“Embedded option in pension funds: the case of conditional indexation policy”

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Abstract

In the last decade the financial crises have led many pension funds to adopt different management approach to overcome the arising difficulties to maintain a solid financial status. Among these, the adoption of an indexation policy, which is now conditional on the solvability of the fund, have been widely adopted. Pension funds recognizing conditional inflation indexation targets are obliged to pay an additional payoff that is linked to the inflation rate through some specific rule. The additional payoff normally takes the form of a contingent claim conditional to a "measure" of sustainability of the payoff itself; in most cases, the measure is linked to an asset and liability ratio able to capture and guarantee the solvability of the fund itself. Therefore, a full valuation of the obligation towards fund's participants and the definition of an optimal investment strategy cannot exclude the proper appraisal of this additional option. The option payoff is conditional to a measurement asset that is different from the reference-underlying asset. This structure recalls a barrier option with different measurement and payoff asset. The paper investigates the opportunity to apply barrier option scheme to the case of a pension fund, whose indexation target is conditional to a specific value of the funding ratio. Results derive from a simulation procedure applied to an exemplar case by means of scenario-based analysis. Numerical results give the opportunity to state the absolute value of the "inflation option" and the relative value with respect to the fund's liabilities. An adequate valuation of this embedded option is important for fund managers to properly adopt hedging strategy of pension fund risks; it can help the corporate sponsors to assess the claim the pension funds has on its balance sheet; the beneficiaries to assess the impact on their pension value of change in policies; finally, it can support the regulator to monitor the solvability of the funds, whereas the embedded options value is substantial relative to the size of the liabilities.

Keywords: pension fund, embedded option, barrier option.

Introduction

The financial crisis of the beginning of the millennium and the recent crisis have led many pension funds to adopt different management approaches to overcome the arising difficulties to maintain a solid financial status. Among these, pension funds have gradually shared risks with various stakeholders, shifting the risks from the fund to stakeholders as retirees, employers and employees. These risksharing agreements can be view as complex options "embedded" in the pension deal, where the roles of writer and holder are taken alternatively by the fund and, by one or more group of stakeholders, according to the characteristics of the agreements. It clears the great importance to estimate the value of these options. It is important for fund managers to properly adopt hedging strategy of pension fund risks which require a complete understanding of the characteristics of these options; for the corporate sponsors to assess the claim the pension funds has on its balance sheet; for the beneficiaries to assess the impact on their pension value of changes in policies. Finally, it can help the regulator to monitor the solvability of the funds whereas the embedded options value is substantial relative to the size of the liabilities. This is particularly significant when one considers that many pension funds have liabilities that equal or even exceed the capitalization of the sponsor company. As observed by Kochen (2009) for the UK and Dutch pension markets, the contingent claims relative to embedded options can easily exceed 20-30% of the total liabilities in a pension fund and dramatically increase in period of financial distress. The embedded option approach, originally introduced by Blake (1998), is gaining importance as a new form of risk management for pension funds, aimed to identify and evaluate the various embedded options by means of market-consistent valuations. The focus of this paper is on a specific type of embedded option deriving from the conditional indexation agreement where the pension scheme is structured such that inflation-linked indexation may be forgone when the funding level falls below a certain threshold. The paper investigates the opportunity to apply barrier option scheme to the case of a pension fund, whose indexation target is conditional to a specific value of the funding ratio, in order to provide a full valuation of the obligation towards participants. The prime result is to determine the value of this option as percentage of the value of the liabilities for a stylized Dutch pension fund.

Numerical results derive from a simulation procedure applied to an exemplar case by means of scenario-based analysis developed for Asset and Liability Management (ALM). The analysis focuses on indexation policy conditional on the level of the funding ratio, as applied in the Netherlands and is under

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consideration for introduction also in other pension systems. Within, this context, the valuation of the embedded option concerning the inflation becomes relevant even in time. The main objective is two-fold: the identification of the more appropriate option scheme to adopt as an efficient replication of the pension fund flows and the selection of an evaluation procedure consistent with the internal management approach. Evidences give the opportunity to state the absolute value of the "conditional indexation option" and the relative value with respect to the fund's liabilities. This valuation technique is an indispensable tool for improving pension fund risk management, for redesigning pension contracts and for supporting decision-making processes. Next section describes the conditional indexation policy in terms of barrier option. Successively the general functioning of the indexation rule adopted in a Dutch based pension funds and the definition of the critical funding ratio are defined. Therefore, the indexation option is evaluated by means of scenario analysis in Asset and Liabilities Management (ALM) framework and results are presented.

1. Conditional indexation as embedded barrier option

Conditional indexation was first introduced in the Netherlands but is gradually being adopted in other European countries with DB systems. The indexation represents a correction of the pension rights aimed at compensating the loss in terms of purchasing power due to inflation rate increases and therefore offers a hedge against the purchasing power risk faced by pension participants (for an example of contingent indexation, see Nijman and Koijen, 2006). The full indexation to inflation of the liabilities has been for decades an undisputed guarantee offered to the participants of a pension fund, but it has become less sustainable for many defined benefit pension funds since the 2000-2003 stock market collapse. Most of them opted to voluntary and conditional/limited indexation policy, depending on the financial position of the fund. It means that the compensation can also be null or only partial when the funding ratio falls below required level. In the UK, indexation cuts are linked to the inflation level itself (indexation is capped at a certain level, e.g., 2.5 per cent or 5 per cent), although introducing conditional indexation is on the UK political pension agenda. In the Netherlands pension funds mostly opted for a solution consisting in a conditional indexation: the decision to grant indexation depends on the nominal funding ratio defined as the ratio of assets to liabilities (Bikker, 2007). If the funding ratio falls below a threshold level, indexation is limited or skipped altogether assuming the features of an option (de Jong, 2008). From a participant's perspective, the conditional indexation implies that the "indexation risk" (or purchasing power risk) partly translates from the pension fund to its participants (Gallo, 2011). From the pension fund management perspective, the solution to offer only conditional indexation has been seen as a good compromise given the adverse financial market conditions. The recent evolution of the full indexation policy towards a conditional indexation policy arises the need for an understanding of impact of these contingent claims on the solvability of the funds. The prospected payoff can be assimilated to an option scheme and should be accurately valuated in the definition of the pension fund's obligation towards its participants.

