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Comparison of the claims reserves methods by analyzing the run-off error

Abstract

The variability of claim costs represents an important risk component, which should be taken into account while implementing the internal models for solvency evaluation of an insurance undertaking. This component can generate differences between future payments for claims and the provisions set aside for the same claims (run-off error).

If the liability concerning the claims reserve is evaluated using synthetic methods, then the run-off error depends on the statistical method adopted; when it is not possible to study analytically the properties of the estimators, methods based on stochastic simulation are particularly effective. This work focuses on measuring the run-off error with reference to claims reserves evaluation methods applied to simulated run-off matrices for the claims settlement development. The results from the numerical implementations provide the authors with useful insights for a rational selection of the statistical-actuarial method for the claims reserve evaluation on an integrated risk management framework.

The setting of the analysis is similar to that adopted in other studies (Stanard, 1986; Pentikainen and Rantala, 1992; Buhlmann et al., 1980), however, it differs for estimation and simulation methods considered and for the statistics elaborated in the comparison.

Keywords: run-off error, outstanding claims reserves, stochastic simulation.

Introduction

The random claim settlement regarding the accident year i (i = 0, 1, ..., t) is given by the sum of a random number of claims, each one subject to a single claim settlement, and it can be represented as follows:

$$\tilde{X}(i) = \sum_{k=0}^{\tilde{N}(i)} \tilde{Y}_k(i), \quad i = 0, 1, \ldots, t, \tag{1}$$

whereas $\tilde{N}(i)$ represents the total number of claims incurred in the year generation i; $\tilde{Y}_k(i)$ is the random settlement for the claim k incurred during the accident year i; t symbolizes both the time of observation of the portfolio and the total number of generations still open.

Since the settlement claimed for every accident usually requires two or more payments, which can take place during the accident year or the subsequent years, the aggregated claims cost for every accident year can be represented as follows:

$$\tilde{X}(i) = \sum_{j=0}^{t} \tilde{X}(i,j), \quad i = 0, 1, \ldots, t, \tag{2}$$

whereas $\tilde{X}(i,j)$ represents the amount paid for settlements regarding claims incurred during the accident year i and settled after j years; t represents the maximum number of deferment years considered for the total settlement of a single claim.

At the time of observation the recorded information from the company regards the amounts:

$$\left\{ X(i,j) : i = 0,1,\ldots,t;\ j = 0,1,\ldots,t - i \right\}, \tag{3}$$

while a forecast of future amounts should be done:

$$\left\{ \tilde{X}(i,j) : i = 1,2,\ldots,t;\ j = t - i + 1,\ldots,t \right\}. \tag{4}$$

The random amount required for future settlements regarding claims not yet settled or reported (and IBNR), for each accident year, is given by:

$$\tilde{R}(i) = \sum_{j=t-i+1}^{t} \tilde{X}(i,j), \quad i = 1, 2, \ldots, t. \tag{5}$$

The aggregate amount required is then given by the sum:

$$\tilde{R} = \sum_{i=1}^{t} \tilde{R}(i). \tag{6}$$

1. The run-off errors

The statistical methods for the outstanding claims reserve evaluation consist in the formulation of a forecasted value of the necessary reserve, based on a projected analysis of the data obtained by the examination of relevant time series. In other words, an evaluation method provides an estimator $\hat{R} = f\left(\tilde{K}_0, \tilde{K}_1, \ldots, \tilde{K}_t\right)$ of the expected value for the outstanding claims reserve[1], which depends on the information at disposal $K = f\left(\tilde{K}_0, \tilde{K}_1, \ldots, \tilde{K}_t\right)$ for each accident year, and of which at the time t there

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[1] In general, for the distribution of the outstanding claims reserve, other than the expected value we can estimate moments of order higher than 1 or even particular quantiles.

are some (partial) determinations (e.g., paid, claim number, closed without payment, re-opened)[2].

The difference between future payments for claims settlements and the amount of the relative outstanding claims reserve, evaluated using a specific estimator, gives us the run-off error. The run-off error for each accident year can be represented as follows:

$$\tilde{e}(i)=\hat{R}(i)-\tilde{R}(i)=\sum_{j=t-i+1}^{t}\left[\hat{X}(i,j)-\tilde{X}(i,j)\right],^{3} \quad i=1,2,...,t, \tag{7}$$

while the run-off error relative to the entire portfolio can be expressed by:

$$\tilde{e}=\hat{R}-\tilde{R}=\sum_{i=1}^{t}\sum_{j=t-i+1}^{t}\left[\hat{X}(i,j)-\tilde{X}(i,j)\right]. \tag{8}$$

The entity of the run-off error depends on the differences between the set of hypotheses on which the estimation model is based and the actual characteristics of the portfolio[4]; such differences condition, evidently, the properties of the estimator of the claims reserve.

The formula measures the run-off error, at the observation period, for all the accident years taken into consideration, compensating the possible differences of opposite sign between the run-off errors of the various accident years.

The estimator $\hat{R}$ is called unbiased if the expected value of the estimator equals the expected value of the outstanding claims reserve for which the estimator is used:

$$E\left[\hat{R}-\tilde{R}\right]=E\left[\hat{R}\right]-E\left[\tilde{R}\right]=0. \tag{9}$$

For an unbiased estimator, the expected run-off error equals 0. You could say that an unbiased estimator provides estimates of the provision for claims that do not contain "loadings" (positive or negative, implicit or explicit). The amplitude of the distortion that characterizes the estimators is, however, only the first criterion of comparison. In fact, a method that can provide estimates with low distortion, but for which the individual forecasts differ considerably from the actual values, may not be an appropriate method for the estimation of reserves. It is useful then to consider other precise indicators such as the mean percentage error, $E\left[\frac{\hat{R}-\tilde{R}}{\tilde{R}}\right]$, and the mean square error, $E\left[\left(\tilde{R}-\hat{R}\right)^2\right]$[5]. Moreover, since it follows:

$$\sigma^2\left[\tilde{R}-\hat{R}\right]=\sigma^2\left[\hat{R}\right]+\sigma^2\left[\tilde{R}\right]-2\sigma\left[\hat{R}\right]\sigma^2\left[\tilde{R}\right]\rho, \tag{10}$$

a good estimation method must provide an estimator with high correlation ρ with the reserve to estimate.

