financial Innovations*, *9*(1)
RELEASED ON	Friday, 30 March 2012
JOURNAL	"Investment Management and Financial Innovations"
FOUNDER	LLC “Consulting Publishing Company “Business Perspectives”

Robert J. Bianchi (Australia), Michael E. Drew (Australia), Adam N. Walk (Australia)

Regimes in Australian pension fund returns: a hidden semi-Markov approach

Abstract

Regimes are of interest to investors as they describe periods of episodic changes in returns and volatility caused by the non-normality and non-linearity characteristics of financial returns. The literature to date has examined regimes in single asset classes with little emphasis on the regime behavior of diversified (i.e. multi-asset investment) portfolios. This study examines whether lowering risk or increasing asset diversification are valid methods for investors to temper the regime behavior of their portfolios. Using a hidden semi-Markov model, the authors analyze the returns of two pension (i.e. superannuation) fund investment portfolios at opposite ends of the risk spectrum, namely a low risk cash-based portfolio and a moderate-to-high risk, but highly diversified, balanced portfolio. The findings show that asset class diversification does not appear to offer any noticeable benefits in relation to managing the regime behavior of investment portfolios. The findings also reveal that risk-reduction towards a cash based investment does not mitigate regimes in diversified portfolios.

Keywords: regimes, pensions, hidden semi-Markov models.
JEL Classification: G11, G23.

Introduction

Since 1991, Australia has legislated a compulsory private retirement savings system (known as 'superannuation') where all workers are required to contribute to a defined contribution pension plan[1]. The current Australian Parliament is considering whether to raise the minimum contribution rate from 9 to 12 per cent of salary, potentially raising the importance of superannuation to Australians even further. As a consequence of this regulated private saving, Investment Company Institute (2011) ranks Australia's pool of funds under management of US$1,456 billion on December 31, 2010 as the fourth largest in the world behind the United States, Luxembourg and France. From this pool of funds, US$974 billion are invested in the long-term superannuation system. Superannuation investors are generally offered a menu of asset allocations within their private retirement account that reflect the wide spectrum within the return-risk relationship in investments. For instance, investors can select from a low risk cash-based portfolio versus a high-risk portfolio that is 100% allocated to stocks. Given this wide range of investment opportunities, it is of interest for investors to understand whether their long-term retirement savings in diversified portfolios are subject to regimes, that is, episodic changes in return and risk.

Previous studies by Bhar and Hamori (2004), Yingjian (2004) and Bulla and Bulla (2006) find regimes in different stock markets indices. Other works by Fabozzi and Francis (1977, 1979) and Chen (1982) inform us that regimes are characterized by periods of different returns and volatility. Hamilton (1989) informs us that non-linearities in financial returns may exist because of discrete shifts in regimes. Whilst the literature provides evidence of regimes in single asset class investments, the reality is that investors hold their long-term retirement savings in multiple asset class portfolios. Furthermore, these diversified portfolios operate with different risk profiles with some funds structured to be low risk while other funds are designed to exhibit moderate or high levels of risk. It is, therefore, important to examine whether regimes in diversified investment portfolios exist and whether these regimes continue to be present as the risk profile of the diversified portfolio changes. To date, there has been little research regarding the presence of regimes in diversified portfolios, or whether regimes can be mitigated by holding low risk or high risk investment portfolios. This gap in the regime literature motivates us to examine these important research considerations. First, does portfolio diversification reduce the presence of regimes? Second, can an investor mitigate the effects of regimes by investing in a low risk diversified portfolio?

This paper employs an emerging quantitative technique known as the hidden semi-Markov model, to analyze the returns from two different types of diversified investment portfolios that are available to Australian superannuation members. The first diversified investment is a low risk 'cash'-based portfolio and the second diversified investment is a moderate-to-high risk 'balanced' portfolio. The 'balanced' portfolio is of particular interest because the design of this fund represents the 'default' investment choice for most Australian superannuation funds, with around 80 per cent of superannuation investors (i.e. members) automatically invested in their fund's 'default'

[1] Whilst defined benefit (DB) plans still exist in Australia, they are a small percentage of total system assets, and new contributions are overwhelmingly directed to defined contribution (DC) plans.

portfolio. This study will demonstrate the presence of regime behavior in both 'cash' and 'balanced' diversified fund returns. This evidence of regimes in these diversified funds demonstrates that neither portfolio diversification nor risk-reduction is of much use in managing the regime behavior of fund returns.

The remainder of the paper is organized as follows. First, we discuss the relevant finance and regime literature. We then present the data employed in this study. The next section introduces the hidden semi Markov model employed to identify regimes in this study. Then we present empirical results and discussion. The final section provides the summary and conclusion.

1. Regimes in financial time series

The finance literature beginning with Merton (1971) has explored portfolio choice in more complex stochastic environments than proposed by Markowitz's (1952) seminal Modern Portfolio Theory, or MPT. Such research has focussed on ways of treating departures from MPT's assumptions whilst still preserving its underlying genius, namely, the benefits of portfolio diversification. Alternative approaches typically fit into one of three classes: firstly, assuming a multivariate normal distribution and accounting for outlying observations as jump processes[1]; secondly, selecting a more appropriate non-normal empirical distribution and explicitly using this distribution in applications (e.g., when constructing portfolios using optimization)[2]; and lastly, assuming the presence of regimes and applying methods robust to regimes. Each of these three methods continue to be developed in the literature and no single approach has emerged as the definitive way of dealing with the complicating features of empirical time series. This paper will focus on the last of these approaches as it leads toward a class of promising quantitative methods, together known as Markov methods, which are capable with dealing with nonlinearities in financial returns.

One of the rich veins of research into time series processes relates to regimes or states[3]. A regime is essentially a sub-sample within a larger data set, with characteristics that set it apart from both the larger data set and other sub-samples from that data set. Much of the research into regimes began in the field of economics where scholars were interested in the cause and timing of abrupt breaks in observed time series. Investigations have examined a variety of phenomena like the nature of the business cycle (Hamilton, 1989), financial crises (Jeanne and Masson, 2000; Cerra and Saxena, 2005; Hamilton, 2005), and the effects of sudden changes in government policy (Hamilton, 1988; Sims and Zha, 2004; Davig, 2004). Facilitating such investigations has been the development of econometric techniques which allow the economist to imply from the available time series, an underlying regime process.

The objective of such techniques is to provide the economist with methods amenable to understanding the complex nature of time series processes as a necessary pre-condition to improved modelling and analysis. Because of the absence of any tested theory about regimes (for example, what causes them or how they are formed), the phenomenon has traditionally been viewed as an empirical problem.

In parallel, finance scholars identified regimes in financial data, which are characterized by differences in returns and risks (Fabozzi and Francis, 1977, 1979; Chen, 1982). For example, many studies characterize a 'normal' regime by a positive mean and moderate volatility, whilst a 'down-market' regime can be defined with a lower (and sometimes negative) mean and noticeably higher volatility. These empirical characteristics of regime behavior are consistent with the positive relationship between return and risk. Intuitively such characterizations also correspond to identifiable market conditions: prior to the Lehman Brothers collapse of 2008 equity market returns were broadly consistent with historical trends (if not a bit higher), whereas the post-Lehman Brothers world has seen negative equity market returns and much higher volatility. Hence, regimes appear to provide a good intuitive explanation of the data, and appear to accommodate the empirical features of time series, like non-normality, volatility clustering and

[1] Modelling asset returns as a jump process in continuous time began with Merton (1976) in which he postulated that the total change in a financial asset price (e.g., a stock price) is made up of two components: (1) "normal vibrations" modelled as a "normal", continuous process using standard geometric Brownian motion; and (2) "abnormal vibrations" which are viewed as a discontinuous "jump" process modelled using the Poisson distribution. What Merton proposed, and the literature has developed (for example, Das and Uppal, 2001), is a method for adjusting classical finance theory to account for the high frequency of outlier price observations in empirical studies, when compared to the frequency expected under an assumption of a log-normal distribution. In addition to jumps in price, studies into the effects of stochastic volatility have been conducted (Liu, 1999; Chacko and Viceira, 2005; Longstaff, 2001). Portfolio choice in the presence of jumps has been investigated in numerous papers (Aase, 1984; Jeanblanc-Picqué and Pontier, 1990; Bardhan and Chao, 1995; Liu, Longstaff and Pan, 2003).

