“A capital structure model with wealth transfers”

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A capital structure model with wealth transfers

Abstract

Perpetuity gain to leverage (G_L) research originating in Modigliani and Miller (1963) and Miller (1977) analyzes the change in value from issuing debt to retire unleveraged equity. Hull (2007, 2010) extends this research by developing the Capital Structure Model (CSM) that demonstrates how the costs of borrowing affect G_L. While the prior perpetuity G_L research is important, its equations are derived for an all-equity or unleveraged firm. Thus, this prior research cannot consider a leveraged situation where wealth transfers between existing equity and debt owners can result from a leverage change. This leads to our research question: "*How will a leveraged situation and the incorporation of a wealth transfer between equity and debt owners affect G_L and thus influence the managerial decision concerning how much leverage is needed to maximize firm value*?" In answering this question, the author incorporates a leveraged situation within the Hull (2010) growth CSM framework and derives G_L equations including those that show how a wealth transfer (linked to a shift in risk) impacts firm value. The latter G_L equations add a 3rd component to the tax-agency and bankruptcy components identified by Hull (2007) in his CSM equations for G_L. This 3rd component captures a wealth transfer between debt and equity owners. With this component in place, the author analyzes three major agency problems from capital structure research: asset substitution, underinvestment, and the relation between an optimal leverage ratio and a wealth transfer.

Keywords: capital structure model, gain to leverage, wealth transfer.
JEL Classification: G32, C02.

Introduction

Perpetuity gain to leverage (G_L) research focuses on an unleveraged firm where only one security type of ownership exists at the time of the debt issuance. This focus cannot consider a leveraged situation and thus is prevented from analyzing the influence of a wealth transfer between security holders when a firm undergoes a leverage change. Building on the most recent capital structure model (CSM) of Hull (2010), this paper assumes a leveraged situation and so can address this research question:

"How will a leveraged situation and the incorporation of a wealth transfer between equity and debt owners affect G_L and thus influence the managerial decision concerning how much leverage is needed to maximize firm value?"

In answering our research question, this paper integrates a number of corporate finance topics. These topics include shift in risk among security holders as considered by Jensen and Meckling (1976) and Masulis (1980); asset substitution as encompassed in Jensen and Meckling (1976) and Leland (1998); underinvestment as discussed by Myers (1977) and Gay and Nam (1998); and, the relation between an optimal leverage ratio and valuation effects as examined by Leland (1998) and Hull (1999).

This paper offers a number of findings that apply to countries with corporate and personal taxes. First, when we incorporate a leveraged situation, the CSM equation for G_L adds a 3rd component to the 1st (tax-agency shield) and 2nd (financial distress) components identified by Hull (2007). This 3rd component captures a wealth transfer between debt and equity owners. While this wealth transfer component is directly linked to the change in the discount rate on the outstanding debt, the tax-agency and financial distress components are heavily associated with the change in the discount rate on equity. Second, this paper's CSM includes G_L equations from equity and debt viewpoints that are formulated by adjusting for wealth transfers between equity and debt owners. Third, we derive equations for an equity-for-debt increment that can also exhibit wealth transfers. These equations are direct polar complements of their debt-for-equity counterparts.

Fourth, recognizing that the asset substitution is a form of wealth transfer, we use a debt-for-equity CSM equation to analyze and illustrate the Leland (1998) claim about the agency costs of debt related to asset substitution being far less than the tax shield. We argue that this claim can be compromised if a firm has a high equity growth rate, a high debt level, or a new debt issue that is small compared to its outstanding debt. Fifth, we use an equity-for-debt CSM equation to investigate an implication of the underinvestment notion that suggests equity's wealth can be enhanced when debt with restrictive covenants is removed. Sixth, we use CSM equations to illustrate how an optimal leverage ratio is diminished when a wealth transfer from debt to equity occurs.

The remainder of the paper is as follows. Section 1 provides the background on perpetuity G_L research and identifies a shortcoming in this research. Section 2 defines and discusses the new variables used in this paper to extend prior CSM research. Section

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3 offers G_L equations for a leveraged firm undergoing a debt-for-equity increment including equations where a wealth transfer can occur. Section 4 looks at G_L equations from equity and debt viewpoints. These equations take into account a wealth transfer linked to a shift in risk. Section 5 offers G_L equations for equity-for-debt transactions. Section 6 uses CSM wealth transfer equations to examine asset substitution, underinvestment, and the optimal leverage ratio. The final section provides conclusions and future research possibilities.

1. Background and shortcoming in prior perpetuity G_L research

The perpetuity gain to leverage (G_L) research can be traced to Modigliani and Miller (1963), referred to as MM. Three key simplifying MM conditions used in formulating G_L include corporate taxes as the only friction, no growth, and an unleveraged situation. These conditions give:

$$G_L = T_C D, \tag{1}$$

where T_C is the exogenous corporate tax rate and D is the value of perpetual riskless debt (D). With no personal taxes and riskless perpetual interest payment (I), we have $D = \frac{I}{r_F}$ where r_F is the exogenous cost of capital on riskless debt. Directly related to the MM equation is the equation by Miller (1977) who expanded on equation (1) by including personal taxes to get:

$$G_L = (1-\alpha)D, \tag{2}$$

where $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$, T_E and T_D are the respective personal tax rates applicable to income from equity and debt, and D now equals $\frac{(1-T_D)I}{r_D}$ where financial distress costs exist (albeit these cost are trivial) so that $r_D > r_F$ can hold.

By focusing only on a tax shield, MM and Miller became subject to theoretical criticism by colleagues who argued for a significant influence from bankruptcy and agency costs (Baxter, 1967; Kraus and Litzenberger, 1973; Jensen and Meckling, 1976; Jensen, 1986). While earlier researchers (Miller, 1977; Warner, 1977) offered evidence that debt-related costs have no real impact on firm value, subsequent researchers (Altman, 1984; Cutler and Summers, 1988; Fischer, Heinkel and Zechner, 1989; Kayhan and Titman, 2007) provided contrary proof. Graham and Leary (2011) have recently reviewed the empirical capital structure research suggesting that this research has explained only part of the observed behavior, studied the wrong models and issues, and improperly measured key variables. This indicates that it is time for new explorative and innovative capital structure research.

To guide capital structure research with a new approach with measurable variables aimed at guiding managerial behavior, Hull (2007, 2010) extended the Miller perpetuity G_L research found in equation (2) by developing the capital structure model (CSM). Maintaining the MM and Miller unleveraged and non-growth conditions, Hull (2007) offered a CSM equation incorporating discount rates capable of overtly capturing negative leverage-related effects. This equation is:

$$G_L = \left[1-\frac{\alpha r_D}{r_L}\right]D - \left[1-\frac{r_U}{r_L}\right]E_U, \tag{3}$$

where r_U and r_L are the unleveraged and leveraged equity rates, E_U is unleveraged equity value, the 1st component, $\left[1-\frac{\alpha r_D}{r_L}\right]D$, represents a positive tax-agency effect, and the 2nd component, $-\left[1-\frac{r_U}{r_L}\right]E_U$, represents non-trivial financial distress costs (captured by increasing r_L values as debt increases) such that its negativity can more than offset the 1st component especially for higher levels of debt.

