“Liquidity of financial options on GARCH option pricing in AMEX option market”

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Liquidity of financial options on GARCH option pricing in AMEX option market

Abstract

This paper examines the pricing efficiency of Heston and Nandi GARCH (HN GARCH) model on financial options in the AMEX option market. A total of eleven major financial options during 2006 are sampled and classified by liquidity, market capitalization, and P/E ratio. The authors find that, while HN GARCH model has smaller valuation errors overall, they appear to be ill-suited for valuation of small market capitalization companies and display notable underpricing for options of low P/E ratio companies. They, however, do a good job modeling the option prices of lower liquidity companies, whose options are much more European in practice.

Keywords: Black-Scholes option pricing model, HN GARCH model, MLE method, liquidity.
JEL Classification: G20, G21.

Introduction

Some of the earlier contributions made by financial researchers regarding the weakness of Black-Scholes model include the displaced-diffusion model, and the flexible binomial model. However, there is one important catch to all of these models – the variance rate is not observable. Latest developments in this field are of two main types: implied volatility and stochastic volatility. The latter, in particular, include continuous-time stochastic models and discrete-time stochastic generalized autoregressive conditional heteroskedasticity (GARCH) models.

While continuous-time stochastic volatility models can price options effectively, they are extremely difficult to implement in practice. Moreover, though both of these models assume that volatility can be observed, it is nonetheless difficult to filter a continuous volatility variable from discrete observations. An alternative to this approach is to use implied volatilities computed from option prices instead. This alternative, however, requires that volatility be estimated for every single trading date of the asset and can be computationally burdensome, if not outright infeasible, for a long time series of option records. Thus, it is important that continuous-time models be augmented with non-trivial volatility estimation techniques.

In addition, the continuous-time model can serve as the limit of a certain GARCH model. Nevertheless, Duan (1995) suggests that most of the existing bivariate diffusion models that has been used to model asset returns and volatility could be represented as limits of a family of GARCH models. In particular, the GARCH option model proposed by Heston and Nandi (2000) is proven to contain Heston's (1993) stochastic volatility model as a continuous time limit. GARCH models have an inherent advantage over the continuous-time models in the sense that the volatility on which they rely can be readily observed from the asset's historical prices. Thus, it is possible to price an option by simply using the information about historical prices of the underlying asset. In contrast, while the continuous-time stochastic models assume volatility to be observable, the volatility cannot be filtered precisely from discrete observations of spot asset prices in a continuous-time stochastic volatility mode. It is, therefore, impossible to price an option by relying on the historical prices of the underlying asset alone.

Given that volatility cannot be observed, an alternative approach is to use the volatility implied from one option to price other options on the same underlying asset. However, such an approach could be error-prone, especially when the trade volume of the options under consideration is low. As a result, GARCH model is preferred over continuous-time models when the performance of stochastic option models and discrete-time models are compared.

Duan (1995) incorporates GARCH into discrete-time model and proposed the GARCH option pricing model to extend the Black-Scholes model. The key hypothesis of the GARCH process is conditional heteroskedasticity, with variance determined by a series of parameters and a sequence of random variables that are noise. Further, nonlinear GARCH, or NGARCH model capture the negative correlation between returns and conditional volatility. The general theory of GARCH option pricing also applies to NGARCH models as well.

Unfortunately, most GARCH models lack closed-form solutions for option prices. As a result, many of these models have been solved empirically by Monte Carlo simulations (for example, Lord et al., 2010; Pitt et al., 2012; Ardia et al., 2012). However, Monte Carlo simulations are often time-consuming and computationally intensive. Ritchken and Trevor (1999) bring forth a lattice approximation to value American options. Duan et al. (1999) suggest a Markov chain approach for GARCH processes with single lags in the variance dynamics. Heston and Nandi (2000) also develop a closed-form solution for European option values (and hedge ratios) in a GARCH model. Heston and Nandi's model, in particular, allows for both multiple lags in the time series dynamics of the variance process and correlation between returns of the spot asset and variance. Hence, it suggests another alternative for option pricing.

This paper tests the empirical implications based on Heston and Nandi (2000) GARCH model on financial options in the American Stock Exchange (henceforth AMEX) by comparing its pricing errors with those of the Black-Scholes model. According to Lehar et al. (2002), Su et al. (2010), and Ekstrom et al. (2011), it may be expected that the GARCH model, with slight modifications, should demonstrate smaller out-of-sample valuation errors compared to the Black-Scholes model.