[2] A further alternative is to use another distribution as a more accurate characterization of returns (instead of assuming normality). Mandelbrot (1963, 1967) and Fama (1965) have suggested that the stable Paretian distribution offers a better representation of financial return series. This is because, compared to the Gaussian distribution, stable distributions have fatter tails (i.e. are leptokurtotic) and a higher peak around the center of the distribution, as well as allowing the modelling of various levels of skewness (Mittnik, Paolella and Rachev, 2000). A range of studies have compared portfolios constructed using the assumption of normality with others which assume the stable Paretian or *t*-distributions (Ortobelli et al., 1999; Tokat et al., 2001; Blake et al., 2001; Tokat and Schwartz, 2002; Ortobelli, Huber and Schwartz, 2002).

[3] Henceforth, the terms regimes and states will be used interchangeably.

return persistence. Importantly, regimes have also been proposed as one of the causes of non-linearity in financial time series: Hamilton (1989) notes that "*non-linearities ... arise if the [time series] process is subject to discrete shifts in regime – episodes across which the dynamic behavior of the series is markedly different*" (p. 358).

Non-normality, non-linearity and potential presence of regimes (or states) are the observed characteristics of time series which motivate the use of the hidden semi-Markov model. From the literature it is apparent that this method has the potential to improve the ability of financial economists to model time series in which signs of non-normality and non-linearity are observed. Improved modelling capability should in turn allow for improvements in financial practice whether that be in the areas of portfolio management, risk management or performance evaluation.

2. Hidden semi-Markov models

The first generation of econometric methods developed to deal with regimes were Markov regime-switching models (Goldfeld and Quandt, 1973; Hamilton, 1989); these models have been widely used in economic and finance research for applications such as dating the business cycle (Hamilton, 1989) and analyzing superannuation fund performance (Roca and Wong, 2008). Regime-switching models have been shown to capture complicated forms of heteroscedasticity, leptokurtosis and skewness in the underlying return distribution (Timmermann, 2000) and have been the subject of a growing number of studies.

More recently, the hidden Markov model (HMM) is a computational method, emerging in finance, which offers a flexible approach to modelling financial time series in the presence of regimes. The statistics literature includes a number of important contributions (MacDonald and Zucchini, 1997) on HMMs, and the model is used widely in fields outside finance, for example, speech recognition (Poritz, 1982; Rabiner, 1989) and weather modelling (Hughes and Guttorp, 1994). There are relatively few examples of the application of HMMs to problems in finance (Shi and Weigend, 1997; Bhar and Hamori, 2004; Yingjian, 2004; Bulla, 2006) although the method appears to be gaining attention in academia because it has the ability to model aspects of time series which other well-known econometric techniques (e.g., ARCH) are unable to identify (Manton et al., 1998). Bhar and Hamori (2004) argue that HMMs have been shown to capture the main characteristics of the data, and Bulla (2006) lauds their computational stability. The principal strength of HMMs is however their acknowledged ability to model non-linear processes (Bhar and Hamori, 2004). Rather than modelling a time series as a single distribution of whatever characterization (e.g., Gaussian, *t*, stable Paretian), the HMM allows the researcher to consider the time series as a mixture of J separate distributions, where J represents the number of regimes or states in the model. The power of the approach is that the state sequence between the J states and the parameters of the J distributions, which are 'hidden' from the researcher prior to estimation, are implied from the financial data.

Whilst the HMM is an improvement on earlier efforts of regime modelling, it does exhibit one significant drawback. Rydén et al. (1998) showed that the HMM is unable to reproduce one of stylized attributes of time series identified by Granger and Ding (1995a, 1995b), namely, a very slowly decaying autocorrelation function[1]. Bulla and Bulla (2006) in their study of European sector indexes, using a generalization of the approach of Rydén et al. (1998), demonstrate that the hidden semi-Markov model (HSMM) captures the slow decaying autocorrelation function in financial returns, thus making it, at face value, a superior framework to the HMM.

The HSMM is a computational method which has only very recently been applied to finance applications, having previously been used in fields as diverse as biology, computer science and meteorology. The HSMM is a generalization of the HMM, and allows more flexibility when specifying the sojourn time distribution (Bulla and Bulla, 2006): the distribution which models the duration the process remains in each regime. The only difference between the HMM and the HSMM is that the latter allows a variety of sojourn time distributions. The HMM implicitly assumes a geometric sojourn time distribution, whereas the sojourn time distribution in the HSMM is a question of model selection on the part of the researcher. Intuitively, specifying an accurate sojourn time distribution should allow the model to better reproduce the persistence in regimes which, at least in the abstract, is a similar concept to autocorrelation. Studies have shown that HSMMs have the ability to improve the fit of the model to empirical autocorrelation functions (Bulla and Bulla, 2006).

The HSMM specification employed in this paper follows that proposed by Bulla and Bulla (2006)[2]. It is an extension of the right-censored model proposed by Guédon (2003) to deal with parametric distributions for both the observations and sojourn times. Guédon's model advances the previous work of Ferguson (1980), whose model assumed that the last

[1] The analysis in Granger and Ding (1995a, 1995b) was extended in Granger, Ding and Spear (2000).
[2] The complete specification is available upon request.

observation coincided with the end of a sojourn in the most recent state. Such an assumption is both unlikely to be true and impossible to confirm in practice. This paper uses the expectations maximization (EM) algorithm outlined in Bulla and Bulla (2006) to estimate the parameters of the HSMM, θ. Because the state sequence is unknown, we employ the EM algorithm to iterate until the likelihood is maximized, i.e. it reaches a stationary point (see earlier references, e.g., Baum et al., 1970)[1]. Guédon's (2003) approach is preferred because of its low complexity and its immunity to numerical underflow[2]. We now turn to the data and methodology employed in this study.

3. Data

This study employs the daily returns of two investment options that are offered by a single superannuation fund. The first time series is a low risk cash-type 'defensive' investment portfolio while the second time series is a moderate-to-high risk 'growth' investment portfolio. The data is sourced from the website of QSuper, one of Australia's largest superannuation funds with more than 538,000 investors (known as "members") and approximately A$27,576 million in funds under management as on June 30, 2010. The sample of daily returns is for the period from July 5, 2000 to September 14, 2010 consisting of 2,660 daily observations for each time series.

The returns of the actively-managed cash fund is labelled as "active cash" (AC) throughout. The AC fund invests in cash securities, and aims to deliver the return of the UBS Bank Bill Index (after fees)[3]. The AC fund therefore represents a defensive asset and would be expected to earn a positive mean return with minimal volatility over all timeframes. It will become apparent later in this study that the AC fund has in fact experienced periods of negative returns resulting from exposures to certain "cash-like" or low risk securities, namely, mortgage-backed securities, which were heavily impacted by the credit crisis of 2007-2009.

The growth time series is the returns of the actively-managed balanced fund, and will be labelled as "active growth" (AG) henceforth. As previously mentioned, this "growth" fund is the default for this superannuation fund. The AG fund is a multi-asset class, diversified portfolio with a significant allocation to growth assets (i.e. stocks and alternatives), and targets a return of the Australian Consumer Price Index (CPI) plus four per cent per annum over rolling five year periods (after fees and tax). As at October 2010, the AG fund's asset allocation was: equities (56 per cent), alternative assets[4] (20 per cent), and defensive assets (24 per cent). Such a portfolio would be expected to achieve its target return over the long term, but with significant volatility over shorter time horizons. For example, the AG fund's material exposure to equities resulted in large negative returns during the 2007-2009 period.

Daily arithmetic returns are calculated as follows:

$$R_t = \left(\frac{P_t}{P_{t-1}} - 1 \right) \times 100, \tag{1}$$

where R_t is the arithmetic return at time t expressed as a percentage, and P_t and P_{t-1} are the fund's published unit prices at times t and t-1, respectively.