A large number of papers devoted to the issue of an accurate valuation of embedded options mainly focuses on life insurance contracts. Since the seminal paper of Brennan and Schwartz (1976) and Boyle and Schwartz (1977) using contingent claims theory, a great prominence has been given in the financial and actuarial literature to the issues of pricing and hedging equity-linked life insurance contracts. Other papers that deal with guaranteed equity-linked contract are Boyle and Hardy (1997), Bacinello and Persson (2002), Schrager and Pelsser (2004). In equity-linked contracts, the minimum return guarantee can be identified as a European put option, and hence the classical Black and Scholes (1973) option pricing formula can be used to determine the value of the financial guarantee.

In pension fund's literature, the seminal paper by Blake (1998) recognize that a defined benefit pension fund's portfolio can be replicated by an investment in a portfolio containing the underlying asset (market value of the asset) plus a put minus a call option on this asset, by adopting a Black and Scholes (1973) pricing. As an appropriate portfolio composed by options can replicate the whole fund, also specific (innovative) features as conditional indexation policy can be interpreted as embedded option. In particular, it can be regarded as a barrier option embedded in the pension contract that the pension fund sells to its participants as suggested by de Jong (2008). Among different types of barrier option, we originally evaluate this Indexation Option (IO) as an outside barrier option call down-and-out. Barrier options are contingent claims that either are born (in barrier or knock in) or expire (out barrier or knock out) when the underlying asset price reaches a specified value h defined as "barrier". Given the presence of the barrier, these options typically exhibit a lower value than corresponding plain vanilla options, with higher prospective expected return. There are put and call, as well as European and American varieties. The common feature is that they become activated or, on the contrary, null and void only if the underlying asset reaches a predetermined level (barrier) and, specifically, "in" options start their lives worthless and only become active in the event a predetermined knock-in barrier price is breached, while "out" options start their lives active and become null and void in the event a certain knock-out barrier price is breached. Outside barrier option are two-asset options where the payoff is defined on one asset (the so called payoff asset) and the barrier is defined on another asset (the so called measurement asset). Several types of barrier options (put and call) can be formulized, but for the case under investigation we will refer to the *down-and-out option*, where the contract expires if the measurement asset price falls below the value barrier at the expiration date. In order to configure the scheme of the conditional indexation policy we will refer to a barrier down-and-out option, characterized by the presence of two underlying assets, since the option payoff (the indexed addendum) is conditional to a special event: the funding ratio has not to fall below a defined minimum level. Therefore, recalling the scheme of the down-and-out outside barrier option, the funding ratio takes the place of the "measurement asset" and sets the condition that eliminates any positive payoff, given a decrease in the value of the measurement itself. Accordingly to this scheme, if the barrier is hit, there is no additional payoff and the option expires. The indexed addendum is the proper "payoff asset", which ultimately defines the positive payoff of the option. This framework, here originally applied to pension funds, exactly portrays the case of the minimum requirement for the funding ratio. In the majority of cases, the funding ratio is higher than the minimum requirement (both institutional and internal) and only if it goes down the minimum, the indexation will not be paid. Consistently with the dynamic of the pension fund the possibility of knocking out depends solely on the fact that the measurement – that is to say the funding ratio – reaches the barrier level at certain times. If the option does not expire, that is to say that if the funding ratio at time t +1 does not fall below the required ratio, the pension fund will recognize the indexation.

2. The indexation policy

The indexation policy depends on the financial status of the fund expressed by the funding ratio at the end of the year t (FR). It is computed using the annual market values for both assets (A_t^U) and liabilities (L_t^U):

$$FR_t^U = \frac{A_t^U}{L_t^U}, \tag{1}$$

where (FR_t^U) – ultimate funding ratio – expresses the financial status of the fund as the capability of the amount of the resources available to cover the related nominal liabilities at the end of the year. It is usually expressed in percentage terms, so that a funding ratio of 105 corresponds to a 5% surplus of assets over liabilities.

In most of the defined benefit pension fund, the indexation rule is defined as follows: if the funding ratio is greater than the required funding ratio, full indexation is granted.

According to the actual Dutch regulation, the required funding ratio is defined by the Pension Law and depends on both the Strategic Asset Allocation (SAA) of the fund and the duration mismatch between pension assets and liabilities. Let us assume that the required funding ratio has to be equal to two exemplar cases: 105 corresponding to the minimum solvency requirement and 115 as the average indexation requirement.

Therefore, if the funding ratio is lower that the threshold values (105; 115) the nominal liabilities at time t + 1 corresponds to the nominal liabilities at time t, without any indexation. Hence, only if the nominal liabilities are counterbalanced in terms of assets, the pension fund will proceed to consider an update of the nominal liabilities to the inflation rate, granting indexation.

