“Long memory in the Ukranian stock market”

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Abstract

This paper examines the dynamics of stock prices in Ukraine by estimating the degree of persistence of the PFTS stock market index. Using long memory techniques the authors show that the log prices series is $I(d)$ with d slightly above 1, implying that returns are characterized by a small degree of long memory and thus are predictable using historical data. Moreover, their volatility, measured as the absolute and squared returns, also displays long memory. Finally, the paper examines if the time dependence is affected by the day of the week; the results indicate that Mondays and Fridays are characterized by higher dependency, consistently with the literature on anomalies in stock market prices.

Keywords: stock market prices, efficient market hypothesis, long memory, fractional integration.
JEL Classification: C22, G12.

Introduction

This paper analyzes the behavior of stock prices in Ukraine by modeling the PFTS stock market index. Specifically, it examines its degree of dependence, noting that if the order of integration of the series is equal to 1, it is possible for the efficiency market hypothesis to be satisfied provided the differenced process is uncorrelated. Moreover, it tests the hypothesis of mean reversion (orders of integration below 1 in prices) or alternatively, long memory returns (orders of integration above 1 in the log prices) by using long memory and fractional integration techniques. These are more general than the standard approaches based on integer degree of differentiation, and provide much more flexibility in modeling the dynamics of the process. Finally, the degree of dependence for each day of the week is investigated in order to establish whether there are any day-of-the-week effects.

We use daily data from January 2007 to February 2013 and the main results in the paper can be summarized as follows. First, we observe that the log-prices series are fractionally integrated or $I(d)$ with an order of integration, d, which is slightly above 1 and thus implying that the underlying returns present a small degree of long memory behavior. The same evidence of long memory is obtained for the absolute and squared returns, which are used as proxies for the volatility. These results are consistent with those obtained in other stock markets. More importantly, we also find evidence of higher degrees of dependency on Mondays and Fridays than during the other days of the week, validating the hypothesis that there is an anomaly related with the "day-of-the-week" effect in the Ukrainian stock market.

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The paper is organized as follows. Section 1 describes the methodology. Section 2 presents the data and the main empirical results, while the final section contains some concluding comments.

1. Long memory and fractional integration

Long memory is a feature of the data that implies that observations far apart in time are highly correlated. There are two main definitions of long memory, one in the time domain and the other in the frequency domain. Starting with the former, given a covariance stationary process $\{x_t, t = 0, \pm 1, \ldots\}$, with autocovariance function $E(x_t - Ex_t)(x_{t-j} - Ex_t) = \gamma_j$, according to McLeod and Hipel (1978), x_t is said to be characterized by long memory if

$$\lim_{T \to \infty} \sum_{j=-T}^{j=T} \left|\gamma_j\right| \tag{1}$$

is infinite. The alternative definition, based on the frequency domain, is the following. Suppose that x_t has an absolutely continuous spectral distribution function, implying that it has a spectral density function, denoted by $f(\lambda)$, and defined as:

$$f(\lambda) = \frac{1}{2\pi} \sum_{j'-\infty}^{j=\infty} \gamma_j \cos\lambda j, \; -\pi < \lambda \leq \pi. \tag{2}$$

Then, x_t displays the property of long memory if the spectral density function has a pole at some frequency λ in the interval $[0, \pi)$, i.e.,

$$f(\lambda) \to \infty, as \;\; \lambda \to \lambda^*, \; \lambda^* \in [0, \pi). \tag{3}$$

The empirical literature has focused on the case where the singularity or pole in the spectrum occurs at the 0 frequency, i.e., ($\lambda^* = 0$). This is the standard case of $I(d)$ models of the form:

$$(1 - L)^d x_t = u_t, \; t = 0, \pm 1, \ldots, \tag{4}$$

where d can be any real value, L is the lag-operator ($Lx_t = x_{t-1}$) and u_t is $I(0)$, defined for our purposes as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency.

Given the parameterization in (4) we can distinguish several cases depending on the value of d. Thus, if $d = 0$, $x_t = u_t$, x_t is said to be "short memory" or $I(0)$, and if the observations are autocorrelated (i.e. AR) they are of a "weakly" form, in the sense that the values in the autocorrelations are decaying at an exponentially rate; if $d > 0$, x_t is said to be "long memory", so named because of the strong association between observations far distant in time. If d belongs to the interval (0, 0.5) x_t is still covariance stationary, while $d \geq 0.5$ implies nonstationarity. Finally, if $d < 1$, the series is mean reverting in the sense that the effects of shocks disappear in the long run, contrary to what happens if $d \geq 1$ when they persist forever.