There are a number of reasons why we employ the returns of the defensive and growth portfolios of this superannuation fund. First, to the authors' knowledge, this is the first study to apply HSMMs to a portfolio of cash-like securities, or to a multi-asset class diversified growth fund. This is one of the contributions of this research. Second, the two portfolio returns allow us to compare the regime behavior of these two actively managed diversified funds. This will allow us to compare whether regime behavior is present in both low risk and high risk diversified portfolios. Using these fund returns will allow us to examine whether portfolio diversification assists in mitigating regimes in financial returns, and whether changes in the risk profile of the fund can eliminate regime behavior in financial returns. In particular, the AG fund is of interest in the Australian superannuation industry because it represents a typical default investment portfolio for the typical superannuation investor. As such, around 80 per cent of Australian workers are invested in a similar fund, so the findings of this study may offer some interesting insights into the design, performance and regime behavior of these default funds.

Table 1. Summary statistics

	AC fund	AG fund
Panel A: Descriptive statistics		
Mean	0.0162	0.0203
Standard deviation	0.0220	0.4852
Skewness	3.05	-1.38
Kurtosis	74.97	31.33

[1] An argument in favor of the use of the EM algorithm is its stability. Using the EM algorithm, Bulla and Bulla (2006) noted a "*stable convergence to the global maximum for all cases investigated except when very poor initial guesses were used*" (p. 2197).

[2] Numerical underflow occurs when the result of an arithmetic operation is so small that it cannot be stored in its intended destination format without suffering a rounding error that is larger than usual (Sun Microsystems, 2008).

[3] The UBS Bank Bill Index is an index of bank-accepted bills of exchange issued by a range of approved Australian domestic banks. The index commenced in 1987 and is the most commonly used benchmark for cash based and short-term money market portfolios in Australia. The index is maintained by the Swiss investment bank UBS.

[4] Alternative assets generally combine both growth and defensive characteristics and include asset classes like real estate, private equity, infrastructure and hedge funds.

Table 1 (cont.). Summary statistics

	AC fund	AG fund
Jarque-Bera test statistic	578,219	89,820
Jarque-Bera p-value	0.000	0.000
Panel B: Autocorrelation of variance		
AC1	0.107**	0.266**
AC2	0.058**	0.096**
AC3	0.039**	0.062**
AC4	0.021**	0.097**
AC5	0.039**	0.155**
AC6	0.039**	0.162**
AC7	0.057**	0.162**
AC8	0.036**	0.066**
AC9	0.009**	0.085**
AC10	0.032**	0.033**
AC11	0.040**	0.064**
AC12	0.087**	0.092**
AC13	0.016**	0.111**
AC14	0.024**	0.087**
AC15	0.027**	0.086**
AC16	0.029**	0.084**
AC17	0.108**	0.060**
AC18	0.168**	0.073**
AC19	0.024**	0.051**
AC20	0.091**	0.051**
AC21	0.042**	0.079**
AC22	0.001**	0.042**

Notes: This table presents the descriptive statistics of the Active Cash (AC) fund and the Active Growth (AG) fund. Panel A reports the descriptive statistics and the Jarque-Bera test statistic and p-values. Panel B reports the autocorrelation of variance of the fund returns from a lag of 1 to 22 business days which represents up to one month. * and ** denote statistical significance at the 5% and 1% levels, respectively.

3.1. Summary statistics. The summary statistics for the daily returns of the two investment portfolios are presented in Table 1. Panel A reports the descriptive statistics and the Jarque-Bera test statistic for normality (Jarque and Bera, 1980; 1987). As expected, Panel A confirms that the return and risk of the AC fund is lower in comparison to the AG fund. Panel A also shows very large Jarque-Bera statistics and *p*-values which demonstrates that the daily returns of the two funds exhibit strong non-normality. Whilst both time series are clearly non-normal, there are a number of contrasting characteristics. The defensive fund, AC, has a mean return closer to that of the growth fund, AG, than one might expect which is testament to the nature of growth asset markets over the sample period studied. Equity markets, the main contributor to total risk in the AG fund, have experienced two separate periods of negative returns. The first negative period occurred in 2002, and the second period was from mid-2007 until early 2009, with the latter period being by far the worst period for equity returns in the data sample.

Panel B of Table 1 reports the Ljung-Box Q-statistic (Ljung and Box, 1978) to measure the statistical significance of the autocorrelation of squared returns. Autocorrelations up to 22 lagged returns (denoted by 'AC22') have been calculated to capture up to one month of daily retuns[1]. Panel B confirms the presence of statistically significant autocorrelation in squared returns. This finding suggests heteroscedasticity (non-constant variance), which in turn is indicative of non-linearity in the daily returns of the AC and AG funds.

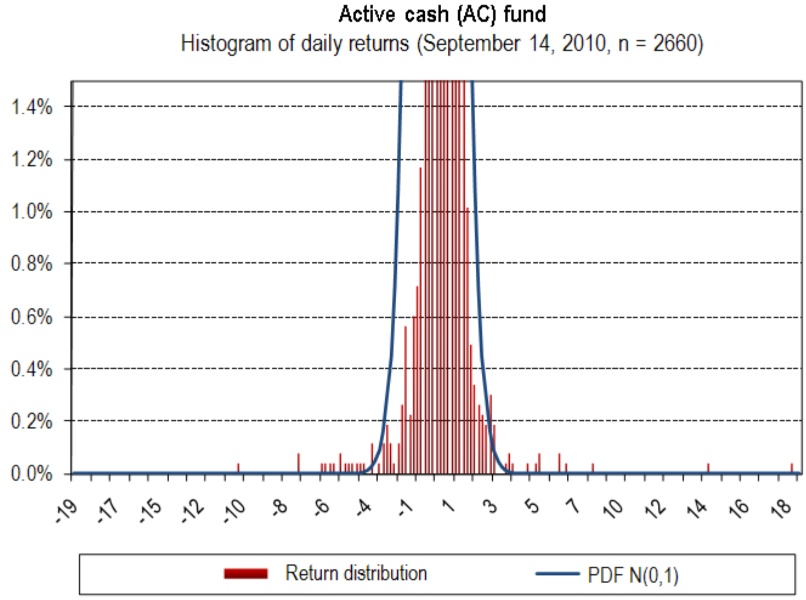


Fig. 1. Empirical distribution magnified (AC fund)

[1] The number of trading days per month typically numbers 22.

The empirical distribution of the AC fund for the period in question is shown in Figure 1, against a standard normal curve, which is magnified in order to highlight the observations in the tails. This provides supporting evidence for the work of Mandelbrot (1963, 1967) and Fama (1963, 1965) who argued against the normality of financial returns. Figure 1 highlights the preponderance of large positive z-scores ($\geq 4\sigma$) and the particularly long right tail with a maximum z-score of more than 18 sigma (versus a minimum z-score of around -10σ). Returns around the center of the distribution also occur more frequently than would occur in the presence of normality. These characteristics together explain the AC fund's positive skew (3.05) and its high calculated kurtosis (74.97). The question about why such a defensive asset class (i.e. cash and short-term money securities) would experience such tail events is beyond the scope of this study, but it is thought to be at least partially due to the presence of mortgage-backed securities within this portfolio at the time of the credit crisis from 2007-2009[1].

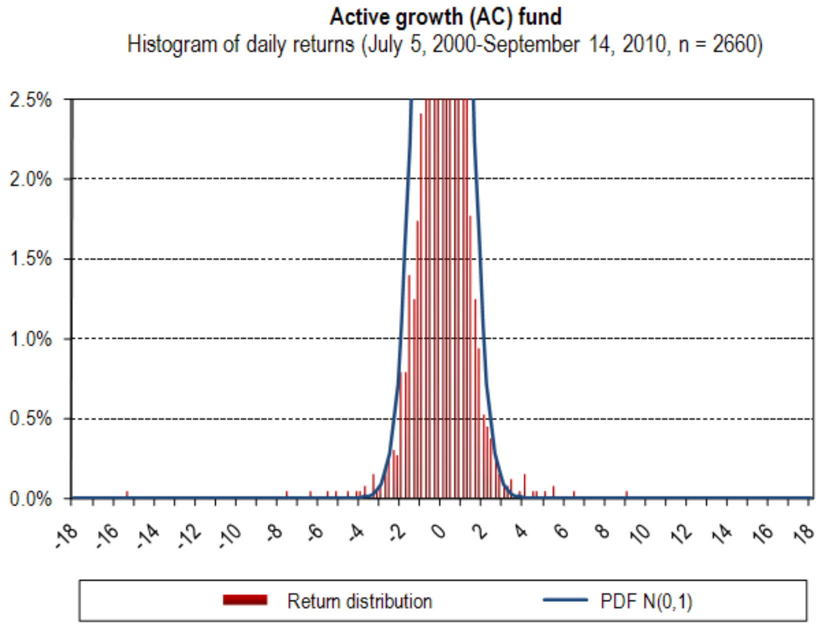


Fig. 2. Empirical distribution magnified (AG fund)

Figure 2 illustrates the non-normality of the AG fund although it is less so than the defensive AC fund. The AG fund is negatively skewed (-1.38) which is a statistical feature consistent with other growth assets with relatively high volatility and heavy left tails. There are also numerous returns which would not be expected to occur in any sample of 2,660 observations in the presence of normality, e.g., an observation of more than -15 sigma. This provides strong statistical support to the critiques of MPT relating to the normality assumption.