To compute the funding ratio, the market value of the assets and liabilities must be computed. At time 0 (evaluation time), the pension fund has a certain current value of the assets ($A_{t=0}$) and liabilities ($L_{t=0}$). The initial funding ratio is defined as:

$$FR_{t=0} = \frac{A_{t=0}}{L_{t=0}}, \tag{2}$$

where $A_{t=0}$ corresponds to the market value of the invested assets and $L_{t=0}$ to the present value of all the future obligation of the fund towards the participants as a whole. For each time t, according to the Liability Driven Investment (LDI) paradigm, the asset portfolio (A_t) is divided into two sections: the matching portfolio ($A_{M,t}$) and the risk-return portfolio ($A_{RR,t}$). The matching portfolio is assumed to earn exactly the liability return to match nominal liabilities as a result of a perfect immunization strategy. The risk-return portfolio consists of different asset classes as equity and alternative assets. It is meant to provide enough resources to grant indexation. The amount invested in each portfolio is defined according the ratio of the matching portfolio to the total value ($w_M = A_{M,t}/A_t$) and of the risk-return portfolio to the total value ($w_M = A_{RR,t}/A_t$) and the portfolio is rebalanced to these pre-defined weights each year. Let us assume, using average percentage concerning the Dutch pension funds, that the percentage of assets invested in the matching portfolio is 37%, while the remaining 63% is invested in the risk-return portfolio.

2.1. The critical funding ratio. To compute the critical funding ratio conditioning indexation, we need to define the market value of asset and liabilities. On the liability side, the value of the liabilities is computed under the hypothesis of the run-off of the pension fund. We set the time t as the moment from which the pension fund is formally closed to new participants and the old ones do not pay any contribution (evaluation time). The pension fund only has annual nominal cash flows (CF) to be paid to the participants at the end of each subsequent year until the definitive closing date (n). The present value of all these future nominal obligations is computed market-to-market as:

$$L_t^U(i_{k,t})\sum_{k=o}^{n}\frac{CF_{t+k}}{(1+i_k)^k}, \qquad (3)$$

where k is the maturity of each residual cash flow and i_k is the spot rate associated to the corresponding node on the interest rate yield curve. The notation $L_t^U(i_{k,t})$ accounts for the fact that the present value is calculated on the basis of a yield curve estimated at time t. The cash flows are computed under usual assumptions about the life expectation of the participants, the expected retirement date and other variables according to a defined actuarial model that takes into account actuarial and longevity risk. We will not investigate these aspects, since we concentrate on the interest rate risk arising from the fair valuation and we define the value in (3) as the present value of an anticipated rent.

The interest rate yield curve is generated by the Nelson and Siegel (1987) model, which has the advantages that it is well-behaved at long maturities, and that its parameters can be set to model virtually any yield curve. The corresponding term structure of interest rate in each year (and next in each scenario) will be determined by combining the values of the three main parameters according to the following relationship:

$$i_k = \beta_0 + (\beta_1 + \beta_2)\frac{1-e^{-k/\tau}}{k/\tau} - \beta_2 e^{-k/\tau} L_t^U(i_{k,t}), \qquad (4)$$

where k is the relevant node; β_0 is an estimate of the long run levels of interest rates; β_1 is the short-term component; β_2 is an estimate of the medium-term component; and τ is the decay factor. Parameters were fitted via least-squares according to a standard procedure defined by Diebold and Li (2006). The yield curve is simulated on the basis of equation (4) and it is used to discount all the future cash flows according to the value of k. We want to remark that the ultimate value of the liabilities at time t is computed as the present value of all the future nominal obligations including the cash flow to be paid at the end of year t (anticipated rent), discounted at the interest rate yield curve estimated according to the formula (4) at time t. Therefore, this value only takes into account the nominal obligation as defined at time t, excluding the eventual increase of the nominal liabilities due to the indexation decision.

From the ultimate value, we derive the corresponding primary value of the liabilities at time t, by subtracting the nominal cash flow to be paid at time t, in order to regard the primary value as the present value of the posticipated rent corresponding to the anticipated one as defined by (3). That is:

$$L_t^P(i_{k,t}) = L_t^U(i_{k,t}) - CF_t. \qquad (5)$$

The primary value of the liabilities $L_t^P(i_{k,t})$ represents the "end of the year" value evaluated on the basis of the yield curve as estimated at time t, and hereafter the initial value of the liabilities at the beginning of the next year filtered by the information available at time t and synthesised in the yield curve. Given these definitions, the "nominal" rate of growth of liabilities is given by:

$$r_{L,t+1} = \frac{L_{t+1}^U(i_{k,t+1})}{L_t^P(i_{k,t})} - 1. \qquad (6)$$

This value gives the increase in the value of the nominal liabilities from their initial value (primary) at the beginning of the year to the end of the same year, only due to the dynamics of cash flows and changes in the interest yield curve from one year to another.

Once the nominal growth of liabilities is computed, every year the primary value of the liabilities at time t, that is to say the initial value of the liabilities at time $t + 1$, is updated by the nominal rate of growth as in formula (6), to obtain the nominal ultimate value at time $t + 1$ as below.

$$L_{t+1}^U(i_{k,t+1}) = L_{t+1}^P(i_{k,t})(1 + r_{L,t+1}). \qquad (7)$$

Then, depending on the value of the funding ratio at time $t + 1$, the indexation decision is taken and applied to the ultimate value in formula (7), to obtain the indexed ultimatevalue of the liabilities, as follows:

$$L_{t+1}^{Uindex} = L_{t+1}^U(i_{k,t+1})(1 + \pi_{t+1}), \qquad (8)$$

where π_{t+1} is the inflation rate as recorded at time $t + 1$. By subtracting the $t + 1$ maturing cash flow (also updated by indexation), we compute a new primary value for the liabilities, which also takes into account the indexation:

$$L_{t+1}^{Pindex} = L_{t+1}^{Uindex} - (CF_{t+1} \cdot (1 + \pi_{t+1})). \quad (9)$$