Hull (2010) expanded on equation (3) by incorporating growth, thus showing how the role of the plowback-payout decision affects the leverage decision. This CSM growth equation is:

$$G_L = \left[1-\frac{\alpha r_D}{r_{Lg}}\right]D - \left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U, \tag{4}$$

where r_{Ug} and r_{Lg} are the growth-adjusted discount rates on unleveraged and leveraged equity, $r_{Ug} = r_U - g_U$ with r_U and g_U the borrowing and growth rates for unleveraged equity, and $r_{Lg} = r_L - g_L$ with r_L and g_L the borrowing and growth rates for leveraged equity. The presence of growth reveals the dangerous nature of choosing too much debt as a positive G_L value can quickly become negative once a firm passes its optimal leverage ratio. The introduction of growth can alter the expected positive and negative values of the components in equation (4) so that they differ from their corresponding components in equation (3)[1].

[1] This alteration tends to occur for a high growth firm that is past its critical point. The critical point is the point where the plowback ratio (*PBR*) using internal equity equals T_C. Due to double taxation on internal equity (and ignoring the marginal flotation costs if external equity was used), Hull (2010) argues that firms can lose value if they cannot sustain a *PBR* of at least T_C. The argument is based on the fact that firms are taxed a first time on internal funds used for growth and then are taxed a second time on the earnings the growth generates for dividends. External equity does not have this double taxation because it is not a source of corporate taxable income until it generates earnings payable as dividends.

The CSM supports trade-off or optimal theorists (Baxter, 1967; Kraus and Litzenberger, 1973; DeAngelo and Masulis, 1980; Hackbarth, Hennessy, and Leland, 2007; Berk, Stanton, and Zechner, 2010) who argue there is an optimal debt-equity mix that maximizes firm value. Consistent with trade-off theory and the CSM, Graham (2000), Korteweg (2010), and Van Binsbergen, Graham, and Yang (2010) collectively show that debt can be expected to enhance firm value by four to ten percent. Consistent with CSM research by Hull (2010), Van Binsbergen, Graham, and Yang (2010) find that the cost of being overleveraged is much higher than the cost of being underleveraged. Also, consistent with Hull (2010), they find that capital structure decisions are most likely made jointly with other corporate policies such as payout.

While supportive of trade-off theory, there is still a shortcoming in CSM research that needs to be addressed to further understand CSM's contribution to the trade-off viewpoint. This shortcoming involves lack of consideration for a leveraged situation. This situation alone can show the impact on G_L of a wealth transfer between securities. Imbierowicz and Wahrenburg (2011) review the wealth transfer literature and examine situations in which a wealth transfer might take place. Of importance, they find that wealth is transferred between security owners independent of the type of event that changes leverage. The challenge of recognizing and incorporating such a wealth transfer effect within a G_L equation motivates this study. With a leveraged situation as our starting point, we are able to derive G_L equations capable of examining how shifts in risk through changes in discount rates lead to a wealth transfer between equity and debt owners. Furthermore, with a leveraged situation, we can analyze both debt-for-equity and equity-for-debt transactions. While a G_L equation for an unleveraged situation shows the impact on unleveraged equity, a G_L equation for a leveraged situation reveals the impact on total firm value (consisting of both equity and debt value) when a wealth transfer occurs. Because both equity and debt owners are impacted by leverage changes, there is an intrinsic need to establish G_L equations for both debt and equity. This paper fulfills this need.

2. Definitions to derive CSM equations for a leveraged situation

This section defines the variables we need to derive perpetual G_L equations within the CSM framework for a leveraged situation. This situation requires us to differentiate between variables by defining them in terms of "before" and "after" a leverage change. With these definitions in place, we can derive G_L equations for both debt-for-equity and equity-for-debt increments as well as G_L equations from equity and debt viewpoints.

2.1. Definitions for debt. We define debt prior to the leverage change as $D_1 = \frac{(1-T_D)I_1}{r_{D_1}}$ where I_1 is the perpetual interest payment before the debt-for-equity increment and r_{D_1} is the debt discount rate. Similarly, for the new debt, we have $D_2 = \frac{(1-T_D)I_2}{r_{D_2}}$ where I_2 is the new interest payment and r_{D_2} is its discount rate. Total debt is $D_1 + D_2$.

We need a way to further identify D_1 if its value is changed because of a change in r_{D_1} stemming from the issuance of D_2. If the value of D_1 decreases because r_{D_1} goes up, then we call D_1 by the name of $D_{1\downarrow}$ and its discount rate by $r_{D_1\uparrow}$. We have: $D_{1\downarrow} = \frac{(1-T_D)I_1}{r_{D_1\uparrow}}$. Albeit less likely, if the value of D_1 increases because r_{D_1} falls from the issuance of D_2, then we call D_1 by the name of $D_{1\uparrow}$ and its discount rate by $r_{D_1\downarrow}$. We have: $D_{1\uparrow} = \frac{(1-T_D)I_1}{r_{D_1\downarrow}}$.

Our expectation concerning the change in r_{D_1} depends on the seniority of claims between D_1 and D_2. Below are three expectations based on the seniority of claims for D_2 being equal, more senior and less senior in claims.

1. If D_1 and D_2 have *equal claims*, we expect $r_{D_2} = r_{D_1}$. If r_{D_1} increases because of the overall level of risk caused by the increase in total debt, then we expect $r_{D_2} = r_{D_1\uparrow}$.
2. If D_2 has *more senior claims* than D_1, we expect $r_{D_1\uparrow} > r_{D_1} > r_{D_2}$ due to dilution of the claims of D_1 such that D_1 falls to $D_{1\downarrow}$. It is also possible that r_{D_1} can remain unchanged.
3. If D_2 has *less senior claims* than D_1, we expect $r_{D_2} > r_{D_1}$ to hold. It is also possible that r_{D_1} can fall to $r_{D_1\downarrow}$ if the claims of D_1 become less risky such that D_1 increases to $D_{1\uparrow}$.

Because r_{D_1} can go up or down, this leads to alternate definitions for total debt (D), which otherwise can be defined as $D = D_1 + D_2$. First, if r_{D_1} rises after the debt-for-equity increment, we have:

$$D = D_{1\downarrow} + D_2 = \frac{(1-T_D)I_1}{r_{D_1\uparrow}} + \frac{(1-T_D)I_2}{r_{D_2}}.$$

Second, if r_{D_1} falls after the increment, we have:

$$D = D_{1\uparrow} + D_2 = \frac{(1-T_D)I_1}{r_{D_1\downarrow}} + \frac{(1-T_D)I_2}{r_{D_2}}.$$

2.2. Definitions for equity. We refer to leveraged equity prior to the debt-for-equity increment as E_{L_1} and after the increment as E_{L_2}. We have:

$$E_{L_1} = \frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg_1}},$$

where C is the before-tax perpetual cash flow earmarked for payout to unleveraged equity owners, r_{Lg_1} is the growth-adjusted leveraged equity discount rate prior to the debt-for-equity increment with $r_{Lg_1} = r_{L_1} - g_{L_1}$, where r_{L_1} and g_{L_1} are equity's discount and growth rates prior to the increment. Similarly, $E_{L_2} = \frac{(1-T_E)(1-T_C)(C-I_1-I_2)}{r_{Lg_2}}$, where r_{Lg_2} is the growth-adjusted leveraged equity discount rate after the debt-for-equity increment with $r_{Lg_2} = r_{L_2} - g_{L_2}$, where r_{L_2} and g_{L_2} are equity's discount and growth rates after the increment.