There exist several methods for estimating and testing the fractional differencing parameter d. Some of them are parametric while others are semiparametric and can be specified in the time or in the frequency domain. In this paper, we use a Whittle estimate of d in the frequency domain (Dahlhaus, 1989) along with a testing procedure, which is based on the Lagrange Multiplier (LM) principle and that also uses the Whittle function in the frequency domain. It tests the null hypothesis:

$$H_o: d = d_o, \tag{5}$$

for any real value d_o, in a model given by the equation (4), where x_t can be the errors in a regression model of the form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, ..., \tag{6}$$

where y_t is the observed time series, β is a (kx1) vector of unknown coefficients and z_t is a set of deterministic terms that might include an intercept (i.e., $z_t = 1$), an intercept with a linear time trend ($z_t = (1, t)^T$), or any other type of deterministic processes. Robinson (1994) showed that, under certain very mild regularity conditions, the LM-based statistic $(\hat{r})$:

$$\hat{r} \rightarrow_d N(0,1) \quad as \; T \rightarrow \infty, \tag{7}$$

where " $\rightarrow_d$ " stands for convergence in distribution, and this limit behaviour holds independently of the regressors z_t used in (6) and the specific model for the $I(0)$ disturbances u_t in (4).

As in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives have the same null and limit theory as the LM test of Robinson (1994). Lobato and Velasco (2007) essentially employed such a Wald testing procedure, even though it requires a consistent estimate of d; therefore the LM test of Robinson (1994) seems computationally more attractive. A semiparametric Whittle approach (Robinson, 1995) will also be implemented in the paper.

2. Data and empirical results

The series examined is the PFTS Ukrainian Stock Index. It is registered by Ukrainian SEC stock exchange, which is in operation since 1997 and currently is the largest marketplace in Ukraine. The PFTS index is calculated based on the results of the trading. The daily trade volume is about $30-60 million. Approximately 220 companies are listed on the PFTS, with a total market capitalisation around $140 billion. We use daily data from January 9, 2007 to February 27, 2013.

Figure 1 (see Appendix) displays the original time series, along with the corresponding returns, obtained as the first differences of the log-transformed data, and also the corresponding correlograms and periodograms. The original series appears to fluctuate throughout the sample period, while the returns are very stable. The correlograms of the returns, however, has many significant values, even for some lags far away from zero, and the periodogram has the highest value at the zero frequency, which suggests some degree of long memory in the return series.

As a first step we estimate a model of the form given by equations (4) and (6), with $z_t = (1,t)^T$, $t \geq 1$, 0, otherwise, i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \quad (1 - L)^d x_t = u_t, \quad t = 1, 2, ..., \tag{8}$$

where y_t is the log-transformed price.

We report in Table 1 the estimates of d in (8) for the three standard cases of no regressors in the undifferenced regression (i.e., $\beta_0 = \beta_1 = 0$ in (8)), an intercept (β_0 unknown and $\beta_1 = 0$), and an intercept with a linear time trend (β_0 and β_1 unknown) along with the 95% confidence interval of the non-rejection values of d using Robinson (1994) parametric approach.

Table 1. Estimates of the fractional differencing parameter in the log of PFTS series

	No regressors	An intercept	A linear time trend
White noise	1.009 (0.979, 1.043)	**1.218** **(1.181, 1.261)**	1.218 (1.181, 1.261)
AR(1)	1.381 (1.321, 1.450)	**1.095** **(1.049, 1.148)**	1.095 (1.049, 1.148)
Bloomfield	1.009 (0.960, 1.068)	**1.101** **(1.060, 1.154)**	1.101 (1.061, 1.154)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of d. The values corresponding to significant deterministic terms are in bold.

The results are reported for the cases of both uncorrelated and autocorrelated errors. In the latter case, we assume first that u_t is an AR(1) process, but then also model the disturbances following the more general specification proposed by Bloomfield (1973).

