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Endogenously unstable demand deposit contracts: triggering bank runs without sunspots

Abstract

The literature on bank runs reduces all coordination mechanisms triggering attacks on banks to exogenous realizations derived from either fundamental or sunspot variables. The authors present a general equilibrium version of these models where the state uncertainty faced by depositors is modeled explicitly, such that bank runs arise as optimal endogenous equilibrium outcomes corresponding to Bayesian coordination games played by rational agents before depositing. Differentials in state information sets between the bank and its depositors lead to rational self-contained equilibrium runs that do not violate the revelation principle. Several numerical simulations illustrating these results are provided.

Keywords: bank runs, self-contained attacks, Bayesian coordination games, revelation principle, information heterogeneity.
JEL Classification: D83, D81.

Introduction

Theoretical motivation and basic stylized facts. There exists a recognized relationship between currency crises and bank runs, the latter preceding the former, defining the most recent financial speculative episodes in economic history, namely the Mexican and Asian crises. Kaminsky and Reinhart (1999) provide ample empirical evidence illustrating this causality. The second generation currency crises literature, see Cole and Kehoe (2000), developed after Obstfeld's (1994) seminal paper, emphasizes the behavior of agents as the driving force behind the attacks in their models. Though multiple equilibria are allowed, and the one leading to an attack is generated by an exogenous sunspot variable, rationality prevails on the expectations formation process of agents and safe areas based on the value of fundamentals are defined. The literature on bank runs, on the other hand, relies heavily on sunspot variables to explain the phenomenon[1], or eliminates it from the equilibrium outcomes[2]. The strategic behavior of agents is generally recognized but not studied explicitly[3].

In order to explain the above empirical relationship, bank runs and currency crises were both initially modeled as sunspot phenomena. Since it seems plausible to assume that most of the agents participating in the currency attacks were also involved in the preceding bank runs, both types of models provided a consistent explanation for the entire cycle defining speculative attacks. An exogenous sunspot affects a set of depositors, who run on their corresponding banks, destabilizing the financial structure of the economy. This is followed by an attack on the currency, by mostly the same agents who run previously on the bank, which further destabilizes the banking system, causing additional runs, and so on. Even though fundamentals play a crucial role modifying the expectations of agents through the different currency attack areas, the attack is ultimately generated by a sunspot variable that coordinates agents on the corresponding equilibrium. This idea gained acceptance due to the fact that recent speculative episodes took place when fundamentals were not bad enough to justify *rational* attacks on the currency, while no attacks had occurred when fundamentals displayed relatively worse values.

The sunspot hypothesis defining sequential self-reinforcing attacks on the banks and currency of a country provided a coherent explanation for both phenomena until Morris and Shin published their paper in 1998. Speculative attacks on currencies were no longer defined uniquely by sunspot variables, but were allowed to be the consequence of rational behavior coupled with imperfect information and noisy signals[4]. This result opened a theoretical paradox that has not been closed yet. Assume that agents are rational utility maximizers who have access to a given information set that is updated at each point in time. These agents, in their role as depositors, run on the bank if, and only if, the attack is coordinated on the realization of an exogenous sunspot variable, for a given value of the fundamentals. Bank runs are followed by speculative attacks on the currency of the corresponding country. However, as shown by Morris and Shin, attacks on the currency are based on signals about the state of the economy received by depositors *after* running on the bank. The financial system will send a negative self-fulfilling signal, which derives directly from the run triggered by the same agents observing the signal, on which the decision of whether or not to at-

[1] See, for example, Diamond and Dybvig (1983), and Peck and Shell (2003).

[2] Green and Lin (2000) and (2003), design sequentially efficient run-proof contracts, but their model is subject to a control the monitor problem in the sense defined by Krasa and Villamil (1992).

[3] Exceptions to this rule are Chari and Jagannathan (1988) and Samartin (2003). However, the set of assumptions needed to generate their results, consisting of confounding multiple equilibria, may seem too restrictive and applicable only in particular cases.

[4] We are obviously referring to the attacks that cannot be justified by the value of fundamentals.

tack the currency will be based. The positive cyclical behavior described above implies that theoretically rational agents stop the attack on the currency to analyze a set of signals that they have just generated by previously running on the bank, based on an exogenous sunspot variable. That is, the current state of the literature defines rational attacks on a currency that cannot be generated by sunspot-based runs on the banks of the corresponding country. The main purpose of the current paper is to solve this paradox by defining runs on the financial intermediaries of a country as rational equilibria independent of any sunspot variable realization. We consider explicitly the information structure defined within the standard bank run model, and allow for beliefs differentials to exist between the bank and its depositors.

Even though the strategic framework used by the second generation currency crises literature is almost identical to the one employed to model bank runs, two important differences must be considered. First, excluding sunspot realizations, the information set of the agents modeled in the bank run literature is fixed through time, while currency speculators are allowed to observe signals before making a decision[1]. Second, the coordination game that speculators play before attacking a currency is ignored by depositors when deciding whether to run on the bank or not.

The second difference deserves some additional comments. Agents do play a coordination game when deciding whether to *deposit* their funds in the bank or to remain in an autarky situation. This game has a unique Nash equilibrium in pure strategies where agents deposit their funds and decide ex-ante not to run on the bank[2]. The existence of a financial intermediary signals that a coordinated no-run equilibrium has been reached among depositors in the *deposit* game. Thus, it should be clear that the same pre-deposit coordination game cannot be used to model the post-deposit run coordination problem to which agents are subject after depositing funds. While this is valid only if the information set of depositors remains unchanged, it is incorrect if agents are allowed to update their information sets by monitoring the bank. In other words, if the equilibrium of the game has been chosen by the agents beforehand, there is no reason to expect a change unless motivated by an exogenous coordination variable. This point has already been made by Alonso (1996) and Jacklin and Bhattacharya (1988), where post-deposit signals defining withdrawal decisions are received by depositors. These models assume that signals reflect the true state of fundamentals, leading to runs caused entirely by negative shocks to the economy. That is, bank runs and the subsequent currency attacks would be mainly due to bad fundamentals, as the first generation currency crises literature predicts, see Krugman (1979). However, this result is at odds with the empirical irrationality exhibited by agents in the crises episodes under consideration, where attacks were coupled with relatively good values of the fundamental variables[3].

We present a general equilibrium model where information spreads regarding the expected state of the economy generate *rational self-contained* bank runs. Information differentials between the bank and its depositors lead to the design of *incentive compatible*, state-contingent, and sequential demand deposit contracts that are subject to self-contained runs by depositors, whose information sets are defined by mean preserving spreads on the true expected state observed by the bank. These contracts dominate the autarky allocation, meaning that agents deposit funds even if they expect an attack to take place in (at least) one of the states of the world. Depositors contain the attack after different payment thresholds, corresponding to the sequential allocations composing the optimal mechanism designed by the bank, are reached. Containment requires the withdrawing line to be observable, which allows depositors to update their beliefs and infer the realized state of the economy. Being able to update their beliefs, agents play a series of signaling Bayesian coordination games after depositing, whose equilibria are defined by a subjective set of certainty equivalent constraints. The equilibria set of these games follows directly from the information and beliefs held by agents *before* depositing. That is, the set of games is played ex-ante, but agents consider all possible (subjective) ex-post realizations to define their strategies.

If a self-contained run takes place, imposing suspension of convertibility would assign an inefficient allocation to a subset of depositors with probability one. However, and restricted to the theoretical framework defined within this paper, suspension constitutes an optimal containment policy if it is

[1] This is not the case in Alonso (1996) and Jacklin and Bhattacharya (1988), where depositors receive signals on which they base their withdrawing decision. In this case, runs are entirely defined by the state of fundamentals. We explain below why we follow a different approach to model bank runs.

[2] The game has an additional equilibrium in mixed strategies as shown by Adao and Temzelides (1998), but it does not prevail if a strong form of forward induction requirement is imposed on the equilibrium. Postlewaite and Vives (1987) showed using a similar game that equilibria involving a positive run probability exist and are unique. They define three different types of agents, as opposed to the general case with only two types, depending on their degree of impatience. The most patient type does not misrepresent her type even though she knows that less patient types will do it with probability one. This is the case since their payoff, though affected by early withdrawals, is still higher under truth telling.

[3] The literature on this subject is quite extensive. See Furman and Stiglitz (1998) for a summary of the main results.

announced after agents deposit. On the other hand, the optimality of suspension does not necessarily hold if a more complex strategic dynamic environment with multiple banks is considered. We provide some basis for further research on this topic, since the current paper aims only to prove the existence of rational bank runs not based on the realization of an exogenous sunspot variable.

The paper proceeds as follows. We start by providing a description and basic analysis of one of the first bank runs to occur at the onset of the current financial crises, that is, the one that took place on Northern Rock. We also explain how the events shaping this run relate to the theoretical structure introduced in the current paper. A description of the strategic environment derived from the optimization problem faced by depositors is given in section 1. Section 2 introduces formally the pre- and post-deposit coordination structures generated by the bank through its sequential mechanism and defines their set of equilibria, which allows for the existence of rational self-contained runs. Section 3 illustrates numerically the theoretical results presented in the previous sections. Section 4 deals with suspension of convertibility and proposes several extensions of the current model. The final section concludes the paper.

A recent and highly relevant case: the run on Northern Rock. Shin (2009) states clearly the specific cause triggering the run on Northern Rock: "the key to the initial 'run' on Northern Rock was the nonrenewal of Northern Rock's short and medium-term paper" by wholesale depositors. Thus, given that short term wholesale funding accounts for funding with a maturity of less than six months raised from sophisticated financial institutions and the fully rational behavior expected from these institutions, one should seriously consider analyzing whether or not wholesale depositors were aware of the run taking place *before* depositing. The standard bank run literature has provided a partially positive answer to this question, relying on either exogenous sunspot variables taking place with a sufficiently low probability, see i.e. Diamond and Dybvig (1983), and Peck and Shell (2003), or signal realizations regarding a stochastic return variable that represents the economy fundamentals, see Jacklin and Bhattacharya (1988) and Alonso (1996).