2. The claim reserve evaluation methods

Between the multiple procedures for the evaluation of outstanding claims reserve proposed in literature, four of them were chosen for this work, considering their widespread utilization in the professional environment: the Chain-Ladder method, the Separation Method (arithmetic and geometric), the Fisher-Lange method and the Bornhuetter-Ferguson method. We give below a concise representation of the content and how they were applied in the analysis.

2.1. The Chain-Ladder method. The Chain-Ladder method considers the run-off triangle of cumulative payments of settlements:

$$\left\{C(i,j): i=0,1,...,t; j=0,1,...,t-i\right\}, \tag{11}$$

whereas $C(i,j)=\sum_{k=0}^{j}X(i,k)$.

The underlying hypothesis is that the distribution of the settlements is constant for each accident year, the development factors are estimated as:

$$\hat{m}_h=\frac{\sum_{i=0}^{t-h-1}C(i,h+1)}{\sum_{i=0}^{t-h-1}C(i,h)}, h=0,1,...,t-1. \tag{12}$$

Then, assuming that development factors remain unaltered also for the future, the cumulative future payments are calculated:

$$\hat{C}(i,t)=C(i,t-i)\prod_{h=t-i}^{t-1}\hat{m}_h, 1=1,2,...,t. \tag{13}$$

[2] For the purpose of the evaluation reliability, the information on which the projected analysis is based should include sufficient, independent and homogeneous data.

[3] The formula measures the run-off error, at the observation period, for the entire accident year, compensating the possible differences of opposite signs during the forthcoming development years. Knowing the gap between expected and actual timing of settlements is of crucial importance for the reinsurance treaties that compensates the reserved claims of the cedent company.

[4] If the amounts of future settlements are discounted at the time of observation, to the error in the forecast of the cash flow of settlements you must add the error relative to the forecasted future rates of return.

[5] Let's recall that, between two biased estimators, $\hat{R}_A$ and $\hat{R}_B$, we will say that $\hat{R}_A$ is more efficient than $\hat{R}_B$ if and only if $E\left[\left(\tilde{R}-\hat{R}_A\right)^2\right]\le E\left[\left(\tilde{R}-\hat{R}_B\right)^2\right]$.

The difference between the ultimate cost and the cumulative cost until the year of observation will provide an estimate of the reserve for a single generation:

$$\hat{R}(i) = \hat{C}(i,t) - C(i,t-i), i = 1, 2, ..., t. \quad (14)$$

The sum of these differences, for all generations, measures the estimated amount of total claims reserve. Among the variants of the method, the one based on the triangles of the relationship between the cumulative average costs theory was considered, which estimates development factors as weighted averages of the ratios observed with the weights obtained by calculating $w_{i,j} = (i+j+1)^2$, that depends on the accident years of claims and the time span of settlement.

2.2. The Taylor separation method. The method of separation (arithmetic) elaborates the triangle of the average costs of claims for the accident year, assuming that each of these costs, net of the random noise term, is, on average, expressed as a product of two factors:

$$E\left[\tilde{X}(i,j)\right] = r_j \lambda_{1+j}. \quad (15)$$

Factor r_j, as a function of only years of development and varies between 0 and 1, is the way in which payments per claim are distributed in time, regardless of generation; while the second factor λ_{1+j}, that depends on both the year of development and the accident year, represents an index of exogeneity, with particular reference to the inflation, extrapolated by log-linear regression. The availability of an adequate information base allows the estimation of the factors $\hat{r}_h$ and $\hat{\lambda}_h$ (h = 0, 1, ..., t), expressing then the "average cost per claim of generation", according to the product of the two factors mentioned above; while the factors $\hat{\hat{\lambda}}_h$ ($h = t + 1$, t + 2,..., 2t) are extrapolated from the factors $\hat{\lambda}_h$ by log-linear regression. It is estimated that in this way the future average costs per claim for each generation, multiplied by the corresponding number of claims, can predict the cumulative amounts of future claims and, subsequently, the total claims reserve.

Among the variants of the described method the so-called separation of geometric type was considered, with extrapolation of the index of inflation, using a log-linear regression.

2.3. The Fisher-Lange method. The Fisher-Lange method is based on the average costs of claims paid in previous generations and their relative settlement speed.

The two key assumptions are: 1) the claim settlement speed is constant over time; 2) the average cost of claims paid is a function of the period between the accident date and the time of actual payment. Hence, starting from the triangle of run-off in the number of claims settled:

$$\{n(i,j) : i = 0, 1,...,t; j = 0, 1,...,t-i\}. \quad (16)$$

The rate of settlement for development year is calculated as:

$$v_j = \frac{1}{t-j+1} \sum_{i=0}^{t-j} \frac{n_{i,j}}{n_{i,j-1}^{(a)}}, j = 1, 2, ..., t-1, \quad (17)$$

with which the number of settled claims is estimated:

$$\hat{n}_{i,j} = \hat{n}_{i,j-1}^{(a)} v_j, j = 1, 2, ..., t; i = t-j+1, ..., t, \quad (18)$$

and the number of claims still outstanding:

$$\hat{n}_{i,j}^{(a)} = \hat{n}_{i,j-1}^{(a)} - \hat{n}_{i,j}, j = 1, 2, ..., t; i = t-j+1, ..., t. \quad (19)$$

Then, we consider the run-off triangle of the average cost paid $\{\bar{X}(i,j) : i = 0,1,...,t; j = 0,1,...,t-i\}$ that produces estimates of future average costs, $\left\{\hat{\bar{X}}(i,j) : i = 1,2,...,t; j = t-i+1,...,t\right\}$, by log-linear regression of the average costs for each development year. Next, multiplying the projected average costs corresponding to the number of claims that are expected to be settled, you get the estimates of the total amounts of claims still outstanding. The sum of all these future amounts represents an estimate of total claims reserve:

$$\hat{R} = \sum_{i=1}^{t} \hat{R}(i) = \sum_{i=1}^{t} \sum_{j=t-i+1}^{t} \hat{n}_{i,j} \hat{\bar{X}}(i,j). \quad (20)$$

2.4. Bornhuetter-Ferguson method. As part of Bornhuetter-Ferguson Method, the run-off triangle of accumulated payments $\{C(i,j) : i = 0, 1,...,t; j = 0, 1,...,t-i\}$ is considered, from which we estimate development factors:

$$\hat{m}_h = \frac{\sum_{i=0}^{t-h-1} C(i,h+1)}{\sum_{i=0}^{t-h-1} C(i,h)}, h = 0, 1, ..., t-1. \quad (21)$$