2.3. Definitions for firm value. Firm value prior to the debt-for-equity increment is: $V_{L_1} = E_{L_1} + D_1$. After the increment, firm value is: $V_{L_2} = E_{L_2} + D$, where $D = D_1 + D_2$ with D_1 able to take on the lower or higher values of $D_{1\downarrow}$ or $D_{1\uparrow}$. Let us assume $D_{1\downarrow}$ occurs. If so, we can use the above definitions for $D_{1\downarrow}$ and E_{L_2} to get:

$$V_{L_2} = E_{L_2} + D = E_{L_2} + D_{1\downarrow} + D_2 = \frac{(1-T_E)(1-T_C)(C-I_1-I_2)}{r_{Lg_2}} + \frac{(1-T_D)I_1}{r_{D_1\uparrow}} + \frac{(1-T_D)I_2}{r_{D_2}}. \quad (5)$$

Let us now assume $D_{1\uparrow}$ occurs. Using the definitions for $D_{1\uparrow}$ and E_{L_2}, we get:

$$V_{L_2} = E_{L_2} + D = E_{L_2} + D_{1\uparrow} + D_2 = \frac{(1-T_E)(1-T_C)(C-I_1-I_2)}{r_{Lg_2}} + \frac{(1-T_D)I_1}{r_{D_1\downarrow}} + \frac{(1-T_D)I_2}{r_{D_2}}. \quad (6)$$

2.4. Definitions for gains to leverage. The total gain to leverage is the gain to leverage before the debt-for-equity increment (which we refer to as $G_{L_1}^{D\to E}$) and the gain from the increment (which we refer to as $G_{L_2}^{D\to E}$). For the latter, we have:

$$G_{L_2}^{D\to E} = V_{L_2} - V_{L_1}, \quad (7)$$

where $V_{L_2} > V_{L_1}$ if the firm moves towards its optimal leverage ratio. Unlike G_L equations for an unleveraged situation, $G_{L_2}^{D\to E}$ no longer has to capture just the change in equity but will capture the changes in both equity and debt if there is a wealth transfer between these two security types.

Suppose the firm is overleveraged and undergoes an equity-for-debt transaction. If so, instead of using $G_{L_2}^{D\to E} = V_{L_2} - V_{L_1}$, we would use:

$$G_{L_2}^{E\to D} = V_{L_1} - V_{L_2}, \quad (8)$$

where now $V_{L_1} > V_{L_2}$ if the firm moves towards its optimal leverage ratio and we replace $G_{L_2}^{D\to E}$ with $G_{L_2}^{E\to D}$ to denote the reversal in the direction of the security exchange.

3. G_L equations for leveraged firms undergoing debt-for-equity increment

Using the definitions just given for a leveraged situation, we now derive three perpetual G_L equations for a debt-for-equity increment using the CSM framework with constant growth and fixed tax rates. These three equations take into account *three possible outcomes*: (1) no wealth transfer associated with r_{D_1} not changing; (2) a wealth transfer from debt to equity associated with r_{D_1} increasing; and, (3) a wealth transfer from equity to debt associated with r_{D_1} decreasing.

3.1. G_L equation for a leveraged firm when r_{D_1} does not change. *For the first outcome*, we assume the discount rate of r_{D_1} for the outstanding interest payment of I_1 remains unchanged when D_2 is issued thus ruling out any possible transfer of wealth from D_1 to equity due to risk shifting. Using (7) and the definitions for D_1, D_2, E_{L_1} and E_{L_2}, while not allowing the value of D_1 to change, Appendix A shows:

$$G_{L_2}^{D\to E} = \left[1 - \frac{\alpha r_{D_2}}{r_{Lg_2}}\right] D_2 - \left[1 - \frac{r_{Lg_1}}{r_{Lg_2}}\right] E_{L_1}. \quad (9)$$

Equation (9) reflects the fact there is no wealth transfer from D_1 to equity that might be associated with a shift in risk from D_1 to equity due to r_{D_1} changing. This means that all of the changes in firm value from the debt-for-equity increment should belong to equity as D_1 is the only other security type that could have been affected by the increment. This is not the case for the next two derivations, where D_1 is affected through the change in its discount rate.

3.2. G_L equation for a leveraged firm when r_{D_1} increases. *For the second outcome*, we assume an increase in r_{D_1}, which is most likely caused by D_2 being senior to D_1 but can also result due to the claims of D_1 being diluted by the new debt. Using the definitions for D_1, $D_{1\downarrow}$, E_{L_1} and E_{L_2} along with equations (5) and (7), Appendix B shows:

$$G_{L_2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1. \quad (10)$$

Equation (10) resembles equation (9) except it has a 3rd component. This component of $-\left[1-\frac{r_{D1}}{r_{D1\uparrow}}\right]D_1$ is negative and identical to the fall in value for D_1 caused when its discount rate increases from r_{D1} to $r_{D1\uparrow}$. The increase in r_{D1} leads to the possibility of a shift in risk that has a downward force on leveraged equity's discount rate (r_L). This downward shift will reduce any increase in r_L caused by the increased leverage. This shift causes a wealth transfer from D_1 to equity because values for securities change when their discounted cash flows have their discount rates change.

Traditional wisdom assumes that the wealth transfer has a zero-sum outcome not affecting firm value. If this is true, then equations (9) and (10) represent the same value. Because the 3rd component in equation (10), $-\left[1-\frac{r_{D1}}{r_{D1\uparrow}}\right]D_1$, is negative and $G_{L2}^{D\to E}$ is assumed to be positive, this means that the 1st and the 2nd components for equation (10) must together represent more positive value than the corresponding two components in equation (9). A comparison of these sets of components reveals that this is likely. First, because we are more apt to have senior claims for D_2 in equation (10) for r_{D1} to increase, this means that r_{D2} in equation (10) is smaller than r_{D2} in equation (9). A smaller r_{D2} translates into a more valuable 1st component in equation (10) compared to equation (9) if $r_{Lg2} > 0$ (which we expect for reasonable debt choices). Second, the downward shift in equity's risk associated with equation (10) means that r_L is less than it would otherwise be. A lower r_L makes cash flows more valuable if $r_{Lg2} > 0$. As derived by Hull (2010), g_{L2} increases as debt increases and thus is capable of negating the positive effect of an increasing r_L such that $r_{Lg2} < 0$ can occur. However, for a typical firm, we only expect $r_{Lg2} < 0$ to occur if the firm takes on too much debt at which point a growth-adjusted CSM breaks down (analogous to how the Dividend Discount Model with growth breaks down).

The enhancement in equity at the expense of D_1 found in equation (10) can be represented by an agency-based theory such as Jensen and Meckling (1976). This theory suggest that increases in debt can transfer wealth from debt to equity due to risk shifting where equity owners' claims become less risky at the expense of debt owners. The risk shifting results when the increase from r_{D1} to $r_{D1\uparrow}$ is accompanied by a value for r_L that is lower than what would otherwise occur when debt is issued (thus making it likely that r_{g2} will be lower). The enhancement in equity can also be represented by the agency prediction of Jensen (1986). This theory claims that debt obligations prevent managers from squandering excess funds on bad projects.