An additional dimension worth investigating about the specifics of the option pricing is also included in this paper. Namely, though it may be expected that GARCH option pricing model would fair more impressively compared to traditional Black-Scholes model, is there any other factor at play here? For one thing, it makes sense to take the liquidity of the underlying asset into consideration; it would seem the higher the liquidity of an asset, the more likely the less information asymmetry there is and hence the smaller the pricing errors expected. Other factors, such as the market capitalization and the P/E ratio of the companies, also deserve some consideration as well. In particular, a greater market capitalization for a company would mean more outstanding shares available for trade and hence probably higher liquidity, while a greater P/E ratio may suggest a higher degree of overpricing; the same argument would also work in reverse for companies with lower P/E ratios as well.

It can be argued that American options, inherently more complex compared to their European counterparts, may be ill-suited for analysis by Heston and Nandi's GARCH model. However, many of the more notable approaches to the pricing of American options, such as the regression-based approach by Longstaff and Schwarz (2001), the Monte Carlo simulation approach by Haugh and Kogan (2004), and the static hedge portfolio approach by Chung and Shih (2009) are extremely computationally intensive. Moreover, as shown by La and Lemieux (2005), for American options whose prices are not extremely close to zero, the variance are not much different from their European counterparts. Thus, this paper makes the connection between the models and applies Heston and Nandi GARCH model to the pricing of American options while noting that such a generalization can have a slight effect on the pricing errors because of how variances are defined.

The rest of this paper proceeds as follows. Section 1 introduces the methodology, including data description and the model applied. Section 2 gives the in-sample estimation through MLE and out-of-sample pricing results. The final section concludes.

1. Data and methodology

For the sake of comparability, all companies selected are from the financial industry. The sample consists of eleven leading financial companies which are, in alphabetical order, Bank of America, Citigroup, Goldman Sachs, ING Group, JP Morgan Chase, Lehman Brothers, Merrill Lynch, Morgan Stanley, Wachovia, Washington Mutual, and Wells Fargo. Moreover, the sample data are collected on Wednesdays during three years for a total of 156 trading days. For the risk free rate, the continuously compounded Treasury bill rates interpolated to match the maturity of the options are used. The samples are obtained from Wharton Research Data Services (WRDS). A brief breakdown of these companies can be found in Table 1.

Table 1. Brief breakdown of the companies analyzed

Company name	Average 3 month trade volume $ million	Market capitalization (billion)	Price to equity ratio 52-week average
Bank of America	48.77	157.30	10.72
Citigroup	115.38	118.24	31.67
Goldman Sachs	14.01	70.06	7.69
ING Group	1.97	80.10	5.62
JP Morgan Chase	42.91	142.15	9.55
Lehman Brothers	30.09	21.73	6.47
Merrill Lynch	30.47	41.83	9.80
Morgan Stanley	18.96	48.07	22.70
Wachovia	36.56	49.91	7.72
Washington Mutual	52.92	9.21	12.20
Wells Fargo	38.54	90.63	11.54

Note: Data courtesy of Forbes.com and Yahoo! Finance.

These companies can be further classified by their average trade volume, market capitalization, and 52-week average P/E ratio. Specifically, companies are considered to be small (large) volume companies if their average 3-month trade volume is less (greater) than or equal to 10 (100) million contracts, while they are considered to be moderate volume companies if their average 3-month trade volume is between 10 and 100 million contracts. Moreover, companies are considered to be small (large) market capitalization companies if their market capitalization is less (greater) than 50 (100) billion dollars, while they are considered to be medium market capitalization companies if their market capitalization is between 50 and 100 billion dollars. Last but not least, companies are considered to have small P/E ratios if their P/E ratios are less than 10; they are considered to have large P/E ratios otherwise.

Table 2 shows how the companies have thus been classified and the criteria for classification. Note that the criteria are rather arbitrary; that is, the boundaries between adjacent classifications are only chosen to facilitate the analyses, whose results will be organized based on this classification. This paper is restricted to the analysis of call options only. Moreover, an option is included in the analysis only if it also fits the following criteria.

First of all, only the call options with moneyness, or K/S, between 0.9 and 1.1 are used for the analysis. The elimination of extremely deep out-of-money and in-the-money options from the analysis that are infrequently traded ensures a certain level of credibility of the results of the analysis. Secondly, only those options with volume greater or equal to 100 contracts were considered for the analysis. This restriction ensures the active trading of the options being considered and thus less information asymmetry between the trading parties that could lead to pricing errors. Last, the call options are eliminated from consideration for the analysis if they do not satisfy the boundary condition below:

$$S(t) \geq C(t,T) \geq Max\left(0, S(t) - PVD - Ke^{-r(T-t)}\right), \quad (1)$$

where $S(t)$ stands for the price of the underlying asset at time t, $C(t,T)$ stands for the price of the call option with time to maturity T at time t, PVD stands for the present value of all dividends on the underlying asset, and $e^{-r(T-t)}$ stands for the present value of the strike price of the call option.