Whilst it has been demonstrated that the daily returns of these two funds are non-normal, the question arises about whether there is any identifiable temporal dimension to the data. Put another way, is there any evidence in the data which supports earlier contentions regarding the non-linearity of the time series, or the presence of regimes in the time series? Recall that regimes are a feature of time series, identified in the literature, which suggest that data can be partitioned into identifiable sub-sets which may be distributionally quite different from the larger time series, or other sub-sets.

A simple or naïve way to identify potential regimes is via a chart of daily returns, shown in Figure 3 for the AC fund and Figure 4 for the AG fund. Please note the seven bands on each of these figures which run parallel to the x-axis, namely, the central band which is the mean return, and the three bands on either side of the mean return which represent ±1, ±2 and ±3 standard deviations from the mean. Observations which fall outside the outermost band are extremely unlikely to occur in the presence of normality.

Figures 3 and 4 assist greatly in building intuition about the time series and in developing possible regime scenarios as a precursor to employ the HSMM. Figure 3, for example, provides strong visual evidence that at least two regimes exist in the data. The first regime is characterized by a moderate mean and muted volatility (i.e., from the commencement of the sample period to mid-2007, and since mid-2009) and a second regime which has a lower mean and greater volatility (i.e., from mid-

[1] As mentioned earlier, the AC fund contained mortgage-backed securities during the global financial crisis. This was confirmed via conversations with the investment staff of the superannuation fund in question.

2007 to mid-2009). A similar set of scenarios could be gleaned from Figure 4 although the evidence isn't compelling. One can only really conclude from Figure 4 that there are at least two volatility regimes, namely, one high (e.g., 2008) and one of a smaller magnitude (e.g., 2003-2006), with a characterization of mean returns which is less obvious from a visual inspection of the figure[1]. The autocorrelations reported in Table 1, Panel B provides statistical support for the presence of heteroscedasticity, thereby recognising the possibility of multiple volatility regimes presence in the data.

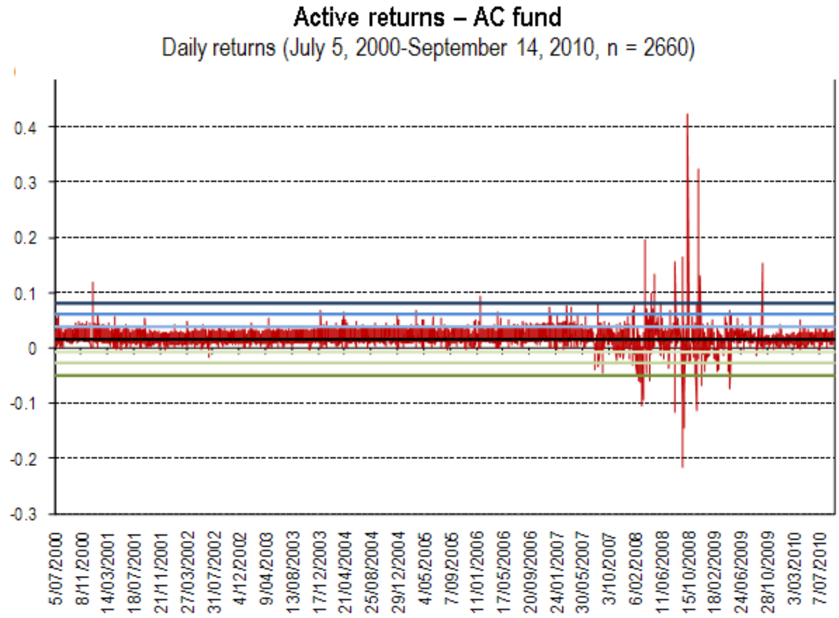


Fig. 3. Daily returns (AC fund)

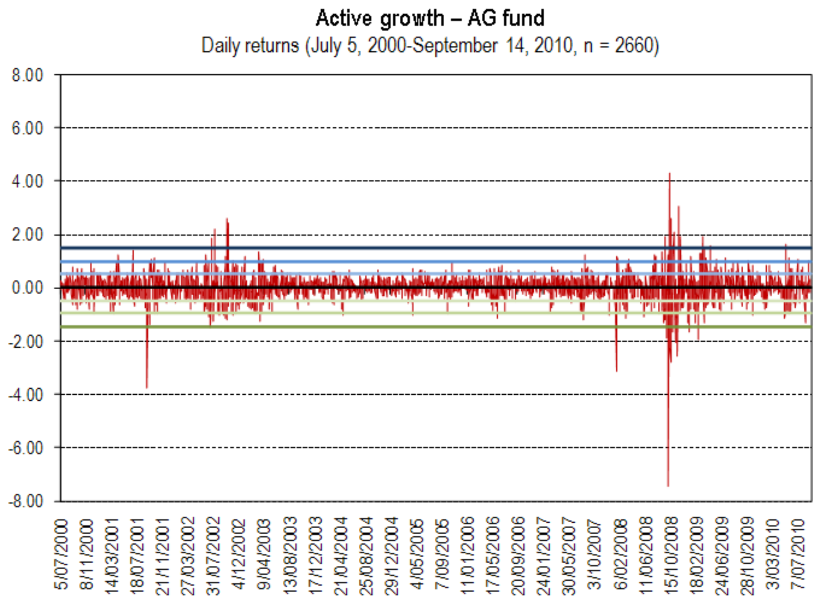


Fig. 4. Daily returns (AG fund)

In summary, through basic visual inspection and analysis of the raw data, it is clear that the two funds in this study exhibit the empirical features of financial time series, namely, non-normality and non-linearity, which are the motivating rationales for the use of regimes in this paper. The statistical properties discussed herein emphasize the need for robust quantitative techniques to deal with these departures from the normality assumption, which underlies much of classical finance theory, in particular MPT. This is the principal reason why Markov methods are considered in this paper.

[1] It is worth noting that, in the literature, intuition or so-called "data-driven arguments" (Bulla, Bulla and Nenadić, 2008) about the number of regimes is offered as a means of model selection alongside statistics such as the Akaike Information Criterion and the Bayesian Information Criterion.

4. Methodology

The HSMM regime methodology employed in this study will be operationalized as follows:

1. Estimate the optimal number of regimes for each time series using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The number of regimes will most likely be limited to two or three, consistent with much of the literature (Rydén et al., 1998; Bulla and Bulla, 2006; Hamilton, 2008; Bulla et al., 2008).
2. Using the preferred number of regimes from step 1, estimate hidden semi-Markov models for both AC and AG funds. Modelling is conducted using *R*, a software environment for statistical computing and graphics (R Development Core Team, 2008). In particular, the R package HSMM is used.
3. For each fitted model, report the estimated model parameters. This will allow a comparison between the model estimate of state parameters and the parameters of the matching empirical distribution.
4. Produce figures which plot the decoded states estimated using the Viterbi algorithm to provide a visual representation of the estimated state process and to test the model output against expectations.

At the end of this analysis, the usefulness of the HSMM in modelling financial returns will be apparent, and we will be able to draw conclusions about regimes in diversified fund returns of varying risk levels. Prior to estimating the HSMMs, it is necessary to outline the model by selecting the number of states, the observation distribution, the initial parameter values and the sojourn time, or run length distribution.