This is a non-parametric approach that approximates ARMA models with only a few parameters. The *t*-values for the deterministic terms (not reported) imply that the model with an intercept is the most adequate specification for all three types of disturbances. The estimated coefficient for the fractional differencing parameter is slightly above 1 in all three cases and, more importantly, the $I(1)$ hypothesis is statistically rejected in favor of higher orders of integration. This implies that the underlying returns are characterized by long memory, with an order of integration of about 0.21 in the case of uncorrelated errors, and slightly smaller if the errors are autocorrelated. This implies that market efficiency does not hold in the Ukrainian stock market since there is some degree of predictability based on historical data.

Next we examine the volatility of the series measured as its absolute and the squared returns[1]. Both series are displayed in Figure 2 (see Appendix) along with their corresponding correlograms and periodograms. We notice that the sample autocorrelation values now decay very slowly, and the periodgrams display large peaks at the zero frequency. This is clearly consistent with the $I(d)$ process presented in section 1 with a positive d.

Table 2. Estimates of the fractional differencing parameter in the absolute returns

	No regressors	An intercept	A linear time trend
White noise	0.256 (0.232, 0.283)	**0.245 (0.222, 0.273)**	0.243 (0.218, 0.271)
AR(1)	0.341 (0.303, 0.382)	**0.326 (0.287, 0.373)**	0.324 (0.283, 0.374)
Bloomfield	0.359 (0.312, 0.417)	**0.343 (0.280, 0.404)**	0.342 (0.281, 0.404)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of d. The values corresponding to significant deterministic terms are in bold.

Table 3. Estimates of the fractional differencing parameter in the squared returns

	No regressors	An intercept	A linear time trend
White noise	0.186 (0.163, 0.211)	**0.183 (1.159, 0.209)**	0.180 (0.157, 0.207)
AR(1)	0.276 (0.241, 0.315)	**0.272 (0.237, 0.312)**	0.270 (0.234, 0.310)
Bloomfield	0.322 (0.271, 0.372)	**0.310 (0.274, 0.367)**	0.310 (0.261, 0.381)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of d. The values corresponding to significant deterministic terms are in bold.

Tables 2 and 3 provide the same information as Table 1 but for absolute and squared returns respectively. The former appear to be characterized by long memory in all cases, with the estimated values of d ranging from 0.245 (with white noise errors) to 0.343 (Bloomfield disturbances). Slightly smaller values are obtained for squared returns (see Table 3), these ranging from 0.183 (white noise u_t) to 0.310 (with Bloomfield autocorrelated errors). This evidence of long memory in the volatility of the series is in line with previous studies of other stock markets and suggests that other approaches based on autoregressive conditional hetero-scedasticity models (ARCH, Engel, 1982; GARCH, Bollerslev, 1986) should be extended to the fractional case (e.g., FIGARCH-type models, Baillie, Bollerslev and Mikkelsen, 1996) when looking at stock market prices.

The results presented so far are based on a parametric approach (though a nonparametric method, Bloomfield, was also implemented for the $I(0)$ disturbances), and should therefore be taken with caution given the possibility of misspecification. Therefore, we also conducted the analysis using a semiparametric method where no functional form is imposed on the $I(0)$ error term. In particular, we used a Whittle approached developed by Robinson (1995) and later extended by Velasco (1999), Velasco and Robinson (2000), Phillips and Shimotsu (2004, 2005), Abadir et al. (2007) and others. This method is essentially a local 'Whittle estimator' in the frequency domain, which uses a band of frequencies that degenerates to zero. The estimator is implicitly defined by:

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2\, d\, \frac{1}{m} \sum_{s=1}^{m} \log \lambda_s \right), \qquad (9)$$

$$\overline{C(d)} = \frac{1}{m} \sum_{s=1}^{m} I(\lambda_s)\, \lambda_s^{2d},\ \lambda_s = \frac{2\,\pi\, s}{T},\ \frac{m}{T} \to 0,$$

where m is a bandwidth number, and $I(\lambda_s)$ is the periodogram of the raw time series, x_t, given by:

$$I(\lambda_s) \;=\; \frac{1}{2\pi T} \left| \sum_{t=1}^{T} x_t\, e^{i \lambda_s t} \right|^2,$$

and $d \in (-0.5, 0.5)$. Under finiteness of the fourth moment and other mild conditions, Robinson (1995) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \text{ as } T \to \infty,$$

where d_o is the true value of d. This estimator is robust to a certain degree of conditional heteroscedasticity (Robinson and Henry, 1999) and is more efficient than other more recent semi-parametric competitors.