Table 2. Classification of the companies analyzed based on average 3 month trade volume, market capitalization and 52-week average P/E ratio

Small volume			
ING Group – medium market capitalization, small P/E ratio			
Moderate volume			
	Small market capitalization	Medium market capitalization	Large market capitalization
Small P/E ratio Small P/E ratio Small P/E ratio	Lehman Brothers Merrill Lynch Wachovia	Goldman Sachs Goldman Sachs Goldman Sachs	JP Morgan Chase JP Morgan Chase JP Morgan Chase
Large P/E ratio Large P/E ratio	Morgan Stanley Washington Mutual	Wells Fargo Wells Fargo	Bank of America Bank of America
Large volume			
Citigroup – large market capitalization, large P/E ratio			

Note: Throughout the paper, Lehman Brothers and Washington Mutual will represent their corresponding categories.

The first inequality must hold because, if not, investors can always earn arbitrage profits by buying one unit of the underlying asset and sell the corresponding call option. In other words, the cost of the right to purchase an asset can never be higher than the cost of the actual asset to be purchased. The second inequality must be satisfied as well to ensure no arbitrage opportunity; if it is not satisfied, an investor can always make an instant profit by buying the call option and exercising it immediately.

Thus, the data set of call options consists of 8773 observations, or about 731 observations per company, with a minimum of 177 and a maximum of 1250. The sample is further categorized into 10 different groups based on their moneyness and time to maturity. In terms of moneyness, the data set is divided into five categories: deep in-the-money call options with $0.9 \leq K/S < 0.95$, in-the-money options with $0.95 \leq K/S < 0.99$, at-the-money options with $0.99 \leq K/S < 1.01$, out-of-money options with $1.01 \leq K/S < 1.05$, and deep out-of-money options with $1.05 \leq K/S < 1.1$. These five categories are then further classified according to their time to maturity: short-term (with time to maturity less than or equal to 30 days) and long term (with time to maturity greater than 30 days). Panel A of Table 3 lists the total number of call option contracts analyzed in this paper by time to maturity and moneyness.

Panel B of Table 3 shows the average implied Black-Scholes volatilities across various time to maturity and moneyness categories from call contracts used for the analysis in this paper. These numbers suggest that implied volatilities are not exactly constant across different times to maturity and moneyness, which may be interpreted as the presence of a mild volatility smirk/smile effect of the sample overall. If the companies that comprise the sample are inspected separately by average trade volume, market capitalization, and

P/E ratio, the results are similar. While these results may seem to violate the assumption of constant volatility under Black-Scholes model somewhat, the effect is relatively mild and will not be taken into consideration in the empirical analysis. In addition, the daily historical closing prices of the stocks of the companies during a year are used to estimate the parameters of the GARCH process.

Table 3. Number of contracts and average implied volatilities of contracts across time to maturity and moneyness

Panel A. Number of contracts			
Moneyness	# of options that will mature in 30 days or less	# of options that will mature in more than 30 days	Total
$0.90 \le K/S < 0.95$	293	861	1154
$0.95 \le K/S < 0.99$	396	1357	1753
$0.99 \le K/S < 1.01$	622	1874	2696
$1.01 \le K/S < 1.05$	569	1595	2154
$1.05 \le K/S < 1.10$	226	980	1206
Total	2106	6667	8773

Panel B. Average implied volatilities of contracts		
	Time to maturity ≤ 30 days	Time to maturity > 30 days
$0.90 \le K/S < 0.95$	0.28305	0.21148
$0.95 \le K/S < 0.99$	0.23372	0.19140
$0.99 \le K/S < 1.01$	0.21802	0.18609
$1.01 \le K/S < 1.05$	0.21624	0.17883
$1.05 \le K/S < 1.10$	0.24593	0.17669

We examine pricing error based on the closed form HN GARCH option valuation model. In general, GARCH models are solved by slow, computationally intensive simulations, making them impractical for empirical analyses. In contrast, HN GARCH model gives an analytical solution and thus is more applicable for the real option market. This model features two assumptions. The first assumption is that the log-spot prices follow a particular GARCH process.

Assumption 1: The spot asset price, $S(t)$ (including accumulated interest or dividends), follows the following process over time steps of length Δ

$$\log\left(S(t)\right) = \log\left(S(t-\Delta)\right) + r + \lambda h(t) + \sqrt{h(t)}z(t),$$
$$h(t) = \omega + \sum_{i-1}^{p}\beta_i h(t-\mathrm{i}\Delta) + \sum_{i=1}^{q}\alpha_i\left(z(t-\mathrm{i}\Delta) - \gamma_i\sqrt{h(t-\mathrm{i}\Delta)}\right),^2$$

where r is the continuously compounded interest rate for the time interval Δ, $z(t)$ is the standard normal disturbance, $h(t)$ is the conditional variance of the log return between t-Δ and t, λ is the risk premium, ω is the mean conditional volatility, γ_i is the asymmetric influence of shock; a large negative shock $z(t)$ raises the variance more than a large positive shock $z(t)$, and α_i is the kurtosis of the distribution; a zero value implies a deterministic time varying variance.