Consistent with much of the regime literature, both two- and three-state models were estimated for the two time series as a prelude to model selection. For each fund, the estimated models were compared using their calculated Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) as suggested in Bulla et al. (2008). Comparisons suggest that the three-state model was superior over two-state models for the daily returns for both the AC (defensive) fund and the AG (growth) fund. Rydén et al. (1998) investigated both two- and three-state models and argued that the results of three-state models seem heavily dependent on outlying observations. The work of Bulla and Bulla (2006) confirms their point. In addressing this point, we note that both of these studies considered pre-global financial crisis data in concluding that three-state models were dependent on outlying observations. Since the onset of the global financial crisis we have many more outlying observations and it would appear appropriate that this data influence model selection and estimation. Indeed, an intuitive argument could be made that the extreme nature of returns during the global financial crisis could have added a new highly volatile regime to the existing dataset. Because of differences in data, we will proceed with considering three-state HSMMs for both time series.

Initial probabilities were set at one-third for each of the three regimes in each time series model. The conditional distribution of the observations is assumed to be normal with only the initial values of the mean and standard deviation requiring estimation. In this study, only a mixture of normal variables will be considered as the primary objective of this paper is the analysis of superannuation fund returns, not perfecting the model specification.

Often it is difficult to resolve initial parameter values because the true parameter values are unknown, and "good" estimates are not intuitively obvious. According to Bulla and Berzel (2008) and Bulla, Bulla and Nenadić (2008), the precision of initial values is not an important element of the model because the stability of the EM algorithm converges to a global maximum. In this study, initial estimates of the observation distribution parameters were set based on a partition of the time series. The analysis looked at two periods in each time series, namely, one from the beginning of the data sample to July 10, 2007, and the second time period from July 11, 2007 to the end of the data sample. This analysis sought to compare the characteristics of the data before the emergence of the global credit crisis (as it is now known) to the period following its outbreak. Therefore, July 10, 2007 may be characterised as an estimate of the beginning of the credit crisis. The initial parameter values for the three regimes were estimated as follows. The mean and standard deviation of returns from July 5, 2000 to July 10, 2007 (regime 2); the mean of *negative* returns and the standard deviation for the period from July 11, 2007 to September 14, 2010 (regime 1); and the mean of *positive* returns and the standard deviation for the period from July 11, 2007 to September 14, 2010 (regime 3).

The initial transition probability matrix (TPM) is set as follows for both fund returns:

$$\Pi = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}. \quad (2)$$

The diagonal of the matrix which represents the same-state probabilities in a three-state model are set as zeros. This is because the same-state process in a HSMM is defined not by the TPM but by the sojourn time distribution.

Bulla et al. (2008) state that "*the selection of the most appropriate run length distribution is a complicated problem*" (p. 5). Non-parametric specifications of the sort considered by Sansom and Thomson (2001) often require a very high number of observations. Sansom and Thomson (2001), for example, worked with datasets where $n \geq 20,000$. A negative binomial sojourn time distribution of the following form was considered by Bulla and Bulla (2006):

$$d_j(u) = \binom{u-2+r_j}{u-1} p_j^{r_j}(1-p_j)^{u-1},\ u = 1,2,\ldots \quad (3)$$

where $r_j = 1, j \in 0,\ldots,J-1$ in this model reduces to a HMM. Such a specification requires additional computation with no assurance that the run length distribution will be any more appropriately specified. So, in the interests of simplicity, the HSMMs considered in this study will assume a logarithmic sojourn time distribution with identical initial parameters estimates. The selection and specification of optimal HSMMs, especially in relation to the sojourn time distribution is certainly an area requiring further research.

5. Empirical results

This section presents the empirical findings for three-state HSMMs on the daily returns of the two funds. The conditional distributions in these HSMMs are assumed to be normal, so estimates of means and standard deviations are shown. The mean and standard deviation for each empirical distribution from Table 1 are again repeated to allow for easy comparison.

5.1. AC fund. The HSMM produced the following parameter estimates and TPM.

Table 2. HSMM parameter estimates (AC fund)

Three-state HSMM				
	Empirical	State 1	State 2	State 3
n	2660	52	2403	205
Mean	0.0162	0.0162	0.0161	0.0207
St. dev.	0.0220	0.0273	0.0114	0.1182

Notes: The column headed "Empirical" gives the sample statistics for the time series. The columns headed "State 1" through "State 3" give the estimated number of observations in each state ("n"), as well as the model parameter estimates ("Mean" and "St. dev.") for each state.

Table 3. HSMM estimated transition probabilities (AC fund)

	To:		
From:	State 1	State 2	State 3
State 1	0	0.8209	0.1791
State 2	0.9709	0	0.0291
State 3	0.5163	0.4837	0

Notes: The table shows the estimated transition probabilities from one state to a different state. Same-state behavior is indicated by the sojourn time distribution parameter shown in Table 4. For the first row denoted "State 1" the estimates should be interpreted thus: when the state process switches from State 1 there is a 82.09 per cent probability it will switch to State 2 and a 17.91 per cent probability that it will switch to State 3.

From Table 2, it is clear that AC fund's natural, or normal, state is State 2 (2403 of 2660 observations), with the lowest mean return and volatility (although the means of regimes 1 and 2 are nearly identical to each other). From Table 3, we learn that if the state process was to transit from State 2 to another state it is most likely to transit to State 1 (p_{21} = 0.9709) which has around the same mean return (0.0162 versus 0.0161) and more than twice the volatility (0.0273 versus 0.0114). Unlikely that the state process transitions from State 2 to State 3 (a 2.09 per cent probability), it enters a higher return (0.0207) state with around ten times the volatility (0.1182 versus 0.0114).

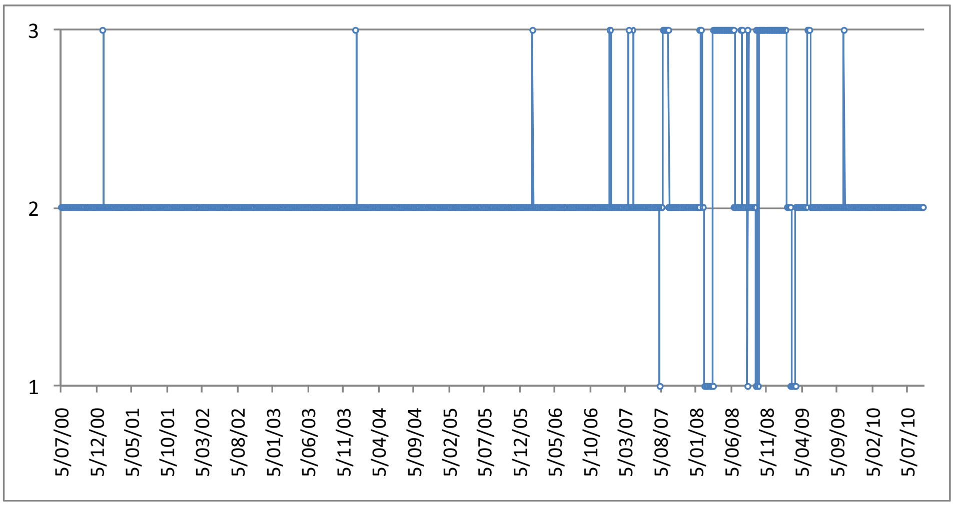
These model results appear to fit finance theory in two respects. Firstly, the results appear to be consistent with classical finance theory, in that return is positively related to risk. For example, regime 3 has the highest return as well as the highest volatility. Secondly, there is evidence of volatility regimes consistent with the work of Manton et al. (1998) who found regimes with similar means but noticeably different volatilities. The reader will note that each regime has approximately the same mean return but with noticeably different volatilities.

Table 4. HSMM estimated sojourn time distribution parameter – logarithmic distribution

	Three-state HSMM		
	State 1	State 2	State 3
p	0.9257	0.9939	0.8617

Notes: The behavior of the HSMM *within each regime* is defined by the estimated parameters for each regime's sojourn time distribution. The estimated parameters in this table should be interpreted as follows: the higher the value, the more persistent the regime.

Consistent with other studies (e.g., Roca and Wong, 2008), we find that the normal state is strongly persistent. State 1 is the next most persistent state, as well as being the most likely state to which the state process switches when it leaves State 2 or State 3. State 3 is the least persistent state.



