“An Empirical Method for Human Mortality Forecasting. An Application to Italian Data”

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An Empirical Method for Human Mortality Forecasting. An Application to Italian Data

Donato De Feo[1]

Abstract

The paper presents an application to Italian data of the Lee-Carter model for forecasting human mortality. Further discussion points out limits of the model and proposes a possible way to improve its performances, in particular over the medium\long run.

Key words: Lee-Carter model, human mortality, reduction factors, life expectancy, mortality trends.

Introduction

The Lee-Carter model represents a fundamental result in the human survival representation and forecasting. Together with an agile data implementation, it presents some aspects making it an important step in the betterment of the demographic framework description, in particular in actuarial valuation field.

As known (cf. Coppola *et al.*, 2002), in the global risk affecting actuarial valuations three fundamental components can be identified: the risk due to the random deviations of the number of deaths from their expected value, the risk due to the improvement in mortality trend and the one due to the volatility in interest rates. Only the first risk component has an accidental nature and can be controlled by means of pooling techniques. In particular, the systematic component of the demographic risk, the longevity risk, involves consistent consequences in actuarial valuations relating to capitals payable in case of life, these effects being increasing when the duration of the contracts increases.

The Lee-Carter model, for its endogenous mechanism of parameter generation year by year and age by age, is able to capture the mortality trend taking place. New data can correct the description of the phenomenon and, consequently, manages to fit the changes in the trend.

In this paper the Lee-Carter model is analysed in its performances when the time horizon varies in a medium-long run perspective; from this point of view, some critical considerations lead to a proposal for improving it.

The layout of the paper is the following: in section 1 the Lee-Carter model is briefly described and in section 2 the reduction factors are considered; in section 3 the model forecast performances are critically analysed and discussed; in section 4 the betterment proposal is presented and, finally, in section 5 some conclusions and further lines of researches are outlined. In the appendix, concluding the paper, the tables of the parameter estimation and the expected life at different ages and time horizon are reported.

1. The Lee-Carter model

The Lee-Carter model is a two dimensional model, as the basic equations show (cf. [Lee and Carter, 1992]):

$$m_{x,t} = \exp(a_x + k_t b_x + e_{x,t}), \tag{1.1a}$$

$$\ln(m_{x,t}) = a_x + k_t b_x + e_{x,t}, \tag{1.1b}$$

where:

- $m_{x,t}$ is the central death rate calculated for an individual with age x at time t

- a_x is the simple average of $ln(m_{x,t})$ along the whole observation period. It describes, on average, the behaviour of the central death rate for every age x
- k_t is the time mortality index. It shows for all ages together how the mortality phenomena have evolved over the past
- b_x is the sensibility parameter. For every age it explains how $ln(m_{x,t})$ reacts when time passes, as the following expression shows: $\frac{d\ln(m_{x,t})}{dt} = \frac{b_x dk}{dt}$. The parameter summarises the strength of the mortality rate decreasing behaviour for every age;
- $e_{x,t}$ represents that part of the mortality which is not caught by the model, with mean equal to zero and variance $\sigma_e<\infty$.

As we can recognize from the simple description of the model, an important feature of the Lee-Carter method is that every parameter brings different information about the phenomena. When we just work out and observe how they change over time and age, we have the opportunity to look at the mortality from an interesting point of view. For example, looking at the sensitivity parameter b_x over the time and working with Italian data, we realize that the mortality rate of post-adolescents is decreasing slower than the rate of the elder population, as shown in the following graph:

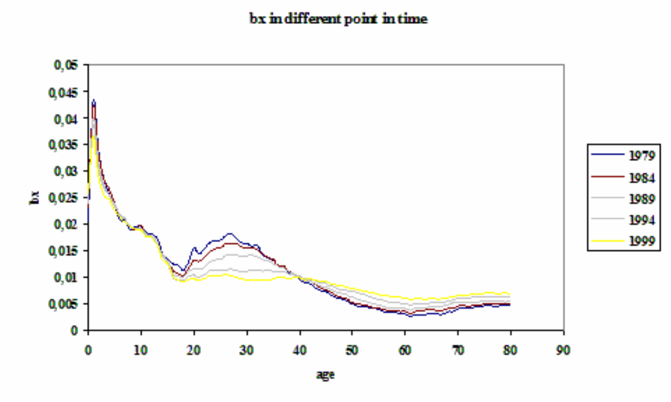

Fig. 1.1. (cf. [De Feo, 2004]): b_x parameter estimated in five point in time

As we can see from equations 1.1a and 1.1b, the model is underdetermined, in the sense that on the right side of the equation we have only parameters to be estimated and the unknown time index.

The first step is to determine the parameter a_x. To do that we normalize the k_t index (cf. Lee and Carter, 1992):

$$\sum_t k_t = 0 \tag{1.2}$$

and we obtain:

$$\sum_{t=t_1}^{t_n} \ln(m_{x,t}) = na_x + b_x \sum_{t=t_1}^{t_n} k_t + \sum_{t=t_1}^{t_n} e_{x,t}.$$

On the basis of formula 1.2 and if $e_{x,t}$ is zero on average, we can write:

$$\frac{\sum_{t=t_1}^{t_n} \ln(m_{x,t})}{n} = \ln\left[\left(\prod_{t=t_1}^{t_n} m_{x,t}\right)^{\frac{1}{n}}\right] = \hat{a}_x. \tag{1.3}$$

Being $m_{x,t}$ observable from the life tables, it is easy to determinate a_x. We suppose that:

$$\sum_x b_x = 1 \tag{1.4}$$

and we write:

$$\sum_{x=0}^{\omega} \ln(m_{x,t}) = \sum_{x=0}^{\omega} a_x + k_t \sum_{x=0}^{\omega} b_x + \sum_{x=0}^{\omega} e_{x,t}.$$

On the basis of equation 1.4 and neglecting the sum of error factors, it follows that the k_t index is very close to the following sum (cf. [Lee and Carter, 1992]):

$$\sum_{x=0}^{\omega} \ln(m_{x,t}) - \sum_{x=0}^{\omega} a_x = \hat{k}_t. \tag{1.5}$$

For what concerns the parameter b_x, we can get it by fitting a simple regression.

Finally the estimated model is the following:

$$\hat{m}_{x,t} = \exp(\hat{a}_x + \hat{k}_t \hat{b}_x), \tag{1.6a}$$

$$\ln(\hat{m}_{x,t}) = \hat{a}_x + \hat{k}_t \hat{b}_x, \tag{1.6b}$$

in which the parameters a_x and b_x do not change over time.