As the α_i and β_i parameters approach zero, the model approaches the Black-Scholes model observed at discrete intervals. This paper focuses on the first-order GARCH process (i.e. $p = q = 1$). The first order GARCH process is stationary with finite mean and variance if $\beta_1 + \alpha_1\gamma_1^2 = 1$.

Assumption 2: The value of a call option one period prior to expiration obeys the Black-Scholes-Rubinstein (hereafter BSR) formula.

The BSR formula makes sense here because the spot price has a conditionally lognormal distribution over one single period. Basically, if the BS formula holds for one single period, the risk neutral distribution of the asset price is lognormal with mean $S(t-\Delta)e^r$. In other words, a random variable $z^*(t)$ with a standard normal distribution under risk-neutral probability can always be found. Follow Heston and Nandi (2000). At time t, an European call option with strike price K with expiration time T is worth:

$$C = e^{-(T-t)}E_t^*\left[Max\left(S(t)-K,0\right)\right] = \frac{1}{2}S(t) + \frac{e^{-r(T-t)}}{\pi}\int_0^\infty Re\left[\frac{K^{-i\varphi}f(i\varphi+1)}{i\varphi f(1)}\right]d\varphi - Ke^{-r(T-t)}\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{K^{-i\varphi}f(i\varphi)}{i\varphi}\right]d\varphi\right), \quad (3)$$

where $E_t^*[\ ]$ denotes the expectation under risk-neutral probability measure.

Thus, once the information of the characteristic function of $\log(S(T))$ becomes available, this proposition allows the expectation to be computed without evaluating two separate integrals. This is different from the approach proposed by Heston (1993) that calls first for two characteristic functions, namely, $f_1(\log S, v, t; \phi)$ and $f_2(\log S, v, t; \phi)$, to be found, which can later be inverted to obtain the desired probabilities as shown in the following formula:

$$P_j\left(\log S, v, T; \log k\right) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{K^{-i\varphi} f_j\left(\log S, v, T; \varphi\right)}{i\varphi}\right] d\varphi. \quad (4)$$

Then, substituting the probabilities into the formula, $C\left(s,v,t\right) = SP_1 - Ke^{-r(T-t)}P_2$, generates the option prices.

Compared with Black-Scholes model, whose emphasis is on the current asset price and its variance, Heston and Nandi GARCH model hinges on both current asset price and the conditional variance. Since the conditional variance is a function of the observed path of asset prices, the option formula becomes a function of current and historical asset prices. The most important difference here is that volatility in Heston and Nandi GARCH model is readily observable in the historical prices of the underlying asset compared to continuous-time stochastic volatility models.

The objectives of study are as follows. This paper examines the valuation performance of GARCH model (both the unrestricted and restricted versions) on the pricing of options of stocks listed on AMEX. Further, we examine the pricing efficiency of HN GARCH model on financial options by liquidity, market capitalization, and P/E ratio. Moreover, we explore which type of options could do a good job modeling the option prices. That is, we shed some light on the effects of liquidity, market capitalization, and P/E ratio on the pricing errors of options by varios models.

Thus, the purpose of research is to explore the following hypotheses.

Hypothesis 1: The pricing errors of HN GARCH model (both the unrestricted and restricted versions) are lower than those in other option pricing models.

Heston and Nandi (2000) develop a closed-form solution for European option values in a GARCH model. HN GARCH model performs well for option pricing in European option. We presume the HN GARCH model also performs well for option pricing in American option.

Hypothesis 2: HN GARCH model could do a better job modeling the option prices of lower liquidity companies.

Since trades happen relatively rarely in companies with smaller liquidity, the options behave more like European options with lower volatilities and hence are more accurately modeled by HN GARCH model.

2. Empirical results

2.1. Estimation. The empirical analysis focuses mainly on the single lag version of GARCH model. Here, Δ is set to 1, and daily stock returns are used to model the evolution of volatility. Unlike the case with continuous time stochastic volatility models, in which the volatilities are unobservable, for the GARCH model, all parameters can be estimated directly from the historical asset prices. The estimation is done with the MLE method. Moreover, in order to bring out the importance of the skewness parameter, γ_1, and its effects, this estimation is run twice, once with an unrestricted model and once with a restricted model with γ_1 set to zero. In the latter case, the model is also called a symmetric GARCH model. This estimation is performed on the data of time series of stock prices during a year.