Figure 3 (see Appendix) displays the estimates of d for the return series and the absolute and squared returns, specifically the whole range of values of the bandwidth parameter along with the

[1] Absolute returns were employed by Ding et al. (1993), Granger and Ding (1996), Bollerslev and Wright (2000) and Gil-Alana (2003), whereas squared returns were used in Lobato and Savin (1998) and Gil-Alana (2005).

95% confidence interval of the $I(0)$ case. It can be seen that the estimated values are slightly above the interval in the case of returns and much higher for the two volatility series. Table 4 displays the estimates for some specific bandwidth parameters – these are significant and positive in all cases.

Table 4. Semiparametric estimates of d

Bandwidth nb.	Stock market returns	Absolute returns	Squared returns
10	0.102	0.215	0.227
20	0.093	0.36	0.306
25	0.194	0.334	0.326
30	0.179	0.267	0.290
35	0.243	0.305	0.319
39***	0.299	0.328	0.317
45	0.299	0.301	0.262
50	0.245	0.339	0.287
60	0.241	0.405	0.324
70	0.192	0.450	0.385
80	0.205	0.492	0.429
90	0.200	0.433	0.334
100	0.161	0.423	0.307

Source: Robinson (1995) and Abadir et al. (2007).
Note: *** Bandwidth number corresponding to $(T)^{0.5}$.

As a final step we examine whether there are any anomalies related to the days of the week, as extensively documented in the financial literature (Osborne, 1962; Cross, 1973; French, 1980; and Gibbons and Hess, 1981). For instance, Osborne (1962) and Cross (1973) using data of the S&P 500 found that returns were lower on Mondays than on Fridays. A similar results was reported by Gibbons and Hess (1981) for the DJIA series and in other studies for a number of countries including Canada, Australia, Japan and the UK (Jaffe and Westerfield, 1985); France (Solnik and Bousquet, 1990); and South Korea, Malaysia, the Philippines, Taiwan and Thailand (Brooks and Persand, 2001).

Figure 4 (see Appendix) displays the PFTS index for each day of the week. It can be seen that the five series display a very similar pattern. Tables 5-7 report the estimates of d for the three cases of white noise, autoregressive and Bloomfield disturbances respect-tively. Consistently with the results reported in Table 1, the estimates are above 1 in all cases. The most interesting feature is that in all three cases the highest degrees of persistence are obtained for Mondays and Fridays, and the lowest for the mid-days of the week. Thus, stock market prices are more persistent on Mondays and Fridays than during the other days of the week, implying a higher degree of predictability of their behavior on these days. The same evidence is obtained when using the semi-parametric approach of Robinson (1995) and Abadir et al. (2007) (see Table 8 for some selected bandwidth parameters).

Table 5. Estimates of the fractional differencing parameter with white noise errors

	No regressors	An intercept	A linear time trend
Monday	1.017 (0.952, 1.100)	**1.187** **(1.124, 1.366)**	1.187 (1.124, 1.365)
Tuesday	1.016 (0.951, 1.099)	**1.144** **(1.085, 1.219)**	1.144 (1.085, 1.218)
Wednesday	1.013 (0.949, 1.096)	**1.135** **(1.077, 1.208)**	1.135 (1.077, 1.208)
Thursday	1.013 (0.948, 1.095)	**1.164** **(1.102, 1.244)**	1.164 (1.102, 1.243)
Friday	1.014 (0.949, 1.097)	**1.212** **(1.146, 1.296)**	1.212 (1.146, 1.295)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of d. The values corresponding to significant deterministic terms are in bold.

Table 6. Estimates of the fractional differencing parameter with AR(1) errors

	No regressors	An intercept	A linear time trend
Monday	1392 (1.280, 1.552)	**1.253** **(1.130, 1.413)**	1.252 (1.130, 1.408)
Tuesday	1.387 (1.266, 1.542)	**1.222** **(1.121, 1.353)**	1.221 (1.121, 1.350)
Wednesday	1.376 (1.258, 1.528)	**1.207** **(1.105, 1.327)**	1.206 (1.105, 1.324)
Thursday	1.375 (1.256, 1.526)	**1.174** **(1.069, 1.293)**	1.173 (1.069, 1.293)
Friday	1.384 (1.266, 1.537)	**1.228** **(1.095, 1.385)**	1.227 (1.095, 1.380)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of d. The values corresponding to significant deterministic terms are in bold.