As a consequence we have to forecast just the time index.

Lee and Carter (cf. Lee and Carter, 1992) for US, Carter and Prskawetz (Carter and Prskawetz, 2001 for Austria find that the ARIMA(0,1,0) well describes the behaviour of the time index.

So for k_t the following model holds:

$$k_t = k_{t-1} - c + e_t,$$

where:

- k_t is the time index at time t;
- c is the ratio between the overall decrement of k_t over the whole observation period and the number of periods where the decrement happened;
- e_t is the error factor at time t.

Concerning now the confidence interval, we define the standard error associated with s forecast periods as follows (cf. Lee and Carter, 1992):

$$\hat{\sigma}_h = \sigma_1 \cdot \sqrt{h} \quad \text{with} \quad 2 \le h \le s \quad \text{and} \quad h \in N, \tag{1.7}$$

where $\hat{\sigma}_1$, the standard error of the estimation, indicates the uncertainty associated with one-year forecast.

From 1.7 we can see that, as the forecast horizon increases, the standard error grows with the time horizon square root.

The following figure shows the value of time index for Italian data from 1947 to 1999 and forecasts from 2000 to 2065 with the relative 95% confidence interval. Data concern the overall population from age 0 to 109 which is assumed to be the limiting age (cf. Cocozza *et al.*, 2005). Data are divided age by age and year by year.

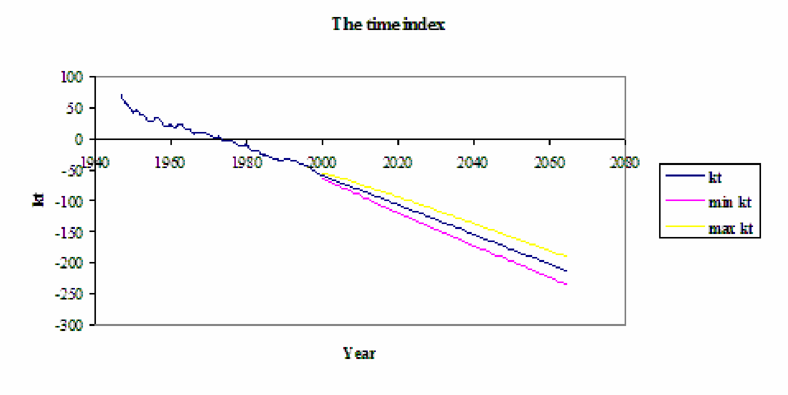

Fig. 1.2. (cf. [De Feo, 2004]): the projection of the time index

2. The application of the model: the introduction of the reduction factors

The Lee-Carter method can be used for generating families of reduction factors, in this way having the advantage to increase the model usability and accuracy of the forecasts (cf. Renshaw and Haberman, 2000).

We begin fixing an appropriate year "zero" and so, by definition of reduction factor, we write:

$$\frac{m_{xt}}{m_{xo}} = RF(x,t)\ . \tag{2.1}$$

On the basis of 1.1b, we have:

$$\ln RF(x,t) = a_x + b_x k_t - a_x - bk_o = b_x(k_t - k_o)$$

and if we suppose $k_t' = k_t - k_0$:

$$\ln RF(x,t) = b_x k_t'$$

Finally, posing $a_{x0} = \ln(m_{x0})$ we have:

$$\ln(m_{xt}) = a_{x0} + b_x k_t'\ . \tag{2.2}$$

In that way the logarithm of the central death rate is represented by a straight line where the inclination depends on b_x and a_{x0}, calculated at the year zero, is the starting point for forecasting.

The use of reduction factors matches practitioners needs: in fact, having determinate parameters of the model, it is easy and quick to obtain forecasts for every age at every time by simply multiplying the value at time zero and the appropriate factor. The choice of a year zero allows us to overcome a problem concerning the position of the forecasts curve in the graph.

It can be observed that a_x includes past data often far over time and, as a consequence, when we calculate the parameter using formula 1.3, the first result can be incoherent with the last available data. Considering an appropriate year "zero", we fix a starting point consistent with recent data, eliminating in this way that distortion. The choice of an opportune year "zero" allows us to overcome problems concerning the position of the forecasting curve in the graph, not influencing the shape of the curve itself.

The problem of the correct choice of the year "zero" can be solved, as suggested in Renshaw and Haberman (2003a), using the last available table or what we think is the most relevant one or we could work out a_{x0} as an average over a certain group of available tables.

Concerning the latter approach, we can write:

$$a_{x,0} = \ln \prod_{t=t_m}^{t_n} m_{x,t}^{1/g}\ , \tag{2.3}$$

where $t_n - t_m = g$, the year zero is $t_m + \left[\frac{g}{2}\right]$ and $\left[\frac{g}{2}\right]$ is the integer part of the ratio.

By definition of the central death rate, we can write (cf. Pitacco, 2000):

$$m(x,x+t) = \frac{\int_0^t \mu(x+u)S(x+u)du}{\int_0^t S(x+u)du}\ .$$

Posing t=1 and approximating the denominator, we have:

$$m_x = \widetilde{m}_x = \frac{S(x)-S(x+1)}{\frac{S(x)+S(x+1)}{2}}$$

from which:

$$\widetilde{m}_x = 2\frac{\frac{S(x)-S(x+1)}{S(x)}}{\frac{S(x)+S(x+1)}{2}} = 2\frac{1-p_x}{1+p_x} = \frac{2q_x}{2-q_x}.$$

It follows:

$$q_x = \frac{2m_x}{2+m_x}.$$

On the basis of what said before and the definition of the central death rate, the probability that an insured aged x at time t survives after j years, can be written as follows:

$$_j p_{x,t} = \prod_{g=0}^{j-1}\left\{1 - \frac{2\cdot e^{\left(a_{x+g,0}+k'_{t+g}b_{x+g}\right)}}{2+e^{\left(a_{x+g,0}+k'_{t+g}b_{x+g}\right)}}\right\}. \quad (2.4)$$

3. Forecasting performances

Now we will have a look at the performances of the model when we forecast the human mortality in different cases: at first we will show forecasts made year by year for twenty years and then we will show forecasts made one time for twenty years ahead.

On the basis of the data available on the website www.mortality.org (Berkeley University of California), we refer to the overall Italian population from 1947 to 1999 and from age 0 to 109, where 109 is the limiting age.