Indeed, the bank run literature has recently assimilated into its strategic structure the unique equilibrium currency crises model of Morris and Shin (1998). Following this paper, Rochet and Vives (2004) and Goldstein and Pauzner (2005) base their works on the existence of strategic complementarities among depositors caused by noisy signals on the realization of the return fundamental variable. That is, these papers rely mainly on the state of fundamentals to justify their corresponding runs. However, as Shin (2009) emphasized, Northern Rock fundamentals were not particularly bad when it was hit by the wholesale run, i.e. "... even though the global credit crunch had a disproportionate effect on Northern Rock, it was not aimed at Northern Rock in particular" or "... an explanation of the run on Northern Rock should make reference to *market-wide* factors and not only to the characteristics of Northern Rock and its creditors viewed in isolation" (cursive in the original)[1].

Furthermore, as Shin (2009) underlines, "the branch deposits were actually the most stable of all deposits, and branch deposits were far more stable than the wholesale funding raised in the capital markets from sophisticated financial institutions". Such behavioral differences must arise from some type of heterogeneity in the defining characteristics of depositors. In particular, as Goldstein and Pauzner (2005) remark, "in many cases, the assumption that investors observe noisy signals is more realistic than the assumption that they all share the very same information and opinions". However, given the relatively short time horizons involving the not renewed deposits and the forecast capabilities of sophisticated financial institutions, there seems to be no apparent reason to assume a significant amount of noise in the determination of the return fundamental variable.

At the same time, it seems plausible to assume that sophisticated financial institutions are able to monitor each other's behavior and withdrawal activities, an aspect that standard bank run models do not account for. Indeed, sophisticated financial institutions would use such information to verify their beliefs regarding the withdrawal strategies of other sophisticated depositors and the ability of the bank to honor the initial demand deposit mechanism. These beliefs, that should account for the heterogeneous behavior displayed by sophisticated depositors and relate directly to their global strategy when projected to a particular bank such as Northern Rock, define a state of the banking system that is not necessarily based on the fundamental return distribution.

The lack of a theoretical strategic environment explaining the events observed in the Northern Rock run obliges us to build our framework on basic demand deposit contract structures while accounting for the characteristics of the run described above. That is, given the evidence presented by O'Connor and Santos Arteaga (2008) and Shin (2009), we ask if fully rational economic agents would be willing to deposit funds in a bank despite knowing that a run, which takes place independently of the fundamental

[1] These points are addressed by O'Connor and Santos Arteaga (2008), who illustrate how it is possible for a system of banks to be subject to an identical shock that affects all banks equally but triggers runs within just a subset of them.

return realization, is endogenously defined within the set of states of the economy, i.e. if depositing funds in such a situation delivers a higher expected welfare than remaining in autarky[1]. We provide a positive answer to this question and illustrate it numerically.

The main differences between our model and the bank run literature based on the second generation currency crises model of Morris and Shin (1998) are outlined below.

First, the strategic environment defining our model is based on a discrete set of depositors, which allows us both to analyze their monitoring abilities and to illustrate numerically their expected welfare induced strategies. These features remain undefined in theoretical frameworks relying on a continuum set of depositors, as Rochet and Vives (2004) and Goldstein and Pauzner (2005).

Second, we do not require a bankruptcy threshold to be surpassed in order to trigger a run on a bank, as is the case in Goldstein and Pauzner (2005). The runs generated within our theoretical setting are self-contained and take place without leading to a general run on the bank, as was the case in the wholesale run on Northern Rock.

Third, we assume a fixed (deterministic) fundamental return. The only source of information (and, therefore, beliefs) heterogeneity is defined in terms of the expected number of interim withdrawals, with *noiseless* state signals being given by the observed withdrawals of depositors[2].

The next section introduces the basic demand deposit contract structure that follows from the existence of financial intermediaries.

1. Theoretical strategic behavior

1.1. Initial assumptions. The basic model follows Peck and Shell (2003), who define an identical environment to the original one described in Diamond and Dybvig (1983), except for the fact that a finite set of identical agents, N, is assumed[3]. This assumption allows for monitoring by depositors through a countable withdrawing line. There are three time periods, $t = 0, 1, 2$. Each agent is endowed with one unit of an homogeneous good in period zero, which can be costlessly stored among periods. There exists a production technology that delivers one unit of output per unit invested if interrupted after one period, and $R > 1$ units of output if the invested unit is kept for two periods[4]. Agents are subject to an exogenous shock in period one defined by the set of possible realizations of a given random variable, Λ^b, with an associated probability function $f(\Lambda^b)$. To simplify notation we refer to:

$$f(\Lambda^b) = f(\lambda_i^b \mid \lambda_i^b \in \Lambda^b).$$

Consider the following probability space (Ω, I, F), where every state in the sample space, $\omega \in \Omega$, determines how many and which agents are affected by the shock. Moreover, I is a *σ-field* defined on Ω, and F is a probability measure on I. Define a random variable λ on the state space as follows:

$$\lambda : \Omega \to \Lambda^b \subseteq \{1,...,N\} : \omega \to \lambda_\omega^b,$$

where $\lambda_\omega^b = \lambda_i^b$ for some $i = 1,...,\# \ \Lambda^b$. The symbol # denotes the cardinality of the set defined after it, i.e. its number of elements. Note that while a given state determines how many and which agents are affected by the shock, the random variable reports only the former component. The number of agents affected by the shock is determined by the realization of the random variable and is Borel measurable with respect to the σ-field I, which allows both agents and the bank to infer the exact realization of the random variable, λ^*, in period one upon observing the length of the withdrawing line.

The agents affected by the shock become impatient, or type 1, and value consumption in period one only. The remaining agents, which we refer to as patient or type 2, value consumption in both periods. Denote by c_{tk}^i the amount of goods received by a type k agent in period t given state i. The state dependent utility of each agent is given by

$$U(c_{11}^i, c_{12}^i, c_{22}^i; \lambda_i^b) = \begin{cases} u(c_{11}^i(\lambda_i^b)) \text{ if the agent is impatient} \\ \rho u(c_{12}^i(\lambda_i^b) + c_{22}^i(\lambda_i^b)) \text{ if the agent is patient} \end{cases},$$

where ρ is the rate of time preference[5], λ_i^b represents a given relative state of the economy, such that $\lambda_i^b \in \Lambda^b$, defining the utility dependence on the realization of the random variable (as well as the sequential nature of the set of allocations offered by the

[1] In O'Connor and Santos Arteaga (2008) we allowed for *noiseless* interim signals on the return realization to be observed by a subset of depositors. It should be noted that such theoretical environment can be easily modified to incorporate the *ex ante* heterogeneity on the state variable realizations described in the current paper.

[2] It should be emphasized that allowing for heterogeneous interim return signals would not modify the results obtained. However, as already stated, our main objective is to verify the intrinsic instability of demand deposit mechanisms that are not subject to exogenous perturbations.

[3] Through the paper N will also stand for the cardinality of the set.

[4] We are assuming that agents can interrupt their investment after one period without suffering any penalty. Jacklin and Bhattacharya (1988) challenge this assumption in a framework with interim, as of period one, signals about the *stochastic* returns obtained in period two. This allows them to design rational, information-based runs dependent on the expected state of fundamentals defining the return.

[5] For simplicity, it is generally assumed that $\rho = 1$.

bank), and $u:\Re_{+}\rightarrow\Re$ is increasing, twice continuously differentiable, and satisfies the Inada conditions, $u'(0) = \infty$ and $u'(\infty) = 0$. We assume the following functional form through the paper

$$u(c)=\frac{c^{1-\beta}}{1-\beta}$$

with $\beta > 1$ everywhere. It is generally assumed that before a financial intermediary is created, agents maximize expected utility conditional on their information set, $\Gamma_a=\{N,\Lambda^b,f(\Lambda^b),T_a\}$, where the subscript refers to agent a, and T_a stands for the realization of the shock corresponding to agent a. The standard assumption regarding the information sets of potential depositors is that *all* of them have identical ones. Thus, if a financial intermediary is formed, it should inherit the homogeneous information set of its depositors regarding the expected state of the system, Λ^b and $f(\Lambda^b)$. That is, the potential bank and its depositors share information sets, as well as beliefs, except for the shock defining the type, which is privately observed by each agent and cannot be verified by the bank, assumed to be the only provider of liquidity in the economy[1]. The main results obtained in this paper derive from relaxing the assumption assigning identical information sets to the bank and its depositors[2].

1.2. Contract-induced coordination games. Consider the optimization problem faced by an *altruistic* bank, given the previous framework and conditional on its information set, $\Gamma_b=\{N,\Lambda^b,f(\Lambda^b)\}$, which is assumed (by the bank) to be identical to the one of its depositors, Γ_a, except for the privately observed type realization of the latter ones. Jointly with a sequential service constraint, the (assumed) altruistic bank maximizes the following value function $V(\Lambda^b, f(\Lambda^b))$:

$$\max_{\substack{c_{11}^i,\,c_{12}^i,\,c_{22}^i \\ i=1,\dots,\#\Lambda^b}} \sum_{\substack{i=1 \\ \lambda_i^b\in\Lambda^b}}^{\#\Lambda^b} f(\lambda_i^b)\left[\sum_{j=1}^{i}(\lambda_j^b-\lambda_{j-1}^b)u(c_{11}^j(\lambda_j^b))+(N-\lambda_i^b)\rho u(c_{12}^i(\lambda_i^b)+c_{22}^i(\lambda_i^b))\right],$$

where $\#\Lambda^b$ denotes the cardinality of the set of possible realizations, $\lambda_0^b=0$, and $(\lambda_j^b-\lambda_{j-1}^b)$ represents the number of agents contained between two consecutive realizations within Λ^b, subject to the set of budget constraints, which must be satisfied $\forall\lambda_i^b\in\Lambda^b$

$$\sum_{j=1}^{i}(\lambda_j^b-\lambda_{j-1}^b)u(c_{11}^j(\lambda_j^b))+(N-\lambda_i^b)\left(c_{12}^i(\lambda_i^b)+\frac{c_{22}^i(\lambda_i^b)}{R}\right)=N, \qquad i=1,\dots,\#\Lambda^b,$$

and a corresponding mechanism compatibility condition. To highlight the difference between the standard models studied in the literature and the current setting, we present the incentive compatibility constraint, *icc* henceforth, defining the optimal mechanism of the bank[3]

$$\sum_{i=1}^{\#\Lambda^b} f_b(\lambda_i^b)\left[u(c_{22}^i(\lambda_i^b))\right]\geq\sum_{i=1}^{\#\Lambda^b} f_b(\lambda_i^b)\sum_{j=1}^{i}\left[\frac{(\lambda_j^b-\lambda_{j-1}^b)}{\lambda_i^b+1}u(c_{11}^j(\lambda_j^b))+\frac{1}{\lambda_i^b+1}u(c_{11}^{i+1}(\lambda_i^b+1))\right], \qquad (1)$$

such that

$$f_b(\lambda_i^b)=\frac{\left[1-(\lambda_i^b/N)\right]f(\lambda_i^b)}{\sum_{\lambda'=0}^{N-1}[1-(\lambda'/N)]f(\lambda')}, \qquad \forall\lambda_i^b,\lambda'\in\Lambda^b,$$

where $f(\lambda_i^b)$ stands for the ex ante (prior) probability assigned to the number of impatient agents being λ_i^b, with $\lambda_i^b\in\Lambda^b\subseteq\{1,\dots,N\}$. This function is generally assumed to be common knowledge among all agents, and therefore, also between depositors and the bank, in period zero. *Patient* agents update this probability in period one, after receiving the type-determining signal, using Bayes' rule[4]. The updating process delivers the subjective probability, as of each agent, of having λ_i^b impatient depositors in the system, conditional on the agent being patient, $f_b(\lambda_i^b)$. These probabilities are used by depositors to calculate the expected payoffs on which to base their strategic behavior in the post-deposit game generated by the bank contract structure that solves the previous optimization problem. Beliefs are a direct function of $f(\lambda_i^b)$, assumed to be uniquely defined with the probability space and known by all agents. This homogeneity assumption implies that depositors share the beliefs of the bank, denoted by