This value represents the initial value of the liabilities for the next year that will be accordingly updated by the nominal growth estimated in equation (7) and eventually by the indexation decision (8). It is denominated "Pindex" to be distinguished by the previously defined primary value, which does not include indexation. However, once the indexation is recognized, it is acquired and guaranteed: it becomes the "nominal" value for the next year. Therefore formula (8) can be timely extended as:

$$L_{t+2}^{U}(i_{k,t+2}) = L_{t+1}^{Pindex} - (1 + r_{L,t+2}). \quad (10)$$

On the other side of the intermediation portfolio, the initial amount of assets at time 0 is invested every year, and therefore A_t, represents the market value of portfolio of the pension fund. The value of the portfolio is the sum of the two parts:

$$A_t = A_{M,t} + A_{RR,t}. \quad (11)$$

The matching portfolio ($A_{M,t}$) is composed by fixed-income assets with duration equal to the duration of the liabilities and that it earns every year a return equal to the nominal rate of growth of the nominal liabilities as defined earlier (equation 6).

$$r_{L,t} = r_{M,t}. \quad (12)$$

where $r_{M,t}$ is the rate of return of the matching portfolio at time t. By means of this position, the interest rate risk is partially offset. Due to the fact that the immunization is only in terms of duration, it only hedges from a parallel shift of the interest rate yield curve. The remaining interest rate risk (convexity risk) and the inflation risk should be hedged by the dynamics of the returns of the other part, the risk-return portfolio ($A_{RR,t}$). This portfolio is composed by: Property, Commodity, Equity Value, Equity Passive, Equity Emerging Market and Equity Growth. It should earn enough to complete the hedging of the nominal liabilities and also provide with extra-return to allow for indexation. The return on the risk-return portfolio of the pension fund is given by:

$$r_{RR,t} = \sum_{j=1}^{z} r_{j,t} \cdot \frac{A_{j,t}}{A_{RR,t}} \text{ with } A_{RR,t} = \sum_{j=1}^{z} A_{j,t}, \quad (13)$$

where $r_{j,i}$ is the rate of return (at time t) of the j-th asset in the risk-return portfolio weighted by the percentage contribution of the j-th asset to the portfolio and where z is the total number of assets or securities in the portfolio itself.

Consistently with the liabilities framework, we define two different values of the assets. The first one, defined as ultimate asset value (A_{t+1}^{U}), is the reference value for the computation of nominal funding ratio. It is computed as:

$$A_{t+1}^{U} = A_{M,t}^{P}(1 + r_{L,t}) + A_{RR,t}^{P}(1 + r_{RR,t}). \quad (14)$$

It expresses the value of the invested assets before the indexation and the payment of the cash flow for the corresponding year, where A_t^P is the primary value for each portfolio. Similarly to the primary value of the liabilities, it is computed as:

$$A_{t+1}^{P} = A_{t+1}^{U} - (CF_{t+1}(1 + \pi_{t+1})). \quad (15)$$

3. The pricing of the conditional indexation option

For the application of the outside barrier option to the indexation case, the recalled Black and Scholes approach above cannot be appropriately used. This is due to the fact that it assumes a continue barrier over the life of the option and a lognormal distribution for both the measurement and payoff asset. To evaluate an outside barrier option analytical solution has been developed (Zhang, 1995). The evaluation of the outside barrier option requires that the density function contain the lognormal distribution of the asset price payoff that is conditional upon the achievement or failure to achieve (depending on if it is knock in or knock out) of the barrier level by the price of the measurement asset *during the life of the option*. The crux is that in this pricing approach the barrier is modelled in a continuous framework. This assumption implies a density function even for the barrier since the option price relies on two defined stochastic process put in a consistent Black and Scholes framework, that is to say respectively for the payoff asset and the measurement asset:

$$\begin{aligned} d\ln(S_t / S_0) &= \mu_1 dt + \sigma_1 dW_t^1, \\ d\ln(R_t / R_0) &= \mu_2 dt + \sigma_2 dW_t^2. \end{aligned} \quad (16)$$

In other words, the price is based on a bi-variate density function, deriving from a lognormal distribution for both the measurement and the payoff assets. The two lognormal distributions are modelled in a stochastic environment by the application of known drift and diffusion coefficients (μ, σ), as well as on the base of a known correlation between the two relevant disturbance dynamics ($dW_t^1 \cdot dW_t^2 = \rho$).

In the pension fund case, the barrier is represented by a specified level of the funding ratio and is not observed continuously, but in a discrete time and on a specific date. Therefore, we will define the indexation option (IO) as an outside barrier option (down-and-out) having a discrete barrier. The observation time is set equal to the last day of each year, when the market value of the assets and liabilities are computed and the inflation rate is observed. For this reason, the lognormal distribution cannot be regarded as anaccurate description of the relevant dynamic. At the same time, the payoff asset is more similar to an interest rate option. As a consequence a numerical approach to the evaluation of the embedded option emerges as an obliged choice. We proceed on by using a scenario-based approach.

The simulation approach gives the opportunity to state simultaneously the value of the barrier and the the value of the payoff. The implementation of this methodology consents the modelling of the relevant values according to correlation factors of the primary risk and value drivers, since these correlations are included in the scenario generation by means of the scenario generation scheme (see *infra*).