Benchmark values are determined according to the cost of generation by multiplying the premiums of each generation for a suitable loss-ratio (Bornhuetter and Ferguson, 1972). In this case, the benchmark value was calculated using the following formula[6]:

$$\hat{\bar{C}}(t) = \frac{\sum_{i=0}^{t} C(i,t-i) \prod_{h=t-i}^{t-1} \hat{m}_h (1+i)^2}{\sum_{i=0}^{t} (1+i)^2} . \quad (22)$$

The estimates of the ultimate cost for each generation are obtained by applying the factors of Bornhuetter-Ferguson to the benchmark values:

$$\hat{C}(i,t) = C(i,t-i) + \hat{\bar{C}}\left(1 - \frac{1}{M_i}\right), \quad (23)$$

whereas $M_i = \prod_{h=t-i}^{t-1} \hat{m}_h$. Then, the estimate of the reserve for each generation:

$$\hat{R}(i) = \hat{C}(i,t) - C(i,t-i), i = 1,...,t. \quad (24)$$

3. The simulation methods of the run-off matrix

In practice, the run-off error can be measured only after the completion of the claims settlement process. In this work, we will quantify the run-off error, simulating the claims settlement process until we obtain all the members of the run-off error formula $\tilde{e}(i), i = 1, 2, ..., t$.

For this purpose, we represent the random settlements, in each cell of the run-off matrix, with the following (collective) model:

$$\tilde{X}(i,j) = \sum_{k=0}^{\tilde{N}(i,j)} \tilde{Y}_k(i,j); i,j = 0,1,...,t , \quad (25)$$

whereas $\tilde{N}(i,j)$ represents the total number of claims for the accident year *i*, settled during the development year *j*; $\tilde{Y}_k(i,j)$ represents the random settlement for the claim *k* incurred during the accident year *i* and settled after *j* years. For simplicity we will exclude the possibility of settlement in installments over several years of development.

For the simulation of the amounts $\tilde{X}(i,j)$ we have considered four methods, which are distinguished for the development rule concerning the claims settlement inside the run-off triangle and are based on probabilistic assumptions regarding both the distribution of the number of claims $\tilde{N}(i) = \sum_j \tilde{N}(i,j)$ and the distribution of the random settlement of each claim $\tilde{Y}_k(i,j)$.

3.1. Method of random development factors. The temporal distribution of the settlements inside the run-off matrix is governed by the development factors, as described in the Chain-Ladder method framework with the exception that the main hypotheses on which the Chain-Ladder method is based upon are not respected in this case[7]. This method simulates the run-off matrix through the following steps (Narayan and Warthen, 1997):

1. A value *n* (*i*) of the random variable $\tilde{N}(i)$, number of claims, is generated from a *Poisson* distribution with a preset parameter λ.
2. *n* (*i*) values, $y_k(i)$, of the random variable $\tilde{Y}(i)$ are generated from a *lognormal* distribution with a preset parameters μ and σ^2 [8].
3. The sum of the claims costs is calculated, obtaining the ultimate cost for each generation:

$$C(i,t) = \sum_{k=0}^{n(i)} y_k(i) . \quad (26)$$

4. *t* Pseudo-random numbers, H_j ($j = 0, 1, ..., t-1$), are generated, calculating:

$$T_j = a + bH_j + cln(1+j), \quad (27)$$

$$U_j = T_1 + T_2 + ... + T_j . \quad (28)$$

5. The cumulative payment for each generation *i*, until the development year *j*, is calculated according to

$$C(i,j) = C(i,t)\left(1 - e^{-U_j}\right). \quad (29)$$

6. The value of the claims reserve for the generation *i* results:

$$R(i) = C(i,t) - C(i,t-i). \quad (30)$$

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[6] The formula modifies that proposed in Bornhuetter and Ferguson (1972), where an arithmetic average of the ultimate cost of the generation is used.

[7] The assumptions implicit in the Chain-Ladder model are: 1) the development of settlement is made according to unknown development factors m_0, m_1, ... m_{t-1}, with $E[C(i,k+1)|C(i,0),...,C(i,k)] = C(i,k)m_k$ and $1 \le i \le t$, $0 \le k \le t$-1; 2) the variables $C(i,0),\dots,C(i,t)$ and $C(i',0),...,C(i',t)$ related to different accident years $i \ne i'$ are independent; 3) there are constant unknowns as $a_0,\dots a_{t-1}$, so $Var[C(i,k+1)|C(i,0),...,C(i,k)] = C(i,k)\alpha_k^2$ with $1 \le i \le t, 0 \le k \le t-1$.

[8] The probability distributions associated with the random quantities are chosen in an arbitrary manner, but they are consistent with the actuarial literature. For the hypothesis of claims frequency with the Poisson distribution, see Buhlmann et al. (1980); for the assumption of log-normal distribution of the amount of settlements – Hewitt and Lefkowitz (1979), Hewitt (1970).

7. For each accident year steps 1 to 6 are repeated, multiplying the ultimate cost of claims by the factor:

$$I(i) = (1 + i_{infl})^i, i = 1, ..., t, \tag{31}$$

whereas $i_{\inf l}$ is an annual inflation rate.

3.2. Method of backward calculated random development factors. This method is similar to the previous one, with the only difference that the development factors are calculated using backward steps. The method simulates the run-off matrices through the following steps (Narayan and Warthen, 1997):

1. A value *n* (*i*) of the random variable $\tilde{N}(i)$, number of claims, is generated from a *Poisson* distribution with a preset parameter λ.
2. *n* (*i*) values, $y_k(i)$, of the random variable $\tilde{Y}(i)$ are generated from a *lognormal* distribution with a preset parameters μ and σ^2.
3. The sum of the claims costs is calculated, obtaining the ultimate cost for the accident year *i*:

$$C(i,t) = \sum_{k=0}^{n(i)} y_k(i). \tag{32}$$

4. *t* random variables are simulated H_j (j = 0, 1, ..., t-1), a normal distribution with parameters

$$\mu_j = \frac{(t-j)+(t-j-1)^2}{d_\mu}; \ \sigma_j^2 = \frac{(t-j)+(t-j-1)^2}{d_{\sigma^2}}. \tag{33}$$

5. Development factors are calculated:

$$m_j = e^{H_j}; \ M_j = \prod_{k=j}^{t-1} m_k, \ j = 0, 1, ..., t-1. \tag{34}$$

The parameters d_μ and d_{σ^2} are assigned values such as to ensure that the factors m_j are greater than 1 with high probability.