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[1] Jacklin (1987) provides a formal justification regarding the inefficiency caused by alternative asset markets introduced within the current context. In short, Jacklin (1987) shows that the demand deposit contracts designed by the bank achieve greater risk sharing than equity shares if trade is restricted among agents.

[2] The lack of financial transparency was presented by Furman and Stiglitz (1998) as one of the main plausible causes triggering the Asian crises.

[3] Obviously, non-negativity constraints are also imposed on all consumption allocations.

[4] Peck and Shell (2003) were the first ones to define explicitly the belief formation process of the agents. Endowing agents with the ability to generate subjective expected payoffs is essential to model strategic interactions among depositors in the post-deposit game.

$f_b(\lambda_i^b)$, the subscript b representing the bank, when defining their optimal strategies.

Compare now the incentive compatibility equilibrium condition to the set of conditions required to guarantee the existence of stable equilibria through the set of post-deposit games defined for each $\lambda^w \le \lambda^b_{\#\Lambda^b}$, where λ^w stands for the number of withdrawing agents observed in line and $\lambda^b_{\#\Lambda^b}$ corresponds to the supremum of Λ^b.

The *icc* condition is based on the expected payoffs that the bank assumes its depositors calculate if they are not able to observe the length of the line before deciding whether or not to withdraw, leading to a static (calculated and fixed as of period zero) inequality in expected payoffs

$$E[u(c_{22}^{i}(\lambda_i^b))] \ge E[u(c_{11}^{i+1}(\lambda_i^b + 1))], \quad (2)$$

which defines a *unique* Bayesian post-deposit game and its corresponding equilibrium[1]. The ability to observe the length of the withdrawing line leads depositors to (subjectively) generate a set of post-deposit games for each $\lambda^w \le \lambda^b_{\#\Lambda^b}$, as well as a set of *certainty equivalent constraints*, *cec* hereafter, defining the respective equilibria

$$E[u(c_{22}^{w}(\lambda^w))] \ge u(c_{11}^{w+1}(\lambda^w + 1)). \quad (3)$$

These equilibrium constraints imply that *patient* agents prefer to misrepresent their type if the utility derived from the allocation offered to the next impatient agent in line is strictly larger than the expected utility from waiting to withdraw until period two. That is, patient agents attack the bank if the allocation received by impatient depositors is larger than the certainty equivalent corresponding to the lottery faced by waiting. The set of lotteries from which the *cec* are derived is based on the set of realizations observed by depositors and their subjective beliefs, updated following Bayes' rule conditional on the number of withdrawing agents in line. Upon direct observation of both post-deposit equilibrium stability requirements it is clear that the set of *cec* imposes a stricter constraint on the mechanism than the *icc*. Thus, the stability conditions (at least a subset of them) defined by the set of *cec* would be violated if the *icc* binds in equilibrium with equality.

It should be emphasized that the entire optimization problem is based on the set of expected realizations of the random variable and its induced probability distribution. The sequential structure of the service constraint defining the solution mechanism is directly generated by these variables through the optimization problem faced by the bank. Moreover, the equilibrium of the model is determined by the effects of Λ^b and $f(\Lambda^b)$ on the set of *cec*. These constraints do not only allow for the implementability of the mechanism obtained as an optimal solution to the optimization problem, but define its stability. As we will show, the set of *cec* guarantees the existence of no-run equilibria within the set of Bayesian games generated by the mechanism, such that we can rely on the revelation principle to eliminate any possible attack from the optimal communication strategies of depositors.

1.3. Updating the Bayesian beliefs of depositors. Assume from now on that the prior set of expected realizations, Λ^b, and the probability function defined on it, $f(\Lambda^b)$, are no longer common knowledge in period zero and may differ between the bank and its depositors. This restriction allows for heterogeneous beliefs among agents, caused by differences between the set of expected realizations (and its associated probability function) defining the optimal mechanism and the set of possible realizations considered by depositors[2]. The latter set equals either $\{1,...,N\}$, if agents have no information about the possible realizations of λ, i.e. agents exhibit total aggregate uncertainty, or Λ^a, if information differentials lead to different expected sets of realizations. We will restrict attention to Λ^a information sets generated by mean preserving spreads on the set of expected realizations observed by the bank. Such a restriction represents a natural economic framework to analyze information differentials. This is not equivalent to affirm that this restriction is either necessary or sufficient to generate rational self-contained runs. Finally, assume that, given the informational advantage that the bank may develop over its depositors, Λ^b and $f(\Lambda^b)$ define the correct expected state of the world.

Consider the set of realizations expected by the agents before depositing, Λ^a, and its associated subjective probability function $f_a(\Lambda^a)$. Define $f(\lambda_i^a \mid p, \lambda^w)$ as the updated Bayesian beliefs of depositors conditional on the agent being patient, p, and the number of withdrawing agents observed, λ^w,

$$f(\lambda_i^a \mid p, \lambda^w) = \frac{f(\lambda_i^a \mid \lambda^w) f(p \mid \lambda_i^a)}{\sum_{\lambda'=1}^{N-1} f(p \mid \lambda') f(\lambda' \mid \lambda^w)}, \qquad \lambda' \in \Lambda^a$$

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[1] It is a known result that the optimal demand deposit contract assigns $c_{21}^i(\lambda_i^b) = c_{12}^i(\lambda_i^b) = 0, \ \forall \lambda_i^b \in \Lambda^b$.

[2] Heterogeneous expectations are justified as a direct consequence of information differentials between the bank and its depositors. Jacklin and Bhattacharya (1988) rely on differences in the risk aversion coefficient of agents to justify heterogeneity in the acquisition of information. We could also assume that financial institutions have easier access to information sources due to scale economies reducing the acquisition costs of information relative to individual agents, or that they have private incentives, either altruistic or egoistic, to develop an informational advantage over their depositors.

with an updated subjective probability for each agent defined by

$$f(\lambda_i^a \mid \lambda^w) = \frac{f(\lambda^w \mid \lambda_i^a) f_a(\lambda_i^a)}{\sum_{\lambda'=1}^{N} f(\lambda^w \mid \lambda') f_a(\lambda')}, \qquad \lambda' \in \Lambda^a,$$

where $\lambda_i^a \in \Lambda^a$. The probability $f(\lambda_i^a \mid \lambda^w)$ represents the private beliefs of depositors about the possible states of the world defined within their subjective set of realizations, $\lambda_i^a \in \Lambda^a$, after observing λ^w agents in line. These beliefs are directly based on the probability function assigned by the agents to the set of possible states, $f_a(\Lambda^a)$.

Beliefs are updated for each value of $\lambda^w = 1,...,\lambda^a_{\#\Lambda^a}$, with $\lambda^a_{\#\Lambda^a}$ representing the supremum of the set Λ^a. From the point of view of a given depositor, the length of the line represents the communication strategies of the remaining agents, since we are considering equilibira in pure strategies and actions are equivalent to messages used as noiseless signal of the state. Without a sunspot variable coordinating all depositors on a run, withdrawals behave as public signals reflecting the strategies of depositors. An observed signal, in the form of λ^w, modifies the beliefs of agents regarding the state of the world conditional on $\lambda_i^a \in \Lambda^a$. Individual agents assume all remaining depositors to share their set of beliefs and act accordingly. Thus, agents expect other depositors to misrepresent their types when there exists an incentive to do so themselves. Agents are, at the same time, aware of the fact that by attacking the bank they will be modifying the final set of realizations, and the behavior of other depositors who react to their signal. The subjectively updated Bayesian probability function of depositors must reflect all the described information properties of the withdrawing line.

The term $f(p \mid \lambda_i^a)$ measures the subjective degree of pessimism inherent to each agent[1]. We assume a neutral distribution through the paper, but the model can easily be extended to analyze the effects of pessimism, and for that matter optimism, on the deposit and withdrawal decisions of agents.

2. Post-deposit coordination games

2.1. Pre- and post-deposit equilibria. Despite the existence of two differentiated coordination games, recognized by Adao and Temzelides (1998), and Peck and Shell (2003), the unique equilibrium (in pure strategies) defined by the depositing decision of agents is assumed to pervade through both games, unless altered by an exogenous sunspot shock in the interim period. There exists a tacit agreement regarding the unique run-free Nash equilibrium of the pre-deposit coordination game. Agents do not deposit funds if they expect a run in the interim period, see Postlewaite and Vives (1987), unless it is caused by a sunspot variable taking place with low enough probability. The post-deposit game is therefore identical to the pre-deposit one, and preserves the same run-free equilibrium. This result relies heavily on the assumed homogeneity of beliefs (and strategies) among depositors, and, particularly, between depositors and the bank. Once information and beliefs are allowed to differ, the resulting equilibrium supports rational self-contained runs expected by the agents *before* depositing, not necessarily generated by an exogenous sunspot variable.