3.3. G_L equations for a leveraged firm when r_{D1} decreases. *For the third outcome*, we assume a decrease in r_{D1}, which is most likely caused by D_1 being senior to D_2. Although D_2 has more risk than D_1, it still enables greater monitoring against risky projects favoring equity and thus can provide addition bond covenants further protecting the claims of D_1. The derivation for this outcome is not shown because it is the same procedure found in Appendix B except we use $r_{D1\downarrow}$ and $D_{1\uparrow}$ instead of $r_{D1\uparrow}$ and $D_{1\downarrow}$. Using the definitions for D_1, $D_{1\uparrow}$, E_{L1} and E_{L2} along with equations (6) and (7), we can get:

$$G_{L_2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1. \quad (11)$$

Equation (11) is like equation (10) except the last component of $-\left[1-\frac{r_{D1}}{r_{D1\downarrow}}\right]D_1$ is now positive because $r_{D1} > r_{D1\downarrow}$. Thus, everything said above when describing the last component in equation (10) is reversed and our description below for equation (11) takes this into account.

The last component of equation (11) mirrors the increase in value for D_1 caused when its discount rate decreases from r_{D1} to $r_{D1\downarrow}$. The decrease in r_{D1} leads to the possibility of a shift in risk where any rise in equity's discount rate is further increased. If the wealth transfer has a zero-sum outcome in terms of the firm's overall value, then equations (9) and (11) represent the same value. This means that the 1st and the 2nd components for equation (11) must represent less value than the corresponding two components in equation (9). A comparison of these sets of components reveals that this is likely. First, we are more apt to have less senior claims for D_2 in equation (11), which means that r_{D2} in (11) is larger than r_{D2} in equation (9). A larger r_{D2} translates into a less valuable 1st component in equation (11) compared to equation (9) if

$r_{Lg_2} > 0$ holds as we expect. Second, the shift in risk associated with equation (11) means that equity's discount rate is greater than it would otherwise be. A greater equity discount rate makes cash flows less valuable if $r_{Lg_2} > 0$.

While agency theory can once again be offered as an explanation for the shift in risk, an agency argument (such as given in section 3.2) is less suitable for equation (11). This is because the expected shift in risk of r_{D_1} to $r_{D_1\uparrow}$ when debt increases is the underlying assumption for the derivation of equation (10) and not for equation (11). In other words, we will not normally find a favorable outcome for outstanding debt (such as D_1 increasing to $D_{1\uparrow}$) when more debt is issued but only when debt is retired.

4. G_L from equity and debt viewpoints

Unlike unleveraged G_L equations that can only capture the change in value for unleveraged equity, G_L equations for a leveraged situation can capture the change in value for all security types that compose firm value at the time of the leverage change. In this section, we break down equations (10) and (11) as these two equations affect two security types. Our dissection yields separate G_L equations for equity and debt.

4.1. Findings of Eisdorfer. Generally speaking, we assume that a wealth transfer (such as results from risk shifting) is a zero-sum game. If there are asymmetry in payoffs, Eisdorfer (2010) argues that equity can costlessly shift risk to debt in a manner that lowers overall firm risk. The end result is that risk-shifting behavior can be more beneficial to equity than currently represented in the literature. This result also suggests that a wealth transfer is not necessarily a zero-sum outcome and equation (9) does not have to equal equations (10) or (11).

Eisdorfer might suggest that a shift upward in debt's risk, such as represent by equation (10), can lower equity's discount rate more than one might expect. The latter possibility is relevant when interpreting a CSM equation that captures a wealth transfer caused by discount rates shifting as the firm's leverage ratio changes. This is particular true when interpreting the impact on each security type brought about by a leverage change.

4.2. G_L equation from equity's and debt's viewpoints when r_{D_1} increases. The last component of equation (10), $-\left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1$, suggests a loss in wealth in D_1 as r_{D_1} increased to $r_{D_1\uparrow}$ when D_2 is issued. If this wealth loss in D_1 accrues to equity through a wealth transfer, we must recognize that equation (10) encompasses different gains to leverage for D_1 and equity. Thus, we need to break down the gain to leverage into both equity and debt components. Doing this gives:

$$G_{L_2}^{D\to E} = G_{L_2}^{Equity} + G_{L_2}^{Debt}, \tag{12}$$

where $G_{L_2}^{Debt}$ and $G_{L_2}^{Equity}$ are the respective gains to leverage for equity and debt.

To get $G_{L_2}^{Equity}$, we have to adjust for the wealth transfer represented by the last component in equation (10) by adding it to the first two components of equation (10). This can be done in two ways. First, we can express $G_{L_2}^{Equity}$, in terms of equation (9) and adjust r_{Lg_2} in a manner that makes equity more valuable by the amount of the wealth transfer (or decrease in D_1), which is $\left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1$. For example, we change r_{Lg_2} to a lower rate referred to as $r_{Lg_2}^{Lower}$ caused by a lower value for r_{L_2}. We have:

$$G_{L_2}^{Equity} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}^{Lower}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}^{Lower}}\right]E_{L_1}, \tag{13}$$

where $\left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}^{Lower}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}^{Lower}}\right]E_{L_1} > \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1}$ by the amount of $\left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1 > 0$ if the zero-sum outcome holds. However, if overall value increases due to a decline in overall risk as suggested by Eisdorfer (2010), then the difference could be greater than just the wealth transfer of $\left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1$ and r_{L_2}, and thus r_{Lg_2}, would be even lower.

Second, if the zero-sum outcome holds, we can express $G_{L_2}^{Equity}$ in terms of equation (9) and adjust by simply adding in the absolute value of the decrease in D_1. Doing this gives:

$$G_{L_2}^{Equity} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} + \left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1, \tag{14}$$

where values for variables in the first two components of equation (14) are identical to those in the same two components of equation (9). If firm value

can increase as Eisdorfer (2010) suggests, then equation (14) would represent even more value due to a lower value for r_{L_2}.

For either equations (13) or (14), the gain to leverage for debt can be represented by the value of the last component of equation (10). Thus, from debt's viewpoint, we have:

$$G_{L_2}^{Debt} = -\left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1, \quad (15)$$

where the "gain" is actually a loss as $G_{L_2}^{Debt} < 0$. It is possible that some of the loss in D_1 enhances D_2. For example, consider the situation where $r_{D_1\uparrow} > r_{D_2}$ because D_2 involves senior debt. In this case, it is less clear that we are able to add in all of the wealth transfer from D_1 to equity as we did in equations (13) and (14). Thus, it is possible that senior debt has been issued in a fashion that may also expropriate value to its advantage.

4.3. G_L equation from equity's and debt's viewpoints when r_{D_1} decreases. The last component of equation (11), $-\left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1$, suggests a gain in wealth to D_1 as r_{D_1} decreases to $r_{D_1\downarrow}$ when D_2 is issued. If this wealth gain in D_1 comes at the expense of equity through a wealth transfer, we must recognize that equation (11) includes different gains to leverage for D_1 and equity. As we just did with equation (10), we will now break down equation (11) once again using (12) where $G_{L2}^{D\to E} = G_{L_2}^{Equity} + G_{L_2}^{Debt}$.

To get $G_{L_2}^{Equity}$, we adjust for the wealth transfer effect represented by the last component in equation (11) by subtracting it from the first two components of equation (11). This can be done in two ways. First, we can express $G_{L_2}^{Equity}$ in terms of equation (9) and adjust r_{Lg_2} in a manner that makes equity less valuable by the amount of the wealth transfer (or increase in D_1), which is $-\left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1$. This can be done by changing r_{Lg_2} to a higher rate referred to as $r_{Lg_2}^{Higer}$ caused by a higher value for r_{Lg_2}. Doing this gives:

$$G_{L_2}^{Equity} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}^{Higher}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}^{Higher}}\right]E_{L_1}, \quad (16)$$

where $\left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}^{Higher}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}^{Higher}}\right]E_{L_1} <$ $\left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1}$ by the amount of $\left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1 < 0$ if the zero-sum outcome holds.