Let us assume to be at time *t'*, with *t'* belonging to the interval [1979, 1999]; at first forecasts concern the following year according to the method explained in §1, using reduction factors and considering data from 1947 until *t'-1*. Finally we assume *g=4*, so that the year zero is *t'-3*.

Figure 3.1a shows actual and forecasted expected life at birth and Figure 3.1b at age 60. Full data are provided in Table A.2a and Table A.2b in the Appendix.

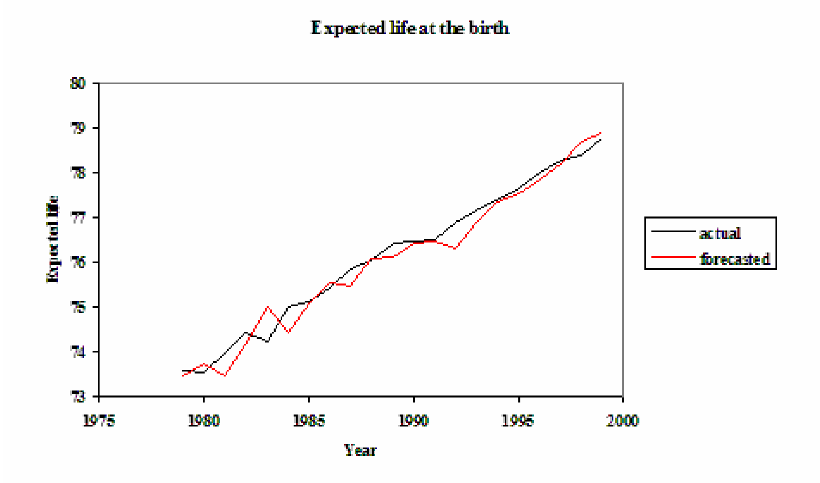

Fig. 3.1a. Expected life at birth, annual forecasts

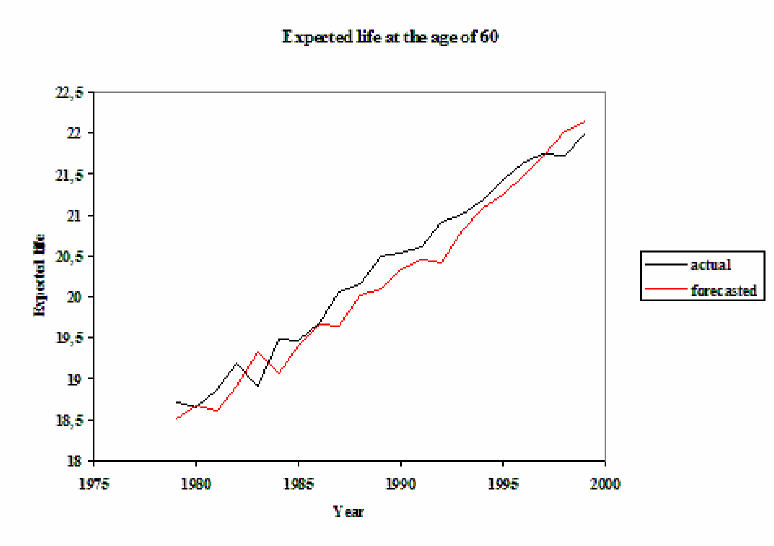

Fig. 3.1b. Expected life at age 60, annual forecasts

We notice that, even if the model is able to follow the mortality trend, it tends to underestimate the expected life. We find more evidence about that tendency in the elderly population, as shown by Figure 3.1b.

We observe now the performances of the model for twenty years forecast. We use data from 1947 to 1978 and forecast mortality until the last available table. Figure 3.2a shows actual and forecasted expected life at birth, Figure 3.2b at age 60. Full data are provided in Tables A.2c and A.2d in the column “Forecast method 1”.

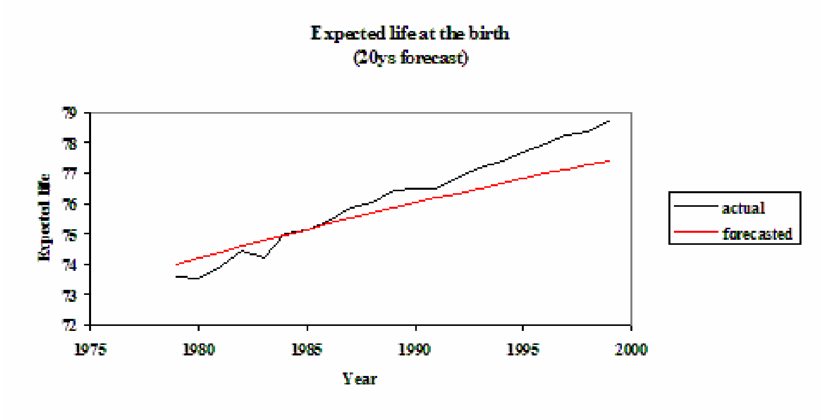

Fig. 3.2a. Expected life at birth, 20 years forecast

In this case the tendency toward underestimation appears more evident; in particular in Figure 3.2b it seems clear that this problem mostly arises from the difficulty of the model to follow the mortality for the elderly population and the idea is that the problem arises from the fact that the parameters a_x and b_x are assumed to be constant over time.

For this purpose, let now observe the real behaviour of the two parameters from 1979 to 1999 using our sample data from 1947 to 1978.

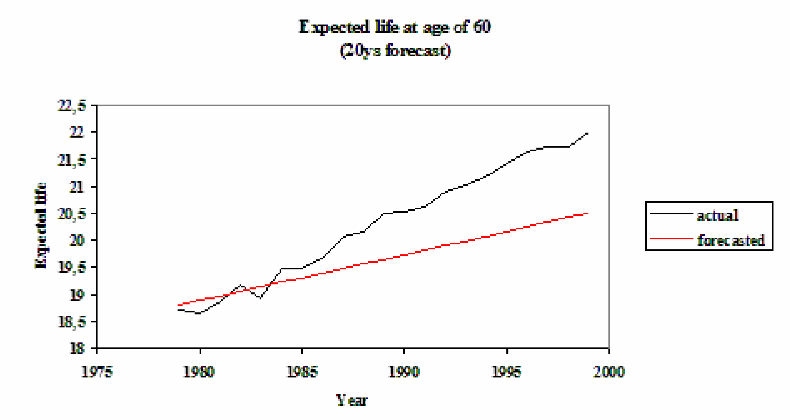

Fig. 3.2b. Expected life at age 60, 20 years forecast

Figures 3.3a and 3.3b show the behaviour of a_x and b_x for some selected ages.