2.2. The set of post-deposit games. The solution to the bank optimization problem is given by a mechanism, $\mathbf{m}(f(\Lambda^b))$, consisting of a vector of optimal and feasible state contingent allocations defined for each value in the set $\Lambda^b = \{\lambda_1^b, \lambda_2^b\}$, and their corresponding probabilities $f(\Lambda^b) = \{f(\lambda_1^b), f(\lambda_2^b)\}$[1]

$$\mathbf{m}(f(\Lambda^b)) = (c_1^1(\lambda_1^b), c_1^2(\lambda_2^b), c_2^1(\lambda_1^b), c_2^2(\lambda_2^b)),$$

where $c_1^1(\lambda_1^b)$ denotes the consumption offered to the first λ_1^b agents in line who declare being impatient, while $c_1^2(\lambda_2^b)$ is allocated to each of the remaining impatient agents in line, in case of an attack up to the point where the bank runs out of reserves[2]. The remaining allocations composing the mechanism vector, $(c_2^i(\lambda_i^b))$, $i = 1, 2$, define the consumption given to patient agents in period two, depending on the number of withdrawing agents in period one.

The strategies of depositors, based on their subjective expected payoffs, defining the Nash equilibria of the set of post-deposit games generated by the bank through $\mathbf{m}(f(\Lambda^b))$, depend on λ^w. That is, a

[1] Neutrality of the distribution is usually assumed under risk, $f(p \mid \lambda_i^a) = [1 - (\lambda_i^a / N)]$. However, in an uncertainty context, associated with a subjective probability function $f_u(\cdot)$, prospect theory suggests pessimistic biases in the subjective expectations of agents, $f_u(p \mid \lambda_i^a) < f(p \mid \lambda_i^a)$, for $\lambda_i^a < \lambda^*$, with $\lambda^* \in [0, N]$ defined by each depositor. In words, agents overvalue the subjective probability of being affected by the shock in relatively good states of the world, i.e. realizations of the random variable below a given cut off point λ^*, and undervalue it in the bad ones. For additional prospect theory results and a quantitative analysis of the subjective parameters see Kahneman and Tversky (1979), and Tversky and Kahneman (1992).

[2] The dimension of Λ^b has been chosen to simplify the presentation without loss of generality. The main results of the paper hold for any n-dimensional, $n \le N$, set of realizations, as long as the mechanism allows for the existence of runs on the bank. At the same time, since optimality requires $c_{21}^i(\lambda_i^b) = c_{12}^i(\lambda_i^b) = 0, \forall \lambda_i^b \in \Lambda^b$, subscripts will denote from now on both the type of depositor and consumption period.

signalling Bayesian game will be played by all patient depositors for each $\lambda^w \le \lambda^a_{\#\Lambda^a}$. The post-deposit game theoretical setting induced by the mechanism is no longer unique, as is the general case considered in the literature, but composed by a set of signalling Bayesian coordination games. Denote this set by Ξ. The dependence of the expected payoffs of depositors, defining each game within Ξ, on the value of λ^w must be reflected in the payoff matrix of the corresponding game, denoted by $\chi(\lambda^w)$, which is given by

		$-(i\text{-}\lambda^w)$	
		Withdraw	Not withdraw
i	Withdraw	$E[u(c_1^r(\lambda^w))]$, $E[u(c_1^r(\lambda^w))]$	$u(c_1^p(\lambda^w+1))$, $E[u(c_2^p(\lambda^w+1))]$
	Not withdraw	$u(c_2^r)$, $E[u(c_1^{r-1}(\lambda^w))]$	$E[u(c_2^{nr}(\lambda^w))]$, $E[u(c_2^{nr}(\lambda^w))]$

where

$$E[u(c_1^r(\lambda^w))] = \left(\frac{\lambda_1^b - \lambda^w}{\lambda^N - \lambda^w}\right) u(c_1^1(\lambda_1^b)) + \left(\frac{\lambda^{N-1} - \lambda_1^b}{\lambda^N - \lambda^w}\right) u(c_1^2(\lambda_2^b)) + \left(\frac{1}{\lambda^N - \lambda^w}\right) u(N - \lambda_1^b c_1^1(\lambda_1^b) - (\lambda^{N-1} - \lambda_1^b) c_1^2(\lambda_2^b)),$$

$$u(c_2^r) = u([N - \lambda_1^b c_1^1(\lambda_1^b) - (\lambda^{N-1} - \lambda_1^b) c_1^2(\lambda_2^b)]R)$$

$$E[u(c_1^{r-1}(\lambda^w))] = \left(\frac{\lambda_1^b - \lambda^w}{\lambda^{N-1} - \lambda^w}\right) u(c_1^1(\lambda_1^b)) + \left(\frac{\lambda^{N-1} - \lambda_1^b}{\lambda^{N-1} - \lambda^w}\right) u(c_1^2(\lambda_2^b)),$$

$$u(c_1^p(\lambda^w + 1)) = \begin{cases} u(c_1^1(\lambda_1^b)) & \text{if } \lambda^w < \lambda_1^b \\ u(c_1^2(\lambda_2^b)) & \text{if } \lambda^w \ge \lambda_1^b \end{cases},$$

$$E[u(c_2^p(\lambda^w+1))] = \sum_{\lambda_i^a \le \lambda_1^b} f(\lambda_i^a \mid p, \lambda^w+1) u\left(\frac{(N - \lambda_i^a c_1^1(\lambda_1^b))R}{N - \lambda_i^a}\right) + \sum_{\lambda_i^a > \lambda_1^b} f(\lambda_i^a \mid p, \lambda^w+1) u\left(\frac{(N - \lambda_1^b c_1^1(\lambda_1^b) - (\lambda_i^a - \lambda_1^b) c_1^2(\lambda_2^b))R}{N - \lambda_i^a}\right),$$

$$E[u(c_2^{nr}(\lambda^w))] = \sum_{\lambda_i^a \le \lambda_1^b} f(\lambda_i^a \mid p, \lambda^w) u\left(\frac{(N - \lambda_i^a c_1^1(\lambda_1^b))R}{N - \lambda_i^a}\right) + \sum_{\lambda_i^a > \lambda_1^b} f(\lambda_i^a \mid p, \lambda^w) u\left(\frac{(N - \lambda_1^b c_1^1(\lambda_1^b) - (\lambda_i^a - \lambda_1^b) c_1^2(\lambda_2^b))R}{N - \lambda_i^a}\right),$$

$E[u(c_1^r(\lambda^w))]$ corresponds to the *expected* payoff, in utility terms, received by an agent if all depositors attack the bank in period one. The superscript in λ^N indicates the number of agents withdrawing funds in period one, i.e. the length of the withdrawing line, when no subscript is used to define the variable. Given N and Λ^b, $\frac{\lambda^{N-1} - \lambda_1^b}{\lambda^N - \lambda^w}$ refers to the probability of reaching the line within the interval $(\lambda_1^b, \lambda^{N-1}]$ if a general run on the bank takes place after observing λ^w agents in line. All agents are assumed identical in this respect. There is no agent with an exogenous a priori advantage allowing him to reach the line before others do.

$u(c_2^r)$ derives directly from the budget constraint of the bank, and represents the utility value of consumption for the last patient agent if all other depositors have declared to be impatient. Note that if

$$\left[N - \sum_{w=1}^{N-1} c_1(\lambda^w)\right] > 0$$

a patient depositor who knows to be the last agent in line has no incentive to misrepresent her type. The last depositor, if patient, receives in period two the return of whatever remains after all the other agents have withdrawn invested in the long run technology. $u(c_1^p(\lambda^w+1))$ is the utility obtained by a patient depositor who withdraws funds in period one after observing a line of length λ^w, while the remaining patient agents do not misrepresent their types simultaneously.

$E[u(c_2^{nr}(\lambda^w))]$ and $E[u(c_2^p(\lambda^w+1))]$ follow from the set of *cec* conditions defining the no-run equilibria of all possible post-deposit games played by patient depositors[1]

$$E[u(c_2^{nr}(\lambda^w))] \ge u(c_1^p(\lambda^w+1)), \forall \lambda^w \le \lambda^a_{\#\Lambda^a}. \quad (4)$$

Differentiating rational self-contained runs from sunspot-based general ones requires the run equilibra of all games, $\chi(\lambda^w)$ with $\lambda^w = 1, ..., \lambda^a_{\#\Lambda^a}$, to be defined on an unilateral basis and not to involve the entire subset of patient depositors. We want to prevent the generation of herds and allow for agents to distribute the attacks depending on the value of λ^w. Thus, the set of run equilibria must consist of self-contained attacks, based on $u(c_1^p(\lambda^w+1)) > E[u(c_2^{nr}(\lambda^w))]$, that do not spread to the entire subset of patient depositors, which is the case as long as

$$E[u(c_2^p(\lambda^w+1))] > E[u(c_1^r(\lambda^w))], \ \forall \lambda^w \le \lambda^a_{\#\Lambda^a}.$$

2.3. Communication strategies. This section introduces formally the communication strategies of patient depositors defining the possible equilibria of each game within the set of post-deposit games generated by each value of $\lambda^w \le \lambda^a_{\#\Lambda^a}$. Agents are able to

[1] When defining $E[u(c_2^{nr}(\lambda^w))]$, $E[u(c_2^p(\lambda^w+1))]$, $E[u(c_1^r(\lambda^w))]$ and $u(c_2^r)$, we have assumed that all impatient agents up to $\lambda^a_{\#\Lambda^a} \le (N-1)$ are paid $c_1^2(\lambda_2^b)$, and that the last remaining depositor receives a strictly positive consumption allocation in case of a general run. While this is not necessarily true, it has been done to simplify the matrix entrances and does not affect the results presented.

calculate in period zero, *before* knowing their type, the set of all possible games they will be facing in period one depending on the number of withdrawing agents in line. This is the case since both the withdrawing *and* depositing decisions are based on the expected set of equilibria subjectively obtained by the agents in period zero for each $\chi(\lambda^w) \in \Xi$, with $\lambda^w = 1,..., \lambda^a_{\#\Lambda^a}$. Formally, every agent defines her set of strategies based on the subjective expected utilities obtained from the mechanism designed by the bank for all possible values of $\lambda^w \le \lambda^a_{\#\Lambda^a}$

$$E[u_a]: M \times \{1,..., \lambda^a_{\#\Lambda^a}\} \times F_a(\Lambda^a) \to$$
$$\to \{E[u(\mathbf{m}_a(f_a(\Lambda^a) \mid \mathbf{m}(f(\Lambda^b)), \lambda^w))]: \lambda^w \le \lambda^a_{\#\Lambda^a})\},$$

where M corresponds to the set of mechanisms offered by the bank, with $\mathbf{m}(f(\Lambda^b)) \in M$, while $F_a(\Lambda^a)$ is the set of subjective probability functions defined by the agents on their expected set of realizations, $f_a(\Lambda^a) \in F_a(\Lambda^a)$. The previous function is a mapping from the set of allocations offered by the bank and the set of subjective beliefs held by agents to the set of expected utilities derived from the subjective mechanisms calculated by the depositors, $E[u(\mathbf{m}_a(f_a(\Lambda^a) \mid \mathbf{m}(f(\Lambda^b)), \lambda^w))]$, that are used to define the entries of the $\chi(\lambda^w)$ payoff matrix, for each $\lambda^w \le \lambda^a_{\#\Lambda^a}$. Every subjective mechanism calculated for a given realization of λ^w, $\mathbf{m}_a(f_a(\Lambda^a) \mid \mathbf{m}(f(\Lambda^b)), \lambda^w)$, defines a corresponding expected game, $\chi(\lambda^w)$, within the set Ξ. Thus, the set of games played by the agents is the image of a function of all pairs of the form $(\mathbf{m}_a(f_a(\Lambda^a) \mid \mathbf{m}(f(\Lambda^b)), \lambda^w), f_a(\Lambda^a))$.