The volatility of volatility is very small (not reported for briefly) for companies of all trade volume, market capitalization, and P/E ratio, being no more than 0.0004 in all cases. It may be worth noting, though, that the company with the greatest α_1, around 0.0004 in the asymmetric GARCH model and 0.0002 in the symmetric GARCH model, is ING Group, an indication that the returns of the company with the smallest trade volume may be the most volatile. Moreover, the parameter used to measure the degree of mean reversion, $\beta_1 + \alpha_1\gamma_1^2$, is between 0.6 and 0.8 for most companies for both versions of the two models, indicating a strong mean reversion. However, mean aversion seems to be almost nonexistent for Bank of America for both models, with values of $1.56*10^{-8}$, $9.10*10^{-8}$, $3.63*10^{-8}$, and $4.00*10^{-9}$ for the models computed. This lack of mean reversion can perhaps be explained by the stock split it executed late in 2004, which greatly increased the volatility of its stock prices. Other than Bank of America, Washington Mutual, another company of large P/E ratio, also exhibited this apparent lack of mean reversion. Yet, the other company of large P/E ratio, Wells Fargo, showed no such result. This seems to cast Washington Mutual as an anomaly, whose small market capitalization and large P/E ratio may be a sign of great information asymmetry causing the significant deviation from the mean. The annualized long-run mean of volatility are relatively low for both models with or without risk.

The parameters obtained from the historical price levels using MLE can then be plugged into option pricing formula to compute the call option prices. The catch, however, is that the information set for the historical price levels are not quite the same as those of the option prices. More specifically, option prices are forward looking, meaning they carry expectations about future price evolutions of the underlying asset. As Su et al. (2010) show, the pricing of options by plugging in parameters obtained from MLE into Heston and Nandi's GARCH pricing model is plagued by significant errors. As a result, some corrections and improvements on this method, to be discussed in the next section, are in order.

2.2. In-sample estimation. Researchers who use time series models prefer longer sample periods; trivially, the more samples points there are available, the better the estimations should be. That said, the distribution of the stock returns do vary with time, and if the sample period is too long, it may contain noise that greatly reduces the volatility clustering and thus compromises the accuracy of volatility estimations. Thus, a rolling window of one year is used to update the parameters of the GARCH model; that is, for every week out of sample, the time series of returns from the previous 252 trading days are used to filter out the variance. As already demonstrated in Su et al. (2010), such a method can help reduce the pricing errors of GARCH option pricing.

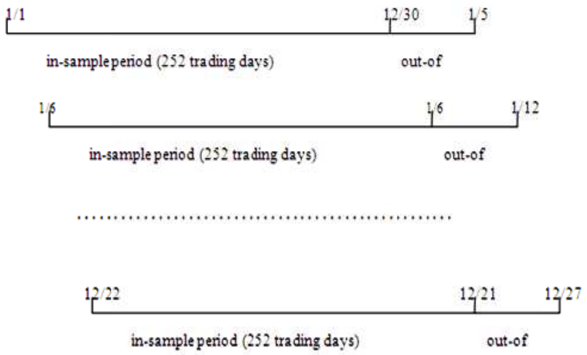

Fig. 1. Out-of-sample comparison

2.3. Out-of-sample comparison. After the GARCH option model pricing parameters have been estimated using the sample period of the 52 weeks of trading days of 2004, they are then used to predict the option prices in the out-of-sample period from January 1 to December 31, 2006. Here, for the GARCH option pricing model, it is assumed that the implied parameters are constant over each week of computation. The traditional Black-Scholes model will serve as the benchmark. To allow for different models' effectiveness to be objectively compared, three loss functions have been computed to assess the merit of the models. The specific loss functions used in this paper are mean absolute errors (MAE), mean percentage errors (MPE), and root mean squared errors (RMSE). None of these loss functions is perfect, however; for example, RMSE tends to assign proportionally greater weights to observations with high volatilities; MPE, on the other hand, tends to assign great weights to short time to maturity and out-of-money options with prices close to zero. Therefore, all three measures are included for fairness to allow their strengths to be maximized and weaknesses be minimized.

The loss functions are defined below mathematically:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|C^i_{model} - C^i_{market}\right|,$$
$$MPE = \frac{1}{N}\sum_{i=1}^{N}\frac{\left(C^i_{model} - C^i_{market}\right)}{C^i_{market}}, \tag{5}$$
$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(C^i_{model} - C^i_{market}\right)^2},$$

where C^i_{model} is the price of call option i as computed from the models, C^i_{market} is the market price of call option I, and N is the number of contracts involved.