Since we concentrate on the "additional" amount paid if the relevant condition holds, we define the option payoff as "$L^U_{t+1}(i_{k,t+1})(\pi_{t+1})$ or nothing". In practice, if the funding ratio at time $t+1$ falls below the minimum requirement (barrier), the pension fund will recognize only the "nominal" liability value $L^U_{t+1}(i_{k,t+1})$. On the other hand, if the funding ratio is equal or higher than the barrier, the pension fund will recognize the indexed value of the liability $L^{UIndex}_{t+1}(i_{k,t+1}) = L^U_{t+1}(i_{k,t+1})(1+\pi_{t+1})$, that is:

$$\begin{cases} L^U_{t+1}(i_{k,t+1}) \ \forall \ FR^U_{t+1} < h, \\ L^U_{t+1}(i_{k,t+1})(1+\pi_{t+1}) \ \forall \ FR^U_{t+1} \geq h; \end{cases} = L^U_t(i_{k,t}) + \underbrace{\begin{cases} 0 \forall FR^U_{t+1} \leq h, \\ L^U_{t+1}(i_{k,t+1})(\pi_{t+1}) \ \forall \ FR^U_{t+1} > h \end{cases}}_{indexation\ option}, \tag{17}$$

where the last addendum is the payoff of the indexation option payoff (IOP) as:

$$IOP_{t+1} = \max[L^U_{t+1}(i_{k,t+1})(\pi_{t+1}),\ 0] \ \forall \ FR^U_{t+1} > h. \tag{18}$$

The previous formulation gives the payoff referred to time $t+1$. The present value at time t of the IOP_{t+1} is calculated using the spot rate referring to the first node of the yield curve and observed in t ($i_{1,t}$) gives the price of the IO. And so on for the residual duration of the pension funds. Therefore, the present value of the whole indexation option payoff ($WIOP_t$) at time t is the sum of n indexation option payoff differing for the time to maturity and discounting for the appropriate spot rate as observe in time t. Formally:

$$WIOP_t = \frac{IOP_{t+1}}{1+i_{1,t}} + \frac{IOP_{t+2}}{(1+i_{2,t})^2} + \frac{IOP_{t+n}}{(1+i_{n,t})^n} = \sum_{k=1}^{n} \frac{IOP_{t+k}}{(1+i_{k,t})}. \tag{19}$$

Given the discretization of the barrier, the present value of the IOP, that is to say that the price/value of the option is estimated by numerical methods, based on scenario analysis as far as the asset and liability values are concerned. More specifically, since each scenario s (with $s = 1, 2, \ldots, q$) gives rise to a different yield curve the expected value of $WIOP_t$ is the present value of n option payoff in q states of the world, as follows:

$$E[WIOP_t] = \frac{1}{q}\sum_{s=1}^{q}\sum_{k=1}^{n} \frac{IOP_{t+k,s}}{(1+i_{k,t,s})^k}, \tag{20}$$

where $i_{k,t,s}$ is the spot rate observed in t referring to period t - ($t+k$) and to scenario s and $IOP_{t+k,s}$ refers to the IOP as it is at time $t+k$ and scenario s.

4. Scenario generation

As in most ALM studies (see Ziemba, 1998), the scenarios for the economic relevant variables are generated by statistical model called Vector Auto Regressive Model (VAR), introduced by Sims (1980). The model is formalized as follows:

$$x_{t+1} = a + Dx_t + \varepsilon_{t+1}, \tag{21}$$

where a denotes a vector of the intercepts, D denotes the matrix of coefficients, x_t is the state vector composed by the economic variables and ε_t is the vector of shocks to the system which is assumed to be normally distributed with zero mean and variance-covariance matrix $\sum_\varepsilon$: ε_t : ~ $N(0, \sum_\varepsilon)$.

This model is preferred to others because it is able to create scenarios that are "in accordance with the past" (Boender, 1997). In particular, if the parameters of the VAR are estimated by Ordinary Least Square (OLS) procedure on a sufficiently long historical period, the long-term averages, standard deviations and (auto-)correlations of the scenarios generated are identical to the observations in the historical period used for the model estimation.

After the estimation of the coefficients D of the VAR model, the scenarios are generated by simulating recursively from the VAR model. For this, the estimated covariance matrix of the residuals $\sum\varepsilon$. is decomposed by means of the Cholesky matrix (Gentle 1998) (C), such that $CC' = \sum\varepsilon$. The decomposition is used to estimate values of ε_t. This is done by sampling a vector u from a standard normal distribution $N(0,1)$ so that $u \sim N(0, 1)$ of which $Cu \sim N(0, CC')$ is derived. By multiplying the Cholesky decomposition with a vector of random numbers from a standard normal distribution, new shocks to the system are generated which give simulations of $\varepsilon = Cu$. The Cholesky matrix permits us to impose the historical covariance structure on the future scenarios. These values are used in the equation (20) in order to generate a fan of scenarios according to the formula:

$$x_{t+1} = a + Dx_t + C\varepsilon_{t+1}, \tag{22}$$

where x_{t+1} is a vector of future values for the variables, D is a matrix with the estimated coefficients, x_t is a vector of values for the variables in the previous node, C is the Cholesky matrix, and ε_{t+1} is a vector of random standard normally distributed innovations.