The bank, endowed with the information set $\Gamma_b = \{N, \Lambda^b, f(\Lambda^b)\}$, designs an optimal incentive compatible mechanism $\mathbf{m}(f(\Lambda^b))$, using the following allocation rule

$$\alpha: F(\Lambda^b) \to [C]^{2(\#\Lambda^b)}: f(\Lambda^b) \to \mathbf{m}(f(\Lambda^b)) = (c^i_k(\lambda^b_i))^{i=1,...,\#\Lambda^b}_{k=1,2},$$

where $F(\Lambda^b)$ is the set of probability functions that may be induced by a given set of realizations, Λ^b. This rule results in the set of optimal (and feasible) state contingent allocations that will be used to define the mechanism. Note that the domains of the allocation rule and the optimal mechanism calculated by the bank have been defined in terms of $F(\Lambda^b)$. This dependence highlights the stochastic nature of the optimal allocation set, the mechanism, and the resulting set of post-deposit games, which are *all* based on the expectations hold by the bank, but not necessaryly by its depositors.

The communication strategy of a *patient depositor*, μ_a, defined for each $\chi(\lambda^w) \in \Xi$, with $\lambda^w \le \lambda^a_{\#\Lambda^a}$, is given by the following map

$$\mu_a : \Xi \to \{w, nw\}, \quad \forall a \text{ patient}$$

that assigns to each game $\chi(\lambda^w)$ within Ξ an equilibrium withdrawing decision, consisting of either withdrawing funds, *w*, or waiting, *nw*. To simplify notation, we will denote the communication strategy corresponding to the *a*-th depositor and game $\chi(\lambda^w)$ by $\mu_a(\lambda^w)$. The communication profile of a patient depositor is given by a vector of messages (communication strategies) defined for each one of the games expected to be played in period one

$$\mu_a(\Xi) = (\mu_a(\lambda^a_1),..., \mu_a(\lambda^a_{\#\Lambda^a})) \in \{w, nw\}^{\lambda^a_{\#\Lambda^a}}, \quad \forall a \text{ patient}$$

The vector of messages derived from the communication strategies of *all* depositors is simply λ^w, since the message space is equivalent to the space of actions. Agents consider a set of optimal communication profiles formed by their optimal vector of messages and those of all remaining depositors

$$\{\mu^*_a(\Xi), \mu^*_{-a}(\Xi)\} = \Phi_a, \quad \forall a \text{ patient}$$

where $\mu^*_{-a}(\Xi)$ assigns an identical optimal communication profile to all patient depositors, given by $\mu^*_a(\Xi)$. This assumption is justified by the lack of communication among depositors, standard to this type of models, and the absence of any additional signals privately received by depositors regarding the beliefs of other agents, a la Morris and Shim, in the interim period. The union set of optimal communication profiles defining the final value of λ^w is given by Φ, and includes the vectors of messages generated by all depositors, allowing for heterogeneous beliefs among them.

2.4. Post-deposit equilibria set. In order to study the interim period general rational expectations equilibrium, we must consider the set of equilibria defined for each and every Bayesian game played by patient agents after depositing, $\chi(\lambda^w) \in \Xi$, for any possible value of $\lambda^w \in \{1,..., N\}$.

Definition 1: Bayesian equilibrium of a post-deposit game. A perfect Bayesian equilibrium of a signalling interim (post-deposit) game, $\chi(\lambda^w) \in \Xi$, is given by a communication strategy, $\mu^*_a(\lambda^w)$, and the corresponding updated beliefs $f(\lambda^a_i \mid p, \lambda^w)$, such that $\forall a \in N \setminus [\lambda^b_j]$, where N defines the set of all agents, and $[\lambda_j^b]$, $j = 1,..., \# \Lambda^b$, stands for the set of agents affected by the shock, and for all

$$\lambda^w = 1,...,\lambda^a_{\# \Lambda^a}, \quad \begin{array}{l} E[u(c(\lambda^w) \mid \chi(\lambda^w), \mu_a^*(\lambda^w))] \geq \\ E[u(c(\lambda^w) \mid \chi(\lambda^w), \mu_a(\lambda^w))] \end{array}$$

which in expected utility terms translates into

$$\sum_{\lambda_i^a \in \Lambda^a} f(\lambda_i^a \mid p,\lambda^w) u_a(\mathbf{m}_a(f_a(\Lambda^a)) \,|\, \mathbf{m}(f(\Lambda^b)), \lambda^w, \mu_a^*(\lambda^w)) \geq$$
$$\sum_{\lambda_i^a \in \Lambda^a} f(\lambda_i^a \mid p,\lambda^w) u_a(\mathbf{m}_a(f_a(\Lambda^a)) \,|\, \mathbf{m}(f(\Lambda^b)), \lambda^w, \mu_a(\lambda^w)).$$

For the entire post-deposit game theoretical structure to hold, there must exist at least one Bayesian Nash equilibrium in pure strategies per game, defined by the optimal communication strategies of patient depositors after observing λ^w agents in line and updating their subjective beliefs.

Runs are eliminated from the communication strategies of depositors if, and only if, we are able to guarantee the existence of a non-run Bayesian Nash equilibrium for each and every post-deposit game expected to be played by the agents. This stability condition follows directly from the application of the revelation principle to the entire set of interim games.

Definition 2: Revelation principle. Define the set of messages sent by the agents to be the set of withdrawing depositors, λ^w, due to the existing equivalence between messages and actions. When an agent withdraws funds is also sending an observable message revealing that is impatient. The message space is then $\{w, nw\}^N$, with $\lambda^w \in \{w, nw\}^N$, and is defined by the communication strategies of depositors for each and every possible value of λ^w. Consider the stochastic allocation rule defined by the bank depending on the set of expected messages, $\alpha(f(\Lambda^b))$. Assume that a mechanism is given, $\mathbf{m}(f(\Lambda^b))$, that both induces and is defined by the expected set of messages Λ^b, with an associated allocation function $\alpha(f(\Lambda^b))$, such that the set of Bayesian post-deposit games generated by $\mathbf{m}(f(\Lambda^b))$ has a set of Bayesian Nash equilibria defined by the communication profile $\mu^*(\lambda^w) = (\mu_1^*(\lambda^w), \mu_2^*(\lambda^w),...,\mu_{N-\lambda^{b*}}^*(\lambda^w))$, for all $\lambda^w \leq \lambda^b_{\#\Lambda^b}$, where all patient agents report truthfully. That is, for a given realization of the random variable defining the true state of the economy $\lambda^{b*} \in \Lambda^b$, we have $\lambda^w = \lambda^{b*}$, and $\mu_a^*(\lambda^w) = nw$, $\forall a \in N \backslash [\lambda^{b*}]$, and $\lambda^w \leq \lambda^{b*}$.

The revelation principle states that, given the previous Bayesian equilibrium, there exists a direct revelation mechanism, $(\mu'(\lambda^w), \alpha')$, with $\alpha' = \alpha(f(\Lambda^b))$, since it is a stochastic rule, such that $\mu'(\lambda^w) = \mu^*(\lambda^w)$, and all the agents accepting the mechanism report truthfully.

In other words, the revelation principle states that runs on the bank can be prevented, except if caused by a sunspot realization, as long as *each and every* post-deposit game played by patient depositors has a no-run Nash equilibrium where agents can coordinate their communication strategies.

We define now the set of rational expectations equilibria induced by $\mathbf{m}(f(\Lambda^b))$. In order to do so, the Bayesian equilibrium definition presented above must be integrated in a setting with multiple games, whose payoffs matrices are defined as functions of the sequential allocations composing the mechanism, and the set of subjective beliefs held by depositors.

Definition 3: Rational expectations equilibria of the deposit games. A set of rational expectations equilibria for the corresponding set of deposit games, both pre and post, is given by an implementable mechanism designed by the bank, $\mathbf{m}(f(\Lambda^b))$, defining a set of allocation vectors for all possible realizations within Λ^b, and a communication profile, $\mu^*(\lambda^w)$, $\forall \lambda^w \leq \lambda^a_{\#\Lambda^a}$, generated by the communication strategies of patient depositors, such that:

1. The set of allocation vectors composing the mechanism maximizes the expected utility of depositors, $\mathbf{m}^*(f(\Lambda^b)) \in \arg\max V(\Lambda^b, f(\Lambda^b))$, and satisfies the budget, certainty equivalent (incentive compatibility), and non-negativity constraints. $V(\Lambda^b, f(\Lambda^b))$ is the true value function, defining the optimization problem faced by agents in period zero for the probability function, $f(\Lambda^b)$, and the set of expected realizations, Λ^b, observed by the bank, which correspond to the true expected state of the economy.
2. The mechanism satisfies a sequential service constraint. The consumption allocation received by an impatient agent in period one, $c_1(\lambda^w)$, depends only on the number of agents standing before him in line, $\lambda^w - 1$. The number of allocations defining the sequential withdrawing intervals of the mechanism, $c_1^i(\lambda_i^b), \forall \lambda_i^b \in \Lambda^b$, is given by the cardinality of the set Λ^b.
3. The set of optimal communication profiles, Φ, based on the private beliefs of depositors for a given (optimal) mechanism $\mathbf{m}^*(f(\Lambda^b))$, with $(\mu_a^*(\lambda_1^a),...,\mu_a^*(\lambda^a_{\#\Lambda^a})) \in \Phi_a$, $\forall a$ patient, where $(\mu_a^*(\lambda_1^a),..., \mu_a^*(\lambda^a_{\#\Lambda^a}))$ represents the vector of optimal communication strategies for agent a given the set of games generated by all the possible values of λ^w that can be observed according to her private set of expected realizations, Λ^a,

defines a set of perfect Bayesian equilibria for the set of signalling games played in the interim period, $\forall a \in N \backslash [\lambda^{b^*}]$, and $\forall \lambda^w \in \{1, \ldots \lambda^b_{\#\Lambda^b}\}$. The homogeneity assumption on the set of private beliefs defined among patient agents, $F_a(\Lambda^a)$, simplifies this requirement to a unique communication profile, since $\mu_a(\Xi) = \mu_{a'}(\Xi)$, for all depositors $a \neq a'$, with $a, a' \in N \backslash [\lambda^{b^*}]$.