6. The cumulative payment at the end of the year *j* (j = 0, 1,..., t – 1), for the generation *i*, is calculated as:

$$C(i,j) = \frac{C(i,t)}{M_j}. \tag{35}$$

7. The claim reserve for the generation *i* results:

$$R(i) = C(i,t) - C(i,t-i) \tag{36}$$

8. For each accident year steps 1 to 7 are repeated, multiplying the cost of settlements for each accident year by the factor:

$$I(i) = (1 + i_{infl})^i, i = 1, \ldots t, \tag{37}$$

whereas i_{infl} is an annual inflation rate.

3.3. Method of single settlements. This simulative method is derived from the estimation models proposed by Stanard (1986) and by Buhlmann, Schnierper and Straub (1980), which consider the settlement of a single claim as a stochastic process depending on three parameters: the incurring year, the reporting year and the settlement year.

In this framework, the simulating model assumes an exponential distribution for the deferment periods regarding the reporting year and the settlement year (McCleanahan, 1975; Weissner, 1978).

Furthermore, the settlement amount varies with the variation of the deferment period between the settlement year and the incurring year.

The method simulates the run-off matrices through the following steps:

1. A value *n* (*i*) of the random variable $\tilde{N}(i)$, number of claims, is generated from a *Poisson* distribution with a preset parameter λ.
2. For each claim *n* (*i*), it is necessary to simulate:
   2.1. the deferment of the time of accident, Δt_1, respect to the beginning of the year of generation; with Δt_1 an uniform random variable (0, 1);
   2.2. the amplitude of the deferral period from the time reported, Δt_2, measured from the time of accident; with Δt_2 exponential random variable with preset mean $\mu_{\Delta t2}$;
   2.3. the amplitude of the deferral period from the time of closing, Δt_3, measured from the time reported; with Δt_3 exponential random variable with preset mean $\mu_{\Delta t3}$;
   2.4. Let's assume $\Delta t_2 = \min(\Delta t_2; t - \Delta t_1)$ and $\Delta t_3 = \min(\Delta t_3; t - \Delta t_1 - \Delta t_2)$.
3. For the settlement related to the single claim a Pareto distribution is assumed with density:

$$f_{\tilde{Y}}(y) = \frac{\alpha(j)[\beta(j)]^{\alpha(j)}}{y^{\alpha(j)+1}}, \ \beta(j) \le y < +\infty; \quad \beta(j) > 0, \ \alpha(j) > 0 \tag{38}$$

whereas: $a(j) = a_a - b_a j$ is the shape parameter and $\beta(j) = [\alpha_\beta + b_\beta j](1 + i_{infl})^j$ scale parameter, dependent on the annual inflation rate i_{infl}. The model used to represent the dynamics of the parameters of the Pareto distribution generates values with increasing settlement parallel to deferment period, this ensures that the cumulative amounts of settlement of a generation, along the rows of the matrix of development, have a positive trend. In practice, the assessment of the amount of a claim can in time have either an increase or a decrease, resulting in a non-monotonic cumulative settlement.

So, for each of the n (i) claims an amount of settlement is associated with $y_k(j), k=1,...,n(i)$, calculated as:

$$y_k(j)=0, \quad se\, j < \Delta t_1+\Delta t_2, \tag{39}$$

$$y_k(p;i,j)=\frac{\beta(j)}{(1-p)^{\frac{1}{\alpha(j)}}}, \tag{40}$$

se $\Delta t_1+\Delta t_2 \le j \le \Delta t_1+\Delta t_2+\Delta t_3$.

$$y_k(p;i,\Delta t)=\frac{\beta(\Delta t)}{(1-p)^{\frac{1}{\alpha(\Delta t)}}}, \tag{41}$$

$se\ \ j>\Delta t_1+\Delta t_2+\Delta t_3$

whereas Δt is the smallest integer greater than or equal to the sum $\Delta t_1 + \Delta t_2 + \Delta t_3$, while p is generated by a random variable with an uniform distribution on (0, 1).

4. Cumulating the settlements observed in each cell, we obtain the aggregate amount X (i, j) while the cumulative payment for the generation i, until the development year j, is given by C (i, j).
5. Cumulating until the last year of development the amount of the final cost for the generation considered C (i, t) is obtained; while the loss reserve is $R(i) = C(i, t) - C(i, t - i)$.
6. For each accident year steps 1 to 5 are repeated, inflating the cost of settlements for each accident year at the annual inflation rate i_{infl}.

3.4. Pentikainen-Rantala method. This method simulates the development of the aggregated settlements for claims incurred during a given year, assuming that the structure function and the inflation rate follow an autoregressive process. This method operates through the following steps:

1. For accident claims during the generation of the most remote (base year, $i = 0$), we choose arbitrarily the average number claims, n, and the first three moments from the origin of the single cost distribution, respectively, $m = a_1$, a_2 and a_3.
2. It simulates the number of claims incurred in the base year, n (0), (using the inverse of the Anscombe tranformation) and its aggregate cost of claims X (0, 0) (using the formula of Wilson-Hilferty, applicable to a *compound Poisson* random variable).
3. The number of claims for subsequent generations is calculated using the following:

$$n(i)=n(0)I_n(i), \tag{42}$$

whereas

$$I_n(i) = (1 + i_n)^i,\ i = 0,1,...,t, \tag{43}$$

while i_n measures the annual rate of growth of the portfolio.

4. Represented, then, the structure function (function that modifies the average frequency of claims paid annually) with the autoregressive process of the first order:

$$\tilde{q}(i,j)=a_q+b_q\tilde{q}(i,j-1)+\tilde{\varepsilon}_q. \tag{44}$$

It simulates $\tilde{\varepsilon}_q=N(0;\sigma_q)$ for each cell (i, j) and then calculates:

$$\{q(i,j): i=1,2,...,t; j=t-i+1,...,t\}, \tag{45}$$

assuming $q(i,0)=1,\ \forall i$.

5. The temporal distribution of the number of claims for each accident year is determined by the following:

$$n(i, j) = n(i)q(i,j)g_n(j), \tag{46}$$

whereas $g_n(j)$ is the function of the temporal distribution of the number of claims (measures the probability an incurred claim in the year i is liquidated after j years). The values of the probability $g_n(j)$ (j = 0, 1, .., t) have been hypothesized independent to the generation year and assumed equal to the components of the vector:

$$g_n=\begin{Bmatrix}0.22;0.18;0.15;0.12;0.10;0.08;0.06;0.04;\\0.027;0.016;0.007\end{Bmatrix}.$$

6. Assumed that the cost of a single claim could grow due to inflation, we can simulate the paths of the inflation rate, modeled with the auto regressive process:

$$\tilde{i}(\tau+1)=max\left\{i_{infl}+b_{infl}\left[\tilde{i}(\tau)-i_{infl}\right]+\tilde{\varepsilon}_{infl};i_{min}\right\}, \tag{47}$$

whereas i_{min} = minimum inflation rate; i_{infl} = average inflation rate; while it is assumed that $\tilde{i}(0)=i_{infl}$ and $\tilde{\varepsilon}_{infl}=N(0;\ \sigma_{infl})$.