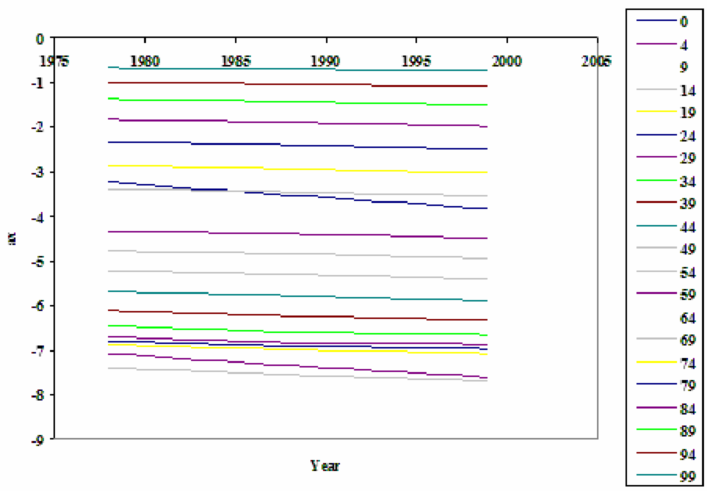

Fig. 3.3a. a_x estimated from 1979 to 1999 for some selected ages

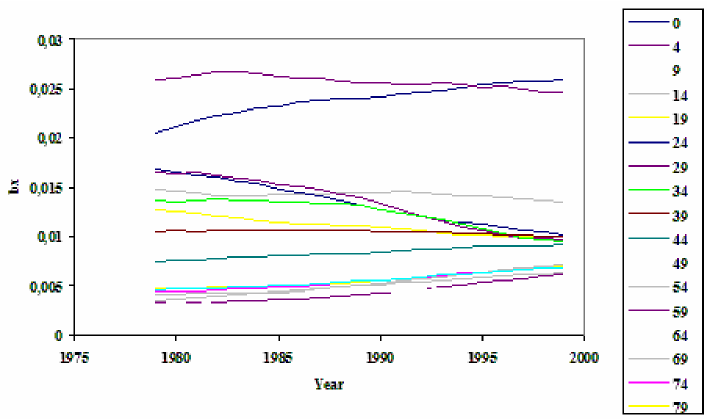

Fig. 3.3b. b_x estimated from 1979 to 1999 for some selected ages

We can observe that a_x decreases for all ages, slightly for almost all except for a few of them, for which the parameter declines in a faster way.

On the contrary, we can recognise for b_x different trends; in particular, it appears to be far from a constant behaviour in the age intervals [20;33] and [44;80+] as the following figures show:

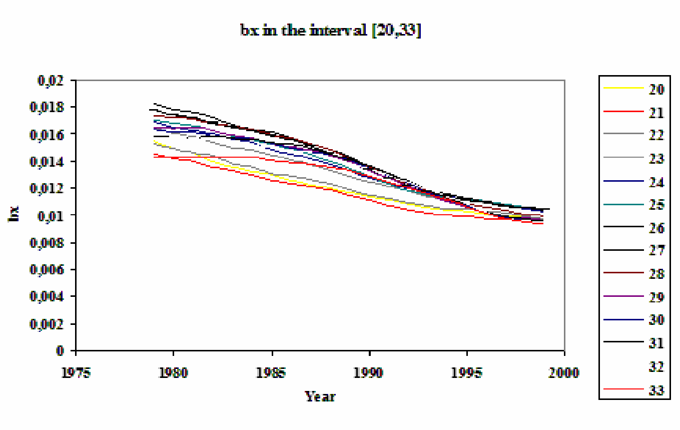

Fig. 3.4a. b_x estimated from 1979 to 1999 in the age interval [20,33]

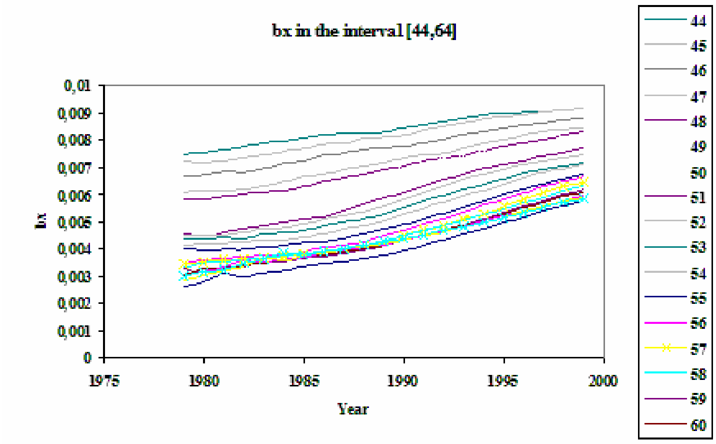

Fig. 3.4b. b_x estimated from 1979 to 1999 in the age interval [44,64]

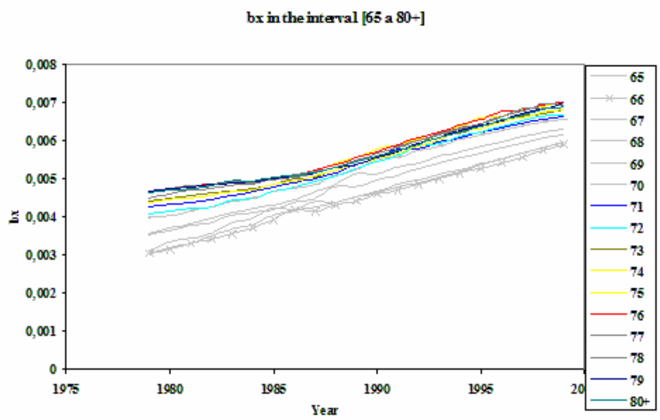

Fig. 3.4c. b_x estimated from 1979 to 1999 in the age interval [65,80+]

The model tends to underestimate the mortality for the ages for which the parameter decreases, because of the constancy assumption, while where b_x increases, it tends to overestimate the mortality, as we can observe comparing the Figure 3.4b and Figure 3.4c with Figure 3.1b and Figure 3.2b.

4. A solution to the overestimation problem

We see that the model furnishes excellent representations of the mortality phenomenon for time intervals of 8-10 years. In the short run, especially when we make one year forecasts, the hypothesis of constancy doesn't have a substantial impact on the performances of the model.

For longer time horizons, on the contrary, it shows both a tendency to overestimate the mortality, in particular for the elderly population, and that the spread between the actual mortality rate and the estimated one tends to increase when the time horizon increases.