4. Agents exhibit Bayesian rationality when updating the set of subjective beliefs used to define their communication strategies in each one of the corresponding post-deposit games, $\chi(\lambda^w) \in \Xi$, with $\lambda^w \in \{1, \ldots, \lambda^a_{\#\Lambda^a}\}$. The set of optimal communication profiles, Φ, gives place to a set of noiseless signals represented by the length of the withdrawing line, λ^w. These signals are used by patient depositors to update their beliefs about the expected state realization and the corresponding consumption received in period two, which derives directly from the set of equilibria generated by the optimal communication profiles within Φ through the expected remaining games in the set, $\chi(\lambda^w) \in \Xi$, for $\lambda^w \in \{1, \ldots, \lambda^a_{\#\Lambda^a}\}$. Subjective beliefs generate the forward induced structure that defines the set of optimal communication profiles of patient depositors, giving place to the set of Bayesian Nash equilibria for the entire set of interim games.
5. The revelation principle holds when active throughout the entire set of post-deposit games. Consider the game $\chi(\lambda^w) \in \Xi$, defined by patient depositors through $\mathbf{m}^*(f(\Lambda^b))$, and $f(\lambda^a_i \mid p, \lambda^w)$, $\forall \lambda^a_i \in \Lambda^a$, after observing λ^w withdrawing agents. Given the *cec* subjectively calculated by depositor for $\chi(\lambda^w)$, $\mu^*_a(\lambda^w) = nw$, $\forall a \in N \backslash [\lambda^{b^*}]$, if there exists a Bayesian Nash equilibrium in this game defined by the truthful communication strategies of all depositors. However, if $u(c^p_1(\lambda^w + 1)) > E[u(c^{nr}_2(\lambda^w))]$, $\mu^*_a(\lambda^w) = w$, $\forall a \in N \backslash [\lambda^{b^*}]$. In this case, the perfect Bayesian equilibrium of the signaling game $\chi(\lambda^w)$ is given by either an unilateral attack, if $E[u(c^p_2(\lambda^w + 1))] \geq E[u(c^r_1(\lambda^w))]$, or a general run on the bank, if $E[u(c^p_2(\lambda^w + 1))] < E[u(c^r_1(\lambda^w))]$. The revelation principle does not hold and cannot guarantee a no-run (truth-telling) equilibrium for any subset of post-deposit games where $u(c^p_1(\lambda^w + 1)) > E[u(c^{nr}_2(\lambda^w))]$.
6. Given $\mathbf{m}^*(f(\Lambda^b))$, the set of subjectively updated beliefs, $f(\lambda^a_i \mid p, \lambda^w)$, for all $\lambda^a_i \in \Lambda^a$, generating the set of optimal communication profiles for all patient depositors, Φ, and the set of Bayesian Nash equilibria defined by the optimal communication profiles (based on Λ^a and $f_a(\Lambda^a)$) for each and every post-deposit game within the set Ξ, with $\chi(\lambda^w) \in \Xi$, $\forall \lambda^w \in \{1, \ldots, \lambda^a_{\#\Lambda^a}\}$, agents prefer depositing funds in the bank to autarky.

The literature on bank runs considers a unique post-deposit game for a given $f(\Lambda^b)$, whose no-run equilibrium is guaranteed by a direct application of the revelation principle. The equilibrium set defined for the subjective set of signalling Bayesian games allows for bank runs that follow from the *optimal* communication strategies of patient depositors.

2.5. On Mechanism monotonicity and self-contained run equilibria. Sunspot variables lead to general runs on the bank if realized, which conditions the depositing decision of agents on the imposition of an exogenous constraint on their realization probabilities. On the other hand, given $f_a(\Lambda^a)$, bank runs must self-contain to be compatible with the rational deposit equilibria definition presented above and for agents to deposit in period zero. A run on the bank can be contained if the mechanism allows for individual incentives to run for a given subset of values defined within the sequential intervals induced by Λ^a, but at the same time keeps all patient depositors from withdrawing simultaneously, given $f_a(\Lambda^a)$ and assuming that all agents share identical beliefs. That is, runs are contained if, after having an incentive to unilaterally misrepresent their types, the allocation adjustment process defined by the mechanism suffices to coordinate the strategies of patient depositors back on the truth telling equilibrium.

The proof of the following theorem, as well as that of the lemma illustrating the decreasing monotonicity of the demand deposit mechanism in λ^b_i, are presented in Di Caprio and Santos Arteaga (2010)[1].

Theorem 1. Assume that the bank does not have enough resources to honor the sequential allocations composing the mechanism in case of a general run. If a mechanism is vulnerable to general runs, there exist subjective sets of random variable realizations, Λ^a, with their associated probability functions, $f_a(\Lambda^a)$, leading to self-contained attacks on the bank.

3. Main results

There are no a priori restrictions on the private information sets acquired by agents in period zero.

[1] The theorem relies both on the dynamic updating process, based on λ^w, defined by patient depositors when calculating their expected payoffs and the decreasing monotonicity of $c^i_1(\lambda^b_i), \forall \lambda^b_i \in \Lambda^b$.

However, we can always define sets that serve exactly our purpose of generating self-contained runs, without imposing any criteria on how information is obtained. The generic quality of the model described allows for the simulation of any agent type fitting precisely our required characteristics. This would present us with a justification problem from an economic perspective. The mean preserving spread restriction on the information sets of depositors will provide us with such intuitive requirement. The less transparent the financial system considered or, equivalently, the higher the information acquisition costs, the larger the information dispersion suffered by depositors, and, therefore, the weaker and more run-prone the financial structure of the system becomes[1].

Communication is not allowed among agents, as is generally assumed in this type of models. However, allowing for agents to communicate does not necessarily eliminate the incentives of depositors to attack the bank. All agents expecting a self-contained run have a clear incentive to report the set Λ^b (together with $f(\Lambda^b)$) as the true expected one. This can be interpreted as an attempt to gain withdrawing advantage by decreasing the expected number of depositors who attack the bank. Adding new levels of uncertainty to the current framework would allow for a more complete, and complex, analysis of the strategic interactions among depositors and their effect on the generation of runs.

The self-fulfilling nature of the attacks does not contradict the rationality assumption imposed on the expectations formation process of depositors. Runs take place before the state of the economy is realized, which prevents them from being noticed by either the bank or its depositors until withdrawals stop. We are applying Adam Smith's invisible hand principle to the depositing decision of agents. Depositors do not consider the real nature of the bank as long as they are able to monitor the withdrawing line and guarantee that the bank honors the mechanism offered, despite knowing that it is not the optimal one according to their information sets. If agents deposit funds and agree to the terms of the mechanism, it is because they *strictly* prefer the expected utility derived from depositing to an autarky situation. The mechanism does *not* transmit information about the true expected state of the economy, since depositors do not know if it has been designed by an altruistic bank. At the same time, agents do not care about this fact, as they are not obliged to deposit, and will do so if, and only if, it is in their own best interest. With this is mind, define a bank run as follows

Definition 4. A bank run is a strategic situation where the unique Nash equilibrium in pure strategies of at least one of the subgames defining the post-deposit coordination environment faced by patient depositors is given by either (w, nw) or (w, w). Alternatively, a bank run is a strategic situation where at least one agent has a rational incentive to deposit funds in period zero and misrepresent her type in period one for a subset of post-deposit Bayesian coordination games, $\chi(\lambda^w) \in \Xi$.

Mean preserving information spreads on the expected state of the economy provide us with a simple setting to study self-fulfilling, as well as self-contained, bank runs. The following section illustrates numerically the main findings of this paper.

3.1. Numerical simulations. This section presents a series of simulations of the pre- and post-deposit game-theoretical strategic structures illustrating the existence of rational self-contained, as well as general, run equilibria for a given set of expected states observed by the bank, Λ^b (jointly with $f(\Lambda^b)$), and different sets of subjective beliefs hold by depositors, $f(\Lambda^b)$. The framework considered is rich enough to include states of the world generating attack prone and safe areas, allowing for a simple comparison with the currency crises literature[2].

We consider a framework with 100 agents (potential depositors), a unique bank, an exogenously fixed (as of period zero) interest rate of 5 percent, i.e. $R = 1.05$, and a coefficient of relative risk aversion given by $\alpha = 2$. It should be noted that none of the main results obtained relies on the particular values assigned to these variables, which remain within the reasonable limits defined in the literature, see Peck and Shell (2003), and Chari and Jagannathan (1988). Unless stated otherwise, the true set of expected states observed by the bank is given by $\Lambda^b = \{30, 50\}$, with its corresponding associated set of probabilities $f(\Lambda^b) = \{0.8, 0.2\}$. These state values are chosen without loss of generality, and any mean preserving spread over the set of expected states defined in a setting with multiple (> 2) sequential allocations gives place to similar attack areas and does not affect the results obtained. In other words, runs do not depend on the number of contracts, but on the information spread suffered by depositors. This is not to say that we can always justify runs based on beliefs differentials independently of the number of allo-

[1] We are not the first ones to remark the importance of information differentials regarding current issues in international economics. Information heterogeneity among agents has been assumed by Morris and Shim (1998) to explain currency crises, Calvo and Mendoza (2000) to analyze the portfolio diversification and financial contagion phenomena, and van Wincoop and Bacchetta (2003) to study the exchange rate determination puzzle.