Table 4 reports the out-of-sample valuation errors for various models. According to the table, the updated GARCH model outperforms the BS and non-updated GARCH models in most of the moneyness and time to maturity categories for almost all the companies. This is especially true for the deep out-of-money (1.05 ≤ K/S < 1.10) and in-the-money (0.90 ≤ K/S < 0.95) options, where BS and non-updated GARCH models have been proven by Su et al. (2010) to fair less impressively. The difference in performance between BS and non-updated GARCH models, however, is not so straightforward; these two models seem to be evenly matched, with each model outperforming the other in different moneyness and time to maturity categories for different companies.

Table 4. The out-of-sample pricing errors

Panel A							
		Lehman Brothers		Washington Mutual		Goldman Sachs	
Moneyness	Model	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. >30
0.90 ≤ K / S< < 0.95	BS	0.274	0.228	0.357	0.235	0.306	0.284
	GARCH	0.162	-0.123	0.327	0.163	-0.294	-0.154
	GARCH updated	0.150	0.159	0.231	0.134	0.290	0.099
0.95 ≤ K / S< < 0.99		0.228	-0.188	0.270	0.229	0.262	0.276
	GARCH	-0.057	0.013	-0.007	0.053	-0.019	-0.168
	GARCH updated	0.045	-0.040	0.114	0.128	0.135	-0.058
0.99 ≤ K / S< < 1.01	BS	0.138	-0.231	0.272	0.254	0.218	0.303
	GARCH	0.078	0.000	0.166	0.152	-0.126	-0.186
	GARCH updated	-0.105	-0.104	-0.104	0.184	-0.004	-0.132
1.01 ≤ K / S< < 1.05	BS	0.214	0.208	0.146	0.265	0.241	0.324
	GARCH	-0.013	0.023	-0.268	0.121	-0.124	0.048
	GARCH updated	-0.060	0.072	-0.140	0.087	0.053	0.052

Table 4 (cont.). The out-of-sample pricing errors

Panel A							
		Lehman Brothers		Washington Mutual		Goldman Sachs	
Moneyness	Model	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. >30
1.05 ≤ K / S< < 1.10	BS	0.321	0.245	0.226	0.244	0.384	0.319
	GARCH	0.309	0.142	0.323	0.275	-0.092	-0.286
	GARCH updated	-0.106	-0.090	0.318	0.091	-0.145	0.094

Panel B							
		Wells Fargo		JP Morgan Chase		Bank of America	
Moneyness	Model	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. >30
0.90 ≤ K / S < 0.95	BS	0.293	0.264	0.302	0.114	0.214	0.327
	GARCH	0.276	0.073	-0.151	0.046	0.069	0.113
	GARCH updated	0.257	0.243	0.196	0.024	-0.051	0.131
0.95 ≤ K / S< < 0.99	BS	0.249	0.241	0.173	0.221	0.199	0.238
	GARCH	-0.197	-0.143	-0.017	0.046	-0.037	0.056
	GARCH updated	0.097	0.173	-0.062	-0.124	-0.066	0.099
0.99 ≤ K / S< < 1.01	BS	0.252	0.220	0.199	0.257	0.242	0.210
	GARCH	0.089	-0.096	-0.113	0.175	-0.100	-0.040
	GARCH updated	0.052	-0.110	-0.102	-0.119	-0.129	0.184
1.01 ≤ K / S< < 1.05	BS	0.276	0.241	0.229	0.303	0.321	0.261
	GARCH	-0.060	-0.108	-0.004	-0.062	0.031	0.080
	GARCH updated	0.179	0.023	-0.061	0.072	0.233	0.224
1.05 ≤ K / S< < 1.10	BS	-0.437	0.246	0.395	0.347	0.471	0.364
	GARCH	-0.380	-0.125	0.334	0.409	-0.360	-0.344
	GARCH updated	0.169	-0.078	-0.230	-0.054	0.332	0.335

Panel C					
		ING Group		Citigroup	
Moneyness	Model	t.t.m > 30	t.t.m. > 30	t.t.m. > 30	t.t.m. > 30
0.90 ≤ K / S< < 0.95	BS	0.237	0.184	6.452	7.905
	GARCH	0.148	0.227	5.496	6.762
	GARCH updated	-0.207	-0.010	4.756	5.565
0.95 ≤ K / S< < 0.99	BS	0.098	-0.064	4.008	5.871
	GARCH	-0.236	-0.046	5.449	5.453
	GARCH updated	0.187	-0.075	2.169	4.286
0.99 ≤ K / S< < 1.01	BS	0.093	0.160	3.741	5.258
	GARCH	-0.046	-0.074	3.662	6.993
	GARCH updated	-0.133	-0.020	3.297	5.291
1.01 ≤ K / S< < 1.05	BS	-0.276	0.276	3.789	7.848
	GARCH	0.254	0.275	4.412	6.818
	GARCH updated	-0.248	0.186	3.528	6.803
1.05 ≤ K / S< < 1.10	BS	0.410	0.351	6.436	9.856
	GARCH	0.362	0.271	5.713	9.053
	GARCH updated	-0.304	0.161	5.578	7.231

Notes: "t.t.m." means time to maturity.