Table 7. Estimates of the fractional differencing parameter with Bloomfield errors

	No regressors	An intercept	A linear time trend
Monday	1.012 (0.911, 1.147)	**1.242** **(1.123, 1.400)**	1.242 (1.123, 1.402)
Tuesday	1.002 (0.901, 1.147)	**1.231** **(1.111, 1.397)**	1.230 (1.111, 1.386)
Wednesday	1.003 (0.902, 1.046)	**1.213** **(1.091, 1.366)**	1.212 (1.091, 1.375)
Thursday	0.991 (0.906, 1.132)	**1.177** **(1.061, 1.321)**	1.177 (1.061, 1.319)
Friday	1.001 (0.894, 1.131)	**1.219** **(1.102, 1.380)**	1.218 (1.101, 1.377)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of d. The values corresponding to significant deterministic terms are in bold.

Table 8. Semiparametric estimates of d

Bandwidth nb.	Monday	Tuesday	Wednesday	Thursday	Friday
5	0.130	0.128	0.138	0.154	0.138
10	0.500	0.500	0.500	0.500	0.500
15	0.101	0.089	0.093	0.106	0.105
18***	0.096	0.093	0.096	0.101	0.097
20	0.084	0.093	0.100	0.095	0.085
25	0.181	0.191	0.100	0.200	0.189
30	0.186	0.182	0.191	0.198	0.192

Source: Robinson (1995) and Abadir et al. (2007).
Note: *** Bandwidth number corresponding to $(T)^{0.5}$.

Finally, the analysis for the absolute and squared returns by day of the week (in Tables 9 and 10) also shows higher estimates of *d* for Mondays and Friday (especially Mondays) than for the other days of the week.

Table 9. Estimates of the fractional differencing parameter in the absolute returns

	No regressors	An intercept	A linear time trend
Monday	0.281 (0.212, 0..363)	**0.255 (0.183, 0.338)**	0.253 (0.180, 0.339)
Tuesday	0.257 (0.181, 0.341)	**0.238 (1.171, 0.322)**	0.235 (0.161, 0.322)
Wednesday	0.245 (0.182, 0.323)	**0.224 (0.162, 0.302)**	0.218 (0.151, 0.300)
Thursday	0.206 (0.143, 0.281)	**0.187 (0.128, 0.261)**	0.182 (0.122, 0.258)
Friday	0.248 (0.182, 0.329)	**0.225 (0.163, 0.305)**	0.221 (0.158, 0.303)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of *d*. The values corresponding to significant deterministic terms are in bold.

Table 10. Estimates of the fractional differencing parameter in the squared returns

	No regressors	An intercept	A linear time trend
Monday	0.245 (0.172, 0.325)	**0.236 (0.166, 0.326)**	0.233 (0.150, 0.326)
Tuesday	0.203 (0.134, 0.291)	**0.198 (1.129, 0.286)**	0.193 (0.122, 0.284)
Wednesday	0.206 (0.147, 0.289)	**0.203 (0.142, 0.283)**	0.198 (0.134, 0.281)
Thursday	0.185 (0.121, 0.260)	**0.181 (0.121, 0.256)**	0.177 (0.111, 0.254)
Friday	0.196 (0.126, 0.289)	**0.191 (0.123, 0.277)**	0.190 (0.1119, 0.276)

Note: The values in parentheses give the 95% confidence band for the non-rejection values of *d*. The values corresponding to significant deterministic terms are in bold.

Conclusions

In this paper we have examined the properties of the Ukranian stock market by estimating the order of integration of the PFTS series, daily, from January 9, 2007 until February 27, 2013. The main findings are the following. First, the log-prices series is highly persistent, with an order of integration significantly above 1, which implies that stock returns are characterized by long memory behaviour. The same feature is detected in the absolute and squared returns which are used as a measure of volatility. Finally, the analysis by day of the week produces evidence of higher degrees of dependence on Mondays and Fridays than on the other days of the week.