[2] Simulation details and Matlab codes are available from the authors upon request.

cations. For instance, not all demand deposit contracts allow for the existence of runs, either general or self-contained. Besides, even though attacks can be generated in a framework with N allocations, one per agent, it is unrealistic to assume that are triggered by information differentials between the bank and its depositors. Sunspots provide a much better justification in this case.

The optimal sequential allocations composing the mechanism for $\Lambda^b = \{30, 50\}$, with $f(\Lambda^b) = \{0.8, 0.2\}$, are given by $c_1^1(\lambda_1^b) = 1.0162$ with associated utility $u(c_1^1(\lambda_1^b)) = -0.984058$ and $c_1^2(\lambda_2^b) = 1.0105$ with associated utility $u(c_1^2(\lambda_2^b)) = -0.989609$.

The expected utility derived from remaining in autarky, if agents use the same information set as the bank to calculate their expected payoffs, equals

$eu(aut) = -0.968571$,

which is lower than the expected utility obtained from depositing

$eu(dp) = -0.968446$.

If no run is expected in period one, or is expected with low enough probability, agents have an incentive to deposit their funds in the bank, given identical information sets.

Assume that agents observe a mean preserving spread state set defined over the true expected state set of the economy. That is, given $f(\Lambda^b)$ and $f(\Lambda^b)$, agents observe Λ^a and $f_a(\Lambda^a)$, such that $\Lambda^b \neq \Lambda^a$, $f(\Lambda^b) \neq f_a(\Lambda^a)$, and

$$\sum_{i=1,\dots\#\Lambda^b} f(\lambda_i^b)\lambda_i^b = \sum_{j=1,\dots\#\Lambda^a} f_a(\lambda_j^a)\lambda_j^a,$$

with $\lambda_i^b \in \Lambda^b, \forall i = 1,\dots,\#\Lambda^b$, and $\lambda_j^a \in \Lambda^a, \forall j = 1,\dots,\#\Lambda^a$.

The subjective payoff structure defined by potential depositors relies heavily on the chosen probability function, $f_a(\Lambda^a)$. While this idea is intuitively clear, the depositing strategic framework conditional on expected self-contained runs is entirely based on the relative realizations and probabilities composing the set of expected states observed by depositors. That is, the existence of self-contained run areas and the respective depositing decisions of agents depend on the *relative* expected state of the economy.

Proposition 1. If a mechanism is vulnerable to general runs[1], there exists an interval of withdrawing depositors, which we refer to as self-contained run area, such that, any mean preserving spread probability function on the state of the economy, whose support includes a subset of expected realizations defined within this interval, may lead to a self-contained run on the bank.

Proof. We calculate the exact values defining the self-contained run area for $\mathbf{m}^*(f(\Lambda^b))$, with $\Lambda^b = \{30, 50\}$ and $f(\Lambda^b) = \{0.8, 0.2\}$. Consider again the utility derived from the optimal consumption allocations composing the mechanism

$u(c_1^1(\lambda_1^b)) = -0.984058$,

$u(c_1^2(\lambda_2^b)) = -0.989609$.

The lower and upper bounds delimiting the interval that gives place to the self-contained run area, $[\lambda^m, \lambda^M]$, are based on the above utility values

$$\lambda^m : \frac{\left(\dfrac{[100 - 30c_1^1(\lambda_1^b) - (\lambda^m - \lambda_1^b)c_1^2(\lambda_2^b)]R}{70 - \lambda^m}\right)^{1-\alpha}}{1-\alpha} = u(c_1^1(\lambda_1^b)),$$

$$\lambda^M : \frac{\left(\dfrac{[100 - 30c_1^1(\lambda_1^b) - (\lambda^M - \lambda_1^b)c_1^2(\lambda_2^b)]R}{70 - \lambda^M}\right)^{1-\alpha}}{1-\alpha} = u(c_1^2(\lambda_2^b)).$$

The solutions to both equations are given by $\lambda^m = 71.3988$ and $\lambda^M = 74.6254$. Note that for an attack to be self-contained the maximum number of patient agents expected to misrepresent their type cannot exceed the width of the self-contained run interval[2]. Any subjective set of beliefs whose support includes a subset of realizations defined within the interval [72, 74], or giving place (after observing λ^w withdrawing agents) to an expected utility (from reporting truthfully) that falls between $u(c_1^1(\lambda_1^b))$ and $u(c_1^2(\lambda_2^b))$, may trigger a self-contained run on the bank.

Theorem 2. (1) If a mechanism is vulnerable to general runs, there exists a set of mean preserving spread probability functions defined on the expected state of the world, Λ^b and $f(\Lambda^b)$, such that the subjective realization sets and beliefs these spreads give place to, Λ^a and $f_a(\Lambda^a)$, lead to self-contained attacks on the bank. (2) Given the expected self-contained run that could take place in the interim period, agents deposit their funds in period zero. That is, the expected utility derived from depositing conditional on the

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[1] That is, there exists a value of λ^w smaller than N, denoted by λ^r, such that the bank does not have enough resources to honor the sequential allocations of the mechanism in case of a run

$$\frac{[N - \sum_{i=1}^{\#\Lambda^b}(\lambda_i^b - \lambda_{i-1}^b)c_1^i(\lambda_i^b) - (\lambda^r - \lambda_{\#\Lambda^b}^b)c_1^{\#\Lambda^b}(\lambda_{\#\Lambda^b}^b)]R}{N - \lambda^r} < c_1^{\#\Lambda^b}(\lambda_{\#\Lambda^b}^b).$$

[2] Though necessary, this condition is clearly not sufficient.

possible self-contained run is strictly higher than the expected utility from remaining in autarky.

Proof. (1) It must be emphasized that, while the self-contained run area is defined by the interval $[\lambda^m, \lambda^M]$ presented in Proposition 1, self-contained runs do not take place within this interval, whose effect is restricted to the updated Bayesian beliefs of depositors, but in the *attack area* defined by $(\lambda_i^a, \lambda_i^b]$ (the equality in subindexes is based on the numerical examples illustrated through the proof). We assume that patient depositors expecting a self-contained run are not able to withdraw as soon as λ_i^a is realized, but must wait to observe $\lambda_i^a + 1$ agents in line before attacking. That is, patient agents cannot anticipate the realization of the state before receiving the corresponding signal through λ^w.

Consider the self-contained run area described in Proposition 1. As already stated, for the run to be self-contained, the maximum number of depositors expected to attack cannot exceed the width of [72, 74]. With this restriction in mind, define the following mean preserving spread on the true expected state

$\Lambda^a = \{28, 72\}$ with $f(\Lambda^a) = \{0.8636, 0.1364\}$.

Given this information set, the *rhs* of the *cec* calculated for the first 28 withdrawing agents delivers an expected utility value of

$$eu(cec(\lambda_1^a)) = E[u(c_2^{nr}(\lambda^w \le 28 \mid f_a(\Lambda^a)))] = f(\lambda_1^a \mid p, \lambda^w \le 28)\, u\left(\frac{[N - \lambda_1^a c_1^1(\lambda_1^b)]R}{N - \lambda_1^a}\right) +$$

$$+ f(\lambda_2^a \mid p, \lambda^w \le 28)\, u\left(\frac{[N - \lambda_1^b c_1^1(\lambda_1^b) - ((\lambda_2^a + 1) - \lambda_1^b) c_1^2(\lambda_2^b)]R}{N - (\lambda_2^a + 1)}\right) = -0.960053,$$

which is higher than $u(c_1^1(\lambda_1^b))$ = -0.984058, so no attack takes place. The second term defined in $eu(cec(\lambda_1^a))$ must account for the effect that a self-contained run would have on the final number of expected withdrawing depositors and the corresponding consumption allocation. However, $f(\lambda_2^a \mid p, \lambda^w \le 28)$ must be calculated for $\lambda_2^a = 72$ to be consistent with the subjective beliefs of depositors.

If an additional agent is observed withdrawing after 28 depositors have withdrawn funds, the subjective beliefs of the remaining patient depositors must be dated accordingly, which modifies the expected utility value obtained from the *rhs* of the *cec* to

$$eu(cec(\lambda_2^a)) = E[u(c_2^{nr}(\lambda^w > 28 \mid f_a(\Lambda^a)))] = -0.986639$$

$$u(c_1^2(\lambda_2^b)) = -0.989609 < eu(cec(\lambda_2^a)) <$$
$$< u(c_1^1(\lambda_1^b)) = -0.984058,$$

triggering a self-contained run on the bank, see the *self-contained attack* figure.

(2) Given the self-contained run that would take place if λ_2^a is realized, agents deposit in period zero if the expected utility derived from depositing

$$eu(dp) = f_a(\lambda_1^a)\left[\left(\frac{\lambda_1^a}{N}\right)u(c_1^1(\lambda_1^b)) + \left(\frac{N-\lambda_1^a}{N}\right)u(c_2^1(\lambda_1^a))\right] + f_a(\lambda_2^a)\left[\left(\frac{\lambda_1^b}{N}\right)u(c_1^1(\lambda_1^b)) + \left(\frac{(\lambda_2^a+1)-\lambda_1^b}{N}\right)u(c_1^2(\lambda_2^b)) + \left(\frac{N-(\lambda_2^a+1)}{N}\right)u(c_2^2(\lambda_2^a+1))\right]$$

is strictly higher than the expected utility from remaining in autarky

$$eu(aut) = f_a(\lambda_1^a)\left[\left(\frac{\lambda_1^a}{N}\right)u(1) + \left(\frac{N-\lambda_1^a}{N}\right)u(R)\right] +$$

$$+ f_a(\lambda_2^a)\left[\left(\frac{\lambda_2^a}{N}\right)u(1) + \left(\frac{N-\lambda_2^a}{N}\right)u(R)\right]$$

This condition is satisfied for Λ^a = {28, 72} and $f(\Lambda^a)$ = {0.8636, 0.1364}, since[1]
eu(dp)=-0.968537 > *eu(aut)*=-0.968572.