So we derive the paths of the inflation factor:

$$\tilde{I}_{infl}(T)=\prod_{\tau=0}^{T}\left(1+\tilde{i}(\tau)\right),\ T=i+j;\ i,j=0,...,t. \tag{48}$$

7. In this way, for each matrix of the run-off, the future settlement flows are obtained:

$$\{X(i,j): i=1,2,...,t; j=t-i+1,...,t\}, \tag{49}$$

with $X(i,j)=X(0,0)I_n(i)I_{infl}(i+j)g_n(j)q(i,j)$. (50)

8. Cumulating until the last year of development, the amount of the ultimate cost for the individual generations is obtained. For each generation, the ultimate cost, deducting the

cumulative cost of the evaluation year, results in the individual reserve. Its total forms the claims reserve for the entire portfolio.

4. Numerical application

A comparative analysis was set up for the examination of the run-off error amplitude regarding each estimating method, considering different sets of parameters, which were recursively modified predicting: a different level of inflation, a higher volatility of the settlement amount, a higher volatility of the disturbing factors characterizing the settlement process, various temporal profiles for the claims development.

For each set of parameters 4.100 settlement matrices were generated with each one of the described simulation techniques. The inferior triangle of the future settlements was obtained from the superior triangle of every simulated matrix. Therefore, gap indicators between estimated reserves and effective (simulated) reserves were calculated.

4.1. Method of random development factors. The numerical values attributed to the parameters were the following:

number of claims: $\lambda = 1000$;

settlement: $\mu = 7.36$; $\sigma^2 = 1.51$; $a = 0.1$; $b = 0.2$; $c = 0.5$;

inflation rate: $i_{infl} = 4\%$.

Table 1 shows the statistics elaborated to analyze the method. These statistics allow to know the sign of the error, and then the tendency of the evaluation methods to overestimate or underestimate the value of the reserve.

Table 1. Method of random development factors

	Chain Ladder v.1	Chain Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
Bias	24.777	313.494	24.195	85.506	124.583	-1.195.325
Mean square error	1.829.637	1.894.680	938.966	1.121.171	711.317	1.106.897
Mean percentage error	0.32%	2.11%	0.26%	0.64%	0.82%	-7.32%
Corr. coeff.	0.016	0.049	0.275	0.26	0.603	0.123

Total claims reserve: mean = 16.037.707; standard deviation = 669.881.

Considering all the accident years, all the methods for outstanding claims reserve prediction provide more or less biased estimators, while showing a restrained mean percentage error (with the exception of the Bornhuetter-Ferguson method). According to the mean square error criterion, the Fisher-Lange method, whose estimator is characterized by an adequate correlation level with the estimated reserves, presents a higher level of preferability.

Analyzing the single accident years, we deduce that the Chain-Ladder method provides a less biased estimator with a lower mean square error, with the relevant exception of the last accident year, which compromises, more than for any other method, the overall efficiency of the estimator.

The following table shows the analysis of the statistics calculated for each generation:

Table 2. Bias method of random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	1	1	-2	0	-2	231
2	-4	-5	3	6	12	939
3	4	5	-90	-88	-67	2.845
4	-35	-38	-150	-136	126	7.781
5	28	33	-919	-810	-6	17.610
6	15	44	-2.328	-1.796	933	32.177
7	342	430	5.824	7.850	14.231	45.557
8	1.574	3.766	-7.062	-735	7.298	-36.458
9	-9.907	4.359	-15.369	1.480	16.743	-333.608
10	32.760	304.898	44.289	79.736	85.315	-932.399
Total	24.778	313.493	24.196	85.507	124.583	-1.195.325

Table 3. Mean square error method of random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	63	63	83	88	52	97
2	253	252	353	363	298	383

Table 3 (cont.). Mean square error method of random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
3	868	866	1.213	1.251	988	1.368
4	2.815	2.816	3.979	4.073	4.129	4.262
5	8.450	8.468	12.877	12.656	11.618	13.546
6	23.415	23.500	32.960	35.360	32.408	36.984
7	59.671	60.255	84.531	91.292	76.758	91.467
8	156.331	157.852	202.370	219.125	175.911	220.458
9	411.758	419.046	382.567	461.876	382.154	440.427
10	1.780.933	1.877.611	736.047	752.026	574.306	744.817
Total	1.829.637	1.894.680	938.966	1.121.171	711.317	1.106.897

Table 4. Mean percentage error method of random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	1.02%	1.02%	1.05%	1.47%	0.28%	50.20%
2	0.57%	0.57%	1.57%	1.66%	1.65%	42.08%
3	0.73%	0.74%	0.26%	0.27%	0.02%	32.60%
4	0.48%	0.47%	0.63%	0.66%	1.20%	25.37%
5	0.55%	0.56%	0.21%	0.25%	0.64%	17.79%
6	0.44%	0.45%	0.12%	0.32%	0.85%	11.02%
7	0.41%	0.42%	1.40%	1.66%	2.11%	5.99%
8	0.41%	0.51%	0.31%	0.62%	0.75%	-1.05%
9	0.11%	0.41%	0.20%	0.65%	0.84%	-6.69%
10	1.03%	4.48%	1.08%	1.45%	1.42%	-11.27%
Total	0.32%	2.11%	0.26%	0.64%	0.82%	-7.32%

4.2. Method of backward calculated random development factors. The numerical values attributed to the parameters were the following:
number of claims: $\lambda = 1000$;
settlement:
$\mu = 7.36$; $\sigma^2 = 1.51$; $d_\mu = 100$; $d_\sigma{}^2 = 500$;
inflation rate: $i_{infl} = 4\%$.

Table 5 shows the statistics elaborated for the second method.