Second, if the zero-sum outcome holds, we can express $G_{L_2}^{Equity}$ in terms of equation (9) and adjust by subtracting out the value of the increase in D_1. Doing this gives:

$$G_{L_2}^{Equity} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} + \left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1, \quad (17)$$

where values for variables in the first two components of equation (17) are identical to those in the same two components of equation (9).

For either equations (16) or (17), the gain to leverage for debt can be represented by the value of the last component of (10). Thus, from debt's viewpoint, we have:

$$G_{L_2}^{Debt} = -\left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1, \quad (18)$$

where $G_{L_2}^{Debt} > 0$.

5. G_L equations for leveraged firms undergoing equity-for-debt increment

The three G_L equations of (9), (10), and (11) are for a debt-for-equity increment. We can get three complementary G_L equations for an equity-for-debt increment. The derivational procedure for these equations is similar to equations (9), (10), and (11) except we reverse the procedure by using $G_{L2}^{E\to D}$ in equation (8) instead of $G_{L2}^{D\to E}$ in equation (7). Below we describe the complementary G_L equations.

First, using the previous definitions for D_1, D_2, E_{L1}, and E_{L2}, while not allowing the value of D_1 to change when D_2 is retired so there is no wealth transfer, we get the complement to equation (9):

$$G_{L2}^{E\to D} = \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2, \quad (19)$$

where we expect $\left[1-\frac{r_{Lg1}}{r_{Lg2}}\right]E_{L1} > 0$ and $-\left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 < 0$ to hold for reasonable debt levels.

Equation (19) differs from equation (9) in that the signs for the two components are reversed along with the order in which they appear in the equation.

Second, in equation (10), we saw the possibility of a wealth transfer from debt to equity that was linked to an increase in the discount rate on D_1. We can get an equation representing this same directional wealth transfer for an equity-for-debt increment if we assume r_{D1} increases to $r_{D1\uparrow}$ when D_2 is retired. Using definitions for D_1, $D_{1\downarrow}$, E_{L1}, and E_{L2} along with equations (5) and (8), Appendix C shows:

$$G_{L_2}^{E\to D} = \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 + \left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1, \quad (20)$$

where $\left[1-\frac{r_{D1}}{r_{D1\uparrow}}\right]D_1 > 0$ because $r_{D1\uparrow} > r_{D1}$. Unlike the last component in equation (10) of $-\left[1-\frac{r_{D1}}{r_{D1\uparrow}}\right]D_1$ that is negative, the last component in equation (20) is positive. One explanation for r_{D1} increasing would be that the bond constraints associated with D_2 prevented managers from squandering funds on bad or ultra-risky projects that made cash flows safer for D_1.

Third, in equation (11), we considered the possibility of a wealth transfer from equity to debt that was linked to a fall in the discount rate on D_1. We can get an equation denoting this same directional wealth transfer for an equity-for-debt increment if we assume r_{D1} decreases to $r_{D1\downarrow}$ when D_2 is retired. The derivation for this outcome is not shown because it is the same procedure found in Appendix C except we use $r_{D1\downarrow}$ and $D_{1\uparrow}$ instead of $r_{D1\uparrow}$ and $D_{1\downarrow}$. Given the definitions for D_1, $D_{1\uparrow}$, E_{L1}, and E_{L2} along with equations (6) and (8), we can get:

$$G_{L_2}^{E\to D} = \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 + \left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1, \quad (21)$$

where $\left[1-\frac{r_{D1}}{r_{D1\downarrow}}\right]D_1 < 0$ because $r_{D1} > r_{D1\downarrow}$. Unlike the last component in equation (11) of $-\left[1-\frac{r_{D1}}{r_{D1\downarrow}}\right]D_1$ that is positive, the last component in equation (21) is negative. The increase in D_1 for an equity-for-debt increment is consistent with the notion that lower levels of debt are safer than higher levels of debt. Elliott, Prevost, and Rao (2009) find a positive debt effect for equity announcements. They discover that this positive debt effect is especially true for firms with lower rated debt indicating a greater fall in r_{D1} for greater overleveraged situations.

While not shown in detail due to space constraints, we could express (20) and (21) from equity and debt viewpoints as was done for equations (10) and (11). To express G_L in terms of both equity and debt viewpoints from the equity-for-debt increment, we would replace $G_{L2}^{D\to E}$ in equation (12) with:

$$G_{L_2}^{E\to D} = G_{L_2}^{Equity} + G_{L_2}^{Debt}. \quad (22)$$

To get $G_{L_2}^{Equity}$ and $G_{L_2}^{Debt}$ expressions, we would have to adjust for the wealth transfer effect and this could be done in the fashion described previously in sections 4.2 and 4.3 when generating equations (13) through (18).

6. Consistency of wealth transfers and CSM equations

Asset substitution and underinvestment are the two major types of wealth transfers discussed in the agency theory literature. The effect of a wealth transfer on an optimal leverage ratio is also an important topic of discussion. In this section, we use this paper's CSM equations to examine the predicted outcomes for these two wealth transfer types and the Leland (1998) finding that the optimal debt level can increase when wealth is transferred.

6.1. Asset substitution. The asset substitution problem identified by Jensen and Meckling (1976) is a form of wealth transfer that occurs when riskier assets are substituted for safer assets. Our situation of a debt-for-equity transaction in equation (10) does not involve issuing new debt to acquire new assets that are more risky thereby "substituting" in that sense more risky assets for less risky assets. Our situation captured by equation (10) is one that involves increasing the proportion of debt ownership thereby diluting the claims of prior debt owners and making these claims more risky. The fewer number of equity owners that exist after the debt-for-equity transaction has a greater proportion of residual ownership and thus more to gain if the firm does well. The situation of diluting debt's claims while giving equity the chance for greater wealth embodies the outcome found in the asset substitution problem in that one security class profits at the expense of another security class through a transfer of risk. If this is true, then asset substitution can be proxied by the wealth transfer component in equation (10) and we are in a position to examine an asset substitution claim from the agency literature.

The claim we will examine is the Leland (1998) claim that a tax shield effect from debt is typically far greater than the agency costs of debt related to asset substitution. To examine this claim, we compare the tax shield component of equation (10), which is $\left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2$, with the asset substitution or wealth transfer component of equation (10), which is $-\left[1-\frac{r_{D1}}{r_{D1\uparrow}}\right]D_1$. Assuming $D_1 = D_2$ so as to equalize the playing field, it can be shown that the advantage to the tax shield component occurs when $\left|\frac{r_{D_1}}{r_{D1\uparrow}}\right| > \left|\frac{\alpha r_{D_2}}{r_{Lg2}}\right|$ where the brackets signify absolute value. Substituting in $r_{Lg_2} = r_{L2} - g_{L2}$, we get $\left|\frac{r_{D_1}}{r_{D1\uparrow}}\right| > \left|\frac{\alpha r_{D_2}}{r_{L2} - g_{L2}}\right|$ as a necessary condition before the Leland claim can hold when $D_1 = D_2$. As seen in the latter inequality (and of no surprise) a larger value for $r_{D1\uparrow}$ will weaken the claim. However, more revealing, we see that a larger value for g_{L2} will also weaken the claim. As suggested by Hull (2010, 2011), g_{L2} is a volatile variable due to its capacity to increase rapidly especially as higher debt levels are reached. If so, the claim will not hold for situations that generate large g_{L2} values.