Notes: The most probable state sequence is determined by the Viterbi algorithm. The y-axis labels correspond to the states in Tables 2 to 4 and the x-axis represents time.

Fig. 5. Three-state HSMM (Viterbi algorithm) – AC fund

Figure 5 shows the estimated state sequence. The most obvious finding from the estimated state sequence in Figure 5 is the noticeable change in the returns of the AC fund from around August 1, 2007[1]. Until this date, the returns of the AC fund remained consistently in regime 2 (the regime with the lowest return and volatility), with only a small number of short switches to regimes 1 and 3. From August 1, 2007, the behavior of the AC fund returns changed dramatically and the state sequence visited regimes 1 and 3 for a significant proportion of days until May 12, 2009 when it returned to the more normal regime 2 sequence. The increase in the sojourn times in regimes 1 and 3 is also a notable feature of this sample period. Out of the 465 observations between August 1, 2007 and May 12, 2009, the returns of the AC fund in regime 2 decreased markedly to 219 daily observations representing only 47 per cent of the total (versus the full sample probability of around 90 per cent). The cause of this change in the data is impossible to confirm without further in-depth analysis but it is thought to include the change in market conditions over this period[2].

These findings demonstrate that regimes can still be observed in the daily returns of a low-risk short-term money market investment portfolio such as the AC fund. While the exact causes of regime changes are beyond the scope of this paper, the HSMM framework provides important information to the investor and offer significant potential from the perspective of the industry practitioner.

5.2. AG fund. The estimated HSMM on the AG fund produced the parameter estimates presented in Table 5, and the estimated TPM reported in Table 6.

Table 5. HSMM parameter estimates (AC fund)

	Three-state HSMM			
	Empirical	State 1	State 2	State 3
N	2660	184	2321	155
Mean	0.0203	-0.2192	0.0637	-0.0236
St. dev.	0.4852	1.6620	0.2501	0.5297

Notes: The column headed “Empirical” gives the sample statistics for the time series. The columns headed “State 1” through “State 3” give the estimated number of observations in each state (“n”), as well as the model parameter estimates (“Mean” and “St. dev.”) for each state.

Table 6. HSMM estimated transition probabilities (AG fund)

	To:		
From:	State 1	State 2	State 3
State 1	0	0.0017	0.9983
State 2	0.0000	0	1.0000
State 3	0.0952	0.9048	0

Notes: The table shows the estimated transition probabilities from one state to a different state. Same-state behavior is indicated by the sojourn time distribution parameter shown in Table 7. For the first row denoted “State 1” the estimates should be interpreted thus: when the state process switches from State 1 there is a 0.17 per cent probability it will switch to State 2 and a 99.83 per cent probability that it will switch to State 3.

[1] Note that this timeframe is consistent with the partition of the time series which was used to estimate the initial parameter values. The partition saw July 10, 2007 as the point at which the returns were expected to change. It is interesting that many financial journalists date the very beginnings of the credit crisis to a liquidity crisis which began on August 9, 2007 (Bajaj, 2007; Elliott, 2008). On this day, the S&P 500 dropped nearly three per cent on news that a French bank had suspended three of its funds because of concerns surrounding troubles in the US market for home loans. The Federal Reserve and the European Central Bank injected liquidity into the markets as a response to tightening credit conditions.

[2] Interestingly this period coincides with the period immediately prior to the failure of a number financial firms in the United States. On September 14, 2008 (US-time), Lehman Brothers declared that it would file for bankruptcy, and on September 16, 2008 the insurer AIG suffered the liquidity crisis which prompted its effective nationalization by the U.S. Federal Reserve. But, perhaps most tellingly, on September 16, 2008 the Reserve Primary Fund, a money market mutual fund very similar in nature to the AC fund in this study, lowered its share price below $1 for the first time in fourteen years due to its exposure to Lehman Brothers debt securities.

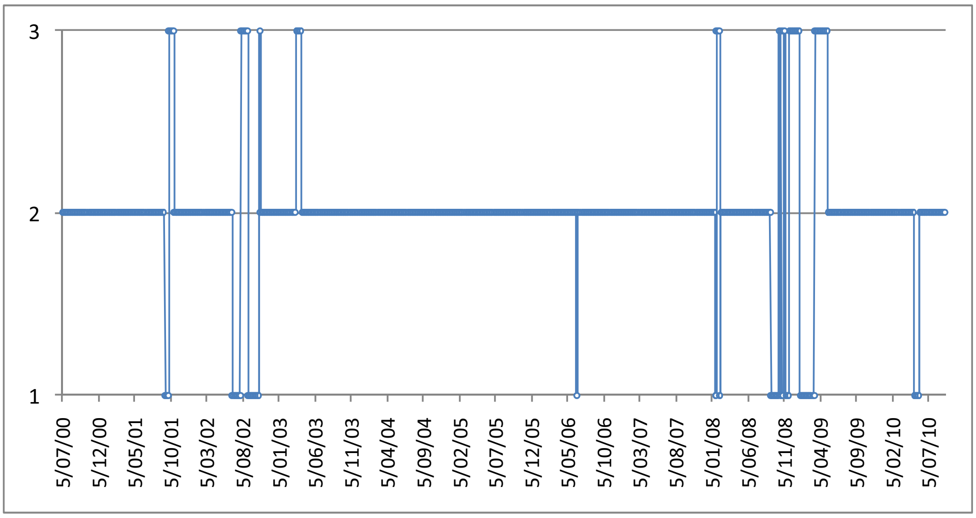
The estimated parameters for each regime's sojourn time distribution are presented in Table 7.

Table 7. HSMM estimated sojourn time distribution parameter (logarithmic distribution)

	Three-state HSMM		
	State 1	State 2	State 3
p	0.9741	0.9893	0.9806

The behavior of the HSMM within each regime is defined by the estimated parameters for each regime's sojourn time distribution. The estimated parameters in this table should be interpreted as follows: the higher the value, the more persistent the regime.

The most probable state sequence, estimated with the Viterbi algorithm, is shown in Figure 6.



Notes: The most probable state sequence is determined by the Viterbi algorithm. The y-axis labels correspond to the states in Tables 5 to 7 and the x-axis represents time.

Fig. 6. Three-state HSMM (Viterbi algorithm) – AG fund

Table 5 shows that the AG fund exhibits the highest mean return and the lowest volatility in regime 2 and is again the most commonly observed regime with 2,321 out of 2,660 observations. Regimes 1 and 3 are both more volatile than regime 2, each with negative returns. Consistent with the previous findings in Ang and Bekaert (2002), the HSMM framework shows evidence of regimes with high volatility and lower means. Table 5 also suggests that regime 1 has the highest volatility (around six times that of regime 2) and the lowest return (four times lower than regime 2).

Table 5 reports that all regimes in the AG fund are differentiated by both mean returns and volatility. The AG fund illustrated in Figure 6 suggests that the HSMM produces a relatively stable state sequence. As discussed earlier, equity market risk is the predominant source of risk in the AG fund. If the HSMM correctly identifies regime changes then known equity market events should be identified from Figure 6. The estimated HSMM appears to capture equity market dynamics. Switches from regime 2 to regimes 1 and 3 are visible around September 11, 2001 as equity markets responded to the World Trade Center attacks, as are more protracted sojourns in regimes 1 and 3 with the 2001 U.S. economic recession and its subsequent slow recovery in 2002. More recently, the global credit crisis is visible from September 8, 2008, where the AG fund leaves regime 2 and doesn't return to it until May 7, 2009, which equates to a run of 173 daily observations[1]. It is important to note that the AG fund model captures the global credit crisis at a later date than the AC fund model (September, 8, 2008 versus early August 2007). This difference in dates is because cash and credit markets were first affected by the emerging liquidity crisis in the middle of 2007, which subsequently developed into a broader economic crisis affecting company prospects and therefore stock markets. Both models suggest the crisis continued for a significant period before "normality" (approximated by regime 2) was restored.