Table 5. Method of backward calculated random development factors

	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
Bias	-12.824	63.746	-66.990	-231.254	117.269	-413.579
Mean square error	809.202	846.605	988.131	865.436	668.263	750.730
Mean percentage error	0.16%	0.90%	-0.37%	-2.01%	1.15%	-3.74%
Corr. coeff.	-0.003	0.006	0.007	0.121	0.619	0.031

Total claims reserve: mean = 10.334.444; Standard deviation = 544.545.

The Fisher-Lange method estimator shows the lower mean square error for both the single accident year estimation and the whole portfolio estimation. The Chain-Ladder estimator results to be the less biased estimator and shows the lower mean percentage error.

The separation methods are characterized by a systematic underestimation of the reserve, which results to be a discriminating characteristic for a method utilized in controlling the reserves set aside by an insurance company. The Bornhuetter-Ferguson method, whose outstanding claims reserve prediction is based upon a benchmark value, which depends on the ultimate cost for each accident year, presents a systematic overestimation (underestimation) of the reserve concerning the first (last 3) accident years, as it is evident from the following analysis carried out for single generation:

Table 6. Bias method of backward calculated random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	-20	-20	-97	-54	-38	5.215

Table 6 (cont.). Bias method of backward calculated random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
2	-82	-76	-96	-184	29	9.226
3	-70	-62	-1.000	-1.698	-803	18.938
4	139	232	-1.005	-3.449	264	37.521
5	2.131	2.429	-2.634	-8.320	-491	57.795
6	2.090	3.096	-6.289	-17.544	1.608	66.220
7	3.776	6.326	4.230	-14.729	20.488	59.336
8	1.135	7.458	-16.111	-46.198	13.763	-31.116
9	-11.327	4.119	-31.717	-73.289	21.730	-199.733
10	-10.594	40.245	-12.243	-65.789	60.719	-436.980
Total	-12.822	63.747	-66.962	-231.254	117.269	-413.578

Table 7. Mean square error method of backward calculated random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	3.886	3.886	4.064	4.123	1.236	5.008
2	5.599	5.606	6.052	6.067	2.419	6.937
3	12.456	12.498	13.799	13.325	5.808	14.391
4	28.861	29.004	30.551	30.496	19.951	31.634
5	58.878	59.235	63.688	62.634	38.121	64.172
6	107.160	108.156	119.705	113.280	70.446	113.479
7	167.381	169.689	170.473	169.346	111.389	165.079
8	266.342	270.585	266.367	258.065	162.574	250.723
9	384.642	391.803	389.864	364.979	248.066	341.587
10	530.841	548.648	480.162	449.923	312.832	405.473
Total	809.202	846.605	988.131	865.436	668.263	750.730

Table 8. Mean percentage error method of backward calculated random development factors

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	7.85%	7.85%	7.74%	8.20%	0.32%	60.26%
2	3.23%	3.25%	3.79%	3.37%	1.38%	45.71%
3	2.98%	3.00%	2.02%	0.73%	-0.43%	35.24%
4	2.93%	3.00%	2.50%	0.94%	1.09%	28.22%
5	3.16%	3.25%	2.07%	0.37%	0.56%	20.46%
6	2.51%	2.67%	1.60%	-0.25%	0.87%	12.99%
7	2.11%	2.33%	2.23%	0.54%	2.35%	7.41%
8	1.65%	2.02%	0.72%	-1.00%	1.20%	-0.09%
9	0.92%	1.50%	0.30%	-1.39%	1.32%	-6.27%
10	0.80%	2.21%	0.72%	-0.87%	1.98%	-11.13%
Total	0.16%	0.90%	-0.37%	-2.01%	1.15%	-3.74%

4.3. Method of single settlements. The numerical values attributed to the parameters were the following:

number of claims: $\lambda = 1000$;
settlement: $a_\lambda = 1000$; $b_\lambda = 50$; $a_\theta = 2.5$; $b_\theta = 0.5$;
inflation rate: $i_{infl} = 4\%$;
deferment: $\mu_{\Delta t2} = 2$; $\mu_{\Delta t3} = 2$.

Table 9 shows the statistics elaborated for the third method.

Table 9. Method of single settlements

	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
Bias	24.777	313.494	24.195	85.506	124.583	-1.195.325
Mean square error	1.829.637	1.894.680	938.966	1.121.171	711.317	1.106.897
Mean percentage error	0.32%	2.11%	0.26%	0.64%	0.82%	-7.329%
Corr. coeff.	0.016	0.049	0.275	0.26	0.603	0.123

Total claims reserve: mean = 5.425.343; Standard deviation = 4.931.581.

The simulating technique appears to be rather coherent in structure, with the claims development model upon which the Fisher-Lange method is based, thus, resulting in an estimator with the lowest estimation gap for both the single accident year estimation and the whole portfolio estimation. All methods provide estimators with high levels of correlation with the estimated reserve.

In addition for more information, the following are the analysis for single generation:

Table 10. Bias method of single settlements

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	4.989	4.989	4.695	4.857	-204	37.263
2	4.957	5.044	4.162	4.124	-886	46.565
3	75	85	-754	-619	-578	50.110
4	-3.614	-3.407	-4.580	-4.396	-711	52.288
5	-3.118	-2.483	-3.700	-3.936	-1.850	55.827
6	-1.650	-1.423	-3.720	-3.256	-39	50.144
7	-234	-281	-5.057	-3.650	-395	29.150
8	8.433	7.308	-3.764	-690	-2.981	-7.172
9	1.432	2.595	-9.268	-4.769	-528	-77.950
10	5.404	75.152	-13.969	-7.676	-3.657	-180.374
Total	16.678	87.574	-35.955	-20.006	-11.829	55.851

Table 11. Mean square error method of single settlements

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	82.883	82.883	86.131	89.781	9.116	203.848
2	63.645	63.682	64.787	67.476	36.431	165.234
3	64.482	64.383	85.689	75.615	51.733	153.741
4	99.431	101.014	126.332	109.475	71.658	157.271
5	118.956	119.744	149.207	132.985	84.632	164.446
6	162.280	165.513	191.740	168.769	119.910	178.214
7	201.054	293.458	215.251	212.772	123.064	218.720
8	129.362	130.634	159.866	127.099	123.208	118.654
9	195.525	192.323	177.777	164.425	99.041	141.318
10	215.568	1.071.072	270.126	205.425	189.500	243.978
Total	966.558	1.319.801	1.258.208	1.093.184	536.283	1.372.551