4. A note on suspension of convertibility

If suspension is announced as part of $\mathbf{m}^*(f(\Lambda^b))$ in the above numerical examples, all agents expecting realizations of the random variable higher than λ_2^b will not deposit funds, since a payment of zero is received with positive probability. However, the mean preserving spread distribution defined on the true set of expected states may not be wide enough to trigger a self-contained run. In this case, agents would deposit if the bank does not announce suspension, but do not expect or generate a (self-contained) run in the interim period. Therefore, given the unique (endogenous) stochastic variable de-

[1] The same analysis can be performed for an alternative expected state of the world, given, for example, by Λ^b={20, 40} and $f(\Lambda^b)$={0.8, 0.2}. The optimal consumption allocations composing the mechanism are $c_1^1(\lambda_1^b)$=1.0187 with associated utility $u(c_1^1(\lambda_1^b))$=-0.981643, $c_1^2(\lambda_2^b)$=1.0137 with associated utility $u(c_1^2(\lambda_2^b))$=-0.986485, defining a self-contained run interval delimited by λ^m= 66.2142 and λ^M = 69.5472. Consider the following mean preserving spreads on the expected state of the economy, and the corresponding depositing decisions of agents.
Λ^a = {18, 67} with $f(\Lambda^a)$ = {0.8776, 0.1224},
eu(*dp*) = -0.963791 > *eu*(*aut*) = -0.963808,
Λ^a = {18, 68} with $f(\Lambda^a)$={0.88, 0.12},
eu(*dp*) = -0.963793 > *eu*(*aut*) = -0.963810.
Once again, agents deposit despite the self-contained run that would take place if λ_2^a is realized.

fined within the homogeneous information spread framework studied, it can be easily shown that

Proposition 2. *In an environment with either one or multiple banks, suspension of convertibility prevents all self-contained runs based on information differentials if it is announced after agents deposit. Moreover, suspension eliminates any run based on the realization of an exogenous sunspot variable if* $c_2^{\#\Lambda^b}(\lambda^b_{\#\Lambda^b}) \geq c_1^1(\lambda_1^b)$.

If suspension is not announced and a sunspot-based general run is expected to take place, the maximum probability of such an event that allows for agents to deposit is given by the solution to

$$f(nr)E[U(nr)] + \\ + f(run)E[U(run)] \geq E[U(aut)], \tag{5}$$

where $f(run)$ defines the subjective probability assigned to the realization of an exogenous run-triggering sunspot variable, and $f(nr) = 1 - f(run)$. Given $N = 100$, $\Lambda^b = \{30, 50\}$ and $f(\Lambda^b) = \{0.8, 0.2\}$, this probability is equal to 0.002641 if both last agents in line are patient, and 0.0025145 if they are both impatient[1]. Any agent expecting a sunspot-based run to take place with a *subjective* probability higher than 0.002641 will not deposit funds, unless suspension is imposed and constitutes an efficient containment policy.

Proposition 3. *In an environment with either one or multiple banks, suspension of convertibility prevents all sunspot-based runs if it is announced before agents deposit and*

$c_2^{\#\Lambda^b}(\lambda^b_{\#\Lambda^b}) \geq c_1^1(\lambda_1^b)$.

On the other hand, any bank imposing an efficient suspension policy as part of $\mathbf{m}^*(f(\Lambda^b))$ attracts all agents whose $f(run)$ is higher than 0.002641, as equation (3) would be satisfied for any subjective value assigned to $f(run)$. Thus, given a unique endogenous stochastic variable, multiple banks, and an efficient suspension policy, banks should coordinate the timing of their announcements in order to implement the optimal mechanism among all agents. The current theoretical framework supports such an optimality requirement due to its simplicity and the fact that it consists of a unique banking cycle.

Assume that a second (endogenous) stochastic variable, given by R, is added to the current model such that the allocations composing the mechanism are also based on its expected value. The realization of R is assumed to be observed in the final (second) period, but interim signals about its expected value are received by a subset of depositors, as proposed by Jacklin and Bhattacharya (1988). Clearly, our theoretical framework is com-patible with this extension, within which suspension does not constitute an efficient containment policy per se, since it could be used to contain runs generated by a fundamental variable. At the same time, modifying the assumed utility function to allow for the existence of *depositors* with highly biased sunspot probabilities would provide us with a formal strategic scenario where the containment of fundamental, sunspot and information based runs determines the dynamic stability of the banking system.

Conclusion

The current paper has illustrated both theoretically and numerically how fully rational economic agents are willing to deposit funds in a bank despite knowing that a run, which takes place independently of the fundamental return realization, is endogenously defined within the set of states of the system. The most important characteristic of the model presented is the assumption of a fixed, i.e. deterministic, fundamental return to which economic agents have access even if they do not deposit their funds and remain in an autarkic state. This assumption highlights the intrinsic instability of demand deposit mechanisms. The resulting system would clearly become much more unstable if returns were stochastic variables and the beliefs of depositors regarding their distribution would have been heterogeneously defined. The only source of information (and, therefore, beliefs) heterogeneity within the current setting has been defined in terms of the expected number of interim withdrawals, with *noiseless* state signals being given by the observed withdrawals of depositors. Differentials in state information sets between the bank and its depositors have been shown to lead to *rational self-contained* equilibrium runs that *do not violate the revelation principle*.

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References

1. Adao, B. and T. Temzelides (1998). Sequential Equilibrium and Competition in a Diamond-Dybvig Banking Model, *Review of Economic Dynamics*, 1, pp. 859-877.

[1] An equal distribution policy between the depositors remaining after λ^Y is reached has been assumed.

2. Alonso, Irasema (1996). Truthful Revelation in the Diamond and Dybvig Banking Environment, *Journal of Monetary Economics*, 37, pp. 73-87.
3. Calvo, G.A. and E.G. Mendoza (2000). Rational Contagion and the Globalization of Securities Markets, *Journal of International Economics*, 51, pp. 79-113.
4. Chari, V.V. and R. Jagannathan (1988). Banking Panics, Information, and Rational Expectations Equilibrium, *Journal of Finance*, 43, pp. 749-761.
5. Cole, H.L. and T. Kehoe (2000). Self-Fulfilling Debt Crises, *Review of Economic Studies*, 67 (1), pp. 91-116.
6. Di Caprio, D. and F.J. Santos Arteaga (2010). Hidden Equilibria in Coordination Games with Heterogeneous Information Sets: the Case of Bank Runs, *International Journal of Applied Mathematics*, 23, pp. 597-627.
7. Diamond, D. and P. Dybvig (1983). Bank Runs, Deposit Insurance, and Liquidity, *Journal of Political Economy*, 91, pp. 401-419.
8. Furman, J. and J.E. Stiglitz (1998). Economic Crises: Evidence and Insights from East Asia, *Brookings Papers on Economic Activity*, 2, pp. 1-135.
9. Goldstein, I. and A. Pauzner (2005). Demand-Deposit Contracts and the Probability of Bank Runs, *Journal of Finance*, 60, pp. 1293-1327.
10. Green, E.J. and P. Lin (2000). Diamond and Dybvig's Classic Theory of Financial Intermediation: What's Missing? *Federal Reserve Bank of Minneapolis Quarterly Review*, 24, pp. 3-13.
11. Green, E.J. and P. Lin (2003). Implementing Efficient Allocations in a Model of Financial Intermediation, *Journal of Economic Theory*, 109, pp. 1-23.
12. Jacklin, Charles (1987). Demand Deposits, Trading Restrictions, and Risk Sharing. In: Contractual Arrangements for Intertemporal Trade, ed. Edward C. Prescott and Neil Wallace, pp. 26-47, *Minnesota Studies in Macroeconomics*, Vol. 1. University of Minnesota Press.
13. Jacklin, C. and S. Bhattacharya (1988). Distinguishing Panics and Information-based Bank Runs: Welfare and Policy Implications, *Journal of Political Economy*, 96, pp. 568-592.
14. Kaminsky, G.L. and C.M. Reinhart (1999). The Twin Crises: The Causes of Banking and Balance-Of-Payments Problems, *American Economic Review*, 89, pp. 473-500.
15. Kahneman, D. and A. Tversky (1979). Prospect Theory – Analysis of Decision under Risk, *Econometrica*, 47 (2), pp. 263-291.
16. Krasa, S. and A.P. Villamil (1992). Monitoring the Monitor. An Incentive Structure for a Financial Intermediary, *Journal of Economic Theory*, 57, pp. 197-221.
17. Krugman, Paul (1979). A Model of Balance-of-Payments Crises, *Journal of Money, Credit, and Banking*, 11, pp. 311-325.
18. Morris, S. and H.S. Shin (1998). Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks, *American Economic Review*, 88, pp. 587-597.
19. Obstfeld, Maurice (1994). The Logic of Currency Crises, *Cahiers Economiques et Monetaires* (*Banque de France*), 43, pp. 189-213.
20. O'Connor, A. and F.J. Santos Arteaga (2008). Information and Intertemporal Decision Processes in Banking: the Run on Northern Rock, *International Journal of Applied Decision Sciences*, 1, pp. 359-396.
21. Peck, J. and K. Shell (2003). Equilibrium Bank Runs, *Journal of Political Econom*y, 111, pp. 103-123.
22. Postlewaite, A. and X. Vives (1987). Bank Runs as an Equilibrium Phenomenon, *Journal of Political Economy*, 95, pp. 485-491.
23. Rochet, J.C. and X. Vives (2004). Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All? *Journal of the European Economic Association*, 2, pp. 1116-1147.
24. Samartin, Margarita (2003). Should Bank Runs be Prevented? *Journal of Banking and Finance*, 27, pp. 977-1000.
25. Shin, Hyun Song (2009). Reflections on Northern Rock: the Bank Run that Heralded the Global Financial Crisis, *Journal of Economic Perspectives*, 23, pp. 101-119.
26. Tversky, A. and D. Kahneman (1992). Advances in Prospect Theory – Cumulative Representation of Uncertainty, *Journal of Risk and Uncertainty*, 5 (4), pp. 297-323.
27. van Wincoop, E. and P. Bacchetta (2003). Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle? NBER Working Paper No. 9498.

Appendix

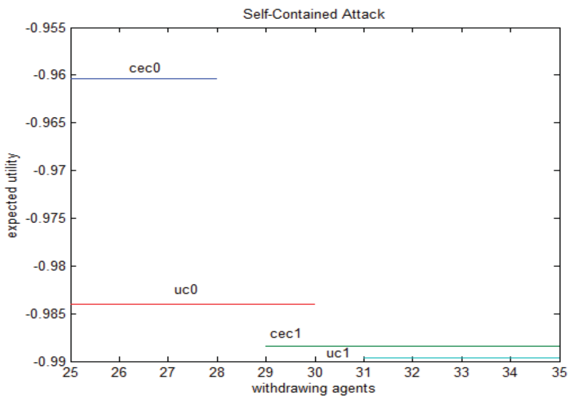

Fig. 1. Self-contained attack game