The improving we propose is referred to the constancy assumption made on a_x and b_x; we will remove it introducing a structure in the two parameters and will analyse as well the consequent changes in the behaviour of the model.

First of all we consider the parameter b_x; calculating it by means of least square method involves the need of the right number of data. In our study case concerning Italian data, we will make the first forecast beginning from 1979 and having at disposal data from 1947 to 1979.

The solution we propose is to use data from 1947 to 1959 figuring out estimations from 1960 to 1978, in this way building up our minimum database in order to make forecasts.

In order to make forecasts from 1979 to 1999 we find evidence of ARMA(1,0) structure.

The model we consider is the following:

$$X_t - AR(1)X_{t-1} = \varepsilon_t, \tag{4.1}$$

where:

- X_t is an ARMA process with mean μ;
- AR(1) is the autoregressive coefficient;
- $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$, with $\sigma_\varepsilon^2 < \infty$ is the error process.

In Table A.3a estimation and additional information about the AR(1) parameter are provided. It can be noticed that for all ages we refuse the hypothesis that the parameter is equal to zero, so AR(1) is always statistically significant.

Forecast of b_x can be obtained by means of the following formula:

$$E(X_{n+h}|F_n) = \mu + AR(1)^h (X_n - \mu), \tag{4.2}$$

where:

- F_n is the set of information available at time n.

Concerning now a_x, at first we note that, using reduction factors, we will speak about a_{x0}. We estimate the parameter from 1950 to 1978 using equation 2.3 with g=4.

On the basis of this set of data, we perform a simple linear regression of a_{x0} against the time.

The assumed model is the following:

$$a_{x0,t+1949} = \hat{a} + \hat{b}t + \varepsilon_{t+1949}, \tag{4.3}$$

where:

- $a_{x0,t+1949}$ is the parameter calculated at time $t+1949$;
- $\hat{a}$ and $\hat{b}$ are the estimated parameters of the regression;
- $\varepsilon_{t+1949} \sim WN(0, \sigma^2)$, with $\sigma^2 < \infty$ is the error made at time $t+1949$;
- finally $t \in [1,50]$, when t belongs to [30,50] we are making forecasts of $a_{x0,t+1949}$.

In Table A.3b the r-squared are provided for every age. We can notice that the model proposed for a_{x0} often can explain more than 90% of the variability of the phenomena and it stays below the 70% only for very extreme ages, which do not have a great impact on the life expectancy.

Forecasts are shown in Figures 4.1a and 4.1b, data are provided in Tables A.2c and A.2d in the column "Forecasts method 2".

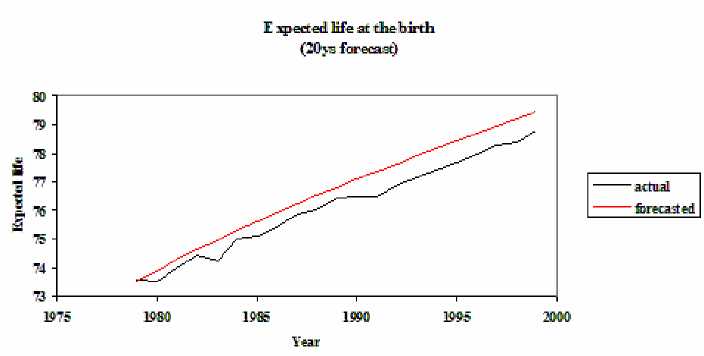

Fig. 4.1a. Expected life at the birth, 20 years forecast according to the adjusted method

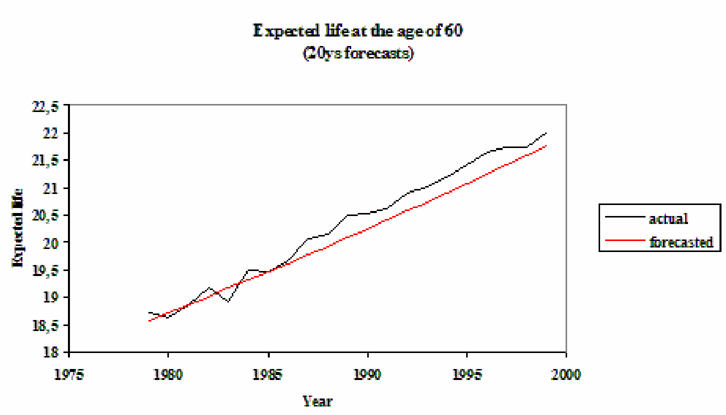

Fig. 4.1b. Expected life at the age of 60, 20 years forecast according to the adjusted method

In particular, comparing Figure 4.1b with Figure 3.2b, we can see an evident betterment in the performances of the model.

Moreover, we can observe that in both cases, referring to the expectancy at birth and at age 60, the proposed adjusted model performs better in term of difference between actual and forecasted data in absolute value and that the new model seems to follow the mortality trend avoiding the problem of the increasing spread between the actual mortality rate and the estimated one when the time horizon increases, observed in the original model and above already noted.

5. Conclusion and further remarks

In this paper an improvement of the Lee-Carter model is proposed, in particular for what concerns its forecasting performances in the medium-long run, time horizons of great importance in actuarial practice, in particular in life annuity portfolio valuations.

Forecast performances obtained by means of the Lee-Carter model are presented and it appears evident that the model tends to the overestimation of the mortality over the long run.

The problem is overcame modifying the constancy assumptions about the two parameters a_x and b_x, assuming respectively an ARMA(1,0) structure and a linear regression estimation.

An increase in the performances of the model over the long run is highlighted. In fact we saw that after twenty years the proposed model underestimates the life expectancy at age 60 for less than a quarter of year and overestimates the life expectancy at birth for a bit more than a half of a year, in this way improving the original model performances.

Possible future researches could examine how this adjusted model works for other countries.

In particular it could be interesting to analyse data for countries where mortality shows different patterns and so for example observing forecast performances in western countries and in developing ones.

Countries like India and China represent, and will represent even more in the future, important markets for life insurances, so if the data availability problem is solved, it could be remarkable to know if in those realities the Lee-Carter adjusted method, proposed in this paper, can be a suitable tool for the achievement of a fair valuation of the insurance policies.