The most important finding from Tables 5 to 7 is the empirical evidence that the AG fund exhibits regimes despite the fact that it is a diversified growth fund. This study demonstrates that the HSMM analysis reports regimes for both diversified funds despite their differing risk profiles. These research findings suggest that portfolio diversification is not

[1] On September 7, 2008, the Federal National Mortgage Association ('Fannie Mae') and the Federal Home Loan Mortgage Corporation ('Freddie Mac'), two large listed U.S. government-sponsored mortgage providers, were placed in 'conservatorship' by the Federal Housing Finance Authority on concerns that neither was financially sound enough to survive the prevailing market conditions. This may have prompted the change in the returns.

effective in removing or eliminating regime changes in diversified fund returns, regardless of whether the funds hold low-risk or growth dominated assets.

Overall, the findings of the HSMM analysis demonstrates that this regime switching framework provides academia and practitioners with a potentially valuable tool for developing a more sophisticated understanding of financial time series than is possible with simple mean-variance analysis, the hallmark of MPT. The HSMM analysis of the AC and AG funds demonstrates that the regime switching in these two funds occurred at different periods in time. The HSMM analysis shows that the low-risk AC fund identified changes in regime behavior as early as August 2007 when the credit crisis began to emerge in global debt markets. Conversely, the changes in regimes for the higher-risk AG fund (with moderate-to-high allocations in stocks) did not exhibit a change in regime until September 2008 which occurred a week before the collapse of Lehman Brothers. The HSMM framework provides investors with knowledge when there are regime changes in fund returns in both low-risk debt portfolios and high-risk equity concentrated portfolios.

An important finding from the HSMM analysis of the AG fund is the presence of three regimes in these multi-asset portfolios. The HSMM framework provides evidence that diversified investment portfolios (such as the AG fund) do not eliminate the effect of regime changes in fund returns[1]. Put simply, investors who wish to avoid regime changes in their returns will not be able to mitigate this risk with portfolio diversification.

Conclusion

This study employed the hidden semi-Markov model (HSMM) on the daily returns of two different types of actively managed diversified pension funds. Previous studies relating to HSMMs (and the less general hidden Markov model) from Bhar and Hamori (2004), Bulla and Bulla (2006) and Yingjian (2004) have considered various equity indexes. The HSMM literature has not considered the analysis of diversified fund returns, defensive asset returns or actively managed fund returns, or any combination of these. The research in this study contributes to the literature because it considers financial returns which combine these attributes. The AC fund is an actively-managed defensive fund and the AG fund is an actively-managed, and highly-diversified, growth fund. Given this unique sample of these diversified pension portfolio returns, this study provided the following contributions to the literature.

First, the findings demonstrated that portfolio diversification does not mitigate regimes in daily financial returns. This study provided evidence that investing in a diversified growth portfolio or in a lower risk cash-based portfolio does not necessarily protect the investor from regimes in financial returns. Regimes can also be observed in defensive funds with significant exposures to relatively safe assets like cash and short-term money market securities. The results provided evidence that investing in a diversified portfolio (regardless of its risk profile) does not necessarily protect the investor from regimes in financial market returns. Most importantly, we demonstrated that regimes can also be observed in defensive funds with significant exposures to relatively safe assets like cash. This finding has important implications for risk-averse investors.

Second, this study provided further evidence of volatility regimes in the data which is consistent with previous literature (e.g., Manton et al., 1998). This is especially the case for the lower risk AC fund comprised of cash-like and short-term debt securities. This is significant as it suggests that investors can experience sufficient differences in the volatility of their returns even though their retirement savings are exposed to low-risk assets.

Third, this paper reported new evidence about the sensitivity of HSMMs to changes in volatility (as in Manton et al., 1998). This study demonstrated that HSMMs can be employed to identify changes in daily financial returns, including potentially dating market events. This would be especially useful for practitioners whose work requires quantitative performance monitoring. For example, fund-of-hedge fund managers could use these methods to monitor hedge funds whose investment processes are unable to be properly understood for reasons of complexity or lack of transparency/disclosure, or where the monitoring of funds is restricted to data analysis.

HSMM analysis provides a richer understanding of financial time series than is possible when operating narrowly in the mean-variance paradigm proposed by Markowitz (1952). The additional insight of HSMM models allows practitioners to make more informed financial decisions. Avenues for future research include the examination of the drivers of regime behavior in these diversified fund returns. Optimal portfolio choice considered from within the HSMM framework is also a natural extension of this empirical research. The investment professional would be particularly interested to know if any sort of active management predisposes a fund to regime changes particularly where the switches are to lower return or more volatile regimes. This is an example of the performance evaluation potential of HSMMs.

[1] Recall the asset allocation of the AG fund is comprised of equities (56 per cent), alternative assets (20 per cent) and defensive assets (24 per cent).

Portfolios containing non-linear payoff assets (like options) would also be interesting to analyze within a HSMM context. From a portfolio construction perspective, it would be interesting to investigate the drivers of regime behavior in diversified portfolios of asset classes, as well as examining whether portfolio choice can be improved using a HSMM framework. The application of the HSMM framework is a relatively new phenomenon in finance and we expect to see an emerging body of new empirical research in the future.

References

1. Aase, K.K. (1984). Optimum portfolio diversification in a general continuous-time model, *Stochastic Processes and Their Applications*, 18, pp. 81-98.
2. Ang, A. and Bekaert, G. (2002). International asset allocation with regime shifts, *Review of Financial Studies*, 15, pp. 1137-1187.
3. Bajaj, V. (2007). Dow Falls 387 Points on New Loan Fears, available at: http://www.nytimes.com/2007/08/09/business/09cnd-stox.html?_r=1.
4. Bardhan, I. and Chao, X. (1995). Martingale analysis for assets with discontinuous returns, *Mathematics of Operations Research*, 20, pp. 243-256.
5. Baum, L.E., Petrie, T., Soules, G. and Weiss, N. (1970). A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains, *The Annals of Mathematical Statistics*, 41, pp. 164-171.
6. Bhar, R. and Hamori, S. (2004). Hidden Markov Models: Applications to Financial Economics, *Advanced Studies in Theoretical and Applied Econometrics*, Vol. 40, Kluwer Academic Publishers, Dordrecht.
7. Blake, D., Cairns, A.J.G. and Dowd, K. (2001). Pensionmetrics: stochastic pension plan design and value-at-risk during the accumulation phase, *Insurance: Mathematics and Economics*, 29, pp. 187-215.
8. Bulla, J. (2006). Application of Hidden Markov Models and Hidden Semi-Markov Models to Financial Time Series, Doctoral Dissertation, Georg-August-Universität, Göttingen.
9. Bulla, J. and Berzel, A. (2008). Computational issues in parameter estimation for stationary hidden Markov models, *Computational Statistics*, 23 (1), pp. 1-18.
10. Bulla, J. and Bulla, I. (2006). Stylized facts of financial time series and hidden semi-Markov models, *Computational Statistics and Data Analysis*, 51, pp. 2192-2209.
11. Bulla, J., Bulla, I. and Nenadić, O. (2008). HSMM – an R package for analysing hidden semi-Markov models, *Computational Statistics and Data Analysis*.
12. Cerra, V. and Saxena, S.C. (2005). Did Output Recover from the Asian Crisis? *IMF Staff Papers*, 52, pp. 1-23.
13. Chacko, G. and Viceira, L.M. (2005). Dynamic consumption and portfolio choice with stochastic volatility in incomplete markets, *Review of Financial Studies*, 18 (4), pp. 1369-1402.
14. Chen, S. (1982). An Examination of Risk Return Relationship in Up and Down Markets Using Time Varying Betas, *Journal of Financial and Quantitative Analysis*, 17 (2), pp. 265-285.
15. Das, S.R. and Uppal, R. (2001). Systemic Risk and Portfolio Choice, Working Paper, London Business School.
16. Davig, T. (2004). Regime-Switching Debt and Taxation, *Journal of Monetary Economics*, 51, pp. 837-859.
17. Elliott, L. (2008). Credit crisis – how it all began, available at: http://www.guardian.co.uk/business/2008/aug/05/northernrock.banking.
18. Fabozzi, F.J. and Francis, J.C. (1977). Stability Tests for Alphas and Betas Over Up and Down Market Conditions, *Journal of Finance*, 32, pp. 1093-1099.
19. Fabozzi, F.J. and Francis, J.C. (1979). Mutual Fund Systematic Risk for Up and Down Markets: An Empirical Examination, *Journal of Finance*, 34, pp. 1243-1250.
20. Fama, E.F. (1963). Mandelbrot and the Stable Paretian Hypothesis, *Journal of Business*, 36, pp. 420-429.
21. Fama, E.F. (1965). The Behavior of Stock-Market Prices, *Journal of Business*, 38, pp. 34-105.
22. Ferguson, J.D. (1980). Variable duration models for speech, *Proceedings of the Symposium on the Applications of Hidden Markov Models to Text and Speech*, Princeton, New Jersey, pp. 143-179.
23. Goldfeld, S.M. and Quandt, R.E. (1973). A Markov Model for Switching Regressions, *Journal of Econometrics*, 1, pp. 3-16.
24. Granger, C.W.J. and Ding, Z. (1995a). Some properties of absolute return: an alternative measure of risk, *Les Annales d'Economie et de Statistique*, 40, pp. 67-91.
25. Granger, C.W.J. and Ding, Z. (1995b). Stylized facts on the temporal and distributional properties of daily data from speculative markets, Unpublished paper, University of California at San Diego.
26. Granger, C.W.J., Ding, Z. and Spear, S. (2000). Stylized facts on the temporal and distributional properties of absolute returns: An update, *Proceedings of the Hong Kong International Workshop on Statistics and Finance: An Interface*, Imperial College Press, pp. 97-120.
27. Guédon, Y. (2003). Estimating hidden semi-Markov chains from discrete sequences, *Journal of Computational and Graphical Statistics*, 12 (3), pp. 604-639.
28. Hamilton, J.D. (1988). Rational-Expectations Econometric Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates, *Journal of Economic Dynamics & Control*, 12, pp. 385-423.
29. Hamilton, J.D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica*, 57, pp. 357-384.