This methodology is applied to our dataset to generate a total number of q scenarios equal to 2500 for the relevant economic time series and the asset classes (j) for the period of 2009-2022 on an annual basis. We use annual data of these series for the period from 1970 to 2006 as the inputs for the estimation of an unrestricted first order VAR model including assets returns, interest rates, and price inflation as endogenous variables. In particular, as inflation rate we consider the annual realized Dutch inflation since the Netherlands is the country where the conditional indexation is mostly adopted. As far as interest rate time series are concerned, starting from the initial estimated parameters of the Nelson & Siegel model, we generate the three main parameters (β_0, β_1, β_2) in each node (s, t) to construct a yield curve for each scenario (s) and each time node in each node (t) to discount the liabilities' cash flow.

On the asset side, the asset returns for Property, Commodity, Equity Value, Equity Passive, Equity Emerging Market and Equity Growth are generated. Commodity dataset is represented by Goldman Sachs Commodity Index (GSCI), a composite index of Commodity sector returns which represents a broadly diversified, unleveraged, long-only position in Commodity futures. ROZ/IPD Dutch Property Index represents property data. This index measures the total returns on directly held real estate investments belonging to institutional investors and real estate funds in the Netherlands. Concerning the investment in equities, Equity Growth is represented by worldwide used Morgan Stanley Capital International World Index (MSCIWI). Equity Value category is represented by MSCISWI hedged, which gives the performances of an index of securities where currency exposures affecting index principal are hedged against a specified currency. Finally Emerging Markets Equity category is represented by MSCI Emerging Markets Index, which is a float-adjusted market capitalization index investing in 26 emerging economies.

On the liability side, we make use of an original dataset provided by a Dutch pension funds composed by all the residual cash flows from 2008 to 2022 in the hypothesis of the closing of the fund in 2022 – that is to say – it is closed to the entry of new participants. It is important to underline that these cash flows are estimated by actuarial simulation that is properly linked to the other simulated economic times series.

The option value at time 0 gives the value of the option written by the pension fund to the participants on the inflation rate. The valuation of the IO is applied to the dataset assuming that the investment horizon (n) is set equal to 14 years, the liabilities are conditionally (only) fully indexed to inflation rate and the barrier (h) is set equal to two exemplar levels: 105 (as minimum solvency requirement) and 115 (as a proxy of the required funding ratio according to the Dutch law).

5. Results

Figure 1 below shows the option payoff (OIP) for each scenario at the evaluation time (in our case 1/1/2009), as a function of the asset payoff, and hence, as a function of the inflation rate. The option payoff has value equal to zero when the option expires because the option in that scenario is knocked out or the payoff asset is not positive (as the case of a negative inflation). On the y-axis there is the histogram of the frequencies associated with each payoff, while on the x-axis there is the histogram represents the distribution frequency of the assets payoff across scenarios.

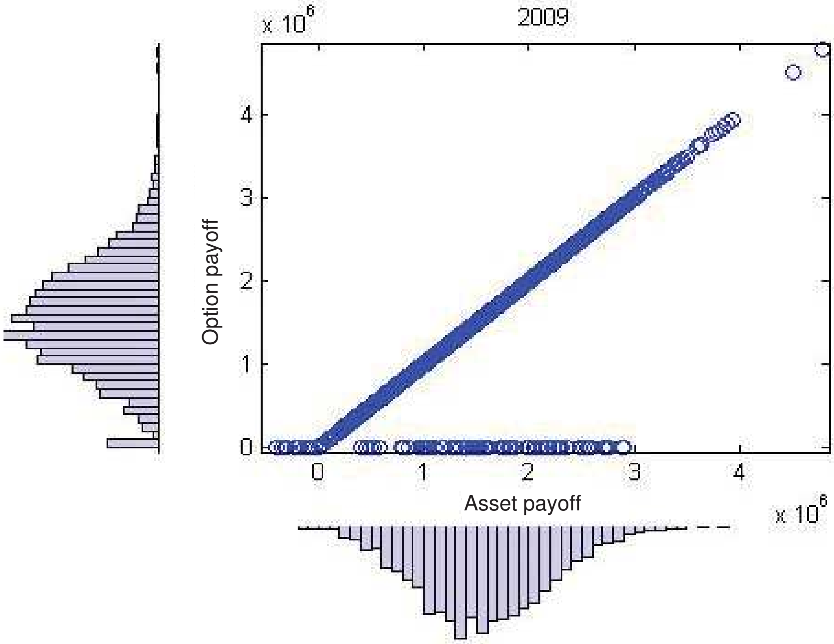

Fig. 1. Option payoff and asset payoff

Figure 2 presents the relation of the option payoff (and the relative frequency distribution) to the funding ratio dynamics at the evaluation time (1/1/2009).