References

1. Boe C., Li N., Tuljapurkar S., (June 2000). A universal pattern of mortality decline in the G7 countries. Nature, 405, 789-92.
2. Carter L.R., Prskawetz A., Rostock – Germany (March 2001). Examining structural shifts in mortality using the Lee-Carter method. Working paper 2001-007. Max Planck institute for demographic research.
3. Coppola M., Di Lorenzo E., Sibillo, (2002). M. Further remarks on risk sources measuring in the case of a life annuity portfolio. Journal of Actuarial Practice, 10, pages 229-242.
4. Cocozza R:, De Feo D:, Di Lorenzo E., Sibillo M., (2005). On the financial risk factor in fair valuation of the mathematical provision. Sent to Astin conference, Zurich.
5. Daykin C.D., (2000). The recent trend of mortality in the United Kingdom. B.A.J., 6, IV, 861-71.
6. De Feo D., (2004). Thesis: Analisi dell'evoluzione del fenomeno della sopravvivenza. Approfondimenti critici del modello Lee-Carter.
7. Himes C.L., Preston S.H., Condran G.A., (1994). A relational model of mortality at older ages in low mortality countries. Population Studies, 48, 269-91.
8. Lee R., Carter L., (September 1992). Modelling and forecasting U.S. mortality. Journal of the Statistical Association, vol. 87, No. 419, 659-75.
9. Lee R., Tuljapurkar S., (1994). Stochastic population forecast for the United States: beyond high, medium and low. Journal of the American statistical association, vol. 89, No. 428, 1175-89.
10. Lee R., (2000). The Lee-Carter method for forecasting mortality, with various extension and applications. NAAJ, 4, 80-93.
11. Lee R., Miller T., (November 2000). Evaluating the performance of Lee-Carter mortality forecast. Demography, 38, 538, 537-49.
12. Lindbergson M., (2001). Mortality among the elderly in Sweden 1988-1997. Scand. Actuarial J., 1, 79-94.
13. Pitacco E., Trieste (2000). Elementi di matematica delle assicurazioni, Lint.
14. Renshaw A.E., (1991). Actuarial graduation practice and generalised linear and non-linear models. J.I.A., 118, II, 295-312.
15. Renshaw A.E., Hatzopoulos P., (1996). On the graduation of "amounts". B.A.J., 2, I, 185-205.
16. Renshaw A., Haberman S., Hatzopoulos P., (June 1996). The modelling of recent mortality trends in U.K. male assured lives. British actuarial journal, vol. 2, part II, No. 7.
17. Renshaw A.E., Haberman S., (July 2000). Modelling for mortality reduction factors. Actuarial research paper, No. 127.
18. Renshaw A., Haberman S., (2003). On the forecasting of mortality reduction factor. Insurance: Mathematics and Economics, vol. 32, 379-401.
19. Renshaw A.E., Haberman S., (2003). Lee-Carter mortality forecasting: a parallel generalized linear modelling approach for England and Wales mortality projections. Applied Statistics, 52, part 1, 119-37.
20. Wilmoth J.R., USA (1993). Computational methods for fitting and extrapolating the Lee-Carter model of mortality change. Technical report, Department of demography – Berkeley University of California.
21. Wilmoth J. R., (17 April 1998). The future of human longevity: a demographer's perspective. Science, vol. 280, 395-97.
22. www.demogr.mpg.de. Max Planck institute for demographic researches.
23. www.mortality.org. Berkeley University of California, Department of demography.

Appendix

A.1 Estimation of the parameters

On the basis of the data available on the website www.mortality.org (Berkeley University of California), we refer to the overall Italian population from 1947 to 1999 and from age 0 to 109, where 109 is the limiting age.

Data are ordered year by year and age by age. In order to make forecasts, in the Table A.1 the estimated parameters of the L-C model are reported.

The parameter a_x was calculated by following formula 2.3, having chosen the last four tables available and 1997 as the year zero. Following Lee and Carter, we assume that b_x is the same from age 80 to 109.

Finally k_t' is the difference between the time index forecasted at time t, with $2000 \le t \le 2065$, and the value of the time index on the year 1997.

Table A.1

Year	k't	Age	ax,0	bx
2000	-10,6703	0	-5,20272	0,026152
2001	-13,0598	1	-7,9378	0,036513
2002	-15,4492	2	-8,39096	0,029475
2003	-17,8387	3	-8,48265	0,025706
2004	-20,2281	4	-8,70828	0,024845
2005	-22,6176	5	-8,81623	0,022741
2006	-25,007	6	-8,82286	0,021298
2007	-27,3965	7	-8,86963	0,02065
2008	-29,786	8	-9,08837	0,020129
2009	-32,1754	9	-8,97217	0,019306
2010	-34,5649	10	-9,02635	0,019121
2011	-36,9543	11	-8,82312	0,01783
2012	-39,3438	12	-8,79847	0,017596
2013	-41,7332	13	-8,65398	0,016409
2014	-44,1227	14	-8,27029	0,013618
2015	-46,5121	15	-8,05735	0,012482
2016	-48,9016	16	-7,90558	0,010566
2017	-51,2911	17	-7,74092	0,009815
2018	-53,6805	18	-7,49726	0,009065
2019	-56,07	19	-7,4743	0,009658
2020	-58,4594	20	-7,43381	0,009617
2021	-60,8489	21	-7,39562	0,009334
2022	-63,2383	22	-7,35037	0,009636
2023	-65,6278	23	-7,32706	0,010168
2024	-68,0172	24	-7,33277	0,010267
2025	-70,4067	25	-7,32943	0,010342
2026	-72,7962	26	-7,34281	0,010403
2027	-75,1856	27	-7,30698	0,010329
2028	-77,5751	28	-7,24483	0,009925
2029	-79,9645	29	-7,24761	0,009723
2030	-82,354	30	-7,16389	0,009578
2031	-84,7434	31	-7,11425	0,009535

Table A.1 (continuius)