In addition, a particular point about Table 4 is that for almost all companies, regardless of their trade volume, market capitalization, or P/E ratio, the volatility of the pricing errors of the options of longer time to maturity (i.e. time to maturity > 30 days) appear to be greater than those of shorter time to maturity (i.e. time to maturity ≤ 30 days) no matter what loss function is used as the basis of comparison. This may be due to the fact that American options are inherently more complex compared to their European counterparts. Whereas European options show more volatility as the exercise date approaches, American options have just the opposite story; the longer the time to maturity, the more possible dates of exercise for American options, and thus the greater the volatility. Another fact about Table 4 is that liquidity does not seem to make a difference at least as far as the loss functions are concerned. Comparing the results of ING Group (small trade volume) and Goldman Sachs (moderate trade volume) as well as Bank of America (moderate trade volume) and Citigroup (great trade volume), no definite conclusion can be drawn between the pricing errors of the pairs. While the pricing errors[1] do appear to suggest that ING Group, the company with the smaller trade volume and hence lower liquidity, is more accurately priced, especially for its options with time to maturity greater than 30 days, the same trend cannot be observed between Bank of America and Citigroup. It is worth noting, though, that in the case of ING Group vs Goldman Sachs, it is actually the company with the lower liquidity that is priced more accurately, an observation that seems to contradict the rationale presented in earlier sections. A possible explanation may be that the call options of lower liquidity, because of the relatively infrequency of their changing hands, behave more like Bermudan, or even European, options, whose volatilities are smaller compared to American ones. The trend becomes more conspicuous for options of longer maturities because of the additional exercise dates that have to be considered. Moreover, there appears to be a greater amount of underpricing for the options of the companies with a lower P/E ratio. From the pairwise comparison of the MPE of Lehman Brothers (low P/E ratio) and Washington Mutual (high P/E ratio), Goldman Sachs (low P/E ratio) and Wells Fargo (high P/E ratio), and JP Morgan Chase (low P/E ratio) and Bank of America (high P/E ratio), it does appear that, in the thirty different scenarios of moneyness, time to maturity, and option pricing models considered, the companies with lower P/E ratios in the respective pairs tend to show negative MPEs more often compared to their higher P/E counterparts. The same could also be said of ING Group, which is another company with a low P/E ratio. This corresponds to the rationale presented earlier about a greater possibility of underpricing errors for companies of lower P/E ratios, whose stocks, and thus options, are of greater value to the investors.

Last but not least, perhaps somewhat surprisingly, Lehman Brothers feature the greatest errors in pricing of all the companies considered in this study. Other than the fact that Lehman Brothers, with a small P/E ratio, may more likely exhibit greater errors because of a significant amount of underpricing, this observation also seems to correspond to the lack of mean reversion for Washington Mutual, another company of small market capitalization. These may combine to suggest that these option price methods are especially ill-suited for the pricing of options of companies with small market capitalizations.

[1] While the pricing errors are measured by MPE, MAE and RMSE, the results are similar. We just present the results of MPE to save space.

Conclusions

This paper presents the valuation performance of GARCH model (both the unrestricted and restricted versions) on the pricing of options of stocks listed on AMEX and compares the pricing errors of this model with other option pricing models. The important observation is that, without updating the parameters on a weekly basis, the GARCH pricing model fairs no better than the traditional Black-Scholes model for the out-of-sample option pricing. That said, the weekly update does afford the GARCH model a significant, perhaps even unfair, edge over the Black-Scholes model. Moreover, the presence of a mild volatility smirk/smile effect may also have complicated the analysis, even with proper adjustments made by using five different implied volatilities to account for differences across moneyness.

Even though the updated GARCH model, as expected, outperforms the other two models, it should still be noted that its valuation errors for extremely out-of-money options are especially high. One possible explanation for this apparent inaccuracy is an inappropriate sample period, which may either have been too short to capture fully the volatilities or too long to include unwarranted noise. Moreover, the analysis presented in this paper relies mainly on MLE, which is inherently backward-looking and may come up considerably short in pricing options that are forward-looking.