Let us illustrate the above comparison of the two components using numbers from the recent pedagogical paper by Hull (2011) where the debt doubles so that $D_1 = D_2$ = \$2.0864B (where B = billions). This doubling involves an increase in the debt level from 20% to 40% where debt level refers to debt as a percentage of unleveraged equity value. The numbers we use are for a situation when the plowback ratio is relatively high at 0.35. This yields an unleveraged growth rate of 4.17% that can reach a leveraged growth rate more than double the 4.17% rate if the firm overshoots its optimal leverage ratio. At the "overshoot" point the CSM with growth yields an untenable situation because such a high growth cannot be sustained over time.

Continuing with our illustration, we gather these values from Hull: $r_{D1} = 0.0530$, $r_{D1\uparrow} = 0.0560$, $r_{D2} = 0.0602$, and $r_{Lg2} = 0.0640$ (where $r_{L2} = 0.1250$; $g_{L2} = 0.0610$). The value for r_{D1} is for the 20% debt level prior to the increment and the values for r_{D2} and r_{Lg2} are for the 40% debt level after the debt-equity increment. We choose the value for $r_{D1\uparrow}$ by using Hull's 30% debt level, while the value for α of 0.7823 is Hull's value for all debt levels as tax rates are assumed to be unaffected by the leverage change. Using the above numbers we get: $\left|\frac{r_{D1}}{r_{D1\uparrow}}\right| = \left|\frac{0.0530}{0.0560}\right| = 0.946$ and $\left|\frac{\alpha r_{D2}}{r_{Lg2}}\right| = \left|\frac{0.7823(0.0602)}{0.0640}\right| = 0.736$. Thus, we see that the Leland claim of $\left|\frac{r_{D1}}{r_{D1\uparrow}}\right| > \left|\frac{\alpha r_{D2}}{r_{Lg2}}\right|$ holds since 0.946 is greater than 0.736 with the difference being 0.946 – 0.736 = 0.210. However, if there is a wealth transfer due to risk shift from debt to equity, then r_{Lg2} would actually be less than the number given by Hull. For example, assume that r_{Lg2} drops by the rise in r_{D1}, which is 0.0560 − 0.0530 = 0.0030. If so, we get $\left|\frac{\alpha r_{D2}}{r_{Lg2} - (\text{rise in } r_{D2})}\right| = \left|\frac{0.7823(0.0602)}{0.0640-0.0030}\right| = 0.772$ giving a gap of 0.946 – 0.772 = 0.174. Thus, the gap has narrowed from 0.210 to 0.174.

While details are omitted for brevity's sake, using the Hull optimal debt level of 50% and extrapolating to get 25% debt level values, we would get: $\left|\frac{r_{D1}}{r_{D1\uparrow}}\right| = \left|\frac{0.0545}{0.0602}\right| = 0.905$ and $\left|\frac{\alpha r_{D2}}{r_{Lg2}}\right| = \left|\frac{0.0518}{0.0574}\right| = 0.902$, which is a difference of only 0.003. Adjusting r_{Lg2} as above, we get a negative difference of −0.097. Thus, we see that it is possible that the Leland claim does not hold as we reach the optimal as we estimate the difference to be as much as −0.097. If we used an $r_{D1\uparrow}$ value of 0.0632 for the 45% debt level, then the difference would be −0.040 even without adjusting for a shift in risk. For a $r_{D1\uparrow}$ of 0.6040 which is slightly past the 40% debt level, the difference would still be negative indicating that the Leland claim would stop holding even without adjusting for a downward shift in equity risk.

From the above analysis, we deduce that a claim giving the tax shield advantage over a wealth transfer effect can be tenuous especially for a firm issuing debt to attain its optimal. We also infer that the increase in the cost for outstanding debt must not be too sensitive to the debt issuance if the Leland claim is to hold. For higher leverage ratios, the firm's leveraged equity growth rates start increasing rapidly indicating that the Leland claim is less likely to hold for firms with higher debt levels that cause higher values for g_{L2} (and thus lower r_{Lg2} values). A higher debt level should not only cause g_{L2} to increase but it should also serve to increase discount

rates on prior debt, which are all factors making it less likely for the claim to hold. Finally, as can be seen by comparing the two components, the claim would be harder to prove if $D_1 > D_2$ as would occur if outstanding debt before the increment was large relative to the new debt and much of this outstanding debt was affected by a loss in value from the new debt issue. This could create a huge advantage for the wealth transfer dominating the tax shield due simply to a relative size factor between D_1 and D_2.

6.2. The underinvestment problem. The underinvestment notion of Myers (1977) suggests that equity would not want to plow additional funds into low risk projects that are more advantageous to debt by creating safer but lower cash flows. Similarly, equity would not want to approve an equity-for-debt transaction if the new equity better served the remaining debt owners by making their cash flows safer at equity's expense. For both cases, the decision to increase equity would not be made if only debt profited. If a firm is facing an underinvestment problem due to too much agency cost from debt, then an equity-for-debt transaction could be profitable for equity by removing undesired debt covenants that favor safer cash flows to debt and lower pay-offs to equity. In terms of equation (21),

$$G_{L_2}^{E\to D} = \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 + \left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1$$

equity owners would pursue an equity-for-debt exchange if $G_{L_2}^{E\to D} > 0$ such as caused by the positive 1st component dominating the negative 2nd and 3rd components.

Let us analyze this underinvestment problem illustrating, once again, with numbers from Hull (2011). For this illustration, the optimal debt-equity ratio of 0.402 is achieved with a 50% debt level. While details are omitted for brevity's sake, using numbers for the 60% to 50% debt levels and assuming the fall in debt's discount rate is from a debt level of 55% to 50% (where we extrapolate Hull's values to get the 55% level), we get

$$G_{L_2}^{E\to D} = \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} - \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 + \left[1-\frac{r_{D_1}}{r_{D_1\downarrow}}\right]D_1 =$$

\$5.854B−\$0.788B + −\$0.250B = \$4.816B.

This answer indicates that if equity experiences a loss in value from restrictive covenants, then removing these covenants frees the firm to invest in projects that are more profitable.

Continuing with our illustration, let us go from a 50% debt level to a 40% debt level with the fall in debt's discount rate from 45% to 40%. For this example, we would not expect to get a positive number because we are moving away from the 50% optimal debt level. This expectation holds as we get $G_{L_2}^{E\to D}$ = −\$0.932B − \$0.102B + (−\$0.208B) = −\$1.242B. The absolute magnitude of \$4.816B is greater than that for −\$1.242B due to asymmetry about the optimal leverage ratio where overshooting the optimal is more costly than undershooting.

The above results using equation (21) are consistent with the empirical study of Hull (1999) who examines 338 equity-for-debt transactions. Hull finds that firms moving toward their optimal leverage ratio experience a superior market reaction compared to those moving away from their optimal. Hull's regression tests indicate that an increase in equity's risk and its lost in value is especially true for higher leveraged firm. This latter finding is consistent with the negative results for the 3rd or wealth transfer component in equation (21) found in the two above illustrations. *Ceteris paribus*, Hull's results are also consistent with the notion that overshooting is more costly than undershooting.