Table 12. Mean percentage error method of single settlements

Generation	Chain Ladder v.1	Chain Ladder v.2	Arithmetic Separation	Geometric Separation	Fisher-Langer	Bornhuette Ferguson
1	8.11%	8.11%	7.28%	7.48%	-0.55%	59.67%
2	5.52%	5.65%	4.65%	4.45%	-0.65%	46.89%
3	1.15%	1.22%	0.75%	0.42%	0.32%	33.54%
4	-0.70%	0.56%	-0.98%	-1.30%	0.54%	23.88%
5	0.06%	0.29%	-0.22%	-0.52%	0.33%	17.72%
6	0.40%	0.50%	-0.18%	-0.35%	0.57%	11.39%
7	0.17%	0.24%	-0.55%	-0.58%	0.04%	4.79%
8	1.18%	1.10%	-0.18%	-0.14%	-0.19%	-0.72%
9	0.16%	0.32%	-0.70%	-0.51%	0.10%	-6.82%
10	0.65%	5.15%	-0.59%	-0.34%	0.15%	-11.96%
Total	0.10%	1.34%	-0.74%	-0.69%	-0.16%	0.49%

4.4. Pentikainen-Rantala method. The numerical values attributed to the parameters were the following: number of claims: λ = 1000; a_q = 0.4; b_q = 0.6; σ_q = 0.05; i_a=1%;

settlement: α_1 = 0.006; α_2 = 0.001; α_3 = 0.0001;

inflation rate: i_{infl}=4%; i_{min}=2%; b_{infl}=0.7;

σ_{infl}=0.015.

Table 13 shows the statistics elaborated for the last method.

Table 13. Pentikainen-Rantala method

	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
Bias	56.909	102.293	-16.565	-26.590	132.006	-139.536
Mean square error	943.396	1.023.314	1.046.517	697.789	1.296.025	926.270
Mean percentage error	1.12%	1.76%	0.07%	-0.07%	2.16%	-1.67%
Corr. coeff.	0.016	0.124	0.127	0.2	0.12	0.134

Total claims reserve: mean = 7.061.123; standard deviation = 458.362.

In this case, according to both the mean percentage error criterion and the dispersion criterion, the geometric separation method presents the higher level of preferability. Moreover, the analysis of individual generations still shows the characteristic underestimation of methods based on the separation.

The following show the analysis for single generation:

Table 14. Bias-Pentikainen-Rantala method

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	28	28	11	-23	56	14.394
2	54	64	-32	-167	253	40.862
3	260	298	-14	-326	817	75.080
4	136	236	-545	-1.122	1.841	108.926
5	509	744	-1.093	-2.019	3.597	137.750
6	1.533	2.116	-2.171	-3.478	6.402	141.394
7	1.452	2.806	-4.546	-6.237	10.674	98.947
8	3.537	6.722	-7.456	-9.522	16.770	-10.590
9	7.488	15.360	-12.063	-13.996	24.627	-217.263
10	41.911	73.919	11.343	10.300	66.968	-529.037
Total	56.908	102.293	-16.566	-26.590	132.005	-139.537

Table 15. Mean square error Pentikainen-Rantala method

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	1.923	1.923	1.936	1.641	1.969	5.594
2	5.336	5.357	6.064	4.797	6.953	14.838
3	11.089	11.166	13.450	10.234	17.137	27.851
4	20.651	20.883	25.879	19.091	33.879	44.926
5	35.586	36.143	45.434	32.315	59.857	67.266
6	59.903	61.212	74.859	52.236	97.263	94.270
7	95.814	99.186	116.416	80.236	148.249	124.900
8	150.595	159.051	173.432	117.311	215.581	157.867
9	241.944	260.946	251.215	168.186	304.459	197.488
10	426.833	472.921	355.287	236.927	420.200	241.784
Total	943.396	1.023.314	1.046.517	697.789	1.296.025	926.270

Table 16. Mean percentage error Pentikainen-Rantala method

Generation	Chain-Ladder v.1	Chain-Ladder v.2	Arithmetic separation	Geometric separation	Fisher-Lange	Bornhuette-Ferguson
1	2.09%	2.09%	1.92%	1.58%	2.03%	150.89%
2	1.29%	1.31%	1.03%	0.65%	1.57%	118.20%
3	1.17%	1.22%	0.82%	0.47%	1.57%	91.46%
4	0.81%	0.87%	0.36%	0.03%	1.62%	67.43%
5	0.78%	0.86%	0.20%	-0.09%	1.62%	47.17%
6	0.84%	0.96%	0.05%	-0.20%	1.66%	29.15%
7	0.67%	0.85%	-0.16%	-0.36%	1.70%	13.29%
8	0.73%	1.01%	-0.29%	-0.45%	1.73%	-0.46%
9	0.85%	1.32%	-0.39%	-0.50%	1.74%	-12.55%
10	2.18%	3.55%	0.79%	0.73%	3.11%	-22.18%
Total	1.12%	1.76%	0.07%	-0.07%	2.16%	-1.67%

The method proposed by Pentikainen and Rantala has been used to test the sensitivity of the estimators when we change the temporal distribution of the settlements.

Figure 2 and Figure 3 show that, under the scenarios described in Figure 1, the precision obviously is higher when the settlements are concentrated in the early development years. The precision becomes very low in the methods when the settlements occur during longer time spans.

The level of precision of the estimators based on geometric separation is superior in all the scenarios outlined. The mean percentage error and the mean square error of the estimators provided by the methods Fisher-Lange and method Bornhuetter-Ferguson achieve elevated values when the temporal distribution of settlements does not assume the canonical forms (scenarios A2 and B1).