Year	k't	Age	ax,0	bx
2032	-87,1329	32	-7,07822	0,009618
2033	-89,5223	33	-6,99265	0,00943
2034	-91,9118	34	-6,95989	0,009612
2035	-94,3013	35	-6,93174	0,009684
2036	-96,6907	36	-6,89803	0,00998
2037	-99,0802	37	-6,82642	0,009966
2038	-101,47	38	-6,77799	0,009765
2039	-103,859	39	-6,77999	0,009973
2040	-106,249	40	-6,68956	0,00988
2041	-108,638	41	-6,61333	0,009634
2042	-111,027	42	-6,57635	0,009762
2043	-113,417	43	-6,48516	0,009508
2044	-115,806	44	-6,37885	0,009225
2045	-118,196	45	-6,31081	0,00928
2046	-120,585	46	-6,20363	0,00892
2047	-122,975	47	-6,11346	0,008638
2048	-125,364	48	-6,02841	0,008503
2049	-127,754	49	-5,93101	0,008327
2050	-130,143	50	-5,82165	0,00812
2051	-132,533	51	-5,73052	0,007909
2052	-134,922	52	-5,60388	0,007645
2053	-137,311	53	-5,51063	0,007389
2054	-139,701	54	-5,42392	0,007274
2055	-142,09	55	-5,31822	0,006894
2056	-144,48	56	-5,24418	0,006838
2057	-146,869	57	-5,16785	0,00671
2058	-149,259	58	-5,06167	0,006559
2059	-151,648	59	-4,96177	0,006429
2060	-154,038	60	-4,8623	0,006372
2061	-156,427	61	-4,75284	0,006003
2062	-158,817	62	-4,66424	0,006249
2063	-161,206	63	-4,56428	0,006168
2064	-163,595	64	-4,45691	0,006073
2065	-165,985	65	-4,348	0,006179
		66	-4,25998	0,006118
		67	-4,15313	0,006162
		68	-4,04814	0,006339
		69	-3,94831	0,006465
		70	-3,85342	0,006748
		71	-3,74823	0,006771
		72	-3,6496	0,006825
		73	-3,55761	0,006954
		74	-3,45824	0,006985
		75	-3,36106	0,007129
		76	-3,22969	0,007153
		77	-3,14135	0,007072
		78	-3,03543	0,007108

Table A.1 (continuius)

Year	k't	Age	ax,0	bx
		79	-2,93488	0,007035
		80	-2,83754	0,007105
		81	-2,70766	0,007105
		82	-2,60584	0,007105
		83	-2,48123	0,007105
		84	-2,37723	0,007105
		85	-2,26327	0,007105
		86	-2,14018	0,007105
		87	-2,03259	0,007105
		88	-1,93079	0,007105
		89	-1,81465	0,007105
		90	-1,71502	0,007105
		91	-1,6195	0,007105
		92	-1,52105	0,007105
		93	-1,43076	0,007105
		94	-1,34638	0,007105
		95	-1,24416	0,007105
		96	-1,15731	0,007105
		97	-1,07387	0,007105
		98	-0,99315	0,007105
		99	-0,91765	0,007105
		100	-0,84401	0,007105
		101	-0,77493	0,007105
		102	-0,71003	0,007105
		103	-0,6501	0,007105
		104	-0,5917	0,007105
		105	-0,54423	0,007105
		106	-0,50158	0,007105
		107	-0,46301	0,007105
		108	-0,40504	0,007105
		109	-0,40547	0,007105

A.2 Tables on the Expected life

Table A.2a

Expected life at birth

Year	Forecasted	Actual
1979	73,47	73,57
1980	73,75	73,54
1981	73,45	73,96
1982	74,15	74,44
1983	75,00	74,24
1984	74,42	75,01
1985	75,08	75,12
1986	75,56	75,43

Table A.2a (continuous)

Year	Forecasted	Actual
1987	75,48	75,86
1988	76,07	76,04
1989	76,11	76,44
1990	76,43	76,49
1991	76,49	76,50
1992	76,30	76,88
1993	76,91	77,16
1994	77,35	77,40
1995	77,56	77,67
1996	77,86	77,99
1997	78,22	78,29
1998	78,70	78,38
1999	78,89	78,78

Table A.2b

Expected life at age of 60

Year	Forecasted	Actual
1979	18,51	18,72
1980	18,68	18,65
1981	18,60	18,87
1982	18,91	19,18
1983	19,33	18,92
1984	19,06	19,49
1985	19,41	19,47
1986	19,66	19,68
1987	19,63	20,06
1988	20,03	20,17
1989	20,09	20,49
1990	20,35	20,53
1991	20,46	20,61
1992	20,41	20,91
1993	20,82	21,02
1994	21,10	21,19
1995	21,26	21,42
1996	21,48	21,65
1997	21,72	21,74
1998	22,03	21,73
1999	22,14	22,01

Table A.2c

Expected life at the birth (20ys forecast)

Year	Forecast method 1	Forecast method 2	Actual
1979	73,98	73,52	73,57
1980	74,19	73,90	73,54
1981	74,39	74,26	73,96
1982	74,59	74,62	74,44
1983	74,78	74,96	74,24
1984	74,97	75,29	75,01
1985	75,15	75,60	75,12
1986	75,33	75,91	75,43
1987	75,51	76,21	75,86
1988	75,69	76,51	76,04
1989	75,86	76,80	76,44
1990	76,03	77,08	76,49
1991	76,19	77,35	76,50
1992	76,35	77,62	76,88
1993	76,51	77,89	77,16
1994	76,67	78,15	77,40
1995	76,83	78,41	77,67
1996	76,98	78,66	77,99
1997	77,13	78,92	78,29
1998	77,28	79,17	78,38
1999	77,43	79,41	78,78

Table A.2d

Expected life at age of 60 (20ys forecast)

Year	Forecast method 1	Forecast method 2	Actual
1979	18,80	18,57	18,72
1980	18,88	18,72	18,65
1981	18,96	18,87	18,87
1982	19,05	19,01	19,18
1983	19,13	19,16	18,92
1984	19,22	19,31	19,49
1985	19,30	19,47	19,47
1986	19,38	19,62	19,68
1987	19,47	19,77	20,06
1988	19,55	19,93	20,17
1989	19,64	20,09	20,49
1990	19,72	20,25	20,53
1991	19,81	20,41	20,61
1992	19,90	20,57	20,91
1993	19,98	20,73	21,02
1994	20,07	20,90	21,19
1995	20,16	21,06	21,42
1996	20,24	21,23	21,65
1997	20,33	21,40	21,74
1998	20,42	21,57	21,73
1999	20,51	21,74	22,01