6.3. Examination of the notion that leverage increases when wealth is transferred. Leland (1998) argues that the optimal leverage ratio may increase when equity has potential for asset substitution by making assets more risky. This notion of an increase in the optimal ratio contradicts the general belief that the optimal ratio will decrease when asset substitution occurs. We will now examine this Leland notion by considering the wealth transfer aspect of asset substitution using CSM equations.

As seen earlier we can decompose equation (10) into (14) for $G_{L_2}^{Equity}$ and (15) for $G_{L_2}^{Debt}$. Together these two equations emphasize that a wealth transfer effect for a debt-for-equity increment increases the value of equity while decreasing the value of debt. An increase in equity value and a decrease in debt value cause the leverage ratio to be lower. Thus, while a debt-for-equity transaction in itself increases a firm's leverage ratio, the wealth transfer serves to reduce this increase. This reduction in leverage caused by a wealth transfer effect supports the belief that leverage will decrease if a wealth transfer from debt to equity occurs. Thus, construing the asset substitution as a form of wealth transfer as argued earlier in section 6.1, equation (10) is inconsistent with the Leland (1998) suggestion that leverage increases when asset substitution results.

Let us input some numbers to illustrate how the wealth transfer increases the leverage by looking at debt-equity ratios using equation (9) where there is

no wealth transfer and equation (10) where there is a wealth transfer. Using numbers from Hull (2011) when the debt level increases from 30% to 50%, equation (9) gives

$$G_{L_2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg_2}}\right]D_2 - \left[1-\frac{r_{Lg_1}}{r_{Lg_2}}\right]E_{L_1} =$$

$$\left[1-\frac{0.7824(0.0662)}{0.0574}\$2.0864\text{B}\right] -$$

$$\left[1-\frac{0.0663}{0.0574}\right]\$8.7135\text{B} =$$

$$\$0.2037\text{B} + \$1.3510\text{B} = \$1.5548\text{B}.$$

The positive contribution of \$1.351B to G_L by the 2nd component occurs because the leveraged growth rate increases with debt such that $r_{Lg1} > r_{Lg2}$. Given a debt value of \$5.2160B and an equity value of \$7.7517B, the debt-equity ratio using equation (9) is $\frac{\$5.2160\text{B}}{\$7.7517\text{B}} = 0.673$. Equation (10) is like equation (9) except it contains the last or wealth transfer component. For this last component, we get $-\left[1-\frac{r_{D1}}{r_{D1\uparrow}}\right]D_1 = -\left[1-\frac{0.0560}{0.0602}\right]3.1296 = -\$0.2183\text{B}$. Let us assume that G_L for equations (10) and (9) are equal due to a zero-sum outcome from the wealth transfer. This means that the first two components in equation (10) equal 1.5548 + 0.2183 = \$1.7731B. This value of \$1.7731B is the value from equity's viewpoint given by $G_{L_2}^{Equity}$ in equation (14). As discussed earlier when presenting equation (13), the 1.7731 can also result by dividing by lower discount rate for r_{Lg2} in the first two components, which would reflect the lower value for leveraged equity discount rate due to the shift in risk from debt to equity. For example, if r_{Lg2} fell about 0.0015 then equation (13) would yield about the same value as (14). Adjusting the leverage ratio of 0.673 for the wealth transfer of −\$0.2183B for debt and \$0.2183B for equity, we get a ratio of $\frac{\$5.2160\text{B} - \$0.2183\text{B}}{\$7.7517\text{B} + \$0.2183\text{B}} = 0.627$ that is below 0.673. Thus, in disagreement with the Leland assertion that an asset substitution increases the optimal leverage ratio, we find just the opposite when using equation (10) and assuming that the wealth transfer component captures an asset substitution effect.

While the use of equation (10) gives results inconsistent with Leland, this is not the end of the conversation. For example, the use of equation (11) would give us the desired Leland result concerning an increase in leverage when wealth is transferred from debt to equity. This is because equation (11) covers the situation of an equity to debt wealth transfer when a firm undergoes a debt-for-equity increment. However, the likelihood of a transfer of wealth from equity to debt is not as great as from debt to equity when we are talking about a leverage increase. Using equation (11), the leverage ratio of 0.673 would increase to 0.721 as predicted by Leland.

Conclusion

This paper treads new ground by incorporating wealth transfers within the capital structure model (CSM) perpetuity gain to leverage (G_L) framework. First, we develop G_L equations for a debt-for-equity transaction including those where there is a wealth transfer. Second, we analyze the impact on equity and debt value when there is a wealth transfer. Third, we develop G_L equations for an equity-for-debt transaction. Fourth, we use CSM equations to illustrate the asset substitution and underinvestment problems as well as the impact of a wealth transfer on the optimal leverage ratio. These illustrations demonstrate the capacity for CSM equations to shed light on major capital structure research issues.

The CSM offers a robust set of G_L equations that provide insight on the major problems uncovered by the capital structure research. This set of equations can help us understand the nature of both growth and shifts in risk when a firm undergoes a capital structure change. While the CSM research may still be relatively new, this paper has attempted to extend Hull (2007, 2010) by further revealing its capacity to provide answers to capital structure questions faced by managers. It is the authorship's hope that the merit of this paper can further promote exposure to the CSM so that it can be enriched by objective scrutiny from corporate finance researchers.

Future G_L research can extend this paper by further exploring the theoretical implications, practical applications, and pedagogical exercises that are inherent in this paper's CSM equations. Extension of CSM research can expand on the wealth transfer aspect of this paper by considering changes in tax rates as the debt-equity ratio changes as this remains one area not yet addressed by the CSM research. Additionally, a practical paper with teaching implications along the lines of Hull (2005, 2008, 2011), but with wealth transfers incorporated, can be developed. This exercise would expand on the illustrations given in section 6.

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Appendix A. Proof of equation (9)

Proof of equation (9) for the situation of a leveraged firm undergoing a debt-for-equity increment with constant growth, fixed tax rates, and no wealth transfer due to risk shifting such that r_{D1} does not change causing D_1 to maintain its value. Using equation (7) where $G_{L2}^{D\to E} = V_{L2} - V_{L1}$ and noting $V_{L1} = E_{L1} + D_1$ and $V_{L2} = E_{L2} + D_1 + D_2$, we have:

$$G_{L2}^{D\to E} = E_{L2} + D_1 + D_2 - E_{L1} - D_1.$$

Noting $E_{L_2} = \frac{(1-T_E)(1-T_C)(C-I_1-I_2)}{r_{Lg2}}$ and canceling out D_1, we have:

$$G_{L2}^{D\to E} = \frac{(1-T_E)(1-T_C)(C-I_1-I_2)}{r_{Lg2}} + D_2 - E_{L_1}.$$

Multiplying out the 1st component and rearranging:

$$G_{L2}^{D\to E} = D_2 - \frac{(1-T_E)(1-T_C)I_2}{r_{Lg2}} - E_{L_1} + \frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg2}}.$$

Multiplying the 2nd component by $\frac{(1-T_D)r_{D_2}}{(1-T_D)r_{D_2}} = 1$ to get $-\left(\frac{(1-T_E)(1-T_C)r_{D_2}}{(1-T_D)r_{Lg2}}\right)\frac{(1-T_D)I_2}{r_{D_2}}$ which is $-\left(\frac{(1-T_E)(1-T_C)r_{D_2}}{(1-T_D)r_{Lg2}}\right)D_2$, factoring out D_2, and setting $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$:

$$G_{L2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg2}}\right]D_2 - E_{L_1} + \frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg2}}.$$