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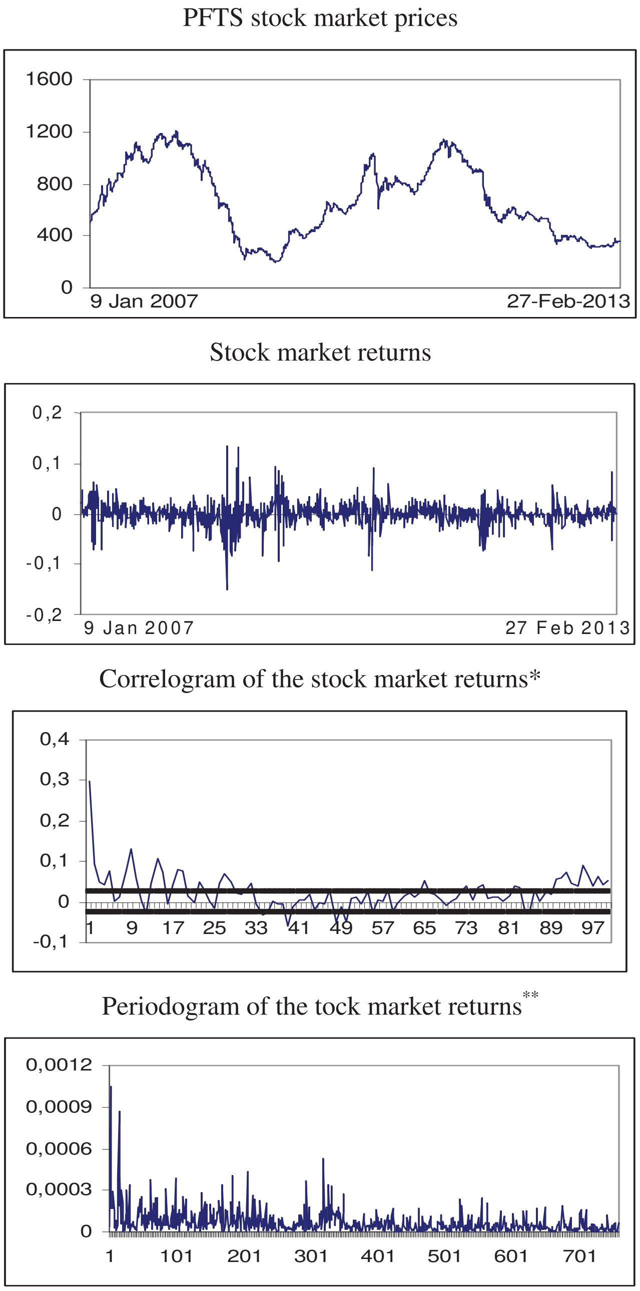
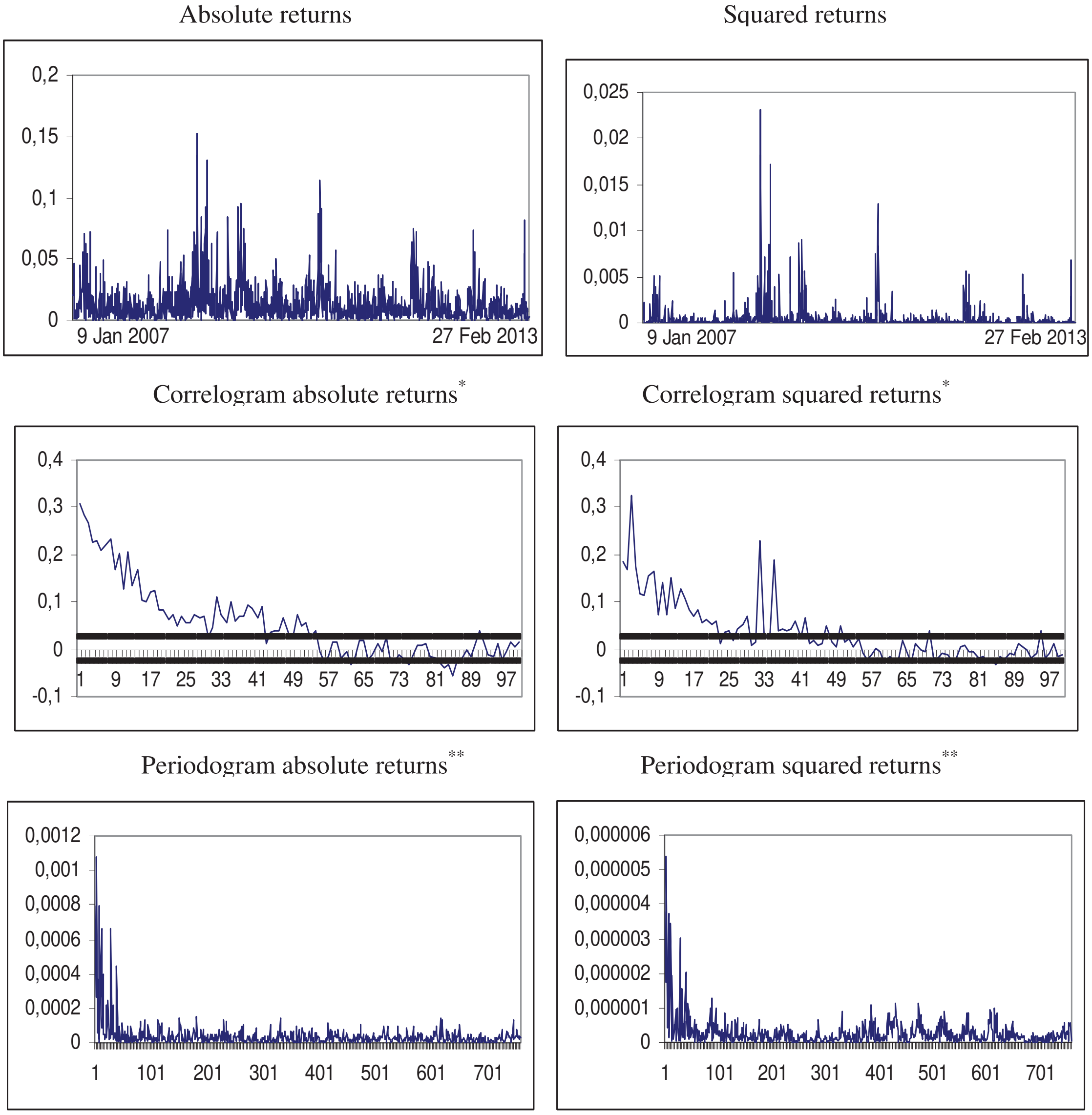
Appendix