A.3 Table on AR(1) and r-squared for a_{x0}

Table A.3a

Age	AR(1) value	Std. Error	t-value	Prob(>|t|)
0	1,025092982	0,098016608	10,45836011	4,00E-09
1	1,019747874	0,105435873	9,671735443	1,26E-08
2	1,013748009	0,035738938	28,36536466	4,44E-16
3	1,019357071	0,055839475	18,25513342	6,61E-13
4	1,01821683	0,022734764	44,78677798	0
5	1,01731168	0,01731939	58,73831029	0
6	1,003045128	0,010829985	92,61740279	0
7	1,007014529	0,007099101	141,8510016	0
8	1,012963825	0,005446664	185,9787599	0
9	0,99648414	0,000528535	1885,369913	0
10	1,001101629	0,001347696	742,824272	0
11	0,99636986	0,000402514	2475,367634	0
12	0,996069682	0,000442679	2250,094932	0
13	0,994467004	0,001771783	561,28029	0
14	0,98650981	0,008687647	113,5531606	0
15	0,99474173	0,014124749	70,425447	0
16	0,983528743	0,034758313	28,29621643	4,44E-16
17	0,973586668	0,038071688	25,57245854	2,66E-15
18	0,969536554	0,043070487	22,51046208	2,15E-14
19	0,973522708	0,032204341	30,22954894	2,22E-16
20	0,99360707	0,006572017	151,187533	0
21	0,980688621	0,014469445	67,77652006	0
22	0,981475145	0,02824523	34,74835042	0
23	0,979432625	0,007408752	132,1994041	0
24	0,992241371	0,004032622	246,0536395	0
25	0,995409078	0,00196597	506,3194803	0
26	0,99562128	0,001457795	682,9638434	0
27	0,996862755	0,001995085	499,6593284	0
28	0,999635475	0,00043468	2299,702927	0
29	0,998189107	0,000198491	5028,878734	0
30	0,998288592	0,001457494	684,9348692	0
31	1,000710553	0,001159668	862,9286933	0
32	1,0013849	0,000272049	3680,902846	0
33	0,996299207	0,000789328	1262,212612	0
34	0,996359833	0,003068053	324,7531042	0
35	0,997874405	0,001126588	885,7495481	0
36	0,995326356	0,000656676	1515,702888	0
37	0,997377269	0,002457857	405,7913724	0
38	1,000054732	0,003378198	296,0320383	0
39	0,998652313	0,005106095	195,5804559	0
40	0,983754812	0,005464911	180,0129764	0
41	0,983515522	0,008628929	113,9788686	0
42	0,982400228	0,007950222	123,5689122	0
43	0,971847528	0,012411051	78,30501309	0
44	0,978092921	0,013850809	70,61630064	0

Table A.3a (continuous)

Age	AR(1) value	Std. Error	t-value	Prob(>|t|)
45	0,9916887	0,012718154	77,97426617	0
46	0,982407076	0,0121898	80,59255385	0
47	0,997682763	0,014343178	69,55799735	0
48	0,982817725	0,006157856	159,6038878	0
49	1,006798146	0,006178277	162,957765	0
50	1,005726938	0,002238369	449,3124277	0
51	0,993365313	0,01871226	53,08633428	0
52	1,028862767	0,020700165	49,70311984	0
53	1,026376345	0,028830745	35,60006265	0
54	1,017364028	0,094605515	10,75374964	2,64E-09
55	1,018055294	0,105869081	9,616172032	1,37E-08
56	1,030134768	0,114597419	8,989162025	3,60E-08
57	1,036575799	0,097605091	10,62009967	3,18E-09
58	1,05323364	0,180345263	5,840095947	9,84E-06
59	1,062866385	0,124389632	8,544654148	7,36E-08
60	1,048285059	0,141000609	7,434613688	4,88E-07
61	1,058238541	0,111035563	9,530627076	1,56E-08
62	1,03304601	0,158852911	6,503160711	2,71E-06
63	1,043276254	0,133080967	7,839409916	2,40E-07
64	1,037666721	0,051660929	20,08610279	1,39E-13
65	1,018036997	0,124536177	8,174628631	1,36E-07
66	1,040815385	0,126890611	8,202461744	1,30E-07
67	1,025805985	0,036716888	27,93826058	6,66E-16
68	1,016694995	0,007695045	132,1233345	0
69	0,993010691	0,006757259	146,9546519	0
70	0,996579767	0,010994624	90,64246375	0
71	1,010898341	0,004077673	247,9105956	0
72	1,017000286	0,007359981	138,1797403	0
73	1,017937493	0,008070099	126,1369254	0
74	1,008278817	0,01289198	78,20977326	0
75	1,016744281	0,014608192	69,60096754	0
76	1,01249474	0,103567076	9,776222138	1,08E-08
77	1,004186089	0,020367079	49,30437506	0
78	1,013185264	0,09056884	11,1869078	1,46E-09
79	1,029408545	0,095063216	10,82867362	2,38E-09
80	1,004978075	0,032265604	31,14704032	1,11E-16

Table A.3b

Age	r-squared	Age	r-squared
0	0,98602803	55	0,964336
1	0,994323138	56	0,96688
2	0,991033818	57	0,928428
3	0,992684179	58	0,918337
4	0,992391552	59	0,836499
5	0,987392237	60	0,862774
6	0,981994765	61	0,772015

Table A.3b (continuouis)

Age	r-squared	Age	r-squared
7	0,97893907	62	0,846079
8	0,972698307	63	0,851965
9	0,967934871	64	0,854882
10	0,967238148	65	0,877699
11	0,951328318	66	0,921025
12	0,959403657	67	0,921115
13	0,939434008	68	0,926956
14	0,872461254	69	0,908151
15	0,836120502	70	0,887383
16	0,789373432	71	0,899872
17	0,761955605	72	0,897409
18	0,705742482	73	0,906477
19	0,78437042	74	0,913814
20	0,869888079	75	0,92483
21	0,811929497	76	0,920726
22	0,84335947	77	0,918361
23	0,881685079	78	0,932226
24	0,923112777	79	0,921216
25	0,928460414	80	0,966598
26	0,937520844	81	0,952325
27	0,936378212	82	0,963258
28	0,956387753	83	0,940805
29	0,95304081	84	0,944461
30	0,962893566	85	0,923541
31	0,970264613	86	0,918362
32	0,965891197	87	0,925011
33	0,959624902	88	0,927457
34	0,974151706	89	0,876468
35	0,95291604	90	0,86589
36	0,948635766	91	0,891213
37	0,931430717	92	0,911712
38	0,91980018	93	0,789909
39	0,895130249	94	0,809469
40	0,912390726	95	0,851083
41	0,887225205	96	0,838907
42	0,897569625	97	0,824072
43	0,854016788	98	0,806464
44	0,825805835	99	0,791258
45	0,824905316	100	0,742365
46	0,856625381	101	0,695145
47	0,821530888	102	0,426702
48	0,862131365	103	0,025768
49	0,850929627	104	0,34644
50	0,920805309	105	0,016883
51	0,927161141	106	0,037818
52	0,921112835	107	0,021821
53	0,966574497	108	0,003579
54	0,981440885	109	0,543515