Nevertheless, the models presented in this paper do shed some light on the effects of liquidity, market capitalization, and P/E ratio on the pricing errors of options by various models. Specifically speaking, companies with smaller liquidity tend to exhibit smaller pricing errors, especially when the options have a long time to go before maturing, as demonstrated by the particular case of ING Group. This may be due to the fact that, because trades happen relatively rarely, the options behave more like European options with lower volatilities and hence are more accurately modeled by Heston and Nandi GARCH model.

Moreover, companies with smaller P/E ratios tend to be underpriced more, though not by a considerable margin. This is in line with the intuition that, the lower the P/E ratio of an asset, the more valuable it is and hence the more underpriced it is relative to other assets. However, since in no company's case does the underpricing dominate (with ING Group being the most conspicuous example, exhibiting underpricing in 13 of the 30 MPEs), the argument is not convincing enough. It should also be noted that, of the companies considered, those with small market capitalizations are the ones with the greatest pricing errors; perhaps the pricing errors as the result of a low P/E ratio appear insignificant compared to the influence exerted by a small market capitalization.

In all, while Heston and Nandi GARCH model appears to be more of a computational convenience than other more optimal models, such as the GJR-GARCH as Lo and Wang (1995) suggested, it does provide some useful insight on the relationship between liquidity, market capitalization, and P/E ratios and their effects on pricing errors. More research will be required to specifically gauge the effects of these factors on the pricing errors.

The above findings have important implications for policy implications. Since HN GARCH model has more of a computational convenience than other more optimal models, market regulators should encourage relevant financial firms to price options HN GARCH model to lower pricing errors. There is a possible direction to suggest for future research. Because global events could change financial environment, there are different extents of influence on different option pricing model. Thus, future research can explore whether the pricing errors of HN GARCH model still lower after global financial events.

References

1. Ardia, D., N., Basturk, L., Hoogerheide and H.K., Van Dijk. A comparative study of Monte Carlo methods for efficient evaluation of marginal likelihood // *Computational Statistics & Data Analysis*, 2012. – №56. – pp. 3398-3414.
2. Black K., M. Scholes. The pricing of options and corporate liabilities // *Journal of Political Economy*, 1973. – №81. – pp. 637-659.
3. Chung S.L., P.T. Shih. Static hedging and pricing American options // *Journal of Banking and Finance*, 2009. – №33. – pp. 2140-2149.
4. Duan J. The GARCH option pricing model // *Mathematical Finance*, 1995. – №5. – pp. 13-32.
5. Duan J., G. Gauthier, and J. Simonato. An analytical approximation for the GARCH option pricing model // *Journal of Quantitative Computational Finance*, 1999. – №2. – pp. 75-116.
6. Ekstrom E., P. Lotstedt, J. Tysk, and L. Von Sydow. Numerical option pricing in the presence of bubbles // *Quantitative. Finance*, 2011.
7. Haugh M., L. Kogan. Pricing American options: A duality approach // *Operations Research*, 2004. – №52. – pp. 258-270.
8. Heston S. A closed-form solution for options with stochastic volatility with application to bond and currency options // *Review of Financial Studies*, 1993. – №6. – pp. 327-343.
9. Heston S., S. Nandi. A closed-form GARCH option valuation model // *Review of Financial Studies*, 2000. – №13. – pp. 585-625.
10. La J., C. Lemieux. A study of variance reduction techniques for American option pricing // Proceedings of the 37th conference on winter simulation, 2005. – pp. 1884-1891.
11. Lehar A., M. Scheicher and C. Schittenkopf. GARCH vs. stochastic volatility: Option pricing and risk management // *Journal of Banking and Finance*, 2002. – №26. – pp. 258-270.
12. Lo A.W., J. Wang. Implementing option pricing formulas when asset returns are predictable // *Journal of. Finance*, 1995. – №50. – pp. 87-130.
13. Longstaff F., E. Schwartz. Valuing American options by simulation: A simple least-squares approach // *Review of Financial Studies*, 2001. – №14. – pp. 307-311.
14. Lord, R., R., Koekkoek and D., Van Dijk. A comparison of biased simulation schemes for stochastic volatility models // *Quantitative Finance*, 2010. – №10. – pp. 177-194.
15. Pitt, M.K., R., Silva, P., Giordani and R., Kohn. On some properties of Markov chain Monte Carlo simulation methods based on the particle filter // *Journal of Econometrics*, 2012. – №171. – pp. 134-151.
16. Ritchken P., R. Trevor. Pricing options under generalized GARCH and stochastic volatility processes // *Journal of Finance*, 1999. – №54. – pp. 377-402.
17. Su Y.C., M.D. Chen and H.C. Huang. An application of closed-form GARCH option pricing model on FTSE 100 option and volatility // *Applied Financial Economics*, 2010. – №20. – pp. 899-910.