Multiplying the 3rd component by $\frac{r_{Lg1}}{r_{Lg1}} = 1$ to get $\left(\frac{r_{Lg1}}{r_{Lg2}}\right)\frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg1}}$ which is $\left(\frac{r_{Lg1}}{r_{Lg2}}\right)E_{L_1}$ and factoring out E_{L_1}:

$$G_{L2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg2}}\right]D_2 - \left[1-\frac{r_{Lg1}}{r_{Lg2}}\right]E_{L_1}. \quad (9)$$

Appendix B. Proof of equation (10)

Proof of equation (10) for the situation of a leveraged firm undergoing a debt-for-equity increment with constant growth, fixed tax rates, and wealth transfer due to risk shifting such that r_{D_1} becomes $r_{D_1\uparrow}$ causing D_1 to become $D_{1\downarrow}$. Using equation (7) where $G_{L2} = V_{L2} - V_{L1}$ and noting $V_{L1} = E_{L1} + D_1$ and substituting for V_{L2} using equation (5):

$$G_{L2}^{D\to E} = \frac{(1-T_E)(1-T_C)(C-I_1-I_2)}{r_{Lg2}} + \frac{(1-T_D)I_1}{r_{D1\uparrow}} + \frac{(1-T_D)I_2}{r_{D2}} - E_{L_1} - D_1.$$

Multiplying out the 1st component, recognizing $\frac{(1-T_D)I_2}{r_{D_2}}$ is D_2, and rearranging:

$$G_{L2}^{D\to E} = D_2 - \frac{(1-T_E)(1-T_C)I_2}{r_{Lg2}} - E_{L_1} + \frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg2}} - D_1 + \frac{(1-T_D)I_1}{r_{D_1\uparrow}}.$$

Multiplying the 2nd component by $\frac{(1-T_D)r_{D_2}}{(1-T_D)r_{D_2}} = 1$ to get $-\left(\frac{(1-T_E)(1-T_C)r_{D_2}}{(1-T_D)r_{Lg2}}\right)\frac{(1-T_D)I_2}{r_{D_2}}$ which is $-\left(\frac{(1-T_E)(1-T_C)r_{D_2}}{(1-T_D)r_{Lg2}}\right)D_2$, factoring out D_2, and setting $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$:

$$G_{L2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg2}}\right]D_2 - E_{L_1} + \frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg2}} - D_1 + \frac{(1-T_D)I_1}{r_{D_1\uparrow}}.$$

Multiplying the 3rd component by $\frac{r_{Lg1}}{r_{Lg1}} = 1$ to get $\left(\frac{r_{Lg1}}{r_{Lg2}}\right)\frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg1}}$ which is $\left(\frac{r_{Lg1}}{r_{Lg2}}\right)E_{L_1}$ and factoring out E_{L_1}:

$$G_{L2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg2}}\right]D_2 - \left[1-\frac{r_{Lg1}}{r_{Lg2}}\right]E_{L_1} - D_1 + \frac{(1-T_D)I_1}{r_{D_1\uparrow}}.$$

Multiplying the last component by $\frac{r_{D_1}}{r_{D_1}} = 1$ to get $\left(\frac{r_{D_1}}{r_{D_1\uparrow}}\right)\frac{(1-T_D)I_1}{r_{D_1}}$ which is $\left(\frac{r_{D_1}}{r_{D_1\uparrow}}\right)D_1$, and factoring out D_1:

$$G_{L2}^{D\to E} = \left[1-\frac{\alpha r_{D_2}}{r_{Lg2}}\right]D_2 - \left[1-\frac{r_{Lg1}}{r_{Lg2}}\right]E_{L_1} - \left[1-\frac{r_{D_1}}{r_{D_1\uparrow}}\right]D_1. \quad (10)$$

Appendix C. Proof of equation (20)

Proof of equation (20) for the situation of a leveraged firm undergoing an equity-for-debt increment with constant growth, fixed tax rates, and wealth transfer due to risk shifting such that r_{D1} becomes $r_{D1\uparrow}$ causing D_1 to become $D_{1\downarrow}$. Using equation (8) where $G_{L2}^{E\to D} = V_{L_1} - V_{L\,2}$ and noting $V_{L_1} = E_{L_1} + D_1$ and substituting for $V_{L\,2}$ using equation (5):

$$G_{L2}^{E\to D} = E_{L_1} + D_1 - \frac{(1-T_E)(1-T_C)(C-I_1-I_2)}{r_{Lg\,2}} - \frac{(1-T_D)I_1}{r_{D1\uparrow}} - \frac{(1-T_D)I_2}{r_{D_2}}.$$

Multiplying out the 3rd component, recognizing $\frac{(1-T_D)I_2}{r_{D_2}}$ is D_2, and rearranging:

$$G_{L2}^{E\to D} = E_{L_1} - \frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg\,2}} - D_2 + \frac{(1-T_E)(1-T_C)I_2}{r_{Lg\,2}} + D_1 - \frac{(1-T_D)I_1}{r_{D1\uparrow}}.$$

Multiplying the 2nd component by $\frac{r_{Lg1}}{r_{Lg1}} = 1$ to get $\left(\frac{r_{Lg1}}{r_{Lg\,2}}\right)\frac{(1-T_E)(1-T_C)(C-I_1)}{r_{Lg1}}$ which is $\left(\frac{r_{Lg1}}{r_{Lg\,2}}\right)E_{L1}$ and factoring out E_{L_1}:

$$G_{L2}^{E\to D} = \left[1-\frac{r_{Lg1}}{r_{Lg\,2}}\right]E_{L\,1} - D_2 + \frac{(1-T_E)(1-T_C)I_2}{r_{Lg\,2}} + D_1 - \frac{(1-T_D)I_1}{r_{D1\uparrow}}.$$

Multiplying the 3rd component by $\frac{(1-T_D)r_{D_2}}{(1-T_D)r_{D_2}} = 1$ to get $\left(\frac{(1-T_E)(1-T_C)r_{D_2}}{(1-T_D)r_{Lg2}}\right)\frac{(1-T_D)I_2}{r_{D_2}}$ which is $\left(\frac{(1-T_E)(1-T_C)r_{D_2}}{(1-T_D)r_{Lg2}}\right)D_2$, factoring out D_2, and setting $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$:

$$G_{L2}^{E\to D} = \left[1-\frac{r_{Lg1}}{r_{Lg\,2}}\right]E_{L\,1} - \left[1-\frac{\alpha r_{D_2}}{r_{Lg\,2}}\right]D_2 + D_1 - \frac{(1-T_D)I_1}{r_{D1\uparrow}}.$$

Multiplying the last component by $\frac{r_{D1}}{r_{D\,1}} = 1$ to get $\left(\frac{r_{D1}}{r_{D1\uparrow}}\right)\frac{(1-T_D)I_1}{r_{D1}}$ which is $\left(\frac{r_{D1}}{r_{D1\uparrow}}\right)D_1$, and factoring out D_1:

$$G_{L2}^{E\to D} = \left[1-\frac{r_{Lg1}}{r_{Lg\,2}}\right]E_{L\,1} - \left[1-\frac{\alpha r_{D_2}}{r_{Lg\,2}}\right]D_2 + \left[1-\frac{r_{D1}}{r_{D1\uparrow}}\right]D_1. \quad (20)$$