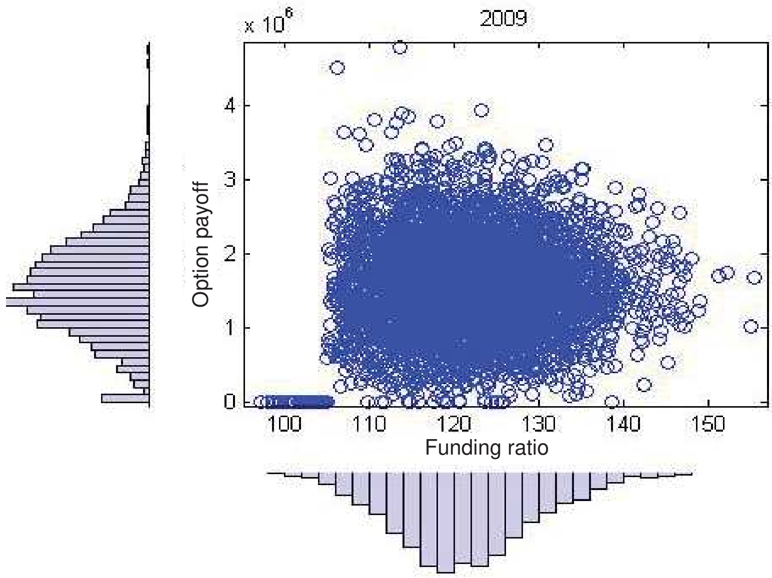

Fig. 2. Option payoff and funding ratio in 2009

Figure 3 shows the distribution of the option payoff (IOP) for each year as a stochastic process. Therefore, for each time node, we can observe the distribution of the annual payoff across scenarios (equation (17)). We notice that the means and the standard deviations of the payoff increase over time according to the increasing volatility of the underlying scenario over time. We can also notice that because of the higher volatility of the funding ratio, the frequency associated with the case where the option is knocked out increases over time. The application of equation (19) gives us the value of the option. Starting from the monetary value, we can deduce the relative value to the nominal liabilities. In this case, the option value at evaluation time (1/1/2009) for the residual 14 years accounts for approximately 27% of the nominal liabilities, that is to say more than 1/4 of the nominal liabilities. It is not an irrelevant percentage of the value of the liabilities and cannot be neglected in a fair valuation.

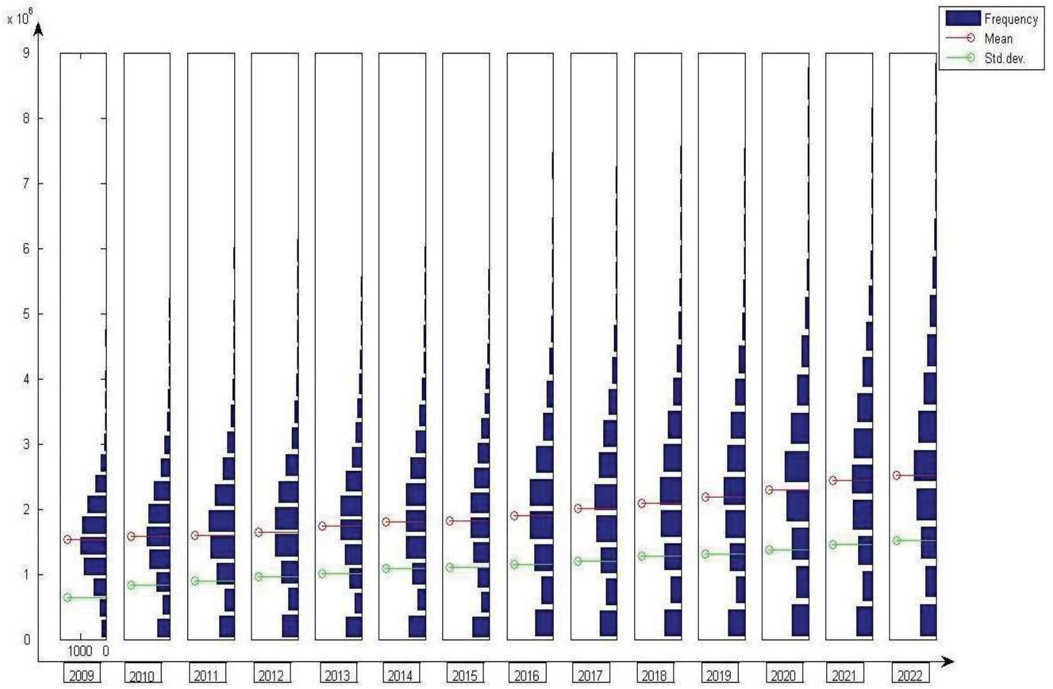

Fig. 3. The distribution frequency of the OIP over time with barrier set at 105

We also develop the same calculus setting the barrier level at 115. As we expected, the option value reaches the value of 22.38% of the liabilities. This is due to the higher level barrier that leads to a higher number of knock-out. As in the preceding case, the graph shows the distribution of the option payoff for each year under consideration as a stochastic process with barrier set at 115. We notice the higher frequency associated with the case where the option is knocked out and a lower means than in the case with barrier set at 105.

As expected the selection of a higher barrier reduces the value (both absolute and relative) of the option, which account for more than 1/5 of the nominal value of the liabilities.