Notes: * The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation. ** The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, $j = 1, \ldots, T/2$.

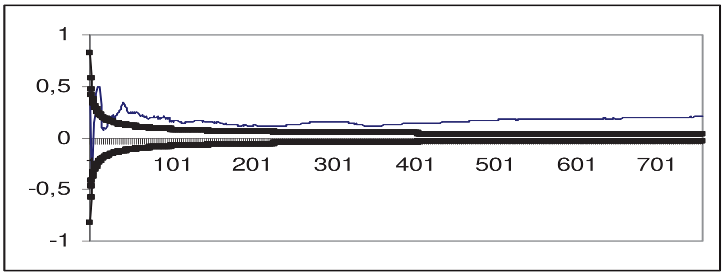
Fig. 1. Time series plots, correlograms and periodograms



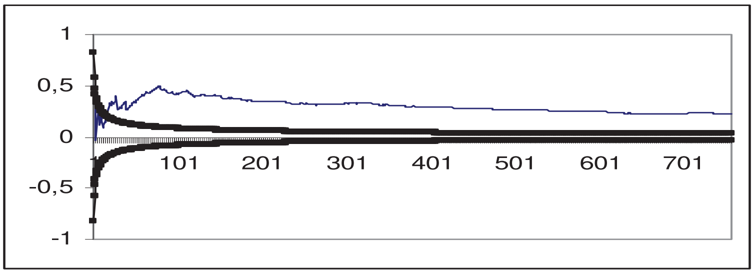
Notes: * The thick lines refer to the 95% confidence band for the null hypothesis of no autocorrelation. ** The horizontal axis refers to the discrete Fourier frequencies $\lambda_j = 2\pi j/T$, $j = 1, \ldots, T/2$.