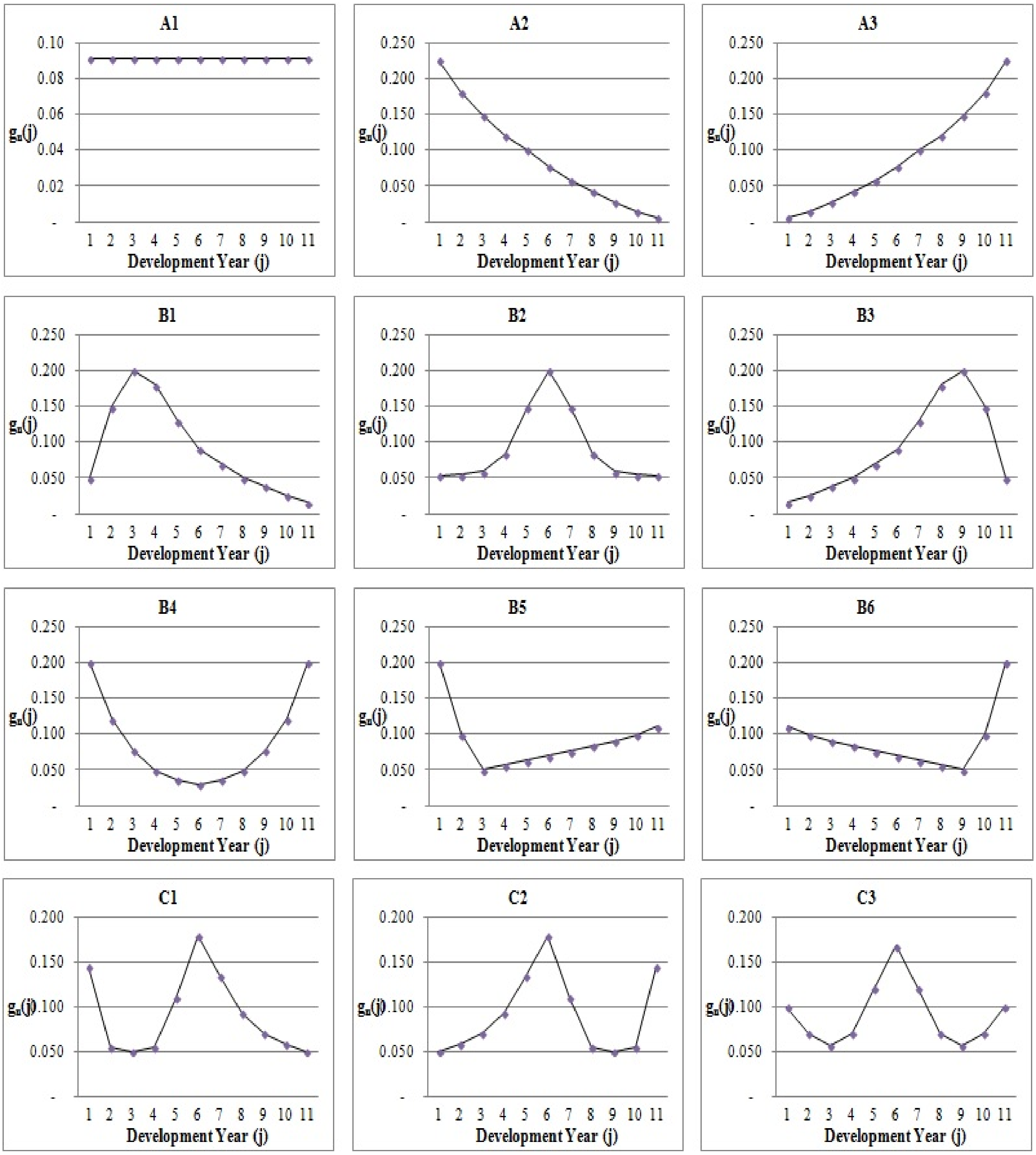

Fig. 1. Scenarios of the temporal distribution of settlements

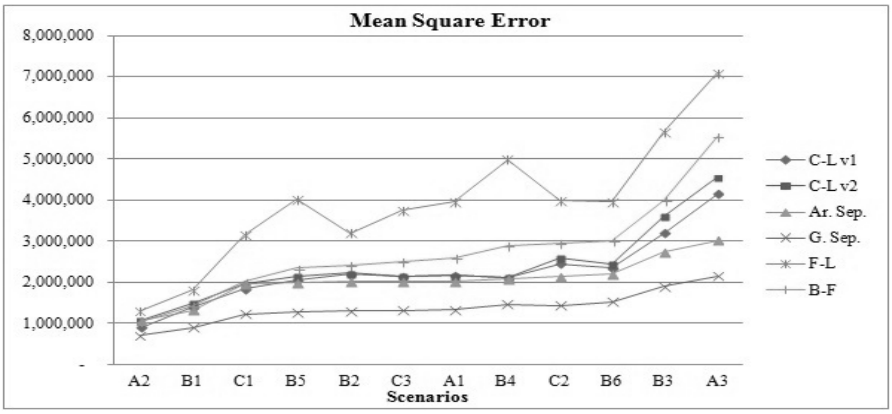

Fig. 2. Mean square error of the estimators by varying the temporal distribution of the number of settlements

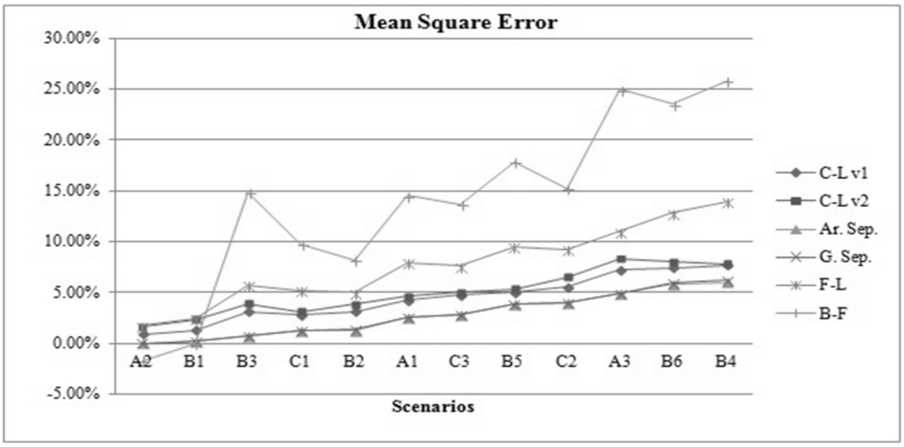

Fig. 3. Mean percentage error estimators produced by varying the temporal distribution of the number of settlements

Conclusion

The numerical implementation results point out the following:

- the estimating methods produce a lower run-off error if applied to a development matrix which, despite not respecting some of the probabilistic hypothesis of the method, provide a settlement distribution according to the mechanism considered by the estimating model;
- the sign of the run-off error may differ from generation to generation and, as a result of compensation, between the individual generations and the entire portfolio;
- some of the estimating methods, despite showing a minor distortion of the reserve estimation for the entire portfolio, result imprecise in the prediction of the run-off for single accident years;
- the preferability of the estimating methods did not show particular sensibility to the choice of numerical values attributed to the parameters.

The analysis has suggested that there isn't a better applicable method for each data set and for each line of business. Therefore, before selecting the more coherent method, it is necessary to examine the dataset, the run-off triangle, the underlying dynamics of the data and the different evolution of the settlement mechanism of different lines of business.

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