30. Hamilton, J.D. (2005). What's Real About the Business Cycle? *Federal Reserve Bank of St. Louis Review*, forthcoming.
31. Hamilton, J.D. (2008). 'Regime switching models', The New Palgrave Dictionary of Economics, Second Edition, Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan. The New Palgrave Dictionary of Economics Online. Palgrave Macmillan, September 8, 2008, http://www.dictionaryofeconomics.com/article?id=pde2008_R000269>.
32. Hughes, J.P. and Guttorp, P. (1994). Incorporating spatial dependence and atmospheric data in a model of precipitation, *Journal of Applied Meteorology*, 33, pp. 1503-1515.
33. Investment Company Institute (2011). *2011 Investment Company Fact Book*, 51st edition, Investment Company Institute.
34. Jarque, C.M. and Bera, A.K. (1980). Efficiency tests for normality, homoscedasticity and serial independence of regression residuals, *Economic Letters*, 6 (3), pp. 255-259.
35. Jarque, C.M. and Bera, A.K. (1987). A Test for Normality of Observations and Regression Residuals, *International Statistical Review*, 55 (2), pp. 163-172.
36. Jeanne, O. and Masson, P. (2000). Currency Crises, Sunspots, and Markov-Switching Regimes, *Journal of International Economics*, 50, pp. 327-350.
37. Jeanblanc-Picqué, M. and Pontier, M. (1990). Optimal portfolio for a small investor in a market model with discontinuous prices, *Applied Mathematics and Optimization*, 22, pp. 287-310.
38. Ljung, G.M. and Box, G.E.P. (1978). On a Measure of a Lack of Fit in Time Series Models, *Biometrika*, 65, pp. 297-303.
39. Longstaff, F.A. (2001). Optimal portfolio choice and the valuation of illiquid assets, *Review of Financial Studies*, 14, pp. 407-431.
40. Liu, J. (1999). Portfolio selection in stochastic environments, Working paper, University of California at Los Angeles.
41. Liu, J., Longstaff, F.A. and Pan, J. (2003). Dynamic Asset Allocation with Event Risk, *The Journal of Finance*, 58, pp. 231-259.
42. MacDonald, I.L. and Zucchini, W. (1997). Hidden Markov and Other Models for Discrete-valued Time Series, *Monographs on Statistics and Applied Probability*, Vol. 70, Chapman & Hall, London.
43. Mandelbrot, B. (1963). The Variation of Certain Speculative Prices, *The Journal of Business*, 36, pp. 394-419.
44. Mandelbrot, B. (1967). The Variation of Some Other Speculative Prices, *The Journal of Business*, 40, pp. 393-413.
45. Manton, J.H., Muscatelli, A., Krishnamurthy, V. and Hurn, S. (1998). Modelling Stock Market Excess Returns by Markov Modulated Gaussian Noise, Working paper, University of Glasgow.
46. Markowitz, H.M. (1952). Portfolio Selection, *The Journal of Finance*, 7, pp. 77-91.
47. Merton, R.C. (1971). Optimum consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory*, 3 (4), pp. 373-413.
48. Merton, R.C. (1976). Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics*, 3, pp. 125-144.
49. Mittnik, S., Paolella, M.S., Rachev, S.T. (2000). Diagnosing and treating the fat tails in financial returns data, *Journal of Empirical Finance*, 7, pp. 389-416.
50. Ortobelli, S.L., Rachev, S.T. and Schwartz, E.S. (1999). The problem of optimal portfolio with stable distributed returns, Working paper, University of California at Los Angeles.
51. Poritz, A.B. (1982). Linear Predictive Hidden Markov Models and the Speech Signal, Acoustics, Speech and Signal Processing, IEEE Conference on ICASSP'82, 7, pp. 1291-1294.
52. Rabiner, L. (1989). A tutorial on hidden Markov models and selected applications in speech recognition, *Institute of Electrical and Electronics Engineers: Transactions on Information Theory*, 77, pp. 257-284.
53. Rachev, S.T., Stoyanov, S.V. and Fabozzi, F.J. (2008). Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization, John Wiley & Sons Inc., Hoboken.
54. Reuters (2007). Bear Stearns halts redemptions in third hedge fund, available at: http://www.reuters.com/article/idUSL0110843220070801.
55. Roca, E.D. and Wong, V.S.H. (2008). An Analysis of the Sensitivity of Australian Superannuation Funds to Market Movements: A Markov Regime Switching Approach, *Applied Financial Economics*, 18, pp. 583-597.
56. Rydén, T., Terasvirta, T. and Asbrink, S. (1998). Stylized facts of daily return series and the hidden Markov model, *Journal of Applied Econometrics*, 13 (3), pp. 217-244.
57. Samuelson, P.A. (1965). Rational theory of warrant pricing, *Industrial Management Review*, 6, pp. 13-31.
58. Samuelson, P.A. (1967). Efficient Portfolio Selection for Pareto-Lévy Investments, *Journal of Financial and Quantitative Analysis*, 2 (2), pp. 107-122.
59. Samuelson, P.A. (1969). Lifetime Portfolio Selection by Dynamic Stochastic Programming, *Review of Economics and Statistics*, 51 (3), pp. 239-246.
60. Samuelson, P.A. (1973). Mathematics of speculative price, *SIAM Review*, 15, pp. 1-42.
61. Sansom, J. and Thomson, P.J. (2001). Fitting hidden semi-Markov models to breakpoint rainfall data, *Journal of Applied Probability*, 38A, pp. 142-157.
62. Shi, S. and Weigend, A.S. (1997). Taking Time Seriously: Hidden Markov Experts Applied to Financial Engineering, *Proceedings of the IEEE/IAFE 2007 Conference on Computational Intelligence for Financial Engineering*, pp. 244-252.
63. Sims, C. and Zha T. (2004). Were There Switches in U.S. Monetary Policy? Working paper, Princeton University.

64. Sun Microsystems (2008). Numerical Computation Guide: Underflow, available at: http://docs.sun.com/app/docs/doc/801-7639/6i1ucu1ug?l=zh_TW&a=view.
65. Timmermann, A. (2000). Moments of Markov switching models, *Journal of Econometrics*, 96, pp. 75-111.
66. Tokat, Y., Rachev, S.T. and Schwartz, E.S. (2003). The stable non-Gaussian asset allocation: a comparison with the classical Gaussian approach, *Journal of Economic Dynamics and Control*, 27 (6), pp. 937-969.
67. Tokat, Y. and Schwartz, E.S. (2002). The impact of fat tailed returns on asset allocation, *Mathematical Methods of Operational Research*, 55, pp. 165-185.