Fig. 2. Absolute and squared returns, correlograms and periodograms

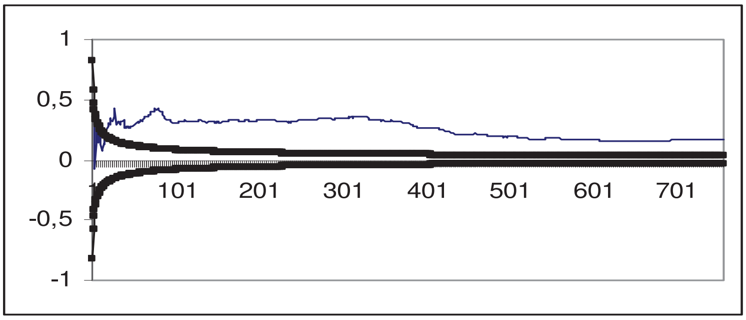
Stock market returns



Absolute returns



Squared returns



Note: The horizontal axis concerns the Bandwidth parameter while the vertical one refers to the estimated value of *d*.

Fig. 3. Estimates of *d* based on the semiparametric approach of Robinson (1995)

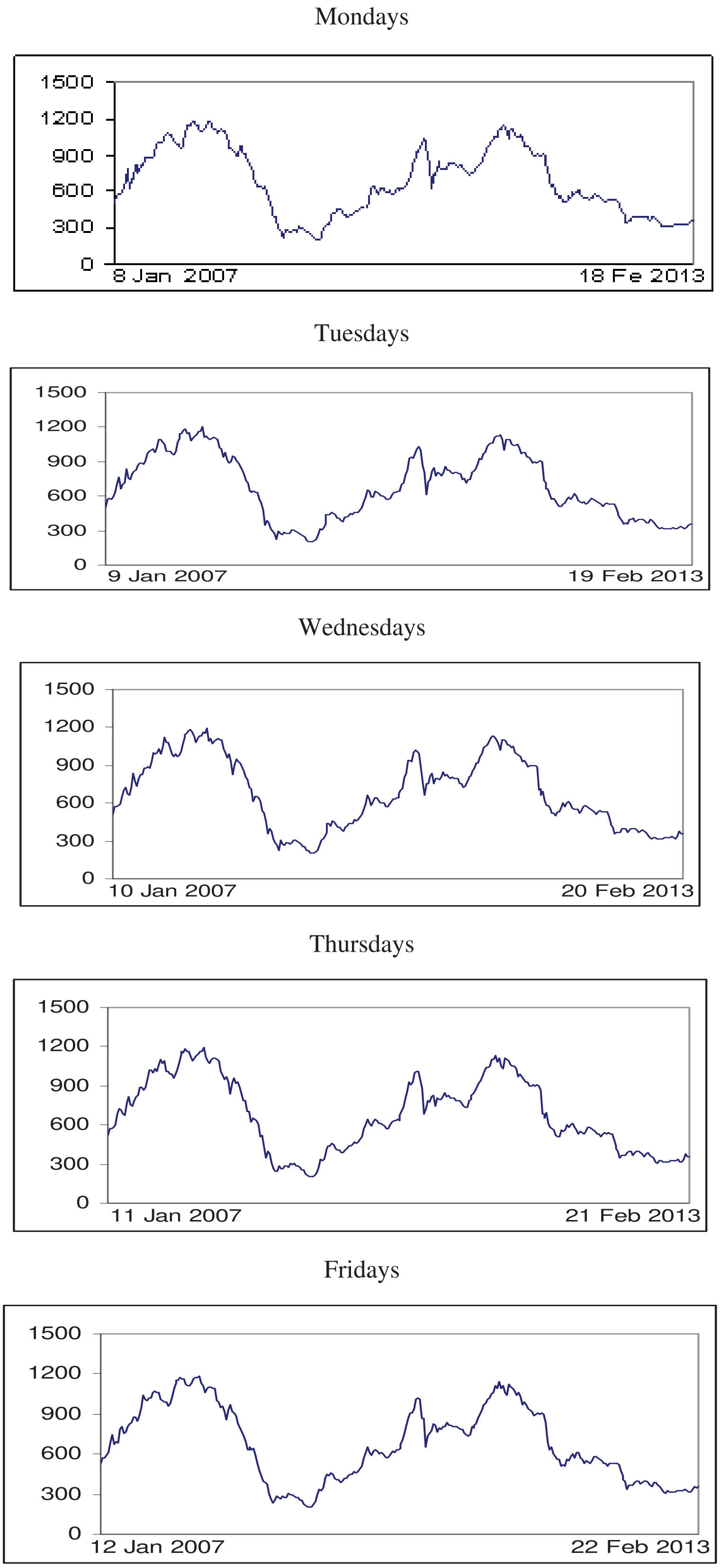


Fig. 4. PFTS by day of the week