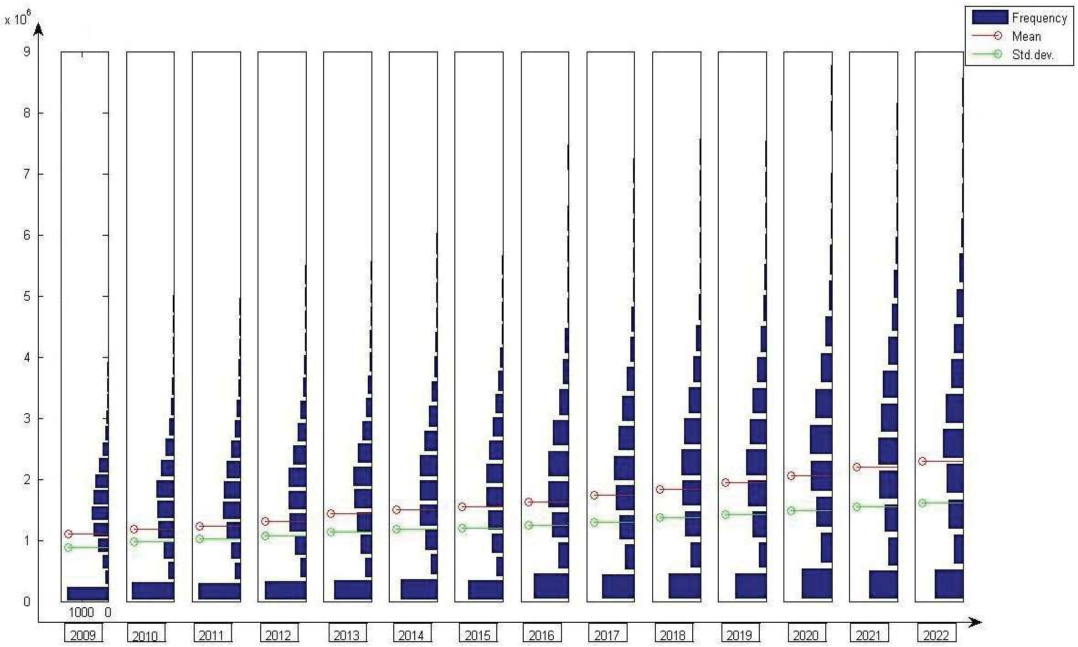

Fig. 4. The distribution frequency of the OIP over time with barrier set at 115

Conclusions

Following the embedded option approach developed for pension funds by Kochen (2009), we originally evaluate the conditional indexation policy adopted by a stylized Dutch pension fund in terms of outside barrier option within a valuation procedure consistent with the ALM features. With respect to these, the outside barrier option is able to depict the full cash flows dynamic and the adoption of a scenario-based analysis allows for a valuation that can be immediately implemented for both managerial targets and accounting reports. In particular, we show that a knock-out call barrier option (with two reference assets) provides with a good framework for this valuation. The option value in 2009 for the following 14 years amounts to 27% of the liabilities value when the funding ratio (barrier) is set at 105 and 22% when the barrier is set at 115. The informational content of this result is crucial for the decision-making process of a pension fund. Further investigations should try to remove several assumptions we impose as the static asset allocation (to account for volatility in the asset mix) or also allow for partial and recovering indexation. Also the definition of an optimal level for the barrier can be considered. This last point is of special interest for regulation and supervision application. The barrier level could be in fact selected so to keep the solvency probability within certain predefined level, in order to assure the survival of the fund. Accounting implementations are even possible, with special reference to those practices where the marking to market require a full unbundling of the basic components of the relevant obligations.

References

1. Bikker, J.A., P.J.G. Vlaar. Conditional indexation in defined benefit pension plans in the Netherlands // *Geneva papers on Risk and Insurance – Issues and Practice*, 2007. – №32. – pp. 494-515.
2. Bacinello, A.R. and Persson, S.A. Design and pricing of equity-linked life insurance under stochastic interest rates // *Journal of Risk and Finance*, 2002. – №3. – pp. 6-21.
3. Black F., M.Scholes. The Pricing of Options and Corporate Liabilities // *The Journal of Political Economy*, 1973. – №81. – pp. 637-664.
4. Blake D. Pension schemes as options on pension fund assets: implications for pension fund management // *Insurance: Mathematics and Economics*, 1998. – №23. – pp. 263-286.
5. Brennan, M.J. and E.S. Schwartz. The pricing of equity-linked life insurance policies with an asset value guarantee // *Journal of Finance Economics*, 1976. – №3. – pp. 195-213.
6. Boender, C.G.E. A hybrid simulation/optimisation scenario model for asset/liability management // *European Journal of Operational Research*, 1997. – №99. – pp. 126-135.
7. Boyle, P.P. and M.R. Hardy. Guaranteed annuity options // *ASTIN Bulletin*, 2003. – №33 (2), pp. 125-152.
8. Boyle, P.P. and E.S. Schwartz. Equilibrium prices of guarantees under equity-linked contracts // *Journal of Risk and Insurance*, 1977. – №44. – pp. 639-660.
9. de Jong F. Pension Fund Investments and the Valuation of Liabilities under Conditional Indexation // *Insurance: Mathematics and Economics*, 2008. – №42. – pp. 1-13.
10. Diebold, F.X.,C. Li. Forecasting the term structure of government bond yields // *Journal of Econometrics*, 2006. – №130. – pp. 337-364.
11. Gallo A. Indexation as primary target for Pension Funds: implication for portfolio management // *Journal of Financial Transformation, Cass-Capco Institute Paper Series on Risk*, 2011. – №31. – pp. 173-183.
12. Gentle, J.E. Numerical Linear Algebra for Applications in Statistics // Springer-Verlag, Berlin, 1998.
13. Kochen, T. Pension Fund Risk Management: Multi-Stakeholders, Risk Management and the Embedded Options Approach // *Journal of Risk Management in Financial Institution*, 2009. – №2. – pp. 130-140.
14. Nelson, C.R., A.F. Siegel. Parsimonious Modelling of Yield Curves // *Journal of Business*, 1987. – №60. – pp. 473-489.
15. Nijman, T.E., R. Koijen. Valuation and Risk Management of Inflation-Sensitive Pension Rights in Fair Value and Pension Fund Management // N. Kortleve, Th.E. Nijman and E. Ponds (eds.), Elsevier Publishers, 2006.
16. OECD (2009). Pensions at a Glance 2009. Retirement-Income Systems in OECD Countries 2009, available at: www.oecd.org/els/social/pensions/PAG.
17. Schrager, D.F. and A. Pelsser. Pricing rate of return guarantees in regular premium unit linked insurance // *Insurance: Mathematics and Economics*, 2004. – №35. – pp. 369-398.
18. Zhang, P.G. A unified pricing formula for Outside barrier options // *Journal of Financial Engineering*, 1995. – №4. – pp. 335-349.
19. Ziemba, W.T., J.M. Mulvey. Worldwide Asset and Liability Modelling // Cambridge University Press, 1998.